The Economics of Bundling Content with Unlicensed Wireless Service *

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Abstract. Adding new unlicensed wireless spectrum is a promising approach to accommodate increasing traffic demand. However, unlicensed spectrum may have a high risk of becoming congested, and service providers (SPs) may have difficulty to differentiate their wireless services when offering them on the same unlicensed spectrum. When SPs offer identical services, the resulting competition can lead to zero profits. In this work, we consider the case where an SP bundles its wireless service with a content service. We show that this can differentiate the SPs' services and lead to positive SP profits. In particular, we study the characteristics of the content services that an SP should bundle with its wireless service, and analyze the impact of bundling on consumer surplus.

Keywords: Unlicensed Spectrum Market · Game Theory · Bundling

1 Introduction

Motivated in part by the success of WiFi, there is an increasing interest in expanding the amount of spectrum available for unlicensed access. Having new unlicensed spectrum can increase competition in the wireless service market and promote the development of new technologies. In addition to the TV white spaces [1] and the Generalized Authorized Access (GAA) tier in the 3.5 GHz band [2], the FCC in the U.S. has recently proposed opening up the 6 GHz band for unlicensed use [3].

Adding unlicensed spectrum promotes competition in that a service provider (SP) can enter the market without paying a license cost. However, the open access of unlicensed spectrum may also lead it to be overcrowded, which results in a "tragedy of commons." Moreover, as shown in [11, 12], competition among SPs in an unlicensed band may result in a "price war," in which no SP makes any profit from the unlicensed wireless service.

To avoid the price war, we propose a market strategy in which an SP can bundle its wireless service with a content service. Based on [6–9], commodity

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bundling is a prevalent marketing strategy that brings cost and information advantages. Furthermore, it enables providers to sort customers into groups with different reservation prices and extract consumer surplus. According to [9], bundling typically reduces the diversity of reservation prices of consumers, and thereby enables sellers to extract more consumer surplus. In this paper, we want to investigate whether a similar phenomenon will happen in a wireless market, which differs from a commodity market in that the SPs utilize a congestible resource to offer service. Furthermore, the reason for zero SP profits in the unlicensed spectrum market is that the SPs offer identical service [11, 12]. Bundling the wireless service with other commodities can differentiate the SPs' services, and potentially lead to positive SP profits. The bundling can be realized through mergers and acquisitions. For example, AT&T has completed the acquisition of Time Warner Inc., and now offers an unlimited data plan bundled with HBO (one of time Warner's leading video services) to its wireless customers [4, 5]. Bundling can also be realized by the cooperation of multiple subsidiaries from the same conglomerate. For example, Google owns both Project-Fi providing wireless service and YouTube Red, a content service.

There have been many recent works studying the competition among wireless service providers on an unlicensed band, e.g., [11–16]. In [11, 12, 15], models of price competition with unlicensed service were studied, which we will adopt in this work. As we have already discussed, [11, 12] showed that price competition can result in zero profits. Reference [13] proposed using contracts to reduce the risk of losing revenue for an incumbent SP on an unlicensed band. Based on [13], reference [14] considered the investment and technology upgrade decisions of new entrant SPs. Reference [16] proposed an alternative market structure based on short-term permits, which was shown to achieve positive profits. As in [13–16], our approach of bundling provides another way to sustain positive profits and is more in line with current practices in the wireless market. We also study the impact of bundling on consumer surplus.

The main questions we want to answer in this paper are as follows:

- Is bundling a promising strategy to use in a wireless market with unlicensed spectrum? Can the SPs achieve an equilibrium that leads to positive profits?
- What are the characteristics (e.g., popularity and value) of the content services that an SP should consider while making the bundling decision?
- How does bundling affect the consumer surplus?

In this paper, we consider two SPs and build a three-stage Stackelberg game to model the bundling decision and competition between the SPs. The main results are as follows:

- The market equilibrium exists and the SPs' profits will increase if an SP bundles its wireless service with a suitable content service.
- The SP should bundle with a content service whose popularity is below an upper bound and value is above a lower bound. Among the services that satisfy these bounds, the SP should choose the one with a high popularity to increase its profit.

– Customer surplus will decrease when an SP chooses to bundle unless the band resource is extremely limited.

2 Model

We use B to denote the unlicensed bandwidth and parameter $x_p < B$ to denote the background traffic on the unlicensed band. We assume that there are two service providers, i.e., SP1 and SP2, who offer wireless service using the unlicensed band. SP1 has the option to bundle its wireless service with the content service, which can differentiate its service from that of SP2. In this work, we do not consider the case where SP2 also has a bundling option. If SP2 bundles its wireless service with the same content service, both SPs still offer the same type of service and achieve zero profits under price competition. If SP2 bundles its wireless service with a different content service, the problem's analysis depends on the customers' valuations on the two content service and we leave such analysis as future work. We formulate the SPs' interactions as the following three-stage Stackelberg game:

– Stage I: SP1 decides whether to bundle its wireless service with the content service. Moreover, SP1 announces p_c, which is the content service's retail price.
– Stage II: SP1 and SP2 decide their prices p_1 and p_2 for wireless customers. If SP1 chooses bundling, p_1 is the price for the bundled service (which contains both the wireless and content service); otherwise, it is the price for the wireless service. Note that p_2 is always SP2's wireless service price.
– Stage III: The customers decide which services to subscribe to, based on p_1, p_2, p_c, and SP1's bundling decision.

2.1 Content Service

In this paper, we assume that the cost of providing the content service does not change with the number of customers using it. Examples are content services provided by companies like HBO, TIDAL and Netflix. We consider two types of customers, who have different valuations for the content service. A fraction α of the customers have high valuations, i.e., θ_h, and a fraction $1-\alpha$ of the customers have low valuations, i.e., θ_l. We assume that $\alpha\theta_h \geq \theta_l$. Intuitively, this means the content service can generate more profit when the content service provider chooses a high price to serve only the customers with high valuations instead of choosing a low price to serve all customers.

2.2 Wireless Service

As in [11, 12, 14], the SPs compete for a common pool of customers to maximize their profits. The customers are modeled as non-atomic users with a total mass of 1. Each customer may choose an SP considering its *delivered price*, which is the sum of the actual price paid and a congestion cost. We use x_1 to denote the total mass of customers that subscribe to SP1's service, and x_2 to denote the total mass of customers that subscribe to SP2's service. When the total traffic on the unlicensed band is x_T, the congestion cost of using the unlicensed band is $\frac{x_T}{B}$. This implies that the congestion is linearly increasing in the traffic [13, 14]. The total traffic x_T includes the background traffic x_p as well as the traffic of the SPs' customers. Then, the delivered price of SP1, denoted as y_1, is $\frac{x_T}{B} + p_1$. Similarly, we use $y_2 = \frac{x_T}{B} + p_2$ to denote the delivered price of SP2.

Each customer is identified as $x \in [0, 1]$. We use $v(x)$ to denote the customer's valuation for the wireless service, and assume that it is given by (similar assumptions have been made in [13, 14]):

$$v(x) = 1 - x.$$

We use θ_x to denote customer x's valuation for the content service. It is a random variable, whose value is given by

$$\theta_x = \begin{cases} \theta_h, & \text{w.p. } \alpha, \\ \theta_l, & \text{w.p. } 1 - \alpha. \end{cases}$$

Recall that α is the fraction of customers with high valuations for the content service.

The traffic generated by a customer on the unlicensed band depends on whether it uses the content service. If a customer does not use the content service, the generated traffic is normalized to 1; otherwise, its generated traffic is $\beta \geq 1$. Note that $\beta > 1$ models a case where subscribing to the content service increases the amount of traffic a customer consumes.

2.3 Customers' Choice

We use a vector $\mathbf{d} \triangleq (d_1, d_2, s)$ to denote a customer's service subscription decision. If the customer subscribes to SP1's service, $d_1 = 1$; otherwise, $d_1 = 0$. If the customer subscribes to SP2's wireless service, $d_2 = 1$; otherwise, $d_2 = 0$. If the customer subscribes to the content service, $s = 1$; otherwise, $s = 0$. A customer decides $\mathbf{d}$ to maximize its welfare. We denote SP1's bundling decision by $b \in \{0, 1\}$, where $b = 1$ means bundling. Thus, if $b = 0$, customer x's welfare given $\mathbf{d}$ would be

$$CW(x, b, \mathbf{d}) = v(x) \cdot \max\{d_1, d_2\} + \theta_x \cdot s - y_1 \cdot d_1 - y_2 \cdot d_2 - p_c \cdot s,$$

where $v(x) \cdot \max\{d_1, d_2\}$ captures that the customer's utility includes $v(x)$ if and only if it subscribes to the wireless service.

If $b = 1$, customer x's welfare is given by

$$CW(x, b, \mathbf{d}) = v(x) \cdot \max\{d_1, d_2\} + \theta_x \cdot \max\{d_1, s\} - y_1 \cdot d_1 - y_2 \cdot d_2 - p_c \cdot s,$$

where $\theta_x \cdot \max\{d_1, s\}$ captures that the customer can use the content service by two different approaches when SP1 chooses bundling. First, the customer can subscribe to SP1's (bundled) service, i.e., $d_1 = 1$. Second, the customer can subscribe to the content service alone, i.e., $s = 1$. It is easy to see that a customer will not choose $d_1 = 1$ and $s = 1$ at the same time when SP1 chooses bundling. Given SP1's bundling decision b, customer x will make decisions to maximize his welfare. Thus, its optimal choice, denoted as $\mathbf{d}^*(x, b)$, is given by:

$$\mathbf{d}^*(x, b) = \arg\max_{\mathbf{d} \in \{0,1\}^3} CW(x, b, \mathbf{d}). \tag{1}$$

3 Wardrop Equilibrium in Stage III

In Sect. 2.3, we discussed an individual customer's choice given SP1's bundling decision, the service prices, and the congestion. In this section, we study the customers' optimal decisions in Stage III and the resulting equilibrium market shares of the SPs. Specifically, we consider the Wardrop Equilibrium [18]. The basic intuition of Wardrop Equilibrium in our model is that, at Wardrop Equilibrium,

- for the customers who only have wireless services, the delivered price they pay should be the same;
- for the customers who have both content service and wireless service, the delivered price they pay should be the same.

Based on the Wardrop Equilibrium conditions, we can analyze the mass of customers choosing different services in equilibrium. Next, we discuss the cases where $b = 0$ and $b = 1$, separately.

3.1 Benchmark Case ($b = 0$)

We denote the case where SP1 does not bundle ($b = 0$) as the *benchmark case*, and the case where SP1 bundles its wireless service with the content service ($b = 1$) as the *bundling case*. In the benchmark case, a customer subscribes to the content service if and only if the content service price p_c is no greater than its valuation. Based on our assumption $\alpha\theta_h > \theta_l$, we can see that SP1 should choose $p_c = \theta_h$ to maximize its profit generated from the content service. Each customer decides which SP's service to subscribe to based on p_1, p_2, and the congestion cost. If $p_1 = p_2$, the Wardrop equilibrium is given by

$$\begin{cases} \frac{\beta\alpha(x_1+x_2)+(1-\alpha)(x_1+x_2)+x_p}{B} + p_1 = 1 - x_1 - x_2, \\ \frac{\beta\alpha(x_1+x_2)+(1-\alpha)(x_1+x_2)+x_p}{B} + p_2 = 1 - x_1 - x_2, \end{cases} \tag{2}$$

where the total traffic x_T equals $\beta\alpha(x_1 + x_2) + (1 - \alpha)(x_1 + x_2) + x_p$ and $\frac{\beta\alpha(x_1+x_2)+(1-\alpha)(x_1+x_2)+x_p}{B}$ is the congestion cost. When SP1 chooses $p_c = \theta_h$, only the customers with high valuations subscribe to the content service. Hence, among the $x_1 + x_2$ customers using the wireless service, a fraction α of them will use the content service on the unlicensed band. Each of them generates β traffic on the unlicensed band. Note that among the $1-x_1-x_2$ customers who do not use the wireless service, a fraction α of them also subscribe to the content service. These customers can use the content service via other approaches (e.g., wire-line networks) and do not generate traffic on the unlicensed band. If $p_1 > p_2$, the equilibrium is given by

$$\begin{cases} x_1 = 0, \\ \frac{\beta\alpha x_2+(1-\alpha)x_2+x_p}{B} + p_2 = 1 - x_2. \end{cases} \tag{3}$$

Similarly, if $p_1 < p_2$, the equilibrium is given by

$$\begin{cases} x_2 = 0, \\ \frac{\beta\alpha x_1+(1-\alpha)x_1+x_p}{B} + p_1 = 1 - x_1. \end{cases} \tag{4}$$

When $p_1 = p_2$, the delivered prices of SP1's and SP2's services are equal. In this case, each customer whose reservation price is greater than the delivered price subscribes to SP1's or SP2's services with an equal probability. An SP can always choose a price that is slightly smaller than the other SP's price to capture the entire market. As a result, this again leads to a price war between the two SPs: they will set prices to zero and have zero profits from the wireless market at the equilibrium (though SP 1 does still have its profit from the content service).

3.2 Bundling Case ($b = 1$)

By considering $CW(x, b, \mathbf{d})$ when $b = 1$, we summarize a customer's optimal choice as follows. Customer x will subscribe to SP1's bundled service ($\mathbf{d}^*(x,1) = \{1,0,0\}$) if

$$\begin{cases} y_1 \le y_2 + \min\{p_c, \theta_x\}, \\ y_1 \le 1 - x + \min\{p_c, \theta_x\}. \end{cases} \tag{5}$$

Recall that y_1 and y_2 are the delivered prices of SP1 and SP2, which are the sum of the congestion cost of the unlicensed spectrum and the service price of each SP. This means that a customer with value $\theta_x > p_c$ will subscribe to SP1's service if the delivered price for SP1's bundled service is smaller than (i) the sum of SP2's delivered price and p_c and (ii) the sum of the customer's value of wireless service and p_c. For a customer with value $\theta_x < p_c$, it will subscribe to SP1's service if the delivered price for SP1's bundled service is smaller than (i) the sum of SP2's delivered price and θ_x and (ii) the sum of the customer's value of wireless service and θ_x.

Similarly, the customer will subscribe to SP2's service and the content service separately ($\mathbf{d}^*(x,1) = \{0,1,1\}$) if

$$\begin{cases} \theta_x \ge p_c, \\ y_1 > y_2 + p_c, \\ y_2 \le 1 - x. \end{cases} \tag{6}$$

The customer will subscribe to the content service only ($\mathbf{d}^*(x,1) = \{0,0,1\}$) if

$$\begin{cases} \theta_x \geq p_c, \\ y_1 > 1 - x + p_c, \\ y_2 > 1 - x. \end{cases} \tag{7}$$

The customer will subscribe to SP2's service only ($\mathbf{d}^*(x,1) = \{0,1,0\}$) if

$$\begin{cases} \theta_x < p_c, \\ y_1 > y_2 + \theta_x, \\ y_2 \leq 1 - x. \end{cases} \tag{8}$$

The customer will not subscribe to any service ($\mathbf{d}^*(x,1) = \{0,0,0\}$) if

$$\begin{cases} \theta_x < p_c, \\ y_1 > 1 - x + \theta_x, \\ y_2 > 1 - x. \end{cases} \tag{9}$$

As we can observe from the conditions above, the customer's choice of services is determined by the delivered prices and the retail price of the content service. There are three possible ranges of p_c (i.e., the content service's price): (a) $\theta_l < p_c \leq \theta_h$, (b) $p_c \leq \theta_l$, (c) $p_c > \theta_h$. We first analyze case (a), and we will later show that SP1 will not choose p_c to be in cases (b) or (c) in equilibrium. Note that in case (a), only the customers with high valuations will pay p_c to subscribe to the content service.

Given that $\theta_l < p_c \leq \theta_h$, we next discuss four cases for the difference between SP1's and SP2's delivered prices: (i) $y_1 - y_2 \leq \theta_l$, (ii) $\theta_l < y_1 - y_2 < p_c$, (iii) $y_1 - y_2 = p_c$ and (iv) $y_1 - y_2 > p_c$. Note that $y_1 - y_2$ is equivalent to $p_1 - p_2$, since the two SPs use the same unlicensed band to serve customers. We assume that $\theta_l = 0$ to simplify the analysis in the rest of the paper.

Case (i) ($y_1 - y_2 \leq \theta_l$): From (6) and (8), we can see that $d_2 = 1$ only if $y_1 - y_2 > \min\{p_c, \theta_x\}$. Hence, if $y_1 - y_2 \leq \theta_l$, no customer will subscribe to SP2's service. From (5), we can see that the customers with $v(x) \geq y_1$ will subscribe to SP1's service regardless of their θ_x. Customers with $v(x) \in [\max\{0, y_1 - p_c\}, y_1)$ will subscribe to SP1's service only if $\theta_x > p_c$. For the customers with $x \in (1 - y_1, \min\{1, 1 - y_1 + p_c\}]$, a fraction α of them subscribe to SP1's service.

As $\theta_l < p_c \leq \theta_h$ is assumed in this case, α fraction of the customers in $(1 - y_1, \min\{1, 1 - y_1 + p_c\}]$ join SP1. Thus, the Wardrop Equilibrium in this case is given by

$$\begin{cases} p_1 < p_2, \\ x_2 = 0, \\ y_1 = \frac{\beta x_1 + x_p}{B} + p_1, \\ x_1 = (1 - y_1) + \alpha \min\{p_c, y_1\}. \end{cases} \tag{10}$$

Case (ii) ($\theta_l < y_1 - y_2 < p_c$): The condition $y_1 - y_2 > \theta_l$ implies $y_1 > y_2 + \min\{p_c, \theta_x\}$ when $\theta_x = \theta_l$. From (5), we can see that the customers with low valuations for the content service will not subscribe to SP1. However, customers will subscribe to SP1's service if $x \leq \min\{1, 1 - y_1 + p_c\}$ and $\theta_x = \theta_h$. We can also see that the customers will subscribe to SP2's service if $x \leq 1 - y_2$ and $\theta_x < y_1 - y_2$. Since $\theta_l \leq y_1 - y_2 < p_c \leq \theta_h$, the condition for a customer to subscribe to SP2's service reduces to $x \leq 1 - y_2$ and $\theta_x = \theta_l$. This indicates that for the customers who subscribe to wireless services, customers with higher value of the content service will subscribe to SP1's service and customers with lower value of the content service will subscribe to SP2's service. As a result, we can compute SP1's and SP2's market shares as $x_1 = \alpha(\min\{1, 1 - y_1 + p_c\})$, and $x_2 = (1 - \alpha)(1 - y_2)$. The Wardrop Equilibrium in this case is given by

$$\begin{cases} 0 < p_1 - p_2 < p_c, \\ \frac{\beta x_1 + x_2 + x_p}{B} + p_2 = 1 - x_1 - x_2, \\ x_1 = \alpha(1 - \max\{0, \frac{\beta x_1 + x_2 + x_p}{B} + p_1 - p_c\}), \\ x_2 = (1 - \alpha)(1 - \min\{1, \frac{\beta x_1 + x_2 + x_p}{B} + p_2\}). \end{cases} \tag{11}$$

Case (iii) ($y_1 - y_2 = p_c$): SP1 and SP2 have the same delivered price for the wireless and content service combination. For the customers with $\theta_x = \theta_h$ and $x \leq 1 - y_2$, they can either (i) subscribe to SP1's bundled service or (ii) subscribe to SP2's service and SP1's content service separately, which lead to the same welfare for the customers. From (8), we can see that the customers with $v(x) \leq y_2$ (i.e., $x \leq 1 - y_2$) will subscribe to SP2's service. We use x_{21} to denote the total number of customers who subscribe to both SP2's service and SP1's content service. Moreover, we use x_{20} to denote the total number of customers who only subscribe to SP2's service (without SP1's content service). Since x_2 is SP2's market share, we have $x_2 = x_{20} + x_{21}$. Based on the analysis above, x_1, x_{21}, and x_{20} satisfy $x_{21} + x_1 = \alpha(1 - y_2)$ and $x_{20} = (1 - \alpha)(1 - y_2)$. The Wardrop Equilibrium in this case is given by

$$\begin{cases} p_1 = p_2 + p_c, \\ \frac{\beta(x_1 + x_{21}) + x_{20} + x_p}{B} + p_2 = 1 - x_1 - x_{21} - x_{20}, \\ x_1 = x_{21}, \\ x_{20} = (1 - \alpha)(x_1 + x_{21} + x_{20}). \end{cases} \tag{12}$$

The condition $x_1 = x_{21}$ implies that when (i) subscribing to SP1's bundled service and (ii) subscribing to both SP2's service and SP1's content service generate the same welfare, a customer will randomly choose one of these two options with an equal probability.

Case (iv) ($y_1 - y_2 > p_c$): From (5), we can see that no customer will subscribe to SP1's bundled service. Thus, the Wardrop Equilibrium is given by

$$\begin{cases} p_1 > p_2 + p_c, \\ x_1 = 0, \\ \frac{\beta\alpha x_2 + (1 - \alpha)x_2 + x_p}{B} + p_2 = 1 - x_2. \end{cases} \tag{13}$$

Combining the analysis of Wardrop Equilibrium in cases (i)-(iv), we can compute SP1 and SP2's market shares as functions of their prices:

$$x_1(p_1,p_2,p_c) = \tag{14}$$
$$\begin{cases} \frac{B(1-(1-\alpha)p_1)-(1-\alpha)x_p}{(1-\alpha)\beta+B}, & \text{if } p_1 \le \min\{p_2,\bar{p_1}^1\},\\ \frac{B(1-p_1+\alpha p_c)-x_p}{\beta+B}, & \text{if } \min\{p_2,\bar{p_1}^1\} \le p_1 \le p_2,\\ \frac{\alpha(B(1-p_1+p_c)-x_p)}{\alpha\beta+B}, & \text{if } p_2 < p_1 \le \min\{\max\{p_2,\bar{p_1}^2\},\bar{p_1}^3\},\\ \alpha, & \text{if } \min\{\max\{p_2,\bar{p_1}^2\},\bar{p_1}^3\} < p_1 \le \max\{p_2,\bar{p_1}^2\},\\ \frac{a((-1+\alpha)(p_1-p_2-p_c)+B(1-p_1+p_c)-x_p)}{1+\alpha(\beta-1)+B}, & \text{if } \max\{p_2,\bar{p_1}^2\} < p_1 < p_2+p_c,\\ \frac{\alpha(B(1-p_2)-x_p)}{2(1+\alpha(\beta-1)+B)}, & \text{if } p_1 = p_2+p_c,\\ 0, & \text{if } p_1 > p_2+p_c; \end{cases}$$

$$x_2(p_1,p_2,p_c) = \tag{15}$$
$$\begin{cases} \frac{B-Bp_2-x_p}{1+\alpha(\beta-1)+B}, & \text{if } p_2 < p_1-p_c,\\ \frac{(2-\alpha)(B(1-p_2)-x_p)}{2(1+\alpha(\beta-1)+B)}, & \text{if } p_2 = p_1-p_c,\\ \frac{(1-\alpha)(B(1-p_2)+\alpha\beta(p_1-p_2-p_c)-x_p)}{1+\alpha(\beta-1)+B}, & \text{if } p_1-p_c < p_2 \le \min\{\max\{\bar{p_2}^1,p_1-p_c\},\bar{p_2}^2\},\\ -\frac{(1-\alpha)(\alpha\beta-B(1-p_2)+x_p)}{1-\alpha+B}, & \text{if } \max\{\bar{p_2}^1,p_1-p_c\} < p_2 < \min\{p_1,\bar{p_2}^3\},\\ 0, & \text{otherwise.} \end{cases}$$

Here, $\bar{p_1}^1 \triangleq \frac{-\beta+(1-\alpha)\beta p_c+Bp_c-x_p}{B}$, $\bar{p_1}^2 \triangleq -\frac{1-p_2-p_c-Bp_c+\alpha(\beta-1+p_2+p_c)+x_p}{1-\alpha+B}$, $\bar{p_1}^3 \triangleq \frac{B(-1+p_2)+\alpha\beta(p_2+p_c)+x_p}{\alpha\beta}$, $\bar{p_2}^1 \triangleq \frac{1+p_1+Bp_1-p_c-Bp_c+a(\beta-1-p_1+p_c)+x_p}{1-\alpha}$, $\bar{p_2}^2 \triangleq \frac{B+\alpha\beta(p_1-p_c)-x_p}{B+\alpha\beta}$, and $\bar{p_2}^3 \triangleq 1-\frac{\alpha\beta+x_p}{B}$.

4 Equilibrium Analysis for Stages II and I

In Sect. 3, we obtained the market shares of the SPs given the prices and bundling choice. Based on this, we can further analyze SP1's and SP2's pricing and bundling decisions. Assuming that SP1 bundles its service, we first compute their profits given the pricing decisions as follows:

$$\pi_1(p_1,p_2,p_c) =$$
$$\begin{cases} x_1(p_1,p_2,p_c)\cdot p_1 + \alpha\cdot\max\{0, y_1(p_1,p_2,p_c)-p_c\}\cdot p_c, & p_1 \le p_2,\\ x_1(p_1,p_2,p_c)\cdot p_1 + (\alpha - x_1(p_1,p_2,p_c))\cdot p_c, & p_2 < p_1 < p_2+p_c,\\ x_1(p_1,p_2,p_c)\cdot p_1 + (\alpha - 2x_1(p_1,p_2,p_c))\cdot p_c, & p_1 = p_2+p_c,\\ \alpha\cdot p_c, & p_1 > p_2+p_c; \end{cases} \tag{16}$$

$$\pi_2(p_1,p_2,p_c) = x_2(p_1,p_2,p_c)\cdot p_2. \tag{17}$$

When SP1 does not bundle its wireless service with the content service, there is a unique price equilibrium in Stage II, which leads to zero wireless service profits for the SPs. When SP1 chooses bundling, there might not exist a price equilibrium in Stage II. In this case, it would be difficult for SP1 to estimate its profit under the bundling choice. Hence, we assume that SP1 will not choose bundling if there does not exist a price equilibrium in Stage II. Next, we show that when SP1 chooses bundling, there exists a price equilibrium in Stage II if and only if the content service's characteristics satisfy certain conditions. Moreover, we will show that when these conditions are satisfied, SP1 should choose bundling in Stage I, which improves SP1's total profit over the benchmark case. We introduce the following theorem.

Theorem 1. *When $b = 1$, there exists a unique price equilibrium in Stage II if and only if the following conditions hold:*

$$\begin{cases} \theta_h > \max\{\frac{(1-\alpha+\alpha\beta)(B-x_p)}{3\alpha\beta(1-\alpha)+4B(1+B-\alpha+\alpha\beta)}, \bar{p}_c\}, \\ \alpha < \frac{1}{1+\beta}, \\ \frac{(1+\alpha(2\beta-1)+2B)}{3\alpha\beta(1-\alpha)+4B(1+B-\alpha+\alpha\beta)} < \frac{1-\sqrt{\alpha}}{\alpha\beta(1+\sqrt{\alpha})+2B}. \end{cases} \tag{18}$$

Here, $\bar{p}_c$ is the larger solution of the two solutions of the quadratic equation[4] $\pi_1(p_2^{PE}(p_c), p_2^{PE}(p_c), p_c) = \pi_1(p_1^{PE}(p_c), p_2^{PE}(p_c), p_c)$.
When these conditions hold, the equilibrium prices are given by:

$$p_c = \theta_h, \tag{19}$$

$$p_1 = \theta_h + \frac{(1+\alpha(2\beta-1)+2B)(B-x_p)}{3\alpha\beta(1-\alpha)+4B(1+B-\alpha+\alpha\beta)}, \tag{20}$$

$$p_2 = \frac{(\alpha\beta+2B+2(1-\alpha))(B-x_p)}{3\alpha\beta(1-\alpha)+4B(1+B-\alpha+\alpha\beta)}, \tag{21}$$

Moreover, when the conditions in (18) hold, SP1 should choose bundling in Stage I, i.e., $b^* = 1$.

We use Fig. 1 to illustrate how the price equilibrium is achieved. The left figure in Fig. 1 shows how SP1's profit changes with p_1 when $p_2 = p_2^*$, and the right one shows how SP2's profit changes with p_2 when $p_1 = p_1^*$. We can observe that both p_1^* and p_2^* are the global optimal solutions. Thus, the price equilibrium is achieved, and it is in Wardrop Equilibrium case (ii).

5 Types of Content Service

For both the benchmark case and bundling case, we have derived the price equilibrium between SP1 and SP2. We can compare SP1's profits under the benchmark case and bundling case, and then analyze the conditions under which bundling improves SP1's profit. This will imply what types of content service SP1 should consider for bundling.

[4] We use the superscript PE to indicate that the corresponding results are derived at price equilibrium.

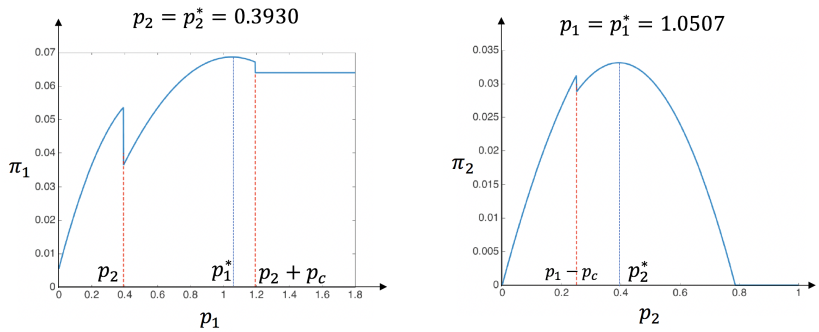

Fig. 1. An example of how price equilibrium is achieved when $B = 0.2$, $\alpha = 0.08$, $x_p = 0$, $\beta = 1$ and $p_c = \theta_h = 0.8$.

In our model, we consider three dimensions of the content service, its popularity (α), value (θ_h) and congestion factor (β). In the conditions needed for SP1 to bundle in (18), these three parameters are closely related to each other. To get a clearer intuition, we analyze these three dimensions separately and focus on one of them in each subsection.

5.1 Popularity

From (18), if θ_h is large enough, α needs to be smaller than a certain upper bound, which is affected by β and B. In Fig. 2, we give the region of α and B that satisfies the feasible condition when $\beta = 1$. This figure shows that, when the band resource (B) increases, SP1 should consider less popular services to bundle. SP1 hopes that with bundling, SP2 will not lower its price to start a price war. Note that when SP1 chooses bundling, the services of SP1 and SP2 are differentiated. In this case, SP2 may also attract customers (i.e., those with low valuations for the content service) and get a positive profit. If α is large, most of the customers have high valuations for the video service and SP2 can hardly attract any customers. This gives SP2 motivation to lower its price, which starts a price war and leads to zero profits to both SPs. The following lemma gives a more general upper bound of α for all possible B.

Lemma 1. *SP1 should bundle its wireless service with a content service when the content service has a popularity less than $\bar{\alpha}$, where $\bar{\alpha} = \min\{\frac{5+2\beta-3\sqrt{1+4\beta}}{2(-2+\beta)^2}, \frac{1}{1+\beta}\}$.*

Note that this result gives a loose upper bound and does not depend on the unlicensed bandwidth B and background traffic x_p. For any given B and x_p, interestingly, SP1 should not bundle its wireless service with a very popular content service. Moreover, given any β, α needs to be smaller than 0.146, which is because $\bar{\alpha} < 0.146$. To get a better intuition of the actual value of α in practice, we give a numerical example here. There are around 400 million wireless

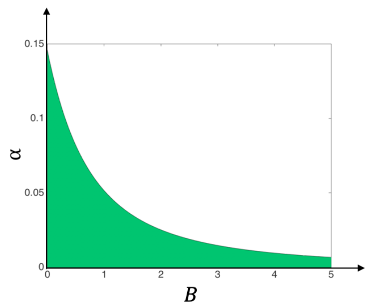

Fig. 2. The region of α and B that satisfies the feasible condition when $\beta = 1$.

subscribers in the U.S. Suppose that there are 600 million potential users (in our model, they correspond to all customers who need to make decisions). According to [17], the video subscription company with the largest market share (i.e., Netflix) has an α of around 0.088, and the company with the second largest market share (i.e., Amazon) has an α of around 0.043. Some content service providers have lower popularity, e.g., HBO Now and YouTube Red have an α of around 0.0083 and 0.0025, respectively.

5.2 Value

From (18), we observe that θ_h needs to be greater than a lower bound. We give an example to show how this bound changes with the popularity α, the band resource B and the amount of primary traffic in the unlicensed band x_p in Fig. 3. The figure shows that, this bound is increasing with B and decreasing with α and x_p. The reasons are as follows. Bundling helps prevent the price war in the sense that if SP1 lowers its price of the bundled service to attract all customers in the wireless market, it will lose a large profit generated from offering the content service. If θ_h is high enough, SP1 will not make its price lower than SP2's price, because the fraction of customers with high valuations for the content service is large and reducing price will greatly reduce SP1's profit from the content service. If B increases, the profit from the wireless market increases, which requires a larger θ_h to hedge SP1's desire to lower its price. On the other hand, the increase of x_p decreases the profit from the wireless market, which in turn lowers the requirement for θ_h. If α increases, the profit from the content service increases, which in turn eases the requirement for θ_h.

5.3 Congestion Factor

From (18), we observe that when β increases, the upper bound of α and the lower bound of θ_h decrease. This means that among bandwidth-consuming services, e.g., video services, the content services that SP1 should bundle its wireless service with should have a larger range of values and a smaller range of popularity.

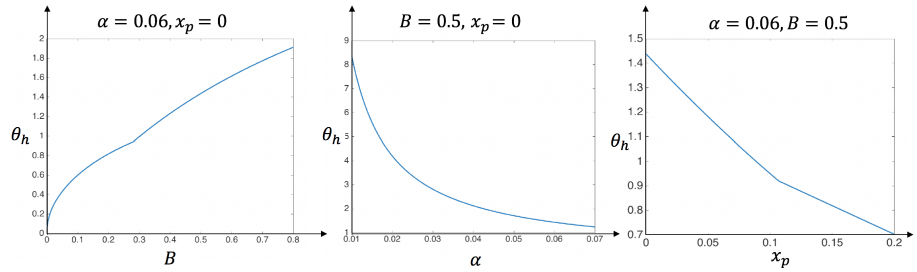

Fig. 3. Impact of α, x_p and B on θ's upper bound when $\beta = 1$.

The reason is that, increasing β decreases the profit from the wireless service, which eases the requirement for θ_h. Increasing β also implies more competition in the wireless market. Thus, a smaller α is required to avoid SP2 decreasing its price.

6 Profits

In this section, we analyze SP1's and SP2's profits in the price equilibrium, compare them in the bundling case and benchmark case, and investigate the impact of the content service's characteristics on the SPs' profits. We first introduce the following theorem.

Theorem 2. *If the conditions in (18) hold, both SP1 and SP2 achieve higher profits in the bundling case, compared to the benchmark case.*

This can be verified by substituting (19) into (16), and comparing it with the SP profits in the benchmark case. We give an example of profit comparison in Fig. 4. In the left figure, the solid lines are the profits of SP1 and SP2 in the bundling case, and the blue dashed line is profit of SP1 in the benchmark case. Note that in the benchmark case, SP1 only generates profit from the content service, and cannot generate profit from the wireless service. It can be observed that bundling improves both SP1's and SP2's profits. In particular, both SPs can generate positive profits from the wireless service. From the left figure, we can also see that SP1's profit in the bundling case increases with α. This implies that SP1 should choose a service with α as large as possible in the feasible region determined by (18).

In the right figure, we compare the increase in profits because of bundling. We define the profit increase of SP1 as $\pi_1(p_1^{PE}(\theta_h), p_2^{PE}(\theta_h), \theta_h) - \alpha\theta_h$ and the profit increase of SP2 as $\pi_2(p_1^{PE}(\theta_h), p_2^{PE}(\theta_h), \theta_h)$ (SP2 has zero profit in the benchmark case). It can be proved that the profit increase of SP1 is always smaller than the profit increase of SP2 in the feasible region, and we can also observe this from the right figure. The insight is that, both SP1 and SP2 might prefer to wait for the other SP to bundle if they both have the bundling option.

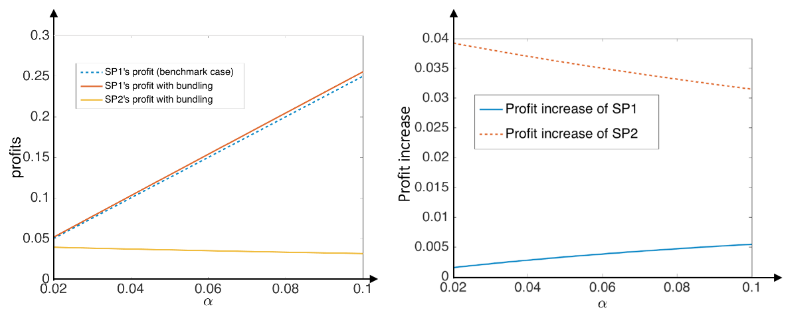

Fig. 4. Profits when $B = 0.2$, $\beta = 1$, and $\theta_h = 2.5$.

However, if both of them wait and do not bundle, they will get zero profit from wireless service.

Theorem 3. *If conditions in (18) hold, $\pi_1(p_1^{PE}(\theta_h), p_2^{PE}(\theta_h), \theta_h)$ is increasing in θ_h, but the profit increase of SP1 is independent of θ_h.*

This theorem indicates that SP1's total profit in the bundling case is increasing in θ_h, which implies that SP1 prefers a content service with high value. However, a high value of θ_h also improves SP1's profit without bundling. The net effect of this is that the increase in profit does not depend on θ_h.

Lemma 2. *If conditions in (18) hold, $\pi_1(p_1^{PE}(\theta_h), p_2^{PE}(\theta_h), \theta_h)$ is decreasing in β.*

This indicates that SP1 should prefer the service with a smaller congestion factor in the feasible region defined in (18). This is intuitive as bundling improves the mass of customers using the content service and generates more congestion on the unlicensed band. This decreases the profits of the SPs. One example is that, when a video subscription service and a music subscription service have similar popularity and values, an SP might prefer the latter service to bundle with.

7 Consumer Surplus

In this section, we compare the consumer surplus, which is the integral of all customers' welfare, in the benchmark case and bundling case. As shown in Fig. 5, bundling decreases the consumer surplus in most cases, expect for the cases when B is extremely small. In the left side of Fig. 5, we consider an extremely small B, i.e., $B = 0.02$. In this case, when α is large, the consumer surplus can be increased by bundling. The intuition is that when the band resource is extremely limited, bundling helps reduce the competition and decrease the congestion, so that consumer surplus increases. However, this effect will be negligible if α is too small and SP2 almost takes the whole market. When B is bigger ($B = 0.07$

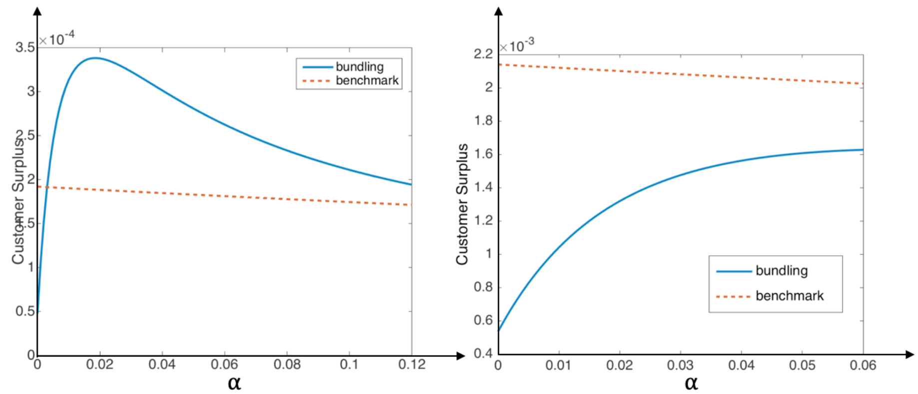

Fig. 5. Two examples of Customer Surplus. Left: $B = 0.02$ and $\beta = 1.5$; Right: $B = 0.07$ and $\beta = 1.5$.

in the right figure), bundling always decreases the consumer surplus. When B is large, the consumer surplus in the benchmark case will be much larger than that in the bundling case.

8 Conclusion

In this paper, we considered the use of bundling as a means of forming a niche unlicensed wireless service market. We studied a case where two SPs compete on an unlicensed band. We showed that an SP can bundle its wireless service with a content service to differentiate the SPs' services. We proved that if the content service has a low popularity, a high value, and a small congestion factor, there exists a unique price equilibrium. Moreover, in this case, both SPs can achieve positive profits and there is no price war between them. We also showed that bundling decreases the consumer surplus, except for the extreme case where the band resource is very limited.

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