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Distributed Consensus-based Kalman Filtering for Estimation with Multiple Moving Targets

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Abstract— **In this paper, we propose a novel distributed consensus-based Kalman filtering (DCKF) with an information-weighted structure for estimation with random mobile targets in continuous-time (CT) systems. First, a novel information-flow structure for the measurement of moving targets is developed based on comprehensive information that includes sensing ranges, target mobility and local information-weighted neighbors. Then, novel necessary and sufficient conditions are given for the convergence of the proposed DCKF. Under these conditions, the estimates of all sensors for multiple targets converge to the consensus values. Finally, comparative simulation studies with the existing Kalman filters demonstrate the superior convergence performance of the new DCKF.**

I. INTRODUCTION

Consensus-based filters have been widely studied [1]–[3]. Olfati [1] proposed the Kalman consensus filter (KCF). However, the consensus estimate is sub-optimal as the cross-covariances between individual estimates are difficult to analyze. Thus, Kamal [2] proposed an information consensus filter (ICF) algorithm to asymptotically achieve the optimal centralized performance. The work of [1] and [3] show that ICF outperforms KCF and generalized Kalman consensus filter (GKCF). Papers [1] and [3] studied the average consensus for a single target. Information weighted consensus for a single target was considered in [2].

Recently, multi-target filtering has attracted wide attention. The work of [4] developed a consensus-based observer for multi-target filtering by using an average-weighted protocol. However, the protocol is not fully distributed and might slow convergence speed. Motivated by these studies, the focus of this paper is to design a fully distributed consensus-based Kalman filter with an information-weighted structure for moving target estimation.

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In real-world settings, the limited sensing range affects the measurement of sensors for moving targets. If targets move into the range, the measurement update equation utilizes the new direct observation from the targets and improves the estimation accuracy [5]. The measurement structure for the filtering studied in the work of [1]–[4], [6], [7] failed to consider the limited sensing range of sensors. The work in [8] developed one parameter to identify whether the target can be directly observed or not. However, this direct/indirect measurement structure is fixed, as the random mobility of targets was not considered either. As a range of tasks (such as search and rescue) are conducted by assuming random and unknown mobility of targets [9]–[12], in this paper we employ random variables to capture the direct observation for multiple randomly moving targets.

The stability and convergence analyses of the estimation filter are necessary to determine the performance of Kalman filtering. The Kalman filters proposed in [2], [3], [7], [13] failed to give rigorous stability and convergence proofs of the algorithms. Zhang [14] provided the necessary condition that if the mean square estimation errors of all sensors are bounded, a special bounded partial weight matrix will exist and guarantee the convergence. This convergence condition requires the knowledge of global topology.

In this paper, we study consensus-based distributed estimation in CT systems. The aim is to develop a novel DCKF for estimation with multiple moving targets. First, we introduce a new distance-based information flow structure for measurement for unknown randomly moving targets. Then, the consensus-based DCKF updates from stochastic direct observation from targets and the weighted information from neighbors. Finally, with a condition guaranteed for the stability, the Kalman filter and estimation show fast convergence speed.

The paper is outlined as follows. Section II develops the distance-based measurement structure and the DCKF for the estimation with multiple moving targets. Section III analyzes the stability of DCKF. Section IV provides comparative simulations. Section V summarizes and concludes the paper.

II. PROBLEM FORMULATION

In this section, some results of graph theory are provided first. Then the novel distance-based information flow structure for measurement is developed. Based on that, the distributed Kalman consensus filter for multiple moving targets is developed.

A. Graph Theory for Sensor Networks

Consider a graph $Gr = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ for a sensor network of N sensors $\mathcal{V} = \{v_1, v_2, \cdots v_N\}$. The set of edges $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ stands for the communication channels, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the associated weighted adjacency matrix. Each entry a_{ij}, graphically represented by an arrow with head sensor i and tail sensor j, is the weight associated with edge (j, i) and denotes the information flow from sensor j to i. $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$, and otherwise, $a_{ij} = 0$. The set of neighbors of sensor i is denoted as $\mathcal{N}_i = \{v_j : (v_j, v_i) \in \mathcal{E}, \forall j \neq i\}$. Define the in-degree of sensor i as $d_i = \sum_{j=1}^{N} a_{ij}$ and the in-degree diagonal matrix $D = \text{diag}\{d_i\} \in \mathbb{R}^{N \times N}$. The graph Laplacian matrix is $L = D - \mathcal{A}$.

B. Distributed Consensus-based Kalman Filtering for Multiple Moving Targets

Consider the k-th target of the linear CT system, which is modeled as

$$\dot{x}^k = A^k x^k + F^k \omega^k \tag{1}$$

where $k = 1, 2, \cdots, M$. $x^k \in \mathbb{R}^n$ denotes the state of target k. $x^k(0) \sim (\bar{x}_0^k, P_0^k)$, where $\bar{x}_0^k$ and P_0^k are the initial value and covariance, respectively. ω^k is a zero-mean Gaussian process noise with covariance matrix W^k. Compared with the non-stationary Wiener process noise [15], this work uses the stationary zero-mean Gaussian noise.

The observation of sensor i for target k is modeled as

$$s_i^k = G_i x^k + \mu_i^k \tag{2}$$

where $s_i^k \in \mathbb{R}^{m_i}$, $i = 1, 2, \cdots, N$ denotes the observation matrices of sensor i for target k. $G_i \in \mathbb{R}^{m_i \times n}$ are distributed observation matrices. $\mu_i^k \in \mathbb{R}^{m_i}$ is a zero-mean Gaussian observation noise with covariance matrix R_i^k. The process noise and observation noise are uncorrelated.

The measurement of sensor i for the moving target k is given by

$$z_i^k = \begin{bmatrix} \lambda_i^k(q,t)(G_i x^k + \mu_i^k) \\ \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} (\hat{x}_j^k + \omega_{ij}^k) \end{bmatrix} \tag{3}$$

where $z_i^k \in \mathbb{R}^{m_i+n}$ is the augmented matrix of two information parts. The two parts consist of direct observation information from targets (first row) and indirect information from neighbors (second row). And $\hat{x}_j^k \in \mathbb{R}^n$ is the estimation of sensor j for target k.

$\lambda_i^k(x,t)$ is denoted as direct distance-based observation coefficient (DDOC). $\lambda_i^k(x,t) = 1$ and $z_i^k \in \mathbb{R}^{m_i+n}$ if target k moves into the sensing range of sensor i. DDOC $\lambda_i^k = 0$ and $z_i^k \in \mathbb{R}^n$, otherwise. ω_{ij}^k is communication channel noise, where $\Xi_{ij}^k = \mathrm{E}\left\{\omega_{ij}^k (\omega_{ij}^k)^T\right\}$, for any i and $j \in \mathcal{N}_i$.

Remark 1. Note that $\lambda_i^k(t)$ is time-varying. In addition, the Fisher information matrix $(P_j^k)^{-1}$ of sensor j [16] increases with increasing accuracy of sensor j's estimates. Thus, the information-flow structure (3) is information-weighted.

Remark 2. Compared with the work of [8] which assumed fixed target states in the measurement structure, i.e. λ_i^k is constant, in this paper, the targets move randomly, and the information flow structure is time-varying, distance-based, and probabilistic. Here, the connection between multiple targets is not the focus. Thus, λ_i^k is independent between targets.

Next, the novel DCKF for estimation with moving targets is presented.

Theorem 1. *Consider the dynamics of multiple targets (1), and the information flow structure for moving targets (3) without communication noises, then one obtains an approximately optimal DCKF for multiple moving targets given by*

$$\begin{aligned} \dot{P}_i^k &= A_i^k P_i^k + P_i^k (A_i^k)^T - \lambda_i^k P_i^k G_i^T (\Omega_i^k)^{-1} G_i P_i^k \\ &\quad - 4 \sum_{j=1}^{N} a_{ij}^k P_i^k Q_i^k P_i^k + F^k W^k (F^k)^T \end{aligned} \tag{4}$$

and the estimation equation is given by

$$\begin{aligned} \dot{\hat{x}}_i^k &= A^k \hat{x}_i^k - \lambda_i^k P_i^k G_i^T (\Omega_i^k)^{-1} (G_i (x^k - \hat{x}_i^k) + \mu_i^k) \\ &\quad + 2 \sum_{j=1}^{N} a_{ij}^k P_i^k Q_i^k (\hat{x}_j^k - \hat{x}_i^k) \end{aligned} \tag{5}$$

where $A_i^k = A^k + \sum_{j=1}^{N} a_{ij}^k I_n$, $\Omega_i^k = E\{\mu_i^k (\mu_i^k)^T\}$, $Q_i^k = [\sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} (P_i^k + P_j^k)]^{-1} \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1}$ *and* $i, j = 1, 2, \ldots N$.

Note. The DKCF is called approximately optimal in the sense clarified by Remark 3 after the proof.

Proof: The distance-based information flow structure for the measurement (3) can be rewritten as

$$\begin{aligned} & z_i^k(t, \lambda_i^k) \\ &= \begin{bmatrix} \lambda_i^k G_i x^k + \lambda_i^k \mu_i^k \\ \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} x^k + \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} (\hat{x}_j^k - x^k + \omega_{ij}^k) \end{bmatrix} \\ &= H_i^k x^k + v_i^k \end{aligned} \tag{6}$$

$$H_i^k = \begin{bmatrix} \lambda_i^k G_i \\ \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} \end{bmatrix},$$

$$v_i^k = \begin{bmatrix} \lambda_i^k \mu_i^k \\ \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} (\hat{x}_j^k - x^k + \omega_{ij}^k) \end{bmatrix}.$$

Define the term consisting of direct observation from target states as

$$\bar{z}_i^k(t, \lambda_i^k) = \lambda_i^k G_i x^k + \lambda_i^k \mu_i^k = \bar{H}_i^k x^k + \bar{v}_i^k \tag{7}$$

where $\bar{H}_i^k = \lambda_i^k G_i$ and $\bar{v}_i^k = \lambda_i^k \mu_i^k$ denote the direct information flow structure and direct measurement noise of sensor i for target k, respectively. Define the indirect observation by using the estimates of information-weighted neighbors of sensor i as

$$\begin{aligned}\widehat{z}_i^k(t,\lambda_i^k) &= \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} x^k \\ &\quad + \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} (\hat{x}_j^k - x^k + \omega_{ij}^k) \\ &= \widehat{H}_i^k x^k + \widehat{v}_i^k \end{aligned} \tag{8}$$

where $\widehat{H}_i^k = \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1}$ and $\widehat{v}_i^k = \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} (\hat{x}_j^k - x^k + \omega_{ij}^k)$.

For the dynamics (1) with (7) and (8) for measurement, the distributed Kalman filtering structure for estimation is given as

$$\begin{aligned}\dot{\hat{x}}_i^k(t,\ \lambda_i^k) &= A^k \hat{x}_i^k + K_i^k (z_i^k - H_i^k \hat{x}_i^k) \\ &= A^k \hat{x}_i^k + \bar{K}_i^k (\bar{z}_i^k - \bar{H}_i^k \hat{x}_i^k) + \widehat{K}_i^k (\widehat{z}_i^k - \widehat{H}_i^k \hat{x}_i^k) \\ &= A^k \hat{x}_i^k + \lambda_i^k \bar{K}_i^k (G_i (x^k - \hat{x}_i^k) + \mu_i^k) \\ &\quad + \widehat{K}_i^k \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} (\hat{x}_j^k - \hat{x}_i^k + \omega_{ij}^k) \end{aligned} \tag{9}$$

where $K_i^k = (\bar{K}_i^k,\ \widehat{K}_i^k)$, $A_i^k = A^k + \sum_{j=1}^{N} a_{ij}^k I_n$.

Define the distributed state synchronization error for target k as $\tilde{x}_i^k = x^k - \hat{x}_i^k$. Then one has error dynamics

$$\begin{aligned}&\dot{\tilde{x}}_i^k(t,\ \lambda_i^k) \\ &= A^k x^k + F^k \omega^k - A_i^k \hat{x}_i^k - \lambda_i^k \bar{K}_i^k (G_i (x^k - \hat{x}_i^k) + \mu_i^k) \\ &\quad - \widehat{K}_i^k \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} (\hat{x}_j^k - \hat{x}_i^k + \omega_{ij}^k) + \sum_{j=1}^{N} a_{ij}^k \hat{x}_i^k \\ &= A_i^k \tilde{x}_i^k + F^k \omega^k - \lambda_i^k \bar{K}_i^k G_i \tilde{x}_i^k - \lambda_i^k \bar{K}_i^k \mu_i^k \\ &\quad + \widehat{K}_i^k \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} (\tilde{x}_j^k - \tilde{x}_i^k + \omega_{ij}^k) - d_i^k \tilde{x}_i^k. \end{aligned} \tag{10}$$

where $d_i^k = \sum_{j=1}^{N} a_{ij}^k$.

With $\omega_{ij}^k = 0$ and $\mu_i^k = 0$, rewrite (10) as

$$\dot{\tilde{x}}_i^k(t,\ \lambda_i^k) = M_i^k \tilde{x}_i^k + N_i^k \tag{11}$$

where

$$M_i^k = A_i^k - \lambda_i^k \bar{K}_i^k G_i - \widehat{K}_i^k \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1}$$

$$N_i^k = -\lambda_i^k \bar{K}_i^k \mu_i^k + \widehat{K}_i^k \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} \tilde{x}_j^k - d_i^k \tilde{x}_i^k + F^k \omega^k.$$

The time signals in this paper λ_i^k, ω^k and μ_i^k are uncorrelated. Define the following covariances $S^k = \mathrm{E}\{\omega^k (\omega^k)^T\}$, $\overline{T}_i^k = \mathrm{E}\{\mu_i^k (\mu_i^k)^T\} = \Omega_i^k$ and $\widehat{T}_i^k = \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1}$.

By building a discrete-time model for (11) and doing approximation with small sampling period γ at time step $h+1$, one has

$$\tilde{x}_i^k(h+1) = e^{M_i^k \gamma} \tilde{x}_i^k + \int_0^{\gamma} e^{M_i^k (\gamma - \tau)} N_i^k(\tau) d\tau \tag{12}$$

$$\tilde{x}_i^k(h+1) \approx (I + \gamma M_i^k) \tilde{x}_i^k(h) + \gamma N_i^k. \tag{13}$$

Substituting (13) into

$$P_i^k(h+1) = E\{\tilde{x}_i^k(h+1) (\tilde{x}_i^k(h+1))^T | \lambda_1^k, \cdots, \lambda_M^k\}$$

leads to the covariance of discrete time as

$$\begin{aligned}P_i^k(h+1) &= (I + \gamma M_i^k) P_i^k(h) (I + \gamma M_i^k)^T \\ &\quad + E\{\gamma (I + \gamma M_i^k) \tilde{x}_i^k (N_i^k)^T\} \\ &\quad + E\{\gamma N_i^k (\tilde{x}_i^k)^T (I + \gamma M_i^T)\} \\ &\quad + \gamma^2 E\{N_i^k (N_i^k)^T\}. \end{aligned} \tag{14}$$

Note that the P_i^k is positive definite from its definition above. Following the Euler approximation of

$$\dot{P}_i^k(t) = \lim_{\gamma \to 0} \frac{P_i^k(h+1) - P_i^k(h)}{\gamma} \tag{15}$$

one has

$$\begin{aligned}\dot{P}_i^k(t) &= M_i^k(t) P_i^k(t) + P_i^k(t) (M_i^k(t))^T \\ &\quad + E\{\tilde{x}_i^k(t) (N_i^k(t))^T\} + E\{N_i^k(t) (\tilde{x}_i^k(t))^T\} \\ &\quad + \gamma E\{N_i^k(t) (N_i^k(\tau))^T\}. \end{aligned} \tag{16}$$

With discrete-time sampling and back to continuous-time domain, one has

$$\begin{aligned}E\{N_i^k(t) (N_i^k(\tau))^T\} &= F^k (W^k/\gamma) (F^k)^T \delta(t - \tau) \\ &\quad + \lambda_i^k \bar{K}_i^k (\bar{T}_i^k/\gamma) (\bar{K}_i^k)^T \delta(t - \tau) \\ &\quad + E\left\{\eta_i^k (\eta_i^k)^T\right\} \end{aligned} \tag{17}$$

where $\eta_i^k = \widehat{K}_i^k \sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} \tilde{x}_j^k - d_i^k \tilde{x}_i^k$. The first and second terms are white noises while the third one is not, i.e. $\delta \neq 0$ when $t = \tau$.

With $\gamma \to 0$, substituting (17) into (16) leads to

$$\begin{aligned}\dot{P}_i^k &= M_i^k P_i^k + P_i^k (M_i^k)^T - \lambda_i^k \bar{K}_i^k \bar{T}_i^k (\bar{K}_i^k)^T \\ &\quad + E\{\tilde{x}_i^k (N_i^k)^T\} + E\{N_i^k (\tilde{x}_i^k)^T\} \\ &\quad + F^k W^k (F^k)^T. \end{aligned} \tag{18}$$

By assuming $\widehat{K}_i^k = (\widehat{K}_i^k)^T$ and letting $E\{\tilde{x}_i^k (N_i^k)^T\} + E\{N_i^k (\tilde{x}_i^k)^T\} = 0$, one obtains suboptimal information gain $\widehat{K}_i^k$ as

$$\widehat{K}_i^k = 2 \sum_{j=1}^{N} a_{ij}^k P_i^k [\sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1} (P_i^k + P_j^k)]^{-1}. \tag{19}$$

To determine the Kalman gain $\bar{K}_i^k$, the value of λ_i^k should be discussed.

1) If $\lambda_i^k = 0$, as it is shown in (10), there are no estimation updates. Thus, one has

$$\bar{K}_i^k = 0. \tag{20}$$

2) If $\lambda_i^k = 1$, the estimation is updated. To obtain $\bar{K}_i^k$, one uses $\partial Tr(\dot{P}_i^k)/\partial \bar{K}_i^k = 0$

$$2\bar{K}_i^k \bar{T}_i^k - 2P_i^k G_i^T = 0 \tag{21}$$

$$\bar{K}_i^k = P_i^k G_i^T (\Omega_i^k)^{-1}. \tag{22}$$

With the combination of two situations (21) and (22), one obtains optimal directive information gains as

$$\bar{K}_i^k = \lambda_i^k P_i^k G_i^T (\Omega_i^k)^{-1}. \tag{23}$$

Substituting direct information gain (23) and indirect gain (20) into (18) and (9), one derives DCKF (4) and (5). □

Remark 3. The DCKF is approximately optimal in the sense that DCKF is based on the Euler approximation process (13) and (15). Also, in (4), the second direct information term is optimal, whereas the third indirect term is suboptimal.

III. Stability and Convergence Analysis

In this section, the convergence of the distributed consensus-based Kalman filtering for estimation is analyzed.

To accomplish this, the statistics of DDOC $\lambda_i^k(t)$ in (3) are needed. Here, the time domain is decomposed into a finite sampling set of s disjoint random intervals $[t_m,\ t_{m+1})$ for $0 \leq m \leq s-1$. In each small interval, independent stochastic process of $\lambda_i^k(t)$ is $\lambda_i^k = 1$ or 0. Then

$$\mathrm{P}\left\{\lambda_i^k(q_i,t) = 1\right\} = p_i^k(q_i,t), \quad \mathrm{E}\left\{\lambda_i^k(q_i,t)\right\} = p_i^k(q_i,t)$$

where $p_i^k(q_i,t)$, the output of PDF, depends on the position of sensor i and time t. $t \in [t_m,\ t_{m+1})$. In addition, $\mathrm{Cov}(\lambda_i^k(t), \lambda_j^k(t)) = 0$, $\mathrm{Cov}(\lambda_i^k(t_1), \lambda_i^k(t_2)) = 0$ and t_1 and t_2 are in different sampling intervals.

$\lambda_i^k(t)$ is time-varying and modulated by the random mobility of targets. To capture random mobility, stochastic models such as Random Mobility Models (RMMs), including Random Walk (RW), Random Direction (RD), Random Waypoint (RWP), and Smooth Turn (ST) are widely used.

Due to the varying $\lambda_i^k(t)$, there exists a different Laplacian matrix L^k, for each $k = 1,\ 2,\ \cdots,\ M$.

Assumption 1. The topology is strongly connected in the sensor networks.

By Assumption 1, the Laplacian matrix L^k has rank $N-1$, i.e. the eigenvalue of L^k which equals zero is not repeated [17], [18].

Lemma 1. [18] Let the Laplacian matrix L^k be a singular M-matrix. Then, there exists a positive vector $q^k = \begin{bmatrix} q_1^k & q_2^k & \cdots & q_N^k \end{bmatrix}^T$ with $q_i^k > 0$, for $\forall i = 1,2,\ldots,N$ such that $q^k L^k \geq 0$.

Lemma 2. [8] Let the Laplacian matrix L^k for target k be a singular M-matrix. Define the vector $q^k = \begin{bmatrix} q_1^k & q_2^k & \cdots & q_N^k \end{bmatrix}^T$ with $q_i^k > 0$ in Lemma 1. Then $q_i^k \sum_{j=1}^{N} a_{ij}^k \geq \sum_{j=1}^{N} q_j a_{ij}^k > 0$ for $\forall i = 1,2,\ldots,N$.

Based on these results, the convergence of the DCKF for estimation with moving targets is proved next.

Theorem 2. *(Proof of convergence of DCKF for estimation with moving targets). Given the information-flow structure (3), the DCKF in Theorem 1 for estimation with multiple moving targets, and process, measurement and communication noise under Assumption 1, the distributed state estimation error $\tilde{x}_i^k(t) = x^k(t) - \hat{x}_i^k(t)$ keeps uniformly ultimately bounded in the mean square as $t \to \infty$ $\forall$ i and k, if only if the pair $(A^k, \bar{H}^k)$ with $\bar{H}^k = \begin{bmatrix} (p_1^k \bar{H}_1^k)^T & (p_2^k \bar{H}_2^k)^T & \cdots & (p_N^k \bar{H}_N^k)^T \end{bmatrix}^T$ is observable.*

Proof: Given the target dynamics (1) and extending from DCKF (4) and (5) by considering process noise W^k, observation noise μ_i^k and communication noise ω_{ij}^k, one has

$$\begin{aligned}
\dot{\tilde{x}}_i^k &= \dot{x}^k - \dot{\hat{x}}_i^k \\
&= (A^k - \lambda_i^k P_i^k \bar{H}_i^T (\Omega_i^k)^{-1} \bar{H}_i)\tilde{x}_i^k + \lambda_i^k P_i^k \bar{H}_i^T (\Omega_i^k)^{-1} \mu_i^k \\
&\quad + 2d_i^k P_i^k Q_i^k (\tilde{x}_j^k - \tilde{x}_i^k + \omega_{ij}^k) + F^k \omega^k
\end{aligned} \tag{24}$$

$$\begin{aligned}
\dot{P}_i^k &= A_i^k P_i^k + P_i^k (A_i^k)^T - \lambda_i^k P_i^k \bar{H}_i^T (\Omega_i^k)^{-1} \bar{H}_i P_i^k \\
&\quad - 4\sum_{j=1}^{N} a_{ij}^k P_i^k Q_i^k P_i^k + F^k W^k (F^k)^T \\
&\quad + 4\sum_{j=1}^{N} a_{ij}^k P_i^k [\sum_{j=1}^{N} a_{ij}^k (P_i^k + P_j^k)]^{-1} \\
&\quad \times \Xi_{ij}^k [\sum_{j=1}^{N} a_{ij}^k (P_i^k + P_j^k)]^{-1} P_i^k.
\end{aligned} \tag{25}$$

The Lyapunov function candidate can be defined as

$$V(\tilde{X}^k(t)) = \mathrm{E}\left\{ \sum_{i=1}^{N} q_i^k (\tilde{x}_i^k)^T (P_i^k)^{-1} \tilde{x}_i^k \right\} \tag{26}$$

where q_i^k is defined in Lemma 2. Under Assumption 1, if the pair $(A^k, \bar{H}^k)$ is observable, $\mathrm{diag}((P_i^k)^{-1}) > 0$ and thus $V(\tilde{X}^k) > 0$ for any $\tilde{X}^k = \begin{bmatrix} \tilde{x}_1^k & \tilde{x}_2^k & \cdots & \tilde{x}_i^k \end{bmatrix}^T \neq 0$.

By deriving the Lyapunov function and substitute (24) and (25) into (26), one obtains

$$\begin{aligned}
&\dot{V}(\tilde{X}^k(t,\ \lambda_i^k)) \\
&= \mathrm{E}\{\sum_{i=1}^{N} q_i^k (\dot{\tilde{x}}_i^k)^T (P_i^k)^{-1} \tilde{x}_i^k + \sum_{i=1}^{N} q_i^k (\tilde{x}_i^k)^T (\dot{P}_i^k)^{-1} \tilde{x}_i^k \\
&\quad + \sum_{i=1}^{N} q_i^k (\tilde{x}_i^k)^T (P_i^k)^{-1} \dot{\tilde{x}}_i^k\} \\
&= -\sum_{i=1}^{N} q_i^k (\tilde{x}_i^k)^T \Sigma_i^k \tilde{x}_i^k - \sum_{i=1}^{N} q_i^k (\tilde{x}_i^k)^T F^k W^k (F^k)^T \tilde{x}_i^k \\
&\quad - \sum_{i=1}^{N} 2q_i^k d_i^k (\tilde{x}_i^k)^T Q_i^k \tilde{x}_j^k - \sum_{i=1}^{N} q_i^k d_i^k (\tilde{x}_i^k)^T (P_i^k)^{-1} \tilde{x}_i^k \\
&\quad - \sum_{i=1}^{N} q_i^k d_i^k (\tilde{x}_i^k)^T (P_i^k)^{-1} \tilde{x}_i^k - \sum_{i=1}^{N} 2q_i^k d_i^k (\tilde{x}_j^k)^T Q_i^k \tilde{x}_i^k \\
&\quad + \sum_{i=1}^{N} q_i^k (\tilde{x}_i^k)^T \phi_i^k + \sum_{i=1}^{N} q_i^k (\phi_i^k)^T \tilde{x}_i^k \\
&\quad - \sum_{i=1}^{N} q_i^k (\tilde{x}_i^k)^T \Theta_i^k \tilde{x}_i^k
\end{aligned} \tag{27}$$

where $\Sigma_i^k = G_i^T(\Omega_i^k)^{-1}G_i$, $\Theta_i^k = 4\sum_{j=1}^{N} a_{ij}^k P_i^k [\sum_{j=1}^{N} a_{ij}^k (P_i^k + P_j^k)]^{-1}\Xi_{ij}^k [\sum_{j=1}^{N} a_{ij}^k (P_i^k + P_j^k)]^{-1} P_i^k$, $Q_i^k = [\sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1}(P_i^k + P_j^k)]^{-1}\sum_{j=1}^{N} a_{ij}^k (P_j^k)^{-1}$ and $\phi_i^k = F^k W^k + p_i^k R_i^k (\Omega_i^k)^{-1} G_i$.

According to Lemma 2, the last term on the right side of the (27) becomes

$$
\begin{aligned}
&-\sum_{i=1}^{N} q_i^k \sum_{j=1}^{N} a_{ij}^k (\tilde{x}_i^k)^T (P_i^k)^{-1}\tilde{x}_i^k \\
&\le -\sum_{i=1}^{N}\sum_{j=1}^{N} (q_j^k a_{ji}^k)(\tilde{x}_i^k)^T (P_i^k)^{-1}\tilde{x}_i^k \\
&= -\sum_{i=1}^{N} q_i^k \sum_{j=1}^{N} a_{ij}^k (\tilde{x}_j^k)^T (P_j^k)^{-1}\tilde{x}_j^k. \qquad (28)
\end{aligned}
$$

Because $0 < 2Q_i^k P_i^k \le I$ and $0 < 2Q_i^k P_j^k \le I$, one has

$$
\begin{aligned}
&-\sum_{i=1}^{N} 2q_i^k d_i^k (\tilde{x}_i^k)^T Q_i^k \tilde{x}_j^k - \sum_{i=1}^{N} q_i^k d_i^k (\tilde{x}_i^k)^T (P_i^k)^{-1}\tilde{x}_i^k \\
&-\sum_{i=1}^{N} q_i^k d_i^k (\tilde{x}_i^k)^T (P_i^k)^{-1}\tilde{x}_i^k - \sum_{i=1}^{N} 2q_i^k d_i^k (\tilde{x}_j^k)^T Q_i^k \tilde{x}_i^k \\
&\le -\sum_{i=1}^{N} 2q_i^k d_i^k (\tilde{x}_i^k)^T Q_i^k \tilde{x}_j^k - \sum_{i=1}^{N} q_i^k d_i^k (\tilde{x}_i^k)^T (P_i^k)^{-1}\tilde{x}_i^k \\
&\quad -\sum_{i=1}^{N} q_i^k d_i^k (\tilde{x}_j^k)^T (P_j^k)^{-1}\tilde{x}_j^k - \sum_{i=1}^{N} 2q_i^k d_i^k (\tilde{x}_j^k)^T Q_i^k \tilde{x}_i^k \\
&\le -\sum_{i=1,j=1}^{N} 2q_i^k a_{ij}^k (\tilde{x}_i^k + \tilde{x}_j^k)^T Q_i^k (\tilde{x}_i^k + \tilde{x}_j^k) \\
&= 0. \qquad (29)
\end{aligned}
$$

Finally, one has

$$
\begin{aligned}
\dot{V}(\tilde{X}^k) \le &-\sum_{i=1}^{N} q_i^k (\tilde{x}_i^k)^T (P_i^k)^{-1} F^k W^k (F^k)^T (P_i^k)^{-1}\tilde{x}_i^k \\
&-\sum_{i=1}^{N} p_i^k q_i^k (\tilde{x}_i^k)^T \bar{H}_i^T (\Omega_i^k)^{-1}\bar{H}_i \tilde{x}_i^k \\
&-\sum_{i=1,j=1}^{N} 2q_i^k a_{ij}^k (\tilde{x}_i^k + \tilde{x}_j^k)^T Q_i^k (\tilde{x}_i^k + \tilde{x}_j^k) \\
&-\lambda_{min}(\Theta^k)\|\tilde{X}^k\|^2 + \|\phi^k\|\|\tilde{X}^k\| \\
\le &-\lambda_{min}(\Theta^k)\|\tilde{X}^k\|^2 + \|\phi^k\|\|\tilde{X}^k\|. \qquad (30)
\end{aligned}
$$

where $\Theta^k = \text{diag}\{q_i^k \Theta_i^k\}$ and $\phi^k = q_i^k \phi_i^k$.

It can be seen that if $\dot{V}(\tilde{X}^k) \le 0$, $\|\tilde{X}^k\| \ge \|\phi^k\|/\lambda_{min}(\Theta^k)$. Therefore, (26) is an appropriate Lyapunov function and the dynamics of state estimation errors are uniformly ultimately bounded if the target k is collectively observable by all sensors, i.e. the pair $(A^k, \bar{H}^k)$ with $\bar{H}^k = \begin{bmatrix}(p_1^k \bar{H}_1^k)^T & (p_2^k \bar{H}_2^k)^T & \cdots & (p_N^k \bar{H}_N^k)^T\end{bmatrix}^T$ is observable. □

Remark 4. The state estimation errors will converge to zero as time goes to infinity without considering the noises in the system.

IV. Simulation Studies

In this section, numerical examples are studied to show the effectiveness of our novel distributed consensus-based Kalman filtering. The comparative study shows that DCKF outperforms the Kalman consensus filter (KCF) [1].

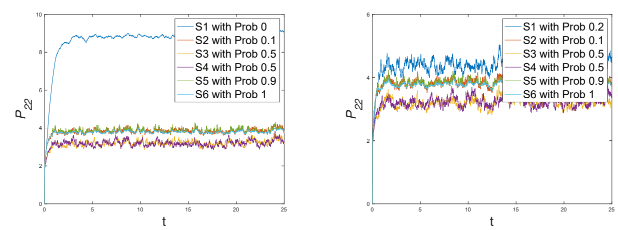
A. Performance Analysis

The system dynamics of three targets are

$$
A^1 = \begin{bmatrix} 0 & -0.5 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \\ 0 & 0 & -0.5 & 0 \end{bmatrix}, A^2 = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
A^3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.
$$

The parameters of these targets are chosen as $\omega^k \sim (0, W)$, where $W = \begin{bmatrix} 2 & 2 & 1 & 1 \end{bmatrix}^T$, $\mu_i^k \sim (0, 0.5\mathbf{1}_4)$, $\omega_{ij}^k \sim (0, 0.5\mathbf{1}_4)$, $F^k = I_4$, $S^k = \text{diag}(2, 2, 1, 1)$, $\Omega_i^k = 0.25I_4$, $\Xi_{ij}^k = 0.5I_4$ and $G_i = I_4$ for $k = 1, 2$ and $i, j = 1, 2, \cdots, 6$. For target 3, $\omega^3 \sim (0, 0.5I_2)$, $\mu_i^3 \sim (0, 0.5\mathbf{1}_2)$, $\omega_{ij}^3 \sim (0, 0.5\mathbf{1}_4)$, $F^3 = I_2$, $S^3 = \text{diag}(2, 1)$, $\Omega_i^3 = 0.25I_2$, $\Xi_{ij}^3 = 0.5I_2$ and $G_3 = I_2$ where $\mathbf{1}_n \in \mathbb{R}^n$ denotes a column vector containing all ones and I_n denotes a $n \times n$ identity matrix.

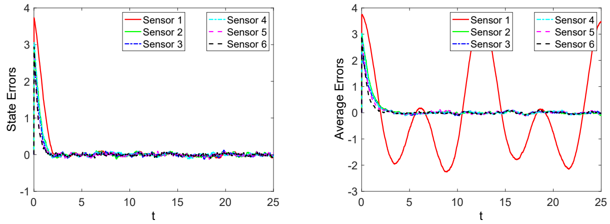


(a) (b)

Fig. 1. The performance of average covariance value P_{22} of DCKF under two observation probability schemes: (a) by Prob scheme 1; (b) by Prob scheme 2. Despite sensor 1 having an observation probability of 0, the uncertainty of the sensor 1s estimate remains bounded. By increasing the observation probabilities, the uncertainty decreases.

The diagonal element P_{22} of the covariance matrix of six sensors for three targets is selected to illustrate the estimation errors. We study two observation probability (Prob) schemes in which Prob Scheme 1: $p_1^k = 0$, $p_2^k = 0.1$, $p_3^k = 0.5$, $p_4^k = 0.5$, $p_5^k = 0.9$, $p_6^k = 1$ for all k; Prob Scheme 2: $p_1^k = 0.2$, $p_2^k = 0.1$, $p_3^k = 0.5$, $p_4^k = 0.5$, $p_5^k = 0.9$, $p_6^k = 1$ for all k. The average covariance values are shown in the Fig. 1(a) and 1(b). From Fig. 1(a). Note that even if $p_1^k = 0$ for sensor 1 (S1), the filter still converges while the errors of the estimation are large. This is because the estimates of S1 only use the information from neighbors. In addition, by increasing the Prob for sensor 1, the error decreases as shown from Fig. 1(a) and 1(b).

Fig. 2(a) gives the average tracking state errors for three targets under the Prob scheme 1. Here, the average tracking

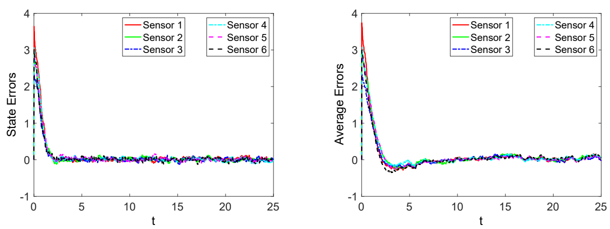


(a) (b)

Fig. 2. The convergence performance of average estimation errors of each sensor for three targets by different filters under Prob scheme 1: (a) by DCKF; (b) by KCF. DCKF is better suited to situations where one of the sensors has an observation probability of 0, as illustrated by the fact that sensor 1s estimation error does not converge when using the KCF, whereas it does for the DCKF.

error is defined as the average value for all states. Although there exists zero observation probability, the errors quickly converge to zero.

B. Comparison Study



(a) (b)

Fig. 3. The convergence performance of average estimation errors of each sensor by different filters under the same Prob 0.2: (a) by DCKF; (b) by KCF. DCKF has better performance of converge speed about estimation errors than KCF.

In this subsection, our novel DCKF for estimation with multiple moving targets is compared with KCF in [1]. Here, two different comparison schemes are studied between two distributed Kalman filters on the convergence rate by considering: 1) the Prob scheme 1 where the zero probability exists; and 2) the Prob scheme 2 where all probabilities are nonzero.

The parameter settings for the distributed KCF are designed as $H_1 = H_2 = I_4$ and $H_3 = I_2$. $\gamma_i^k = \varepsilon/(1+||P_i^k||_F)$ with $\varepsilon = 0.04$ for $i = 1, 2, 3$. The other settings are the same as those described in Section IV A.

First, under the Prob scheme 1 for sensors in Section IV A, the convergence of KCF for the estimation is illustrated in Fig. 2(b). It is hard to say that all estimation errors converge to zero within sampling time.

Second, assume $p_i^k = 0.2$ for any $i = 1, 2, \cdots 6$ and $k = 1, 2, 3$. Figures 3(a) and 3(b) present the average estimation errors for each state of each target using the two algorithms. As it is shown, the proposed DCKF has a faster convergence rate and a smaller estimation error.

Therefore, the proposed DCKF for estimation with random moving targets has superior convergence performance compared with KCF in [1].

V. Conclusion

In this paper, the distributed consensus-based Kalman filtering (DCKF) for estimation with random moving targets is studied in continuous-time dynamics. A novel distance-based information flow structure was developed by considering limited sensing range and stochastic moving targets. Based on the probabilistic measurement model, the new DCKF for estimation also uses information-weighted neighbors. The simulation and comparative studies show the effectiveness and superiority of the DCKF. In the future, the connection between targets can be studied.

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