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Product innovation and process innovation in a dynamic Stackelberg game

Yufang Wang[a], Xinyu Wang[b], Shuhua Chang[b,*], Yun Kang[c]

[a] School of Statistics, Tianjin University of Finance and Economics, Tianjin 300222, China
[b] Coordinated Innovation Center for, Computable Modeling in Management Science, Tianjin University of Finance and Economics, Tianjin 300222, China
[c] Science and Mathematics Faculty, Arizona State University, Mesa, AZ 85212, USA

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ABSTRACT

Under a Stackelberg game structure, we investigate the optimal R&D portfolio of a single-product monopolist investment in product and process innovations of a South-country firm. The research is conducted using a dynamic game with knowledge accumulation. The South-country firm is a manufacturer (the Stackelberg leader) with a two-market framework, in which it supplies products for its domestic market and a North-country firm (the Stackelberg follower). Consumers in the two markets have different green preferences and price sensitivities. Two conventional results reveal that: (a) the investment decisions in product and process innovations are complementary; and (b) the optimal investment efforts positively respond to a learning rate and a knowledge accumulation rate. Specifically, we find that: (a) the optimal innovation efforts of the manufacturer are more heavily affected by the green product preferences of the Stackelberg follower than that of itself; and (b) when consumers' marginal willingness to pay for green products is sufficiently high, optimal investment efforts are higher in the profit-seeking optimum than that in social welfare seeking optimum. In addition, we demonstrate that under both the monopolist optimum and the social optimum, a unique saddle steady-state equilibrium exists.

1. Introduction

Innovation is crucial for the sustainable development and competitiveness of a firm and it can boost the products' profit of the firm. For instance, a power-conserving electrical system can help a firm attract more users, and a more efficient machine helps produce more products in less time. Statistically, in 2005, the G20 accounted for 92% of global spending on research. Canada, the United States, Germany, South Korea, Japan, and China devote 1.71%,2.79%, 2.93%, 4.23%, 3.29%, 2.07% of their GDP on R&D expenditure, respectively. In recent years, many companies have paid increasing attention to R&D expenditure. According to the latest available data from the UNESCO Institute for Statistics, global spending on R&D has reached a record high of almost \$1.7 trillion. About 10 countries account for 80% of the total spending. To pursue sustainable development goals, countries have pledged substantially increased public and private R&D spending as well as the number of researchers by 2030. According to data from OECD, Amazon invested "\$"17.4 billion in 2017, which is almost 30 times its investment in 2004. In this paper, considering the direct impact of product and process innovations on a firm's performance, we focus on technological innovations. Specifically, product innovation is an effort that aims to make products better, for example, higher quality or greener (Lambertini & Mantovani, 2009; Pan & Li, 2016). Process innovation represents an effort to reduce the production cost and leads to change in production function, allowing a firm to place the product at a competitive price (Lambertini & Orsini, 2015; Li & Ni, 2016). For instance, designing more efficient machines may save more human labor and materials during production.

Firms' incentives to innovate in product and process have received much attention in the existing literature. However, most studies are related to optimal problems (Cohen & Levinthal, 1989; Hatch & Mowery, 1998; Xing, 2014; Lambertini & Orsini, 2015; Li & Ni, 2016; Lambertini, Orsini, & Palestini, 2017). In this article, we consider the R&D portfolio of a manufacturer in a more complex and realistic circumstance, where the complexity originates from three aspects.

(1) We discuss the product and process innovation in a game structure. Specifically, the manufacturer is a South-country firm (a firm from a South country) under a two-market framework, and it will produce products for two different markets: for its domestic market and for a North-country firm (a firm from a North country). What we want to emphasize is not the optimal strategies of an individual manufacturer, but that of a system in which the manufacturer will interact with the North-country firm in the view of the supply

* Corresponding author.
E-mail addresses: wangyufangminshan@163.com (Y. Wang), xywang@stu.tjufe.edu.cn (X. Wang), Shuhua55@126.com (S. Chang), yun.kang@asu.edu (Y. Kang).

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chain. Therefore, the scenario described by game structure is closer to the reality. A tariff applied to exported goods will increase the product cost. As in Aparicio, González-Esteban, Pinilla, and Serrano (2018), importing governments encourage foreign commodity imports through relevant economic and administrative measures. For instance, the Ministry of Finance of China and the Ministry of Commerce of China have jointly issued import discount interest funds for encouraging import (http://en.pkulaw.cn/Display.aspx?lib=law&Cgid=223938). Yiwu Municipal Government of China has offered some specific subsidy measures for promoting the development of import trade (http://www.yw.gov.cn/11330782002609848G/a/04/03/02/201801/t20180108_2860079_2.html). Therefore, in this work government subsidy will be considered for the imported goods for the North-country firm. Further, consumers in North and South markets have different green product preferences and degrees of price sensitivities. The manufacturer, as the Stackelberg leader, pursues two goals: maximizing the economic surplus consisting of the product's net revenue from domestic and foreign markets, and minimizing the production cost and tariff cost by adopting two kinds of innovations. The Stackelberg follower is the North-country firm, which has only one goal, that is, maximizing its product's revenue. A possible situation is that when process innovation is involved, the Stackelberg follower is allowed to be able to commit to a non-Stackelberg future action, for example, in the cases where customer adjustment costs are a significant concern McElheran (2015) or process innovation is considered as a form of countering firms' weak internal capabilities Hervas-Oliver, Sempere-Ripoll, and Boronat-Moll (2014). However, in this work, our setting is that the manufacturer produces a product with product and process innovations, and sells it both to its domestic market through itself and to overseas markets through the North-country firm. The North-country firm in our model plays the role that a downstream retailer in a traditional supply chain plays. The manufacturer's goal of product and process innovations is to cater to the needs of two markets.
For example, as a global home appliance manufacturer, Haier is the leader in household appliances in China and number four in the world (Van Agtmael, 2007). As illustrated in Andersson and Wang (2011), Haier has stepped out of China to the world and its globalization does not simply involve selling its products to overseas markets, but occupying markets in developed countries such as Europe, America, Japan, relying on its high-quality products. In this case, the manufacturer has more dominance in the game. Therefore, it's the Stackelberg leader.

(2) The manufacturer makes its pricing decision and innovation investments not only related to the interplay with the Stackelberg follower but also concerned with the impacts of its knowledge accumulation resulting from learning-by-doing on innovation investments. Learning-by-doing is the result of knowledge accumulation generated by experience in the production process Arrow (1962). However, the existing literature has seldom addressed the dynamic role of learning-by-doing on product and process innovation. We consider the effect of learning-by-doing on the two kinds of innovation investments in this paper.
(3) We introduce a more general-state dynamic of production cost function, which relates to the product's green level. This approach depicts a more real production scenario. André, González, and Porteiro (2009) and Lambertini and Tampieri (2012) demonstrate that green high-quality goods explicitly involve higher marginal production cost. In Lambertini et al. (2017), an example about the car industry is listed. Green hybrid cars are costlier (to producers and consumers alike) than brown ones, all else equal, for two reasons: more production cost is needed for a manufacturer to allow the manufacturing firm to abate large initial R&D investments required for investing, designing, putting into production electrical power units; the marginal production cost of hybrid propulsion is largely affected by the marginal production costs of the assembled final products.

The goal of this study is to explore the optimal R&D portfolio investment in cost reduction and quality improvement as a Stackelberg leader. We discuss and compare two equilibria of the manufacturer, namely, the profit optimum and the social welfare optimum. The gap between the optimal innovation levels of the two different equilibria can be used to measure the efficiency of the monopolist's R&D choice of the South-country firm. We will address the following questions:

(1) In a Stackelberg game structure, do the manufacturer's (the leader's) optimal investment levels of product and process innovations rise, drop, change nonlinearly compared with the scenario without considering the interplay with the Stackelberg follower? How do the green product preferences of different kinds of consumers affect the optimal R&D portfolio of the manufacturer, and which country's consumers more heavily affect the manufacturer's R&D portfolio? We answer these questions in Propositions 3 and 7.
(2) Lambertini and Orsini (2015) obtain that the two kinds of innovation are independent of each other. Furthermore, Lambertini et al. (2017) state that both of the innovations are complementary, based on the assumption that the state functions of the production cost and the green level of products are not coupled. In this paper, we couple the state functions of production cost and the green level of products. In this case, are the two kinds of innovations still complementary? Or can we reveal other new insights? We give answers in Propositions 2, 6, 7.
(3) How does the knowledge accumulation of learning-by-doing affect the optimal innovation investments? We answer this question in Propositions 1 and 5.

In our analysis, we first explore the North-country firm's (follower's) optimal response to the South-country firm's pricing policy, and then embed this response into the South-country firm's problem to derive its optimal pricing and innovation-level strategies. Specifically, we introduce a more general-state function of production cost and compare the variation of the optimal innovation levels in different optimum-seeking scenarios: profit maximum and social welfare maximum. Our analysis generates some core insights.

Insights for the Stackelberg game. We analytically obtain the unique saddle point equilibrium of the Stackelberg game where both sides seek profit maximum. The analytical saddle point equilibrium for a profit-maximizing northern firm (the follower) and a social welfare-maximizing southern firm (the leader) are also easier to obtain by using the same method. We have omitted it for simplicity.

Insights for the Stackelberg leader. Our core result is that contrary to the conventional wisdom (as expressed, e.g., in Li & Ni, 2016) that the investment levels of both product and process innovations are lower under the monopolist optimum than under the social welfare optimum, in a Stackelberg game structure, the Southern firm's innovation levels may be higher in profit-seeking optimum than under social welfare seeking optimum. The case will occur when the Stackelberg members have higher green preferences, especially for the Stackelberg followers. When the South-North markets are sufficiently rich, this case is prevalent. Our other first-proposed result is that the optimal innovation levels of the Stackelberg leader are more heavily affected by the green product preference of the Stackelberg follower than that of the Stackelberg leader itself. The essential reason for the phenomenon may lie in the tariff cost from the exported goods. In addition, although we extend the state function of production cost into a more general case, as defined in (3), we obtain the same results as those described as conventional wisdom (Li, 2018; Lambertini & Mantovani, 2009; Chenavaz, 2012): process innovation and product innovation are complementary and promote each other; the learning rate and the growth rate of knowledge accumulation in learning-by-doing are substitutable on the innovation investment and they both promote the relatively optimal investment level.

The remainder of the paper is organized as follows: The relevant literature is reviewed in Section 2. The basic setting of models are presented in Section 3. The Stackelberg game models are presented in Sections 4 and 5, where the South-country firm seeks profit maximum and social welfare maximum, respectively, and the North-country firm pursues its profit maximum. Section 6 summarizes and discusses future research.

2. Literature review

Our study is related to two streams of literature: operations management and economics.

In the operations literature, several studies address the relationship between product and process innovations or focus on firms' dynamic R&D investment activities. Utterback and Abernathy (1975) empirically argue that the product innovation necessarily precedes process innovation through a product's life cycle model, and this perception is reinforced by the work of Adner and Levinthal (2001) and Damanpour and Gopalakrishnan (2001). Bonanno and Haworth (1998) answer whether the regime of competition affects a firm's choice between product and process innovations within a model of vertical differentiation. Lambertini and Mantovani (2009) investigate the substitutability and complementarity of product and process innovations in a multi-product monopolist model. Lambertini and Orsini (2015) investigate the R&D portfolio of a monopolist investment in cost reduction and quality enhancement R&D, and find that the two kinds of innovation are independent of each other. Xing (2014) investigates the optimal choices of R&D risk in a market exhibiting network externalities of a Hotelling spatial model. Lambertini et al. (2017) contribute to the related theoretical debate, focusing on the possibility of having superior product quality levels at lower marginal production cost over time by investigating the optimal R&D portfolio of product and process innovations; they demonstrate that R&D efforts are complementary in the neighborhood of the steady-state equilibrium. Li and Ni (2018) model production decision and product innovation under quality authorization. They find that jumps may occur in the innovation investment and production rates over time. In addition, some works study the green product and process innovations based on the game theory model, such as Guo, Qu, Tseng, Wu, and Wang (2018), Yang, Shi, and Li (2018), Madani and Morteza (2018). All the above works focus on designing investment activities to obtain more economic revenue, and these works are limited to optimal control problems. As mentioned before, our work differs from the above in several dimensions; most importantly, the question we address is broader. Our model is similar to the above in that we also discuss operational details of the manufacturer's investment portfolio and study the relationship between them. However, we extend the manufacturer's optimal control problem to its optimal decision problem as a Stackelberg leader, facing different green-sensitive consumers and the tariff cost of the exported goods.

In the economic literature, many works have demonstrated that innovation is a major driver for strengthening a firm's competitive ability. However, few works emphasize the influence of knowledge accumulation on the investment of product and process innovations. Knowledge accumulation from learning-by-doing affects a manufacturer's strategies in product and process innovations. Cohen and Levinthal (1989) reveal that R&D investment and learning are mutually reinforcing. Gopalakrishnan, Bierly, and Kessler (1999) reexamine the characteristics of product and process innovations and find some strategic implications in knowledge-based dimensions. Some scholars of information system agree that IT positively affects corporate innovation, specifically by supporting knowledge-intensive activities relative to innovation (Bardhan, Viswanathan, & Shu, 2013; Kane & Maryam, 2007). Hatch and Mowery (1998) analyze the relationship between process innovation and learning-by-doing in the semiconductor industry. Pan and Li (2016) demonstrate that the accumulation from learning-by-doing promotes the optimal innovations. Trantopoulos, Krogh, Wallin, and Woerter (2017) offer a theoretical model explaining how a firm's IT and external search activity jointly influence process innovation performance, thereby lowering the cost of producing a good or service. Zhang, Tang, and Zhang (2018) consider the influence of learning effect on product innovation in a differential game model. The questions we discuss in this paper are similar to those addressed by the above works, but the framework is different. A rather common assumption in the economics literature is that a firm's cost in production is related to the process innovation and the obsolescence (or decay) rates of productive efficiency; for example, see the work of Li and Ni (2016, 2018) or the recent treatment by Lambertini et al. (2017). In many cases, however, increasing product quality (referring to the green level in this article) brings up production cost. This case is underlined in our modeling framework.

To some extent, our work can be viewed as an extension of Dai and Zhang (2017), which ignores the production cost and investigates the green process innovation and differential dynamic pricing strategies for a Southern firm under a two-market framework. We consider the factor of production cost in our model. Our paper can also be viewed as an extension and continuation of the work of Lambertini and Orsini (2015) and Li and Ni (2016), which discuss the relationship of process and product innovations. In addition, in their work, production cost and product green performance are decoupled. Compared with the work of Dai and Zhang (2017), Lambertini and Orsini (2015), and Li and Ni (2016), in this paper, we add the following:

(i) We consider the interplay of the supply members. Specifically, the monopolist's instantaneous decision relies on the Stackelberg follower.
(ii) We consider product and process innovations in a product life cycle, and propose a more general instant production cost function, which is affected by production cost, the product's green level, and the green product innovation.
(iii) Most importantly, we discuss the effects of knowledge accumulation and different green preferences on the optimal innovation decisions. Furthermore, we compare the optimal R&D portfolio of product and process innovations for the profit maximum and the social welfare maximum.

3. Basic setting

In a typical South-North framework, the South countries (e.g., China) (denoted as "s") are regarded as developing countries while the North countries (e.g., the US) (denoted as "n") are regarded as developed countries (Masoudi & Zaccour, 2013). In this paper, we describe a transparent production problem where the South-country firm manufactures the same kind of products for two different markets: the domestic market and the Northern market. Consumers in different markets have different levels of green preferences and pricing sensitivities. Specifically, the South-country firm not only produces and sells products in its domestic market, but also exports products to the North-country firm. The southern and northern firms play a Stackelberg game over continuous time $t \in [0, +\infty)$ where the former is the leader and the latter acts as the follower. In this circumstance, at any instant, the Stackelberg leader makes pricing decisions and chooses the investment levels of process and product innovation under learning-by-doing. Meanwhile, the Stackelberg follower decides its optimal retailer price. Assume that no stock for products exists; all the demand is satisfied and all production is sold. Following Dai and Zhang (2017), the demand function of the South and North countries can be written as

$$D_i(t) = \alpha_i - \beta_i p_i(t) + r_i x(t),\ i = s,\ n, \tag{1}$$

where α_i describes the basic market potential; the negative impact of product price on demand is traced by parameters $-\beta_i$ and $\beta_i > 0$;$x(t)$

denotes the green level of products arising from the product innovation; and $r_i > 0$ is the positive impact of green level of the products on demand, as illustrated in Liu, Anderson, and Cruz (2012) and in Zhang, Wang, and You (2015). In this paper, we assume $\beta_s > \beta_n$ and $r_s < r_n$ to distinguish the South and North markets. This assumption depicts a South market where consumers are more price-sensitive but less green-conscious, and the North market is the opposite case, as described in Copeland and Taylor (1994), Dai and Zhang (2017).

In the cross-border production of South and North countries, the carbon tariff is a policy intended to address carbon emission associated with production of the exports. The carbon emission of the exports from the South to the North is taxed by the Northern government. Following Dai and Zhang (2017), the instantaneous cost of tariffs on the unit exported good for the South-country firm is

$$C_e(t) = r(e_0 - gx(t)),$$

where $e_0 > 0$ represents a basic unit of emissions if no innovation is exerted; parameter $g > 0$ captures the contribution of the green innovation to cut down emission; therefore, $e_0 - gx(t)$ is the actual emission amount of per-unit product; and r is a constant carbon tariff rate per unit of emission. As in Krueger (1989), Brammer and Walker (2016), Simula and Consultant (2014), Aparicio et al. (2018), importing governments encourage foreign commodity imports through relevant economic and administrative measures. Specifically, Krueger (1989) explains why import-competing industries tend to be more highly protected than industries producing exportables; Brammer and Walker (2016) shows that governments are increasingly concerned with ensuring that the way firms or consumers buy goods and services has beneficial impacts on economic, social, environmental sustainability. This approach has been termed sustainable procurement, which is the pursuit of sustainable development objectives through the purchasing and supply process. Therefore, governments will provide some public procurement policies for green products (Simula & Consultant, 2014). In this paper, on the basis of the above discussion, we make the assumption that North-country government totally subsidizes the tariff profits to the North-country firm.

At any instant $t \in [0, +\infty)$, the South-country firm chooses the investment levels of product green innovation $k_1(t)$ and production process innovation $k_2(t)$ through learning-by-doing. In most of the previous studies (Lambertini & Orsini, 2015), for simplicity, the green performance $x(t)$ and production cost $c(t)$ are decoupled; i.e., each state $x(t)$ or $c(t)$ only appears in its own dynamic equation. Lambertini et al. (2017) demonstrate that although the above two states are decoupled, they simultaneously appear in both control equations and the two kinds of innovation efforts $k_1(t)$ and $k_2(t)$ are not separable. In addition, as discussed in André et al. (2009) and Lambertini and Tampieri (2012), green high-quality goods explicitly involve higher marginal production cost than brown low-quality ones. Therefore, to be more general, we define the state dynamics describing the evolutions of $x(t)$ and $c(t)$ over time as

$$\dot{x}(t) = k_1(t) - \delta_1 x(t), \tag{2}$$

$$\dot{c}(t) = -k_2(t) + \delta_2 c(t) + hx(t), \tag{3}$$

where δ_1 and δ_2 are two exogenous obsolescence (or decay) rates of quality and productive efficiency, and they are both positive and time-invariant, and $h \geqslant 0$ describes the effects of the green level of products on production cost. The special case of $h = 0$ has also been considered in many studies for different problems (Chenavaz, 2012; Xing, 2014; Li & Ni, 2016; Lambertini et al., 2017; Li, 2018). In this paper, we will discuss the cases of $h > 0$ and $h = 0$, and find some new insights.

We define the cost functions of investing product and process innovations are $\frac{1}{2}bk_1^2(t)$ and $\frac{1}{2}sk_2^2(t)$, respectively, which measure the instantaneous costs of producing a product using machinery and/or skilled labor operating at decreasing returns. In our model, we also consider the effects of learning-by-doing on the costs of the above two techniques. Following the work of Thompson (2010) and Li and Ni (2016), we model the knowledge accumulation in product innovation or process innovation with an exponential smoothing process of the corresponding historical investment as

$$A_1(t)=e^{-\theta_1 t}[A_{10} + \mu\int_0^t e^{\theta_1\tau}k_1(\tau)d\tau],$$
$$A_2(t)=e^{-\theta_2 t}[A_{20} + \xi\int_0^t e^{\theta_2\tau}k_2(\tau)d\tau],$$

where μ and ξ are the growth rates of knowledge accumulation of product and process innovations, which reflect the positive effect of the accumulated innovation efforts. The assumptions about $A_1(t)$ and $A_2(t)$ are consistent with empirical studies of learning-by-doing, in which experience is generally measured as cumulative production (Argote & Dennis, 1990). According to Arrow (1962), innovation cost function decreases in knowledge accumulation. Now, the instantaneous cost functions of product and process innovation $C_x(t)$ and $C_c(t)$ with corresponding knowledge accumulation are given by the following forms, respectively:

$$C_x(t)=\tfrac{1}{2}bk_1^2(t) - c_1(A_1(t) - A_{10}),$$
$$C_c(t)=\tfrac{1}{2}sk_2^2(t) - c_2(A_2(t) - A_{20}),$$

where $c_1 > 0$ and $c_2 > 0$ are the learning rates of innovations on the product and process, which reflect the positive rate of knowledge accumulation for cost reduction. Also, as we have made the assumption that North-country government totally subsidize the tariff profits to the North-country firm, the North-country firm's instantaneous profit is given by

$$D_n(t)(p_n(t) - p_{sn}(t) + r(e_0 - gx(t))),$$

where $p_n(t)$ is the product's retail price in the northern market, and $p_{sn}(t)$ is the wholesale price decided by the South-country firm. The total instantaneous profit of the South-country firm is given by

$$D_n(t)(p_{sn}(t) - r(e_0 - gx(t)) - c(t)) + D_s(t)(p_s(t) - c(t)) - C_x(t) - C_c(t),$$

including the net revenue from the North market, the domestic market, the costs of technology innovations.

4. Equilibrium analysis of the profit-optimal strategies in a Stackelberg game with South-North framework

In this section, we analyze the optimal strategies taken by the South-country firm (the Stackelberg leader) and the North-country firm (the Stackelberg follower) in the case that both sides seek profit maximum. In particular, in a more general case as defined in (3), we seek to answer whether the two forms of innovation efforts are complementary or substitutable in the steady state. States and controls are labeled by "*pp*" when they are in steady state.

On the basis of the aforementioned description and assumptions, South-North country firms in the Stackelberg game must solve the following infinite horizon problems to make their profit maximal. For the North-country firm, the optimal problem is

$$\max_{p_n(t)} \int_0^{+\infty} e^{-\rho t}D_n(t)(p_n(t) - p_{sn}(t) + r(e_0 - gx(t)))dt, \tag{4}$$

s.t.

$$\dot{x}(t) = -\delta x(t) + k_1(t), x(0) = x_0. \tag{5}$$

For the South-country firm, the optimal problem is

$$\max_{p_{sn}(t),p_s(t),k_1(t),k_2(t)} \int_0^{+\infty} e^{-\rho t}\{D_n(t)(p_{sn}(t) - r(e_0 - gx(t)) - c(t)) + D_s(t)(p_s(t) - c(t)) - \left(\tfrac{1}{2}bk_1^2(t) - c_1(A_1(t) - A_{10})\right) - \left(\tfrac{1}{2}sk_2^2(t) - c_2(A_2(t) - A_{20})\right)\}dt, \quad (6)$$

s.t.

$$\dot{x}(t) = k_1(t) - \delta_1 x(t),\ x(0) = x_0, \quad (7)$$

$$\dot{c}(t) = -k_2(t) + \delta_2 c(t) + hx(t),\ c(0) = c_0, \quad (8)$$

$$\dot{A}_1(t) = \mu k_1(t) - \theta_1 A_1(t),\ A_1(0) = A_{10}, \quad (9)$$

$$\dot{A}_2(t) = \xi k_2(t) - \theta_2 A_2(t),\ A_2(0) = A_{20} \quad (10)$$

We use backward induction to solve the corresponding dynamic game between the two multinational corporations. Specifically, we first solve the retailer's problem for given wholesale price, and then solve the manufacturer's problem by taking the retailer's reaction function into consideration.

For the follower, the Hamiltonian of (4) with state (5) is

$$H_n = D_n(t)(p_n(t) - p_{sn}(t) + r(e_0 - gx(t))) + \lambda_n(t)(-\delta x(t) + k_1(t)), \quad (11)$$

where $\lambda_n(t)$ is the dynamic costate of $x(t)$. Using Pontryagin's maximum principle, a necessary condition for solving the above optimal control problem is that the $p_n(t)$ satisfies $\frac{\partial H_s}{\partial p_n} = 0$, that is,

$$p_n^*(t) = \frac{1}{2}p_{sn}(t) - \frac{1}{2}r(e0 - gx(t)) + \frac{1}{2\beta_n}(\alpha_n + r_n x(t)). \quad (12)$$

For the leader, the Hamiltonian of (6) with states (7)–(10) is

$$H_s = D_n(t)(p_{sn}(t) - r(e_0 - gx(t)) - c(t)) + D_s(t)(p_s(t) - c(t)) - \left(\tfrac{1}{2}bk_1^2(t) - c_1(A_1(t) - A_{10})\right) - \left(\tfrac{1}{2}sk_2^2(t) - c_2(A_2(t) - A_{20})\right) + \lambda_1(t)(k_1(t) - \delta_1 x(t)) + \lambda_2(t)(-k_2(t) + \delta_2 c(t) + hx(t)) + \lambda_3(t)(\mu k_1(t) - \theta_1 A_1(t)) + \lambda_4(t)(\xi k_2(t) - \theta_2 A_2(t)), \quad (13)$$

where $\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t)$ are the dynamic costates of $x(t), c(t), A_1(t), A_2(t)$, respectively. After inserting (12) and (13), the resulting first-order conditions on controls are

$$\frac{\partial H_s}{\partial p_{sn}} = -\beta_n p_{sn}(t) + r\beta_n(e_0 - gx(t)) + \frac{1}{2}(\alpha_n + r_n x(t)) = 0, \quad (14)$$

$$\frac{\partial H_s}{\partial p_s} = -2\beta_s p_s(t) + \alpha_s - \beta_s c(t) + r_s x(t) = 0, \quad (15)$$

$$\frac{\partial H_s}{\partial k_1} = -b_1 k_1(t) + \lambda_1(t) + \mu\lambda_3(t) = 0, \quad (16)$$

$$\frac{\partial H_s}{\partial k_2} = -b_2 k_2(t) - \lambda_2(t) + \xi\lambda_4(t) = 0. \quad (17)$$

Furthermore, based on (14)–(17), the costate equations are

$$\dot{\lambda}_1(t) = \rho\lambda_1(t) - \tfrac{\partial H}{\partial x} = (\rho + \delta_1)\lambda_1(t) - h\lambda_2(t) + c(t)\left(\tfrac{1}{2}r_s - \tfrac{3}{4}r_n\right) - \tfrac{r_n}{4\beta_n}(\alpha_n + r_n x(t)) - \tfrac{r_s}{2\beta_s}(\alpha_s + r_s x(t)), \quad (18)$$

$$\dot{\lambda}_2(t) = \rho\lambda_2(t) - \tfrac{\partial H}{\partial c} = (\rho + \delta_2)\lambda_2(t) - \tfrac{1}{4}(\beta_s + \beta_n)c(t) + \tfrac{1}{4}(\alpha_n + r_n x(t)) + \tfrac{1}{2}(\alpha_s + r_s x(t)) + r_n x(t), \quad (19)$$

$$\dot{\lambda}_3(t) = \rho\lambda_3(t) - \frac{\partial H}{\partial A_1} = (\rho + \theta_1)\lambda_3(t) - c_1, \quad (20)$$

$$\dot{\lambda}_4(t) = \rho\lambda_4(t) - \frac{\partial H}{\partial A_2} = (\rho + \theta_2)\lambda_4(t) - c_2, \quad (21)$$

with the transversality conditions

$$\lim_{t\to\infty}\lambda_1(t)x(t)e^{-\rho t} = 0,\ \lim_{t\to\infty}\lambda_2(t)c(t)e^{-\rho t} = 0,$$

$$\lim_{t\to\infty}\lambda_3(t)A_1(t)e^{-\rho t} = 0,\ \lim_{t\to\infty}\lambda_4(t)A_2(t)e^{-\rho t} = 0.$$

Thus, imposing steady conditions on states (7) and (8) yield

$$k_1^{pp} = \delta_1 x^{pp}, \quad (22)$$

$$k_2^{pp} = \delta_2 c^{pp} + hx^{pp}. \quad (23)$$

We now proceed to impose steady-state conditions on (18), (19), (20), (21), and we can easily obtain

$$\begin{cases} \lambda_3^{pp} = \frac{c_1}{\theta_1 + \rho}, \\ \lambda_4^{pp} = \frac{c_2}{\theta_2 + \rho}, \\ \lambda_1^{pp} = b_1 k_1^{pp} - \frac{\mu c_1}{\theta_1 + \rho}, \\ \lambda_2^{pp} = -b_2 k_2^{pp} + \frac{\xi c_2}{\theta_2 + \rho}. \end{cases} \quad (24)$$

From (16), (18), (20), we know that

$$\dot{k}_1(t) = \tfrac{1}{b_1}\left[(\rho + \delta_1)\lambda_1(t) - h\lambda_2(t) + c(t)\left(\tfrac{1}{2}r - \tfrac{3}{4}r_n\right) - \tfrac{r_n}{4\beta_n}(\alpha_n + r_n x(t)) - \tfrac{r_s}{2\beta_s}(\alpha_s + r_s x(t)) + \mu(\theta_1 + \rho)\lambda_3(t) - \mu c_1\right].$$

Imposing stationary conditions on the above equation and using (22), we obtain

$$k_1^{pp} = \tfrac{1}{b_1(\rho + \delta_1)}\left[\tfrac{h}{\rho + \delta_2}\left(-\tfrac{1}{4}(\alpha_n + r_n x^{pp}) - \tfrac{1}{2}(\alpha_s + r_s x^{pp}) + \tfrac{1}{4}(\beta_n + \beta_s)c^{pp} - r_n x^{pp}\right) + c^{pp}\left(\tfrac{3}{4}r_n - \tfrac{1}{2}r_s\right) + \tfrac{r_n}{4\beta_n}(\alpha_n + r_n x^{pp}) + \tfrac{r_s}{2\beta_s}(\alpha_s + r_s x^{pp})\right] + \tfrac{\mu c_1}{b_1(\theta_1 + \rho)}. \quad (25)$$

Similarly,

$$k_2^{pp} = \frac{1}{b_2(\rho + \delta_2)}\left[\frac{1}{4}(\alpha_n + r_n x^{pp}) + \frac{1}{2}(\alpha_s + r_s x^{pp}) - \frac{1}{4}(\beta_n + \beta_s)c^{pp} + r_n x^{pp}\right] + \frac{\xi c_2}{b_2(\theta_2 + \rho)}. \quad (26)$$

The last parts of (25) and (26) reflect the effects of learning-by-doing on South-country's decision, that is,

Proposition 1. *The equilibrium investment efforts of product and process innovations rely on their relative learning rates and growth rate of knowledge accumulation: (a) the learning rate c_1 and the growth rate of knowledge accumulation μ (or c_2 and ξ) are substitutable on the innovation investment, and they both promote the relative optimal investment level; (b) the learning rate of one kind of innovation has no effect on the investment level of the other kind of innovation.*

Proposition 1 shows that the South-country firm does not need to make efforts on the same dimension. This outcome is in accordance with Li (2018), and it is a valuable complement to the results of standard models about product and process innovation (Lambertini & Mantovani, 2009; Chenavaz, 2012). In the work of Lambertini and Mantovani (2009), only the cases in which innovation investments affect the shadow prices of product and marginal production cost, are discussed. The results of Proposition 1 demonstrate that $\frac{\mu c_1}{b_1(\theta_1 + \rho)}$ and $\frac{\xi c_2}{b_2(\theta_2 + \rho)}$ represent the effects of learning-by-doing on the two kinds of innovation investments, respectively.

Given that we investigate the product and process innovations in a game structure, in the equilibria, the complementarity or substitutability between the two kinds of innovation efforts is a matter of great concern. When $h = 0$ in (3), from (2) and (3), product and process innovations seem to be independent, and they have no influence on each other. When $h > 0$, from (2) and (3), production cost $c(t)$ seems to be likely to increase with the increase in product green level $x(t)$. In the next, we perform a strict deduction to scientifically answer these questions.

On the basis of (23) and (26), we obtain

$$c^{pp} = \frac{1}{\delta_2 + \frac{1}{4b_2(\rho+\delta_2)}(\beta_n+\beta_s)}\Bigg[-\frac{1}{b_2(\rho+\delta_2)}\Big(-\frac{1}{4}(\alpha_n + r_n x^{pp}) - \frac{1}{2}(\alpha_s + r_s x^{pp}) - r_n x^{pp}\Big) - hx^{pp} + \frac{\xi c_2}{b_2(\theta_2+\rho)}\Bigg] \equiv f_1(x^{pp}). \tag{27}$$

Therefore,

$$\frac{\partial k_1^{pp}}{\partial k_2^{pp}} = \frac{\partial k_1^{pp}/\partial x^{pp}}{\partial k_2^{pp}/\partial x^{pp}} = \frac{\delta_1}{h + \delta_2\frac{\partial f_1}{\partial x^{pp}}} = \frac{\delta_1}{\frac{\frac{5}{4}r_n + \frac{1}{2}r_s}{b_2(\rho+\delta_2)+\beta_n+\beta_s} + h\left(1 - \frac{1}{1+\frac{\beta_n+\beta_s}{4b_2(\rho+\delta_2)}}\right)} > 0. \tag{28}$$

As seen from (28), both innovation efforts are positive. Therefore, each one boosts the other in the neighborhood of the steady state. From (22) and (28), we can observe that the product's green level also affects the efforts of process innovation investment. All these results are summarized in Proposition 2.

Proposition 2. *No matter whether $h = 0$ or $h > 0$ (h is defined in (3)) and in the neighborhood of equilibrium: process and product innovations are complementary; the efforts of process innovation investment not only depend on the production cost, but also depends on the green level of products. In fact, they are positively correlated (even if $h = 0$).*

This finding shows the South-country firm that the equilibrium is a global innovative content and the two forms of efforts are complementary. This outcome is a valuable complement to the results of Lambertini et al. (2017), which is a simplified case of $h = 0$. The synergy highlighted in Proposition 2 is also in line with the empirical evidence in Cohen and Klepper (1996) and Damanpour and Gopalakrishnan (2001).

Intuitively, the green product innovation effort of the South-country firm will increase with higher green demand from consumers. In the following, from (26) and (28), we can immediately derive

$$\frac{\partial k_2^{pp}}{\partial r_n} = \frac{5x^{pp}}{4b_2(\rho+\delta_2)} > \frac{x^{pp}}{2b_2(\rho+\delta_2)} = \frac{\partial k_2^{pp}}{\partial r_s} > 0$$

and therefore,

$$\frac{\partial k_1^{pp}}{\partial r_n} = \frac{\partial k_1^{pp}}{\partial k_2^{pp}}\frac{\partial k_2^{pp}}{\partial r_n} > \frac{\partial k_1^{pp}}{\partial k_2^{pp}}\frac{\partial k_2^{pp}}{\partial r_s} = \frac{\partial k_1^{pp}}{\partial r_s} > 0.$$

The above two formulas reflect that as preferences are greener, the efforts of product and process innovations rise. Given that r_n and r_s represent the green preferences of consumers in South and North markets and because $r_n > r_s$ as we assumed above, the two kinds of innovation efforts are more sensitive to r_n. These results are summarized in the proposition below.

Proposition 3. *In the neighborhood of equilibrium, the two kinds of innovation efforts improve with the increase of green preferences (r_n and r_s) and they are more sensitive to the green preference of consumers in the North market than the consumers in the South market.*

The above phenomenon can be explained as follows: in the Stackelberg supply chain, all exports will be taxed for the carbon emissions in their manufacturing; therefore, for the South-country firm, exporting products means more costs. Facing higher green demand, a South-country firm adapts more innovation efforts, which can not only expand the foreign markets, but also decrease the total carbon tariffs on the exports. Therefore, innovation efforts are more sensitive to the higher green preference of consumers in the North country than that in the South country.

Now, we analyze the stability properties of the steady-state equilibrium of the Stackelberg game.

Using (25) and the steady-state condition (22), x^{pp} can be written as follows:

$$x^{pp} = \frac{1}{b_1\delta_1(\rho+\delta_1) - w_1 - w_0w_2w_3}\Bigg(w_0w_2\Big(\frac{\alpha_n+2\alpha_s}{4b_2(\rho+\delta_2)} + \frac{\xi c_2}{(\rho+\theta_2)b_2}\Big) - \frac{h}{\rho+\delta_2}\Big(\frac{1}{4}\alpha_n + \frac{1}{2}\alpha_s\Big) + \frac{r_n\alpha_n}{4\beta_n} + \frac{r_s\alpha_s}{2\beta_s}\Bigg) \tag{29}$$

where

$$w_0 = \frac{1}{\delta_2 + \frac{1}{4b_2(\rho+\delta_2)}(\beta_n+\beta_s)}, \tag{30}$$

$$w_1 = -\frac{h}{\rho+\delta_2}\left(\frac{5}{4}r_n + \frac{1}{2}r_s\right) + \frac{r_n^2}{4\beta_n} + \frac{r_s^2}{2\beta_s}, \tag{31}$$

$$w_2 = \frac{h(\beta_n+\beta_s)}{4(\rho+\delta_2)} + \frac{3}{4}r_n - \frac{1}{2}r_s, \tag{32}$$

$$w_3 = \frac{1}{b_2(\rho+\delta_2)}\left(\frac{5}{4}r_n + \frac{1}{2}r_s\right) - h. \tag{33}$$

According to (27) and (29), the analytical expression of c^{pp} can be easily obtained. Furthermore, from (14), (15), (22), (23), we obtain the optimal strategies of the South-country firm as follows:

$$p_{sn}^{pp} = r(e_0 - gx^{pp}) + \frac{1}{2}c^{pp} + \frac{\alpha_n + r_n x^{pp}}{2\beta_n},$$
$$p_s^{pp} = \frac{1}{2}c^{pp} + \frac{\alpha_s + r_s x^{pp}}{2\beta_s},$$
$$k_1^{pp} = \delta x^{pp},$$
$$k_2^{pp} = \theta c^{pp} + hx^{pp}.$$

From (12), the North-country firm's strategy in the steady state is

$$p_n^{pp} = \frac{1}{2}p_{sn}^{pp} - \frac{1}{2}r(e_0 - gx^{pp}) + \frac{1}{2\beta_n}(\alpha_n + r_n x^{pp}). \tag{34}$$

Now, we analyze the stability properties of the steady-state equilibrium $\{x^{pp}, c^{pp}, A_1^{pp}, A_2^{pp}\}$. For the Stackelberg leader, we construct the Jacobian matrix associated with the canonical system (7)–(10), (18)–(21):

$$\begin{bmatrix} -\delta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h & \delta_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\theta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\theta_2 & 0 & 0 & 0 & 0 \\ -\frac{r_n^2}{4\beta_n} - \frac{r_s^2}{2\beta_s} & \frac{r_s}{2} - \frac{3r_n}{4} & 0 & 0 & \rho+\delta_1 & -h & 0 & 0 \\ \frac{5r_n}{4} + \frac{r_s}{2} & -\frac{1}{4}(\beta_s+\beta_n) & 0 & 0 & 0 & \rho+\delta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho+\delta_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho+\delta_2 \end{bmatrix}. \tag{35}$$

Obviously, the characteristic equations of the Jacobian matrix (35) yield five positive eigenvalues and three negative eigenvalues. Therefore, the equilibrium $\{x^{pp}, c^{pp}, A_1^{pp}, A_2^{pp}\}$ is a saddle point. For the Stackelberg follower with canonical system (5), obviously, $\{x^{pp}\}$ is a saddle point. Accordingly, we have the following Proposition 4.

Proposition 4. *Under the profit optimum, a steady-state equilibrium exists for* $\{x^{pp}, c^{pp}, A_1^{pp}, A_2^{pp}\}$ *for the Stackelberg members. Such a steady-state point is the saddle-point equilibrium.*

5. Equilibrium analysis of social welfare optimum of the South-country firm in a Stackelberg game

To complement the analysis, in this section, we will discuss another case. Here, the North-country firm is still the Stackelberg follower, aiming to obtain its maximal net revenues, while the South-country firm, instead of seeking to maximize individual profit, aims to maximize the social welfare. This discussion helps assess the efficiency of the monopolist's R&D choices of the South-country firm. In this scenario, all the states and controls are labeled by "*sp*" when they are in the steady states.

For the Stackelberg follower, the North-country firm's objective function is unchanged, just as (4) with state (5). As consumer surplus, also known as the net income of a consumer, reflexes the difference between the highest total price, in which a consumer is willing to pay when purchasing a certain quantity of a certain commodity, and the total price actually paid, (Turnovsky, Shalit, & Schmitz, 1980), we define the instantaneous consumer surplus of the South country as

$$CS = \int_{p_s(t)}^{(\alpha_s + r_s x(t))/\beta_s} (\alpha_s - \beta_s z + r_s x(t))dz = \frac{1}{2\beta_s}(\alpha_s + r_s x(t) - \beta_s p_s(t))^2. \tag{36}$$

This definition is also applied in Li (2018). The social welfare function is given as

Social Welfare = Profit of Southern Firm + Consumer Surplus,

thus, the objective function of the South-country firm is

$$\begin{aligned}\max_{p_{sn}(t),p_s(t),k_1(t),k_2(t)} \int_0^{+\infty} e^{-\rho t}\{&D_n(t)(p_{sn}(t) - r(e_0 - gx(t)) - c(t))\\ &+ D_s(t)(p_s(t) - c(t))\\ &-\left(\tfrac{1}{2}bk_1^2(t) - c_1(A_1(t) - A_{10})\right)\\ &- \left(\tfrac{1}{2}sk_2^2(t) - c_2(A_2(t) - A_{20})\right)\\ &+\tfrac{1}{2\beta_s}(\alpha_s + r_s x(t) - \beta_s p_s(t))^2\Big\}dt.\end{aligned} \tag{37}$$

The state equations concerning the dynamics of $x(t)$, $c(t)$, $k_1(t)$, $k_2(t)$ are the same as (7)–(10), respectively. Imposing steady conditions on these state functions, we obtain

$$k_1^{sp} = \delta_1 x^{sp}, \tag{38}$$

$$k_2^{sp} = \delta_2 c^{sp} + hx^{sp}. \tag{39}$$

Similar to the analysis in Section 4, we can also obtain that the optimal investments in steady state are

$$\begin{aligned}k_1^{sp}=&\tfrac{1}{b_1(\rho+\delta_1)}\Big[\tfrac{h}{\rho+\delta_2}\Big(-\tfrac{1}{4}(\alpha_n + r_n x^{sp}) - \tfrac{1}{2}(\alpha_s + r_s x^{sp}) + \tfrac{1}{4}(\beta_n + \beta_s)c^{sp} - r_n x^{sp}\Big)\\ &+\Big(\tfrac{3}{4}r_n - \tfrac{1}{2}r_s\Big)c^{sp} + \tfrac{r_n}{4\beta_n}(\alpha_n + r_n x^{sp}) + \tfrac{r_s}{2\beta_s}(\alpha_s + r_s x^{sp}) - r_s c^{sp}\Big]\\ &+\tfrac{\mu c_1}{b_1(\theta_1+\rho)},\end{aligned} \tag{40}$$

$$\begin{aligned}k_2^{sp}=&\tfrac{1}{b_2(\rho+\delta_2)}\Big[\tfrac{1}{4}(\alpha_n + r_n x^{sp}) + \tfrac{1}{2}(\alpha_s + r_s x^{sp}) - \tfrac{1}{4}(\beta_n + \beta_s)c^{sp} + r_n x^{sp}\Big]\\ &+\tfrac{\xi c_2}{b_2(\theta_2+\rho)},\end{aligned} \tag{41}$$

and the corresponding states are

$$\begin{aligned}c^{sp}=&\tfrac{1}{\delta_2 + \frac{1}{4b_2(\rho+\delta_2)}(\beta_n+\beta_s)}\Big[-\tfrac{1}{b_2(\rho+\delta_2)}\Big(-\tfrac{1}{4}(\alpha_n + r_n x^{sp}) - \tfrac{1}{2}(\alpha_s + r_s x^{sp})\\ &- r_n x^{sp}\Big)\\ &-hx^{sp} + \tfrac{\xi c_2}{b_2(\theta_2+\rho)}\Big] \equiv f_2(x^{sp}),\end{aligned} \tag{42}$$

$$\begin{aligned}x^{sp}=&\tfrac{1}{b_1\delta_1(\rho+\delta_1) - w_1 - w_0 w_2' w_3}\Big(w_0 w_2'\Big(\tfrac{\alpha_n + 2\alpha_s}{4b_2(\rho+\delta_2)} + \tfrac{\xi c_2}{(\rho+\theta_2)b_2}\Big) - \tfrac{h}{\rho+\delta_2}\Big(\tfrac{1}{4}\alpha_n + \tfrac{1}{2}\alpha_s\Big)\\ &+\tfrac{r_n\alpha_n}{4\beta_n} + \tfrac{r_s\alpha_s}{2\beta_s}\Big),\end{aligned} \tag{43}$$

where w_2' is

$$w_2' = w_2 - r_s, \tag{44}$$

and w_0, w_1, w_2, w_3 are defined by (30)–(33), respectively.

It follows from (40) and (41) that.

Proposition 5. *Under the social optimum of the South-country firm, learning rate* c_1 *and growth rate* μ *of the knowledge accumulation (or* c_2 *and* ξ*) are substitutable and* $\frac{\partial k_1^{sp}}{\partial \mu} > 0$, $\frac{\partial k_2^{sp}}{\partial \xi} > 0$.

This interpretation for Proposition 5 is merely analogous to that for Proposition 1.

Similar to the deduction of (28), we can obtain

$$\begin{aligned}\frac{\partial k_1^{sp}}{\partial k_2^{sp}} &= \frac{\partial k_1^{sp}/\partial x^{sp}}{\partial k_2^{sp}/\partial x^{sp}} = \frac{\delta_1}{h + \delta_2\frac{\partial f_2}{\partial x^{sp}}}\\ &= \frac{\delta_1}{\frac{\frac{5}{4}r_n + \frac{1}{2}r_s}{b_2(\rho+\delta_2)+\beta_n+\beta_s} + h\left(1 - \frac{1}{1+\frac{\beta_n+\beta_s}{4b_2(\rho+\delta_2)}}\right)} > 0,\end{aligned} \tag{45}$$

which produces the following same results as those in Proposition 2.

Proposition 6. *Process and product innovations are complementary in the neighborhood of the Stackelberg optimum where the North-country firm obtains its maximal net profit and the South-country firm obtains its maximal social welfare optimum.*

Next, we compare the effects of different optimum objectives (profit optimum and social welfare optimum) on the innovation investments of the South-country firm. Consumers in a northern market are much more green-conscious, namely, $r_n > r_s$. Here, we make a reasonable assumption that $\frac{3}{4}r_n > \frac{1}{2}r_s + b_2\delta_2 h$, and then we can obtain Proposition 7.

Proposition 7. *For the South-country firm in the Stackelberg game, if* $h = 0$ *(defined in (3)) or if* $h > 0$ *and* $\frac{5}{4}r_n + \frac{1}{2}r_s > b_2(\rho + \delta_2)h$, *the profit incentives bring a upward perturbation of the optimal investment efforts of product and process innovations, namely,* $k_1^{sp} < k_1^{pp}$, $k_2^{sp} < k_2^{pp}$.

This concept is in striking contrast to Proposition 11 of Li and Ni (2016), which states that the investment efforts of product and process innovations are both lower under the profit optimum than under the social welfare optimum.

Proposition 7 implies that under-investment or over-investment largely depends on the green preferences (represented by r_n and r_s) of the markets of the Stackelberg game, especially that of the North country (represented by r_n), and the parameters in production cost (represented by δ_2, b_2). When the green preferences and production cost satisfy the conditions in Proposition 7, the South-country firm will invest more innovations for profit maximum than for social welfare maximum. Given that the two models in the Stackelberg game have an infinite horizon, if the markets are sufficiently rich, or to say if the richest consumers' marginal willingness to pay for green products is sufficiently high, then the case described by Proposition 7 may occur prevalently.

For the South-country firm, when it makes strategies for profit optimum or for social welfare optimum, the southern firm will also consider the Stackelberg follower, namely, the North-country firm's decision. This idea is perhaps the reason for the difference between our conclusion and Proposition 11 of Li and Ni (2016). In reality, individuals will consider their own optimum, by considering their players in the game. Therefore, we think Proposition 7 is a more general case. Certainly, when the conditions of $\frac{3}{4}r_n > \frac{1}{2}r_s + b_2\delta_2 h$ and $\frac{5}{4}r_n + \frac{1}{2}r_s > b_2(\rho + \delta_2)h$ are not fully satisfied, it will be a more complex situation; to explore this problem, we will conduct further studies in our future work.

Below is the specific demonstration of Proposition 7.

Proof of Proposition 7. We rewrite (29) as

$$x^{pp} = \frac{1}{b_1\delta_1(\rho+\delta_1) - w_1 - w_0w_2w_3}\left(\frac{\alpha_n + 2\alpha_s}{4(\rho+\delta_2)}\left(\frac{w_0w_2}{b_2} - h\right) + w_0w_2\frac{\xi c_2}{(\rho+\theta_2)b_2} + \frac{r_n\alpha_n}{4\beta_n} + \frac{r_s\alpha_s}{2\beta_s}\right) \equiv \frac{1}{B}A, \tag{46}$$

in which

$$\frac{w_0w_2}{b_2} - h = \frac{4(\rho+\delta_2)(\frac{3}{4}r_n - \frac{1}{2}r_s - b_2\delta_2 h)}{(\beta_n + \beta_s) + 4(\rho+\delta_2)b_2\delta_2}.$$

It follows from the assumption $\frac{3}{4}r_n > \frac{1}{2}r_s + b_2\delta_2 h$ that $\frac{w_0w_2}{b_2} - h > 0$. Thus, $A > 0$. As $x^{pp} > 0$, $B > 0$. We can easily see that $w_0 > 0$ (as defined in (30)). Moreover, from (46) we get

$$\frac{\partial x^{pp}}{\partial w_2} = \frac{1}{B^2}\left[w_0\left(\frac{\alpha_n + 2\alpha_s}{4b_2(\rho+\delta_2)} + \frac{\xi c_2}{b_2(\rho+\theta_2)}\right)B + w_0w_3A\right].$$

Case 1: When $h = 0$ in (33), then $w_3 > 0$, so $\frac{\partial x^{pp}}{\partial w_2} > 0$. By comparing (29) and (43), we know that x^{pp} and x^{sp} have the same expression except for w_2' and w_2, $w_2' = w_2 - r_s < w_2$. Therefore, we have

$$x^{sp} < x^{pp}.$$

Then,

$$k_1^{sp} = \delta_1 x^{sp} < \delta_1 x^{pp} = k_1^{pp}.$$

From $c^{pp} = f_1(x^{pp})$ (defined in (27)) and $c^{sp} = f_2(x^{sp})$ (defined in (42)), we gain $f_1(x) = f_2(x) \equiv f(x)$. As $k_2^{pp} = \delta_2 c^{pp} + hx^{pp}$ (defined in (26)) and $k_2^{sp} = \delta_2 c^{sp} + hx^{sp}$ (defined in (41)), we define $k_2 = \delta_2 c + hx = \delta_2 f(x) + hx$. Obviously, there holds

$$\frac{\partial k_2}{\partial x} = \frac{\frac{5}{4}r_n + \frac{1}{2}r_s}{b_2(\rho+\delta_2) + \beta_n + \beta_s} + h\left(1 - \frac{1}{1 + \frac{\beta_n + \beta_s}{4b_2(\rho+\delta_2)}}\right) > 0,$$

which, together with $x^{sp} < x^{pp}$, leads to

$$k_2^{sp} < k_2^{pp}.$$

Case 2: When $h > 0$, only when $\frac{5}{4}r_n + \frac{1}{2}r_s > b_2(\rho+\delta_2)h$, there is $w_3 > 0$ (w_3 is defined in (33)), then we obtain the same result as that in Case 1.

6. Conclusion

In this paper, we propose a more general dynamic description of the production cost, and then in a Stackelberg frame, we discuss manufacturer's decision-making behaviors, especially process and product innovations. Specifically, we develop a Stackelberg differential game that consists a manufacturer, the South-country firm as the leader, and a retailer, the North-country firm as the follower. Consumers in the North–South markets in this paper have different preferences for the green level of products and product price. Therefore, the South-country firm needs to consider the investment efforts of product and process innovations under learning-by-doing to meet consumers' needs. The North-country firm makes decisions to obtain its private profit optimum, while the South-country firm makes decisions from two perspectives: a personal representative (the profit maximum) and a social planner (the social welfare maximum).

Several results of our analysis are worth emphasizing. First, higher learning rate increases the investment efforts of process and product innovations. Second, two kinds of innovations are complementary at equilibrium, thus boosting each other. The above two results are in line with those in Lambertini and Mantovani (2009), Lambertini and Orsini (2015) and Li and Ni (2016), but the third result is converse to that in Lambertini and Mantovani (2009), Lambertini and Orsini (2015, 2016), Lambertini et al. (2017). In the framework of a Stackelberg game, where consumers in the North country have a much greater green-product preference, we obtain that for the South-country firm, the social incentive toward both kinds of innovations is weaker than the private profit-seeking incentive.

In the future, we will conduct the same analysis in an oligopoly model or extend the model to a duopoly case with multi-product production and compare the possible difference from the monopolist case in this paper.

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