Mixed gluinos and sgluons from a new $SU(3)$ gauge group

Stephen P. Martin

Department of Physics, Northern Illinois University, DeKalb IL 60115

I study supersymmetric models in which the QCD gauge group is the remnant diagonal subgroup from the spontaneous breaking of an $SU(3) \times SU(3)$ gauge group at a multi-TeV scale. In renormalizable models with soft supersymmetry breaking, the scalar potential is shown to have global minima with the required gauge symmetry breaking pattern. In addition to a massive color octet vector boson, this framework predicts 3 color octet spin-0 sgluons, and 4 color octet gluinos with both Dirac and Majorana mass terms. One of the gluino mass eigenstates typically has a coupling to quark-squark pairs that is at least as large as the prediction of minimal supersymmetry, but it need not be the lightest one.

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I. INTRODUCTION

The Large Hadron Collider (LHC) has discovered the Higgs boson associated with electroweak symmetry breaking, but so far has not provided any insight into the hierarchy problem. Instead, it has imposed significant lower limits on the masses of supersymmetric particles, and more generally on any new particles that could be involved in symmetries or dynamics that might explain why the electroweak scale is so much lighter than the Planck scale and other high mass scales associated with new physics. At the same time, a mass near 125 GeV for the lightest Higgs boson, within the context of supersymmetric extensions of the Standard model, suggests that the top squark and other superpartner masses could very well lie at a characteristic scale $M_{\rm SUSY}$ in the multi-TeV range, beyond the reach of the 14 TeV LHC.

One possibility is that supersymmetry really is the essential part of the explanation for the big hierarchy problem $M_Z^2 \ll M_{\rm Planck}$, but that there is some other subsidiary principle,[†] not yet understood, that could explain the little hierarchy problem $M_Z^2 \ll M_{\rm SUSY}^2$. A common feature to be expected in that case is that the minimal supersymmetric standard model (MSSM) should be extended beyond the minimal particle content in the multi-TeV mass range or below. Attempts along these lines are far too numerous to review here.

The MSSM already contains one vectorlike combination of fields, the Higgs supermultiplets H_u and H_d, which obtain a bare supersymmetry-preserving mass term, μ. Whatever mechanism is responsible for ensuring that μ is non-zero but also not far above the TeV scale could plausibly also be responsible for placing other vectorlike chiral supermultiplets at the TeV scale. In the same spirit, one can also suppose that there are other gauge supermultiplets that obtain masses at the TeV scale, where the corresponding gauge symmetries are spontaneously broken. In this paper, I will consider one such possibility that has already been widely considered [1–22] in the non-supersymmetric context: that the QCD $SU(3)_C$ gauge group of the MSSM and the Standard Model is the remnant of a spontaneous breaking of the type:

$$SU(3)_A \times SU(3)_B \;\rightarrow\; SU(3)_C. \tag{1.1}$$

The gauge bosons associated with the diagonal subgroup of $SU(3)_A \times SU(3)_B$ are the massless gluons of the Standard Model. The remaining 8 vector bosons have been variously referred to in the literature as axigluons [1–6] or topgluons [7–11] or colorons [12–22], depending on how the Standard Model fermions are assigned to $SU(3)_A$ and $SU(3)_B$ representations. Here, I will study the possibility of realizing this symmetry breaking consistently in a renormalizable softly broken supersymmetric model. This requires the presence of two chiral superfields that transform as the fundamental and anti-fundamental representations of both gauge groups, to be denoted in this paper as:

$$\Phi_j^k \sim (\mathbf{3}, \overline{\mathbf{3}}), \qquad\qquad \overline{\Phi}_k^j \sim (\overline{\mathbf{3}}, \mathbf{3}). \tag{1.2}$$

[†] It is also possible that the little hierarchy is just the result of a coincidence. This should be taken seriously because it is enormously less severe than the big hierarchy problem. However, there is no objective, scientific way of deciding how much of a coincidence is too severe; in my view this is a personal and inherently subjective choice that scientists must nevertheless make in order to decide how to allocate limited resources such as time and money.

A lowered index corresponds to a fundamental **3** representation of $SU(3)$, and a raised index to an anti-fundamental $\overline{\mathbf{3}}$. Thus, in both instances in eq. (1.2), j is an $SU(3)_A$ index, and k is an $SU(3)_B$ index.

This supersymmetric model then predicts the existence of, in addition the coloron vector boson X, four gluino mass eigenstates (including an admixture of what can be regarded as the MSSM gluino) with both Dirac and Majorana mass contributions, three color octet scalars (sgluons), two color singlet fermions (singlinos) and four real scalar singlets, in addition to the usual superpartners of the MSSM. Models with Dirac mass terms for gauginos have a long history [23–44]. The present paper is an alternative to models where Dirac gaugino masses arise due to supersoft [28] supersymmetry breaking following from D-term breaking and feature a continuous R symmetry [25, 28, 31], where the gauge supermultiplet sector can be considered as $N = 2$ supersymmetry multiplets. Instead, the Dirac gluino mass parameters here arise from an additional gauge group and the chiral fermions associated with its breaking. There are also Majorana gluino masses, so that the gluinos are mixed.

I now discuss some other conventions and notations to be used below. Adjoint representation indices of $SU(3)$ are represented by letters $a, b, c, \ldots$. The generators of the fundamental representation are T_j^{ak}, and obey the general $SU(N_c)$ trace, commutator, anti-commutator, and Fierz identities:

$$\mathrm{Tr}[T^a T^b] = \frac{1}{2}\delta^{ab}, \tag{1.3}$$

$$[T^a, T^b]_j{}^k = if^{abc}T_j^{ck}, \tag{1.4}$$

$$\{T^a, T^b\}_j{}^k = \frac{1}{N_c}\delta^{ab}\delta_j^k + d^{abc}T_j^{ck}, \tag{1.5}$$

$$T_j^{ak}T_l^{am} = \frac{1}{2}\delta_j^m\delta_l^k - \frac{1}{2N_c}\delta_j^k\delta_l^m. \tag{1.6}$$

Here eq. (1.3) establishes the usual normalization of the generators, while eq. (1.4) defines the anti-symmetric structure constants f^{abc} and eq. (1.5) defines the symmetric anomaly coefficients d^{abc}. There follows:

$$\mathrm{Tr}[T^a T^b T^c] = \frac{1}{4}\left(d^{abc} + if^{abc}\right), \tag{1.7}$$

$$T_j^{ak}T_k^{al} = \frac{N_c^2 - 1}{2N_c}\delta_j^l. \tag{1.8}$$

Also, for $N_c = 3$ only (as assumed from now on), there are the anti-symmetric tensor invariant symbols ϵ^{jkl} and ϵ_{jkl}, which by convention are taken here to have

$$\epsilon^{123} = \epsilon_{123} = 1. \tag{1.9}$$

Then one also has the useful identity:

$$\epsilon_{jlm}\epsilon^{knp}T_n^{al}T_p^{bm} = d^{abc}T_j^{ck} - \frac{1}{6}\delta^{ab}\delta_j^k. \tag{1.10}$$

The notations and conventions for supersymmetry and 2-component fermions follow those in [45].

For an appropriate choice of potential parameters, as demonstrated below, the scalar components of Φ and $\overline{\Phi}$ will acquire vacuum expectation values (VEVs) of the form

$$\langle \phi_j^k \rangle = \delta_j^k v, \qquad\qquad \langle \overline{\phi}_j^k \rangle = \delta_j^k \overline{v}. \tag{1.11}$$

In that case, the massless gluon field G and the massive color octet vector field X are related to the $SU(3)_A$ and $SU(3)_B$ gauge vector fields by:

$$\begin{pmatrix} G_\mu^a \\ X_\mu^a \end{pmatrix} \; = \; \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} A_\mu^a \\ B_\mu^a \end{pmatrix}, \tag{1.12}$$

where

$$\sin\theta \; = \; g_A/\sqrt{g_A^2 + g_B^2}, \qquad\qquad \cos\theta \; = \; g_B/\sqrt{g_A^2 + g_B^2}, \tag{1.13}$$

and X obtains a squared mass:

$$M_X^2 = (g_A^2 + g_B^2)(|v|^2 + |\overline{v}|^2). \tag{1.14}$$

The QCD coupling is related to the original gauge couplings by

$$g_3 \; = \; g_A g_B/\sqrt{g_A^2 + g_B^2}, \tag{1.15}$$

and fields that transform as (R_A, R_B) under $SU(3)_A \times SU(3)_B$ will transform as the (reducible, in general) representation $R_A \times R_B$ of $SU(3)_C$. In particular, for quarks originally in the fundamental **3** representation of the $SU(3)_A$ gauge group, the covariant derivative is:

$$D_\mu q_j = (\partial_\mu q_j - i g_3 G_\mu^a T_j^{ak} q_k) + i g_3 \tan\theta \, X_\mu^a T_j^{ak} q_k. \tag{1.16}$$

On the other hand, for quarks originally in the fundamental representation of $SU(3)_B$, then we have

$$D_\mu q_j = (\partial_\mu q_j - i g_3 G_\mu^a T_j^{ak} q_k) - i g_3 \cot\theta \, X_\mu^a T_j^{ak} q_k. \tag{1.17}$$

In the following, I will assume that all of the Standard Model quarks and their superpartners live in the fundamental representation of $SU(3)_A$, although this is not inevitable. There can also be additional vectorlike quarks and squarks transforming under $SU(3)_B$, and these will indeed play a role in section IV. These can mix with the usual quarks by Yukawa couplings to the Φ and $\overline{\Phi}$ fields, breaking flavor symmetries and thus allowing the vectorlike quarks to decay. For simplicity, it is assumed that these Yukawa couplings are non-zero but very small, as is technically natural.

In the remainder of this paper, I will explore one possible supersymmetric setup that is renormalizable and avoids fundamental singlets (with potentially dangerous tadpoles). I will show that the symmetry breaking pattern given above can indeed be realized in a stable vacuum that is the global minimum of the potential. In similar non-supersymmetric models, the minimization of the potential has been analyzed in ref. [19]. However, the softly broken supersymmetric case is quite different because two Higgsing fields $\Phi, \overline{\Phi}$ are required by the anomaly cancellation associated with the fermionic components, and because the structure of the dimensionless couplings in the scalar potential is constrained as dictated by supersymmetry. The resulting theory naturally includes Dirac masses for the MSSM gluinos along with the usual Majorana masses. The lightest of the mixed gluino states can be significantly lighter than the color octet vectors X and the spin-0 sgluons. There are also inevitably new color singlet scalars and fermions. The phenomenology of these states will be briefly considered in section V.

II. SUPERSYMMETRIC MODELS WITH $SU(3)_A \times SU(3)_B \to SU(3)_C$

Consider a model consisting of the MSSM and the fields Φ and $\overline{\Phi}$. The most general renormalizable superpotential of this theory is:

$$W \;=\; \frac{1}{6}\epsilon^{jkl}\epsilon_{mnp}\left(y\,\Phi_j^m\Phi_k^n\Phi_l^p + \overline{y}\,\overline{\Phi}_j^m\overline{\Phi}_k^n\overline{\Phi}_l^p\right) - \mu_\Phi\Phi_j^k\overline{\Phi}_k^j + W_{\rm MSSM}, \tag{2.1}$$

where y and $\overline{y}$ are Yukawa couplings and μ_Φ is a mass term, which is analogous to the μ term of the MSSM, and can be presumed to have the same sort of origin. As a very rough estimate, μ_Φ can therefore be taken to be of order a multi-TeV scale. The existence of y and $\overline{y}$ relies on the fact that the gauge groups are $SU(3)$, because only in this case among the special unitary groups does the invariant symbol ϵ^{jkl} exist, corresponding to the group theory fact that the antisymmetric product of fundamental representations $\mathbf{3}\times\mathbf{3}\times\mathbf{3}$ contains a singlet. As a convention, y and $\overline{y}$ can be taken real and positive without loss of generality; then the phase of μ_Φ is physical. The soft supersymmetry breaking Lagrangian is:

$$\begin{aligned}\mathcal{L}_{\rm soft} \;=\; &\left[-\frac{1}{2}M_A\lambda_A^a\lambda_A^a - \frac{1}{2}M_B\lambda_B^a\lambda_B^a - \frac{1}{6}\epsilon^{jkl}\epsilon_{mnp}\left(a\,\phi_j^m\phi_k^n\phi_l^p + \overline{a}\,\overline{\phi}_j^m\overline{\phi}_k^n\overline{\phi}_l^p\right) + b_\phi\phi_j^k\overline{\phi}_k^j\right] + \text{c.c.} \\ &-m^2(\phi_j^k)^*\phi_j^k - \overline{m}^2(\overline{\phi}_j^k)^*\overline{\phi}_j^k,\end{aligned} \tag{2.2}$$

where λ_A^a and λ_B^a are the gauginos for the $SU(3)_A$ and $SU(3)_B$ gauge groups respectively.

One can now expand the scalar fields around diagonal VEVs:

$$\phi_j^k \;=\; \delta_j^k\left(v + \frac{\phi_0}{\sqrt{3}}\right) + \sqrt{2}\,T_j^{ak}\phi^a, \tag{2.3}$$

$$\overline{\phi}_j^k \;=\; \delta_j^k\left(\overline{v} + \frac{\overline{\phi}_0}{\sqrt{3}}\right) + \sqrt{2}\,T_j^{ak}\overline{\phi}^a, \tag{2.4}$$

where ϕ_0, $\overline{\phi}_0$ and ϕ^a, $\overline{\phi}^a$ are complex scalar fields with canonically normalized kinetic terms, which

live in the singlet and adjoint representations of $SU(3)_C$. Similarly, the fermionic components of Φ and $\overline{\Phi}$ can be expanded as:

$$\psi_j^k = \frac{1}{\sqrt{3}}\,\delta_j^k\,\psi_0 + \sqrt{2}\,T_j^{ak}\psi^a, \tag{2.5}$$

$$\overline{\psi}_j^k = \frac{1}{\sqrt{3}}\,\delta_j^k\,\overline{\psi}_0 + \sqrt{2}\,T_j^{ak}\overline{\psi}^a, \tag{2.6}$$

where ψ_0 and ψ^a and $\overline{\psi}_0$ and $\overline{\psi}^a$ are 2-component fermion fields with canonically normalized kinetic terms.

The interactions of the new fermions with the scalars and their VEVs are

$$\begin{aligned}\mathcal{L} = \Bigg[&(\overline{v}+\overline{\phi}_0/\sqrt{3})^*(g_A\lambda_A^a\overline{\psi}^a - g_B\lambda_B^a\overline{\psi}^a) + (v+\phi_0/\sqrt{3})^*(g_B\lambda_B^a\psi^a - g_A\lambda_A^a\psi^a)\\ &+\frac{1}{\sqrt{2}}(d^{abc}+if^{abc})\left(g_A\overline{\phi}^{a*}\overline{\psi}^b\lambda_A^c - g_A\phi^{a*}\lambda_A^b\psi^c + g_B\phi^{a*}\psi^b\lambda_B^c - g_B\overline{\phi}^{a*}\lambda_B^b\overline{\psi}^c\right)\\ &+\frac{g_A}{\sqrt{3}}\left(\overline{\phi}^{a*}\lambda_A^a\overline{\psi}_0 - \phi^{a*}\lambda_A^a\psi_0\right) + \frac{g_B}{\sqrt{3}}\left(\phi^{a*}\lambda_B^a\psi_0 - \overline{\phi}^{a*}\lambda_B^a\overline{\psi}_0\right)\Bigg] + \text{c.c.}\end{aligned} \tag{2.7}$$

from gaugino-fermion-scalar interactions, and

$$\begin{aligned}\mathcal{L} = \Bigg\{&y\left[-(v+\phi_0/\sqrt{3})\psi_0\psi_0 + \frac{1}{2}(v+\phi_0/\sqrt{3})\psi^a\psi^a + \frac{1}{\sqrt{3}}\phi^a\psi_0\psi^a - \frac{1}{\sqrt{2}}d^{abc}\phi^a\psi^b\psi^c\right]\\ &+\overline{y}\left[-(\overline{v}+\overline{\phi}_0/\sqrt{3})\overline{\psi}_0\overline{\psi}_0 + \frac{1}{2}(\overline{v}+\overline{\phi}_0/\sqrt{3})\overline{\psi}^a\overline{\psi}^a + \frac{1}{\sqrt{3}}\overline{\phi}^a\overline{\psi}_0\overline{\psi}^a - \frac{1}{\sqrt{2}}d^{abc}\overline{\phi}^a\overline{\psi}^b\overline{\psi}^c\right]\\ &+\mu_\Phi\left[\psi_0\overline{\psi}_0 + \psi^a\overline{\psi}^a\right]\Bigg\} + \text{c.c.}\end{aligned} \tag{2.8}$$

from the superpotential. The mass eigenstates are then obtained as follows. There are four 2-component $SU(3)_C$-octet fermions (gluinos), with mass matrix in the basis $\tilde{g}^a = (\lambda_A^a,\, \lambda_B^a,\, \psi^a,\, \overline{\psi}^a)$:

$$M_{\tilde{g}} = \begin{pmatrix} M_A & 0 & g_A v^* & -g_A\overline{v}^* \\ 0 & M_B & -g_B v^* & g_B\overline{v}^* \\ g_A v^* & -g_B v^* & -yv & -\mu_\Phi \\ -g_A\overline{v}^* & g_B\overline{v}^* & -\mu_\Phi & -\overline{y}\,\overline{v} \end{pmatrix}. \tag{2.9}$$

This can be diagonalized by a unitary matrix U to obtain the mass eigenvalues:

$$M_{\tilde{g}}^{\text{diag}} = U^* M_{\tilde{g}} U^\dagger. \tag{2.10}$$

There are also two gauge-singlet 2-component fermions, which in the basis $\tilde{\chi} = (\psi_0,\, \overline{\psi}_0)$ have a

mass matrix

$$M_{\tilde{\chi}} \;=\; \begin{pmatrix} 2yv & -\mu_\Phi \\ -\mu_\Phi & 2\overline{y}\,\overline{v} \end{pmatrix}, \tag{2.11}$$

with squared mass eigenvalues

$$|\mu_\Phi|^2 + 2|yv|^2 + 2|\overline{yv}|^2 \pm 2\sqrt{|yv\mu_\Phi^* + \overline{y}^*\overline{v}^*\mu_\Phi|^2 + (|yv|^2 - |\overline{yv}|^2)^2}. \tag{2.12}$$

In order to obtain the new scalar mass eigenvalues, one can proceed by first obtaining the scalar potential

$$V = V_D + V_F + V_{\rm soft} \tag{2.13}$$

as a function of the canonically normalized fields. The supersymmetric D-term contribution is

$$V_D \;=\; \frac{1}{2}(D_A^a D_A^a + D_B^a D_B^a), \tag{2.14}$$

where

$$\begin{aligned} D_A^a \;=\; & \frac{g_A}{\sqrt{2}}\Big[(\overline{v} + \overline{\phi}_0/\sqrt{3})\overline{\phi}^{a*} + (\overline{v} + \overline{\phi}_0/\sqrt{3})^*\overline{\phi}^a - (v + \phi_0/\sqrt{3})\phi^{a*} - (v + \phi_0/\sqrt{3})^*\phi^a \\ & + \frac{1}{\sqrt{2}}\left(d^{abc} + if^{abc}\right)\left(\overline{\phi}^{b*}\overline{\phi}^c - \phi^b\phi^{c*}\right)\Big], \end{aligned} \tag{2.15}$$

$$\begin{aligned} D_B^a \;=\; & \frac{g_B}{\sqrt{2}}\Big[(v + \phi_0/\sqrt{3})\phi^{a*} + (v + \phi_0/\sqrt{3})^*\phi^a - (\overline{v} + \overline{\phi}_0/\sqrt{3})\overline{\phi}^{a*} - (\overline{v} + \overline{\phi}_0/\sqrt{3})^*\overline{\phi}^a \\ & + \frac{1}{\sqrt{2}}\left(d^{abc} + if^{abc}\right)\left(\phi^{b*}\phi^c - \overline{\phi}^b\overline{\phi}^{c*}\right)\Big]. \end{aligned} \tag{2.16}$$

The supersymmetric F-term contribution is

$$V_F \;=\; |F_0|^2 + |F^a|^2 + |\overline{F}_0|^2 + |\overline{F}^a|^2, \tag{2.17}$$

where

$$F_0^* \;=\; \mu_\Phi(\sqrt{3}\,\overline{v} + \overline{\phi}_0) - y(\sqrt{3}\,v + \phi_0)^2/\sqrt{3} \,+\, y\phi^a\phi^a/2\sqrt{3}, \tag{2.18}$$

$$\overline{F}_0^* \;=\; \mu_\Phi(\sqrt{3}\,v + \phi_0) - \overline{y}(\sqrt{3}\,\overline{v} + \overline{\phi}_0)^2/\sqrt{3} \,+\, \overline{y}\overline{\phi}^a\overline{\phi}^a/2\sqrt{3}, \tag{2.19}$$

$$F^{a*} \;=\; \mu_\Phi\overline{\phi}^a + y(v + \phi_0/\sqrt{3})\phi^a - yd^{abc}\phi^b\phi^c/\sqrt{2}, \tag{2.20}$$

$$\overline{F}^{a*} \;=\; \mu_\Phi\phi^a + \overline{y}(\overline{v} + \overline{\phi}_0/\sqrt{3})\overline{\phi}^a - \overline{y}d^{abc}\overline{\phi}^b\overline{\phi}^c/\sqrt{2}. \tag{2.21}$$

Finally, the expansion of the soft supersymmetry-breaking part $V_{\rm soft}$ is

$$\begin{aligned} V_{\rm soft} \;=\; & \Big\{ a\left[(v+\phi_0/\sqrt{3})^3 - \frac{1}{2}(v+\phi_0/\sqrt{3})\phi^a\phi^a + \frac{1}{3\sqrt{2}}d^{abc}\phi^a\phi^b\phi^c\right] \\ & + \overline{a}\left[(\overline{v}+\overline{\phi}_0/\sqrt{3})^3 - \frac{1}{2}(\overline{v}+\overline{\phi}_0/\sqrt{3})\overline{\phi}^a\overline{\phi}^a + \frac{1}{3\sqrt{2}}d^{abc}\overline{\phi}^a\overline{\phi}^b\overline{\phi}^c\right] \\ & - b_\phi\left[(\sqrt{3}\,v+\phi_0)(\sqrt{3}\,\overline{v}+\overline{\phi}_0) + \phi^a\overline{\phi}^a\right]\Big\} + \text{c.c.} \\ & + m^2(|\sqrt{3}\,v+\phi_0|^2 + |\phi^a|^2) + \overline{m}^2(|\sqrt{3}\,\overline{v}+\overline{\phi}_0|^2 + |\overline{\phi}^a|^2). \end{aligned} \tag{2.22}$$

Isolating the quadratic parts of V, the squared masses for the real scalar fields in Φ, $\overline{\Phi}$ are as follows. Writing $\phi^a = (R^a + iI^a)/\sqrt{2}$ and $\overline{\phi}^a = (\overline{R}^a + i\overline{I}^a)/\sqrt{2}$ and $\phi_0 = (R_0 + iI_0)/\sqrt{2}$ and $\overline{\phi}_0 = (\overline{R}_0 + i\overline{I}_0)/\sqrt{2}$, the singlet spin-0 squared mass matrix in the basis $\varphi = (R_0, \overline{R}_0, I_0, \overline{I}_0)$ is:

$$M^2_\varphi \;=\; \begin{pmatrix} U+4|yv|^2-2X_1 & -2X_2-\text{Re}[b_\phi] & 2Y_1 & -2Y_2+\text{Im}[\text{b}_\phi] \\ -2X_2-\text{Re}[b_\phi] & \overline{U}+4|\overline{yv}|^2-2\overline{X}_1 & 2Y_2+\text{Im}[b_\phi] & 2\overline{Y}_1 \\ 2Y_1 & 2Y_2+\text{Im}[b_\phi] & U+4|yv|^2+2X_1 & -2X_2+\text{Re}[b_\phi] \\ -2Y_2+\text{Im}[b_\phi] & 2\overline{Y}_1 & -2X_2+\text{Re}[b_\phi] & \overline{U}+4|\overline{yv}|^2+2\overline{X}_1 \end{pmatrix}, \tag{2.23}$$

where

$$U \;=\; |\mu|^2 + m^2, \qquad \overline{U} \;=\; |\mu|^2 + \overline{m}^2, \tag{2.24}$$

$$X_1 + iY_1 \;=\; y(\mu\overline{v} - yv^2)^* - av, \tag{2.25}$$

$$\overline{X}_1 + i\overline{Y}_1 \;=\; \overline{y}(\mu v - \overline{yv}^2)^* - \overline{a}v, \tag{2.26}$$

$$X_2 \;=\; \text{Re}[\mu^*(yv + \overline{yv})], \qquad Y_2 \;=\; \text{Im}[\mu^*(yv - \overline{yv})], \tag{2.27}$$

and the octet spin-0 (sgluon) squared mass matrix in the basis $S^a = (R^a, \overline{R}^a, I^a, \overline{I}^a)$ is:

$$\begin{aligned} M^2_S \;=\; & \begin{pmatrix} U+|yv|^2+X_1 & X_2-\text{Re}[b_\phi] & -Y_1 & Y_2+\text{Im}[\text{b}_\phi] \\ X_2-\text{Re}[b_\phi] & \overline{U}+|\overline{yv}|^2+\overline{X}_1 & -Y_2+\text{Im}[b_\phi] & -\overline{Y}_1 \\ -Y_1 & -Y_2+\text{Im}[b_\phi] & U+|yv|^2-X_1 & X_2+\text{Re}[b_\phi] \\ Y_2+\text{Im}[b_\phi] & -\overline{Y}_1 & X_2+\text{Re}[b_\phi] & \overline{U}+|\overline{yv}|^2-\overline{X}_1 \end{pmatrix} \\ & + (g_A^2+g_B^2)\begin{pmatrix} v_R^2 & -v_R\overline{v}_R & v_Rv_I & -v_R\overline{v}_I \\ -v_R\overline{v}_R & \overline{v}_R^2 & -\overline{v}_Rv_I & \overline{v}_R\overline{v}_I \\ v_Rv_I & -\overline{v}_Rv_I & v_I^2 & -v_I\overline{v}_I \\ -v_R\overline{v}_I & \overline{v}_R\overline{v}_I & -v_I\overline{v}_I & \overline{v}_I^2 \end{pmatrix}, \end{aligned} \tag{2.28}$$

where

$$v_R + iv_I = v, \qquad \overline{v}_R + i\overline{v}_I = \overline{v}. \tag{2.29}$$

The real symmetric squared mass matrices M_S^2 and M_φ^2 can be diagonalized by orthogonal transformations to obtained the squared mass eigenvalues for the real color octet and singlet spin-0 particles. One of the octet spin-0 eigenvectors is the would-be Goldstone boson of the symmetry breaking, which is absorbed as the longitudinal mode of the massive vector. In the limit of vanishing v_I, $\overline{v}_I$, Im$[b_\phi]$, Y_1, and Y_2 (i.e., no CP-violating phases), the diagonalizations separate into 2×2 blocks corresponding to scalar and pseudo-scalar states. In that case, there are two scalar and one pseudo-scalar sgluons, and two scalar and two pseudo-scalar singlets.

III. MINIMIZATION OF THE SCALAR POTENTIAL

A. The supersymmetric limit

As a warm-up example and a useful limiting case, consider the supersymmetric limit in which the soft parameters a, $\overline{a}$, b_ϕ, m^2, and $\overline{m}^2$ are all set to 0. Then the scalar potential as a function of v and $\overline{v}$ becomes simply:

$$V(v, \overline{v}) = 3|yv^2 - \mu_\Phi \overline{v}|^2 + 3|\overline{y}\,\overline{v}^2 - \mu_\Phi v|^2, \tag{3.1}$$

as this is a D-flat direction. This has distinct minima at $v = \overline{v} = 0$, where the gauge symmetry is unbroken, and at

$$v \;=\; \mu_\Phi/(y^2\overline{y})^{1/3}, \qquad\qquad \overline{v} \;=\; \mu_\Phi/(y\overline{y}^2)^{1/3}, \tag{3.2}$$

where the gauge symmetry is broken to $SU(3)_C$. These are degenerate global minima, with $V = 0$ in both cases, so that supersymmetry is not spontaneously broken. They can be checked to be minima of the full potential eqs. (2.13)-(2.22), by evaluating the scalar squared masses and noting that they are non-negative, other than the octet of vanishing eigenvalues corresponding to the would-be Goldstone bosons of the spontaneously broken gauge symmetry in the case of eq. (3.2). The mass spectrum of the theory contains a massive vector supermultiplet, consisting of the vector bosons, a Dirac fermion (two 2-component fermions), and a real scalar, all of which are octets of $SU(3)_C$ with squared masses $M_X^2 = (g_A^2 + g_B^2)(|v|^2 + |\overline{v}|^2)$. There is also a massive color octet chiral supermultiplet (one 2-component fermion and a complex scalar) with squared masses

$$M^2_{\text{octet}} = R^2|\mu_\Phi|^2, \tag{3.3}$$

and two singlet chiral supermultiplets with squared masses

$$M^2_{\text{singlets}} = \left(2R^2 - 3 \pm 2R\sqrt{R^2 - 3}\right)|\mu_\Phi|^2, \tag{3.4}$$

where

$$R = |y/\overline{y}|^{1/3} + |\overline{y}/y|^{1/3}\,. \tag{3.5}$$

Because $R \geq 2$ (with equality if $|\overline{y}| = |y|$), these squared masses are always positive. One of the singlet chiral supermultiplets is always lighter, and one always heavier, than the octet chiral supermultiplet. There is also an $SU(3)_C$ octet massless vector supermultiplet (the MSSM gluon and gluino), and one massless $SU(3)_C$ octet real scalar would-be Goldstone boson which is absorbed by the massive color octet vector boson, becoming its longitudinal mode.

B. Realistic examples with supersymmetry breaking

Now consider the realistic case that supersymmetry breaking is included. It is useful to take a more general form for the possible scalar field expectation values, to include the possibility that the remnant gauge symmetry is not $SU(3)_C$:

$$\langle \phi_j^k \rangle = \delta_j^k v + \delta_{j3}\delta^{k3} s, \qquad\qquad \langle \overline{\phi}_j^k \rangle = \delta_j^k \overline{v} + \delta_{j3}\delta^{k3} \overline{s}. \tag{3.6}$$

Now if $s = \overline{s} = 0$ and $v, \overline{v}$ are non-zero, the unbroken gauge symmetry will be $SU(3)_C$. If $v = \overline{v} = 0$ and $s, \overline{s}$ are non-zero, then the unbroken gauge symmetry is $SU(2) \times SU(2) \times U(1)$. For general $v, \overline{v}, s, \overline{s}$, the unbroken gauge symmetry would be $SU(2) \times U(1)$. I do not consider even more general VEVs, for which the unbroken gauge symmetry would be even smaller. This is because both the D-term and F-term contributions to the potential are non-negative, and they favor the larger unbroken symmetries $SU(3)_C$ or $SU(2) \times SU(2) \times U(1)$. As found [19] in the non-supersymmetric case with one $(\mathbf{3}, \overline{\mathbf{3}})$ scalar field, no local minimum is expected in the case of a $SU(2) \times U(1)$ or smaller residual symmetry, and I have confirmed this in numerical examples, although I have not attempted a formal or general proof.

The scalar potential D-term, F-term, and soft contributions are then:

$$\begin{aligned}
V_D &= \frac{1}{6}(g_A^2 + g_B^2)\left(|v+s|^2 - |\overline{v}+\overline{s}|^2 - |v|^2 + |\overline{v}|\right)^2, && (3.7)\\
V_F &= |yv^2 - \mu_\Phi(\overline{v}+\overline{s})|^2 + |\overline{y}\overline{v}^2 - \mu_\Phi(v+s)|^2 + 2|yv(v+s) - \mu_\Phi\overline{v}|^2 \\
&\quad + 2|\overline{y}\overline{v}(\overline{v}+\overline{s}) - \mu_\Phi v|^2, && (3.8)\\
V_{\rm soft} &= \left(av^2(v+s) + \overline{a}\overline{v}^2(\overline{v}+\overline{s}) - b_\phi\left[(v+s)(\overline{v}+\overline{s}) + 2v\overline{v}\right]\right) + \text{c.c.} \\
&\quad + m^2\left(|v+s|^2 + 2|v|^2\right) + \overline{m}^2\left(|\overline{v}+\overline{s}|^2 + 2|\overline{v}|^2\right). && (3.9)
\end{aligned}$$

There is a D-flat direction $|\overline{s}| = |s|$ when $\overline{v} = v = 0$. This is unaffected by the $y, \overline{y}, a, \overline{a}$ couplings, but it is lifted by the F-term contribution $V = |\mu_\Phi|^2(|s|^2 + |\overline{s}|^2)$, so it is not a minimum of the supersymmetric limit of the previous section. However, it can be favored by the soft supersymmetry breaking squared mass terms, leading to a runaway unbounded from below (UFB) direction, in which $|\overline{s}| = |s|$ becomes arbitrarily large and the phase of $s\overline{s}$ is the same as that of b_ϕ^*. This will

occur unless

$$|b_\phi| \;<\; |\mu_\Phi|^2 + (m^2 + \overline{m}^2)/2. \tag{3.10}$$

This UFB solution can be separated by a barrier from other local minima with non-zero v, $\overline{v}$, which could therefore in principle be viable if the tunneling rate is small enough.

Next, take the possibility that $v = \overline{v} = 0$ with $|s| \neq |\overline{s}|$, which would lead to the symmetry breaking pattern $SU(3)_A \times SU(3)_B \to SU(2) \times SU(2) \times U(1)$, and consider

$$V(s,\overline{s}) \;=\; \frac{1}{6}(g_A^2 + g_B^2)(|s|^2 - |\overline{s}|^2)^2 + (m^2 + |\mu_\Phi|^2)|s|^2 + (\overline{m}^2 + |\mu_\Phi|^2)|\overline{s}|^2 - (b_\phi s\overline{s} + \text{c.c.}). \tag{3.11}$$

Assuming $m^2 \leq \overline{m}^2$ without loss of generality [otherwise the discussion goes through with $(s, m^2) \leftrightarrow (\overline{s}, \overline{m}^2)$], the possible nontrivial stable minimum of $V(s,\overline{s})$ is at:

$$|s|^2 \;=\; \frac{3}{4(g_A^2 + g_B^2)d}\left[(|\mu_\Phi|^2 + \overline{m}^2)^2 - (|\mu_\Phi|^2 + m^2 + d)^2\right], \tag{3.12}$$

$$|\overline{s}|^2 \;=\; \frac{3}{4(g_A^2 + g_B^2)d}\left[(|\mu_\Phi|^2 + \overline{m}^2 - d)^2 - (|\mu_\Phi|^2 + m^2)^2\right], \tag{3.13}$$

where

$$d = \sqrt{(2|\mu_\Phi|^2 + m^2 + \overline{m}^2)^2 - 4|b_\phi|^2}. \tag{3.14}$$

To satisfy the necessary conditions that d and $|s|^2$ and $|\overline{s}|^2$ are real and positive, $|b_\phi|$ must satisfy:

$$(|\mu_\Phi|^2 + m^2)(|\mu_\Phi|^2 + \overline{m}^2) \;<\; |b_\phi|^2 \;<\; (|\mu_\Phi|^2 + m^2)(|\mu_\Phi|^2 + \overline{m}^2) + \frac{1}{4}(m^2 - \overline{m}^2)^2, \tag{3.15}$$

where the right inequality coincides with the no-UFB condition eq. (3.10), and the left inequality coincides with the destabilization of the trivial vacuum with $s = \overline{s} = 0$. In practice, this is usually a very narrow range of allowed $|b_\phi|$; in particular, it vanishes in the limit $m^2 = \overline{m}^2$. Also, while the condition eq. (3.15) is necessary and sufficient for a non-trivial minimum of $V(s,\overline{s})$, it is far from sufficient to guarantee that eqs. (3.12)-(3.14) provide a local minimum of the whole potential (not restricted to the $s, \overline{s}$ subspace). The sufficient conditions follow from also requiring the positivity of the $36 - 9 = 27$ non-Goldstone squared mass eigenvalues, of which 8 are distinct. These depend on the other parameters in a more complicated way, and can be evaluated on a case-by-case basis.

Now consider the D-flat direction defined by $\overline{s} = s = 0$ and non-zero $v, \overline{v}$, which gives $SU(3)_A \times SU(3)_B \to SU(3)_C$ as desired. For simplicity, consider first a special case that has $\Phi \leftrightarrow \overline{\Phi}$ and CP symmetries, where $\overline{y} = y$ is real and positive by convention, μ_Φ is chosen to be real and positive, b and $\overline{a} = a$ are chosen to be real but not necessarily positive, and[†] $\overline{m}^2 = m^2$, which must be real (by the reality of the Lagrangian) but not necessarily positive. For convenience, define real

[†] This choice precludes the possibility of a $SU(2) \times SU(2) \times U(1)$-preserving minimum, as just discussed.

dimensionless supersymmetry breaking parameters

$$A = a/(y\mu_\Phi), \tag{3.16}$$
$$B = b/\mu_\Phi^2, \tag{3.17}$$
$$C = m^2/\mu_\Phi^2, \tag{3.18}$$

in terms of which eq. (3.10) becomes the requirement

$$|B| < 1 + C \tag{3.19}$$

to avoid an UFB runaway solution. Then one can look for minima

$$v = (\mu_\Phi/y) x e^{i\alpha}, \qquad \overline{v} = (\mu_\Phi/y) x e^{i\beta}, \tag{3.20}$$

where x is real, non-negative, and dimensionless, and α and β are phases. By examining the first derivatives of the potential, one finds that a minimum with $x \neq 0$ that satisfies eq. (3.19) must have $\beta = \alpha$. The potential then becomes simply

$$V(x,\alpha) = \frac{6\mu_\Phi^4}{y^2} x^2 \left(x^2 + \left[\left(\frac{8}{3}\cos^2\alpha - 2 \right) A - 2 \right] x \cos\alpha + B\left(1 - 2\cos^2\alpha\right) + C + 1 \right). \tag{3.21}$$

Minimizing the restricted potential $V(x,\alpha)$ gives a necessary condition, but one must also check using the full scalar potential that at any putative local minimum, all of the 36 real scalar squared masses are non-negative, including an octet of vanishing scalar squared masses for the would-be Goldstone bosons.

For $A = B = C = 0$, one recovers the supersymmetric limit of the previous section, with a minimum at $x = 1$, $\alpha = 0$. More generally, the supersymmetric part of the scalar potential clearly favors $\alpha = 0$ when $x \neq 0$. The A term also favors $\alpha = 0$ for large negative A. However, for large positive A, the A term part favors symmetry breaking with $\cos^2\alpha = 1/2$. The B term favors $\alpha = 0$ if $B < 0$, but $\alpha = \pi$ if $B > 0$. The tension between these contributions means that even though all potential parameters were chosen to be real, the VEV can be forced to be complex at a local minimum if A is positive and sufficiently large. There are thus two types of possible local symmetry breaking minima, which from now on are parameterized by $x e^{i\alpha} = x_R + i x_I$. Without loss of generality, one can take x_I to be non-negative.

For the first type, the VEV is real and satisfies the stationary condition

$$2x_R^2 + (A-3)x_R + 1 + C - B = 0, \tag{3.22}$$

leading to

$$x_R = \frac{3-A}{4}\left(1 + \sqrt{1 + 8(B - C - 1)/(3-A)^2}\right), \tag{3.23}$$
$$x_I = 0, \tag{3.24}$$

For this to be a local minimum, it is necessary but not sufficient that the argument of the square root is positive:

$$(3-A)^2 \;>\; 8(1-B+C). \tag{3.25}$$

From requiring positivity of the $36-8=28$ non-Goldstone scalar squared masses, one finds the other necessary conditions:

$$(1+nx_R)^2 + C \;>\; |B + nx_R(A+x_R-1)| \tag{3.26}$$

to be imposed for each of $n = 1, 2, -2$, and

$$(g_A^2+g_B^2)x_R^2 + (1-x_R)^2 \;>\; -C. \tag{3.27}$$

Together, the five conditions (3.25)-(3.27) are sufficient to guarantee the existence of this local minimum. The constraint (3.27) is the only one that depends on the gauge couplings, and it rarely comes into play; it is automatic unless $C < 0$, and even then it is always satisfied for sufficiently large gauge couplings. If the no-UFB condition eq. (3.19) is also imposed, then the three conditions of eq. (3.26) can be simplified to:

$$2x_R + B \;>\; 0, \tag{3.28}$$

$$(1-3A)x_R + 2B \;>\; 0, \tag{3.29}$$

$$(11-A)x_R - 2 + 4B - 2C \;>\; 0. \tag{3.30}$$

For eq. (3.23) to be the global minimum, it is necessary but not sufficient (because of the possibility of the second type of solution below) that eq. (3.19) is also satisfied as well as $V \leq 0$, which yields

$$(3-A)^2 \;\geq\; 9(1-B+C), \tag{3.31}$$

which is slightly stronger than eq. (3.25).

The second type of local minimum has a complex VEV, with stationary conditions

$$x_I^2 + x_R^2 - (1+A)x_R + (1+B+C)/2 \;=\; 0, \tag{3.32}$$

$$4Ax_R^2 - [(1+A)^2+2B]x_R + (1+A)(1+B+C)/2 \;=\; 0, \tag{3.33}$$

leading to

$$x_R \;=\; \frac{(1+A)^2+2B}{8A}\left[1+\sqrt{1-8A(1+A)(1+B+C)/[(1+A)^2+2B]^2}\right], \tag{3.34}$$

$$x_I \;=\; \sqrt{x_R(1+A-x_R) - (1+B+C)/2}. \tag{3.35}$$

As necessary but not sufficient requirements, both square roots must have positive argument, so

$$[(1+A)^2+2B]^2 \;>\; 8A(1+A)(1+B+C), \tag{3.36}$$
$$2x_R(1+A-x_R) \;>\; 1+B+C, \tag{3.37}$$

The remaining necessary conditions, coming from positivity of the four distinct non-Goldstone scalar boson squared mass eigenvalues, are:

$$(1+nx_R)^2+n^2x_I^2+C \;>\; \sqrt{[B-nx_I^2+nx_R(A+x_R-1)]^2+n^2x_I^2(1+A-2x_R)^2}, \tag{3.38}$$

again for each of $n = 1, 2, -2$, and

$$(g_A^2+g_B^2)(x_I^2+x_R^2)+(1-x_R)^2+x_I^2 \;>\; -C. \tag{3.39}$$

Together, the six conditions eqs. (3.36)-(3.39) are sufficient to guarantee the existence of this local minimum. Again, eq. (3.39) can only come into play if $C < 0$, and even then it is automatically satisfied if the gauge couplings are sufficiently large. For a local minimum of this type to be the global minimum, it is necessary but not sufficient (because of the possibility of the first type of solution described above) that eq. (3.19) is satisfied as well as $V \leq 0$, a constraint that can be written as

$$(x_R^2+x_I^2)^2+(2B-8Ax_R/3)x_R^2 \;\geq\; 0. \tag{3.40}$$

The implications of the preceding results are illustrated in Figure 3.1, which shows a phase diagram for symmetry breaking in the $B = b_\phi/\mu_\Phi^2$ vs. $A = a/y\mu_\Phi = \overline{a}/\overline{y}\mu_\Phi$ plane, for the choices $C = m^2/\mu_\Phi^2 = \overline{m}^2/\mu_\Phi^2 = 0$ (left panel) and 0.5 (right panel). As noted above, there can be no $SU(2)\times SU(2)\times U(1)$-preserving vacuum here, because of the choice $m^2 = \overline{m}^2$. The red shaded regions on the left and right sides of each plot have UFB runaway solutions because $|B|$ is too large. In the central unshaded regions, there are no symmetry breaking local minima. The green region shows the points where the global minimum of the potential breaks $SU(3)_A \times SU(3)_B \to SU(3)_C$, and the blue region shows where there is at least one such local minimum with no UFB runaway. These are the regions that could be our world. At $A = B = C = 0$, the supersymmetric limit is realized, so that this point is on the border between the local and global minimum regions in the left panel. A dotted curve separates the region where the lowest symmetry breaking local minimum has a real VEV from the region where it has a complex VEV (which occurs for A positive and not too small), given our choice of all real input parameters.

In view of the rather complicated set of requirements given above even in the simplifying case of assumed real parameters with a $\Phi \leftrightarrow \overline{\Phi}$ symmetry in the Lagrangian, I have not attempted to characterize the necessary and sufficient conditions in the general case. However, using numerical methods I have checked that in generic cases, for large areas in a general parameter space, there are global minima that realize the $SU(3)_A \times SU(3)_B \to SU(3)_C$ breaking. For example, Figure 3.2 shows phase diagrams for the case that there is no symmetry between Φ and $\overline{\Phi}$, for $\overline{y} = 0.5y$

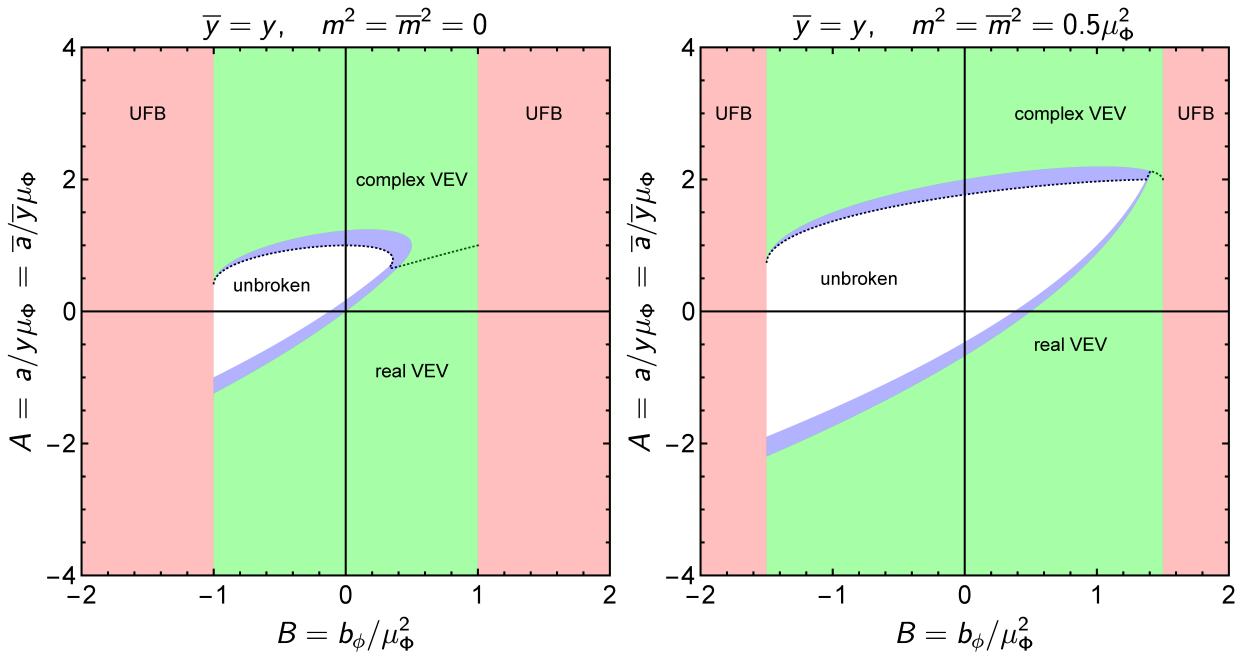

FIG. 3.1: Phase diagrams for symmetry breaking in the case $\overline{y} = y$ and $\overline{a} = a$, for $m^2 = \overline{m}^2 = 0$ (left panel) and $m^2 = \overline{m}^2 = 0.5\mu_\Phi^2$ (right panel), with the input parameters μ_Φ and y real and positive, and a, b_ϕ chosen real, so that the discussion of eqs. (3.19)-(3.40) applies to the minimization of the scalar potential. In the red shaded regions on the left and right sides of each plot, the scalar potential has an unbounded from below direction. In the unshaded central region, the $SU(3)_A \times SU(3)_B$ gauge symmetry is not broken at any local minimum of the potential. The symmetry breaking $SU(3)_A \times SU(3)_B \to SU(3)_C$ occurs at a global minimum of the potential in the large green shaded regions, and at only a local minimum in the thin blue shaded regions. The supersymmetric limit occurs at the origin $(b_\phi, a) = (0, 0)$ in the left panel; there the local symmetry breaking minimum is degenerate with the local non-symmetry breaking minimum. In each panel, the lowest $SU(3)_C$-symmetric minimum has complex $v = \overline{v}$ above the dotted curve.

real and positive and with $m^2 = 0$, $\overline{m}^2 = 0.5\mu_\Phi^2$ (left panel) and with $m^2 = 0.25\mu_\Phi^2$, $\overline{m}^2 = \mu_\Phi^2$ (right panel). The axes of the plots are b_ϕ/μ_Φ^2 and $a/y\mu_\Phi = \overline{a}/\overline{y}\mu_\Phi$, which are assumed to be real but can have either sign. In this example, because $m^2 \neq \overline{m}^2$, there are very small regions where minima with unbroken gauge group $SU(2) \times SU(2) \times U(1)$ can exist, depending on the other parameters. From eq. (3.15), these occur within the narrow ranges adjacent to the UFB region, $1.2247 < |B| < 1.25$ (left panel) and $1.5811 < |B| < 1.625$ (right panel). The exact extents of these small regions depend on other parameters besides the plot axes, so they are not shown. I have also checked in other examples that global minima with unbroken gauge group $SU(3)_C$ do occur in large regions of generic parameter space, including where μ_Φ and the soft input parameters are allowed to have complex phases, and that smaller residual gauge symmetries like $SU(2) \times U(1)$ generally do not occur.

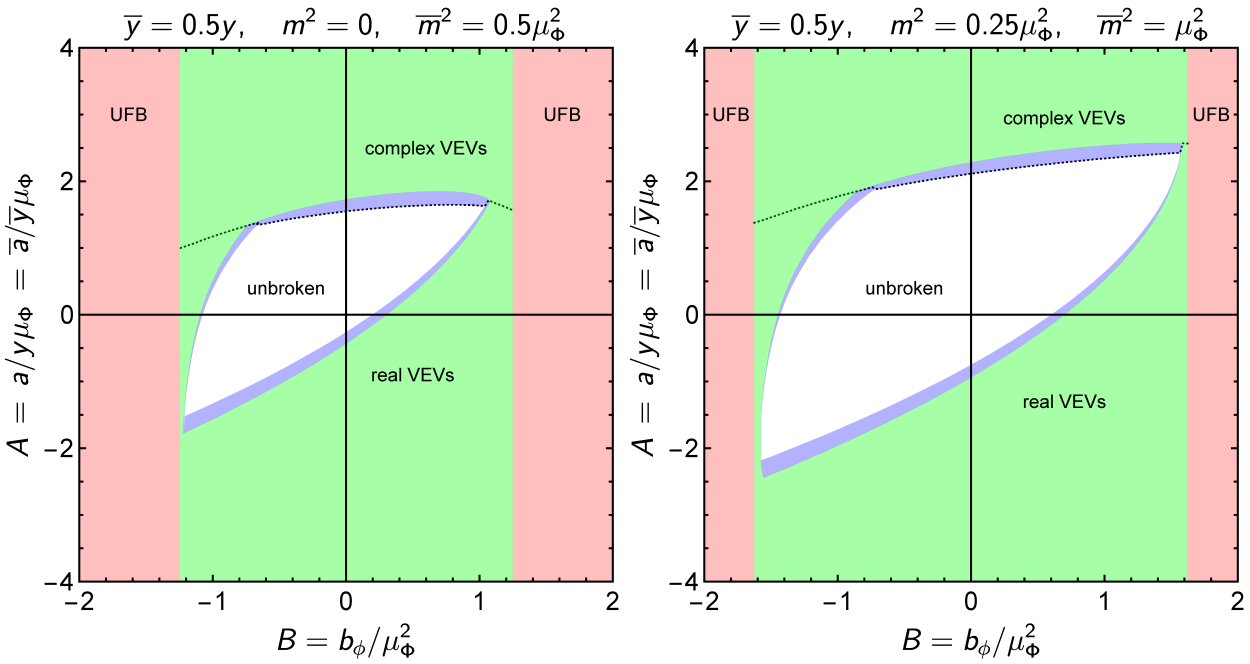

FIG. 3.2: Phase diagrams for symmetry breaking in the case $\overline{y} = 0.5y$ real and positive, with $m^2 = 0$, $\overline{m}^2 = 0.5\mu_\Phi^2$ (left panel) and with $m^2 = 0.25\mu_\Phi^2$, $\overline{m}^2 = \mu_\Phi^2$ (right panel). The axes are $B = b_\phi/\mu_\Phi^2$ and $A = a/y\mu_\Phi = \overline{a}/\overline{y}\mu_\Phi$, which are assumed to be real but can have either sign. In the red shaded regions on the left and right sides of each plot, the scalar potential has an unbounded from below direction. In the unshaded central region, the $SU(3)_A \times SU(3)_B$ gauge symmetry is not broken at any local minimum of the potential. The symmetry breaking $SU(3)_A \times SU(3)_B \to SU(3)_C$ occurs at a global minimum of the potential in the large green shaded regions, and at only a local minimum in the thinner blue shaded regions. In each panel, the lowest $SU(3)_C$-symmetric minimum has complex VEVs v and $\overline{v}$ above the dotted curve. Minima with unbroken gauge group $SU(2) \times SU(2) \times U(1)$ can also occur, but only in very small regions that are subsets of thin strips adjacent to the UFB region, namely $1.2247 < |B| < 1.25$ (left panel) and $1.5811 < |B| < 1.625$ (right panel). These small regions are not shown because they depend on the other parameters.

IV. MODEL REALIZATION WITH GAUGE COUPLING UNIFICATION

A. Renormalization group running

One aspect of low-energy supersymmetry that has often been touted as an attractive feature is the apparent unification of gauge couplings above 10^{16} GeV. In the case that $SU(3)_C$ is the remnant of two independent $SU(3)$ gauge groups, this is certainly no longer automatic (but as we will see it can at least be accommodated). Furthermore, given the Standard Model value of α_S, the formula eq. (1.15) implies that both g_A and g_B must be fairly strong at the multi-TeV scale, since g_3 is necessarily smaller than both of them, and if they were equal, $g_3 \approx g_A/\sqrt{2}$. Assuming that the MSSM quark supermultiplets live in the $SU(3)_A$ representation, then the presence of 6 additional triplets Φ and $\overline{\Phi}$ means that the 1-loop β function for $SU(3)_A$ must be non-negative, and the 2-loop β function is positive, so that g_A cannot be asymptotically free as in the MSSM. To unify with g_B, additional fields charged under $SU(3)_B$ must be included.

There are many ways to include chiral superfield representations that are charged under $SU(3)_B$. Suppose that there are additional vectorlike quark and lepton supermultiplets in representations

(and their conjugates) as in the MSSM, but with color charges only under $SU(3)_B$. This will ensure that the new fields are not exotic and none of them need be stable, since they can decay by mixing with MSSM states. In particular, consider possible chiral supermultiplets in the following vectorlike representations of $SU(3)_A \times SU(3)_B \times SU(2)_L \times U(1)_Y$:

$$n_Q \times \Big[(\mathbf{1},\mathbf{3},\mathbf{2},\frac{1}{6}) + (\mathbf{1},\overline{\mathbf{3}},\mathbf{2},-\frac{1}{6})\Big], \tag{4.1}$$

$$n_d \times \Big[(\mathbf{1},\mathbf{3},\mathbf{1},-\frac{1}{3}) + (\mathbf{1},\overline{\mathbf{3}},\mathbf{1},\frac{1}{3})\Big], \tag{4.2}$$

$$n_u \times \Big[(\mathbf{1},\mathbf{3},\mathbf{1},\frac{2}{3}) + (\mathbf{1},\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})\Big], \tag{4.3}$$

$$n_L \times \Big[(\mathbf{1},\mathbf{1},\mathbf{2},-\frac{1}{2}) + (\mathbf{1},\mathbf{1},\mathbf{2},\frac{1}{2})\Big], \tag{4.4}$$

$$n_e \times \Big[(\mathbf{1},\mathbf{1},\mathbf{1},-1) + (\mathbf{1},\mathbf{1},\mathbf{1},1)\Big], \tag{4.5}$$

for integers n_Q, n_d, n_u, n_L, and n_e. These fields are supposed to have weak isosinglet bare masses in the multi-TeV range, due to whatever mechanism also provides for the MSSM μ term. They can also mix with the MSSM quarks and leptons, in the case of quarks through Yukawa couplings to Φ and $\overline{\Phi}$. That mixing is assumed here to be too small to affect anything else significantly. In the following, beta functions will be denoted in the general loop expansion form

$$\beta_X = \sum_{n\geq 1} \frac{1}{(16\pi^2)^n} \beta_X^{(n)}. \tag{4.6}$$

Then at 2-loop order,[†] the gauge couplings in a Grand Unified Theory (GUT) normalization have beta functions:

$$\beta_{g_A}^{(1)} = 0, \tag{4.7}$$

$$\beta_{g_A}^{(2)} = g_A^3\Big(48g_A^2 + 16g_B^2 + 9g_2^2 + \frac{11}{5}g_1^2 - 6y^2 - 6\overline{y}^2 - 4y_t^2 - 4y_b^2\Big), \tag{4.8}$$

$$\beta_{g_B}^{(1)} = g_B^3(-6 + 2n_Q + n_u + n_d), \tag{4.9}$$

$$\beta_{g_B}^{(2)} = g_B^3\Big([-20 + \frac{34}{3}(2n_Q + n_d + n_u)]g_B^2 + 16g_A^2 + 6n_Q g_2^2 + \frac{2}{15}[n_Q + 2n_d + 8n_u]g_1^2 - 6y^2 - 6\overline{y}^2\Big), \tag{4.10}$$

$$\beta_{g_2}^{(1)} = g_2^3\,(1 + 3n_Q + n_L)\,, \tag{4.11}$$

$$\beta_{g_2}^{(2)} = g_2^3\Big(24g_A^2 + 16n_Q g_B^2 + [25 + 21n_Q + 7n_L]g_2^2 + \frac{1}{5}[9 + n_Q + 3n_L]g_1^2 - 6y_t^2 - 6y_b^2 - 2y_\tau^2\Big), \tag{4.12}$$

$$\beta_{g_1}^{(1)} = \frac{g_1^3}{5}\,(33 + n_Q + 2n_d + 8n_u + 3n_L + 6n_3)\,, \tag{4.13}$$

[†] In all numerical results below, the full 3-loop beta functions are used to run all supersymmetric parameters and the 2-loop results are used for soft parameters. These can be straightforwardly obtained from the general expressions in refs. [46–60], so only the partial 2-loop or 1-loop formulas are shown here for illustration.

$$\beta_{g_1}^{(2)} = \frac{g_1^3}{5}\Big(88g_A^2 + \frac{16}{3}[n_Q + 2n_d + 8n_u]g_B^2 + [27 + 3n_Q + 9n_L]g_2^2 + \frac{g_1^2}{15}[597 + n_Q + 8n_d + 128n_u + 27n_L + 216n_e] - 26y_t^2 - 14y_b^2 - 18y_\tau^2\Big). \quad (4.14)$$

The $SU(3)_A$ coupling does not run in the 1-loop approximation, but this is an accident, violated by 2-loop effects. The Yukawa couplings y and $\overline{y}$ will be assumed not to be small in the following, and so their running is important, and given by:

$$\beta_y^{(1)} = y(6y^2 - 8g_A^2 - 8g_B^2), \quad (4.15)$$

$$\beta_y^{(2)} = 8y\Big(\frac{8}{3}g_A^4 + \frac{16}{3}g_A^2 g_B^2 + \left[2n_Q + n_d + n_u - \frac{10}{3}\right] g_B^4 + 2(g_A^2 + g_B^2)y^2 - 3y^4\Big), \quad (4.16)$$

with the same equations for $y \to \overline{y}$. The beta functions for the top-quark, bottom-quark, and tau-lepton Yukawa couplings are obtained from the MSSM results with the replacement $g_3 \to g_A$.

There are several choices for the integers n_Q, n_d, n_u, n_L, and n_e that can lead to approximate gauge coupling unification. In the following, I will simply choose one that seems interesting, with no claim or expectation of uniqueness:

$$n_Q = 1, \qquad n_d = 3, \qquad n_u = 0, \qquad n_L = 0, \qquad n_e = 1. \quad (4.17)$$

It is also possible, for example, to include a chiral supermultiplet which would transform as an octet under $SU(3)_B$; this would also lead to three new possible Yukawa couplings. One reason for the choice made here is that one can arrange for gauge coupling unification at high scales while having $g_B > g_A$ at the symmetry breaking scale, with g_3 consistent with the Standard Model QCD coupling.

Since $\beta_{g_A}^{(1)} = 0$ and $\beta_{g_B}^{(1)} = -g_B^2$ are both accidentally small in magnitude due to the choice of chiral superfield representations, and $\beta_{g_A}^{(2)}$ and $\beta_{g_B}^{(2)}$ both have large positive contributions, the RG running can have a character similar to the Caswell-Banks-Zaks infrared fixed point [61, 62], although here the conformal regime is not actually reached. In the following, I consider a case that realizes approximate gauge coupling unification through y and $\overline{y}$ that are large at the TeV scale. This is natural in the sense that the negative contributions proportional to g_A^2 and g_B^2 in eq. (4.15) will drive y and $\overline{y}$ to be larger in the infrared, since the $SU(3)$ gauge couplings are necessarily large. However, when y and $\overline{y}$ themselves become sufficiently large, the terms proportional to y^2 in eq. (4.15) and proportional to $\overline{y}^2$ in its counterpart for $\overline{y}$ will put the brakes on, leading to a quasi-fixed point behavior. (This is not a true fixed point, because g_A and g_B are still running, and thus provide a moving target.)

As an illustration, Figure 4.1 shows a sample 3-loop RG trajectory, starting with an assumption that at the low-energy threshold scale $Q = 7.5$ TeV, where they are taken to match onto the Standard Model,

$$g_B/g_A = 1.5, \qquad g_3 = 0.96171, \quad (4.18)$$

$$g_2 = 0.628645, \qquad g_1 = 0.366436, \quad (4.19)$$

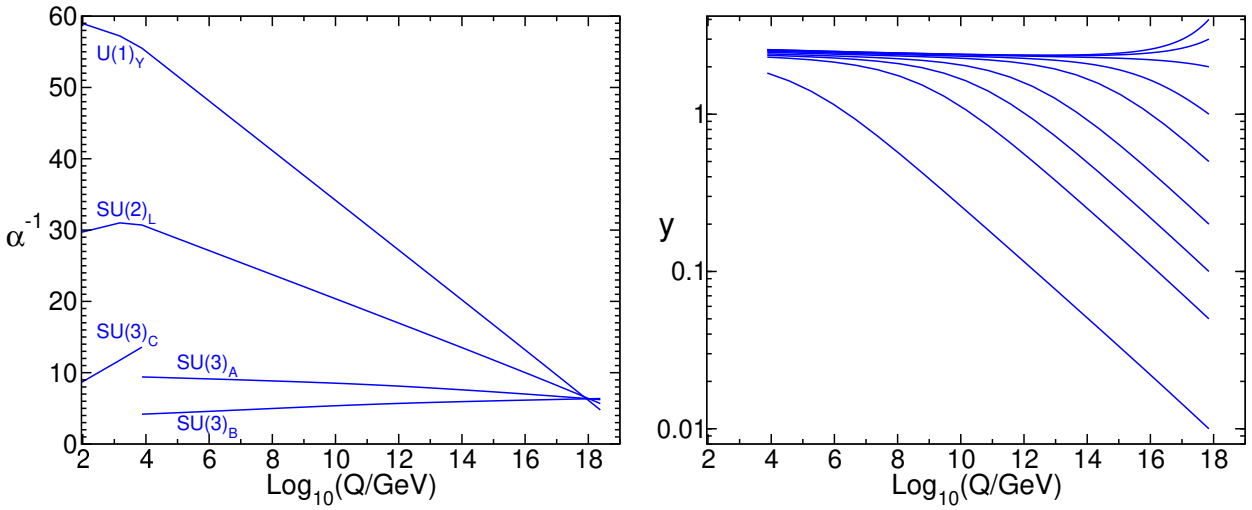

FIG. 4.1: Three-loop renormalization group running of supersymmetric couplings for the example model defined by eqs. (4.18)-(4.21) and (4.37)-(4.41), as a function of the renormalization scale Q. The inverses of the gauge couplings $\alpha_a = g_a^2/4\pi$ are shown in the left panel. The right panel shows a variety of renormalization group trajectories for the Yukawa coupling $y = \overline{y}$, obtained by taking different boundary conditions at the unification scale, illustrating the strongly attractive infrared quasi-fixed point behavior, with power-law–like running for small y due to the influence of large g_A and especially g_B.

$$y = \overline{y} = 2.38, \tag{4.20}$$

$$y_t = 0.783363, \qquad y_b = 0.012305, \qquad y_\tau = 0.010205, \tag{4.21}$$

with the Standard Model Yukawa couplings chosen to correspond to $\tan\beta = 10$. In the left panel, the gauge couplings are seen to nearly unify at a scale 7.1×10^{17} GeV, much closer to the reduced Planck scale than the unification scale found in the MSSM. The $SU(3)_B$ coupling increases in strength in the infrared, but does not hit a pole, with $\alpha_B = 0.239$ at $Q = 7.5$ TeV; this is comparable to α_S evaluated at 3.5 GeV in the Standard Model. The chosen value of $y = \overline{y}$ is near the 3-loop quasi-fixed point value for the RG equation system. This is illustrated in the right panel of Figure 4.1, which shows the running for a variety of different input values. Note that even if y and $\overline{y}$ start at much lower values (say, of order 0.1) at the apparent unification scale, they are efficiently driven with power-law-like running in the infrared to the quasi-fixed point regime due to the influence of the large, and slowly running, gauge couplings g_A and especially g_B.

The beta functions for dimensionful parameters can also be obtained from the general results in refs. [46–60]. For the supersymmetric parameter μ_Φ, one has

$$\beta^{(1)}_{\mu_\Phi} = \mu_\Phi \left[2y^2 + 2\overline{y}^2 - \frac{16}{3}(g_A^2 + g_B^2) \right], \tag{4.22}$$

$$\beta^{(2)}_{\mu_\Phi} = \mu_\Phi \Big[\frac{128}{9} g_A^4 + \frac{256}{9} g_A^2 g_B^2 + \Big(-\frac{160}{9} + \frac{16}{3}[2n_Q + n_u + n_d] \Big) g_B^4 + \frac{16}{3}(g_A^2 + g_B^2)(y^2 + \overline{y}^2) - 8y^4 - 8\overline{y}^4 \Big]. \tag{4.23}$$

As long as y and $\overline{y}$ are small, this provides for μ_Φ to grow rapidly in the infrared, but this running

slows as y and $\overline{y}$ approach the quasi-fixed point regime. This makes it plausible that if μ_Φ has an origin similar to that of the MSSM μ parameter, that μ_Φ should be larger than μ at the low scale. In any case, it is technically natural for it to have any value; in the following it is assumed to be of the same order of magnitude as the soft supersymmetry breaking masses, as could follow for example from the Kim-Nilles [63] or Giudice-Masiero [64] mechanisms. For the gaugino masses:

$$\beta^{(1)}_{M_A} = 0, \tag{4.24}$$

$$\beta^{(2)}_{M_A} = g_A^2\Big([192g_A^2 - 12y^2 - 12\overline{y}^2 - 8y_t^2 - 8y_b^2]M_A + 32g_B^2[M_A + M_B] + 18g_2^2[M_A + M_2] + \frac{22}{5}g_1^2[M_A + M_1] + 12ay + 12\overline{ay} + 16a_t y_t + 16a_b y_b\Big), \tag{4.25}$$

$$\beta^{(1)}_{M_B} = (-12 + 4n_Q + 2n_d + 2n_u)\, g_B^2 M_B, \tag{4.26}$$

$$\beta^{(2)}_{M_B} = g_B^2\Big(\{-80 + 136(2n_Q + n_d + n_u)/3]g_B^2 - 12y^2 - 12\overline{y}^2\}M_B + 32g_A^2[M_A + M_B] + 12n_Q g_2^2[M_2 + M_B] + \frac{4}{15}(n_Q + 2n_d + 8n_u)g_1^2[M_1 + M_B] + 12ay + 12\overline{ay}\Big), \tag{4.27}$$

$$\beta^{(1)}_{M_2} = (2 + 6n_Q + 2n_L)g_2^2 M_2, \tag{4.28}$$

$$\beta^{(2)}_{M_2} = g_2^2\Big([(100 + 84n_Q + 28n_L)g_2^2 - 12y_t^2 - 12y_b^2 - 4y_\tau^2]M_2 + 48g_A^2[M_2 + M_A] + 32n_Q g_B^2[M_2 + M_B] + \frac{2}{5}(9 + n_Q + 3n_L)g_1^2[M_2 + M_1] + 24a_t y_t + 24a_b y_b + 8a_\tau y_\tau\Big), \tag{4.29}$$

$$\beta^{(1)}_{M_1} = \frac{2g_1^3}{5}\,(33 + n_Q + 2n_d + 8n_u + 3n_L + 6n_3)\, M_1, \tag{4.30}$$

$$\beta^{(2)}_{M_1} = \frac{g_1^2}{5}\Big(\Big[(\frac{796}{5} + \frac{4}{15}n_Q + \frac{32}{15}n_d + \frac{512}{15}n_u + \frac{36}{5}n_L + \frac{288}{5}n_e)g_1^2 - 52y_t^2 - 28y_b^2 - 36y_\tau^2\Big]M_1 + 176g_A^2[M_1 + M_A] + \frac{32}{3}g_B^2(n_Q + 2n_d + 8n_u)[M_1 + M_B] + (54 + 6n_Q + 18n_L)g_2^2[M_1 + M_2] + 104a_t y_t + 56a_b y_b + 72a_\tau y_\tau\Big), \tag{4.31}$$

and for the soft supersymmetry breaking parameters associated with the $\Phi, \overline{\Phi}$ sector:

$$\beta^{(1)}_a = 18y^2 a + 8g_A^2(2yM_A - a) + 8g_B^2(2yM_B - a), \tag{4.32}$$

$$\beta^{(1)}_{\overline{a}} = 18\overline{y}^2\overline{a} + 8g_A^2(2\overline{y}M_A - \overline{a}) + 8g_B^2(2\overline{y}M_B - \overline{a}), \tag{4.33}$$

$$\beta^{(1)}_{b_\phi} = 2b_\phi\Big[y^2 + \overline{y}^2 - \frac{8}{3}(g_A^2 + g_B^2)\Big] + 4\mu_\Phi\Big[ay + \overline{a}\,\overline{y} + \frac{8}{3}(g_A^2 M_A + g_B^2 M_B)\Big], \tag{4.34}$$

$$\beta^{(1)}_{m^2} = 12y^2 m^2 + 4|a|^2 - \frac{32}{3}(g_A^2|M_A|^2 + g_B^2|M_B|^2), \tag{4.35}$$

$$\beta^{(1)}_{\overline{m}^2} = 12\overline{y}^2\overline{m}^2 + 4|\overline{a}|^2 - \frac{32}{3}(g_A^2|M_A|^2 + g_B^2|M_B|^2). \tag{4.36}$$

A consequence of these results is that if the gaugino masses are taken to be positive and large, then a/y, $\overline{a}/\overline{y}$, and b/μ_Φ tend to run to negative values in the conventions used here.

The special case that will be adopted as an example here is the "no-scale" limit, which presumes that at very high scales the supersymmetry breaking is dominated by gaugino masses, with all other soft supersymmetry breaking parameters arising from them due to renormalization group running. A nice feature of this limit is that it automatically provides for nearly flavor-blind first and second

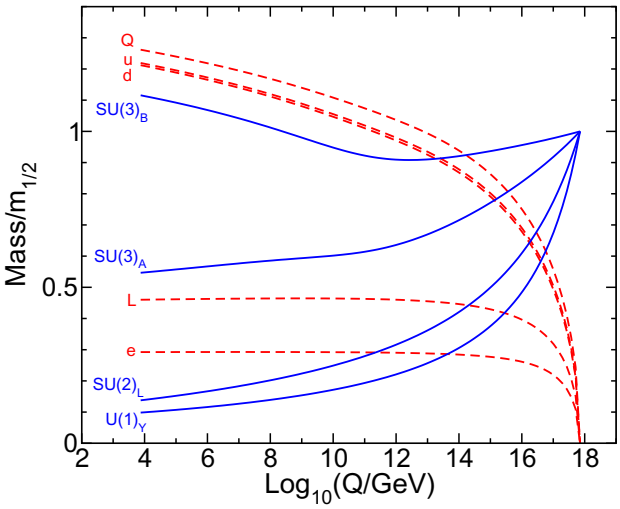

FIG. 4.2: Renormalization group running gaugino masses (solid lines) and MSSM squark and slepton masses (dashed lines), as a function of the renormalization scale Q, for the example model defined by eqs. (4.18)-(4.21) and (4.37)-(4.41), with vanishing scalar masses and unified gaugino masses $m_{1/2}$ at the high scale.

family squark and slepton masses, due to the observed fact that the corresponding Yukawa couplings are small. As an illustration, Figure 4.2 shows the running of the gaugino masses and the MSSM squark and slepton masses as a function of the renormalization scale Q, for the case of a common gaugino mass $m_{1/2}$ at the apparent unification scale. Note that all of the sleptons are significantly heavier than the wino and bino, in contrast to the no-scale limit of the usual MSSM. This is due to the couplings g_1 and g_2 being much larger at high RG scales than is the case in the MSSM. It is also worth noting that the 1-loop approximation is not very good, notably for M_A, which has an accidentally vanishing beta function at 1-loop order, but is seen to decrease significantly in the infrared due to 2-loop and higher order effects. The squark masses are larger than both M_A and M_B, again in contrast to the no-scale limit in the MSSM. Of course, these expectations could easily be modified if the high-scale boundary conditions are different, for example due to non-universal gaugino masses.

B. Mass spectrum for an example model line

As an illustration of the possibilities for masses in the $SU(3)_A \times SU(3)_B$ gauge/gaugino and Φ, $\overline{\Phi}$ sector, consider an example model defined by the parameters of eqs. (4.18)-(4.21) and the results following from renormalization group evolution as described in the previous subsection starting with a universal gaugino mass parameter $m_{1/2}$:

$$a = \overline{a} = -2.381\, m_{1/2}, \tag{4.37}$$

$$b_\phi = -0.6669\, m_{1/2}\mu_\Phi, \tag{4.38}$$

$$m^2 = \overline{m}^2 = (0.30806\, m_{1/2})^2, \tag{4.39}$$

$$M_A = 0.5467\, m_{1/2}, \tag{4.40}$$

$$M_B = 1.1156\, m_{1/2}, \tag{4.41}$$

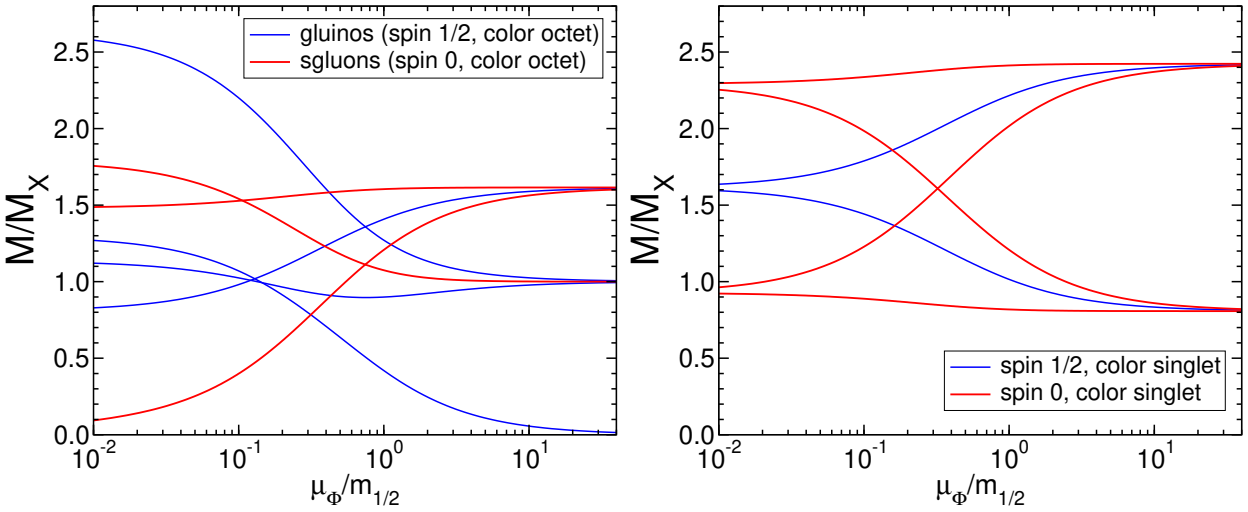

FIG. 4.3: The ratios of fermion and scalar masses to the mass of the color octet vector boson M_X, as a function of $\mu_\Phi/m_{1/2}$, for the example model line defined by eqs. (4.18)-(4.21) and (4.37)-(4.41). The left panel shows the masses of the gluinos (octet fermions) and sgluons (octet scalars). The right panel shows the masses of the color-singlet scalars and fermions from the Φ and $\overline{\Phi}$ multiplets. The right side of each plot approaches the supersymmetric limit, with masses as discussed in subsection III A with $R = 2$.

where μ_Φ is the value at the low renormalization scale. Although these values were obtained at $Q = 7.5$ TeV, I will not commit to a particular overall mass scale for the superpartners or the new states in the results shown below, but instead show mass ratios normalized to the octet vector boson mass.

The potential minimization is then found to be of the type with a real VEV given by eq. (3.23), with $v = \overline{v} = x_R$, where x_R then depends on the ratio $r = \mu_\Phi/m_{1/2}$. I vary this ratio to obtain a one-parameter model line. The numerical values of the dimensionless supersymmetry breaking parameters defined in eqs. (3.16)-(3.18) are $A = -1.00034/r$, $B = -0.6669/r$, $C = 0.0949/r^2$. These obey each of the constraints in eqs. (3.25)-(3.31) for all r, and therefore yield a global minimum of the potential at which the breaking $SU(3)_A \times SU(3)_B \to SU(3)_C$ occurs, except for the range $0.2058 < r < 0.4611$ where it is only a local minimum due to a UFB solution, see eq. (3.19). Even in that range of r, the $SU(3)_C$-preserving vacuum is separated from the UFB by a barrier, making it potentially viable despite the UFB, if the tunneling rate is acceptably small. In any case, that range of r will be included in the following plots, for the sake of continuity. (Note that a slight decrease in $|B|$ would ensure that the whole range of r would be a global minimum for the $SU(3)_C$ preserving vacuum.)

In Figure 4.3, I show the masses of the four gluinos (spin-1/2 color octets), the three sgluons (real spin-0 octets), the two singlinos (spin-1/2 color singlets) and the four spin-0 color singlets, all normalized to the vector (coloron) mass M_X. Note that large $r = \mu_\Phi/m_{1/2}$ corresponds to the supersymmetric limit, in which one gluino is much lighter than the other new states whose masses are then given by eqs. (1.14), and (3.2)-(3.5), with $R = 2$ in the present case, which leads to $M_{\rm singlets} = \mu_\Phi$ and $3\mu_\Phi$ and $M_{\rm octets} = 2\mu_\Phi$, where $\mu_\Phi = yM_X/\sqrt{2(g_A^2 + g_B^2)} \approx 0.808 M_X$. In the opposite limit of small $\mu_\Phi/m_{1/2}$, the lightest of the new particles is the pseudo-scalar sgluon. More generally, everywhere along the model line there is always at least one gluino state and one sgluon state and one singlet scalar with mass below or close to the octet vector boson mass.

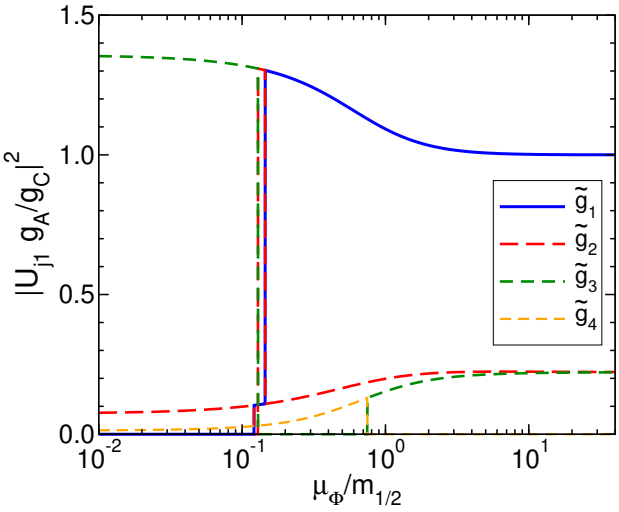

FIG. 4.4: The ratio of the squared magnitude of the coupling of each gluino mass eigenstate $\tilde{g}_j$ to quark/squark pairs to the corresponding MSSM coupling, $|U_{j1}g_A/g_3|^2$, as a function of $\mu_\Phi/m_{1/2}$, for the example model line defined by eqs. (4.18)-(4.21) and (4.37)-(4.41) in the text. The right side of the plot approaches the supersymmetric limit. There is always a gluino mass eigenstate with coupling to quark/squark pairs at least as large as in the MSSM, with the ratio of couplings for the lightest gluino approaching 1 in the supersymmetric limit.

In Figure 4.4, for each of the four gluino mass eigenstates ($\tilde{g}_j$, $j = 1, 2, 3, 4$, in increasing order of mass) I show the square of the ratio of the coupling to MSSM squark/quark pairs to the corresponding coupling that occurs in the MSSM. This is given by $|U_{j1}g_A/g_3|^2$, in terms of the unitary matrix U defined in eq. (2.10) and the gauge couplings g_A and g_3, governed by eq. (1.15). The result is that there is always a gluino mass eigenstate with coupling to quark/squark pairs at least as large as in the MSSM, with the ratio of couplings for the lightest gluino approaching 1 in the supersymmetric limit. However, if $\mu_\Phi/m_{1/2}$ is small, then the lightest gluino mass eigenstate is not MSSM-gaugino-like and has essentially no tree-level coupling to quark-squark pairs. The second lightest gluino state in that regime does couple to quark-squark states, but with a strong suppression. The MSSM-gluino-like state that has enhanced couplings to quark/squark pairs can be up to about 1.6 times heavier than the lightest gluino state, and 1.3 times heavier than the X vector boson. Also, in that case of small $\mu_\Phi/m_{1/2}$, the lightest new state by far is one of the sgluons; it is possible that this would be the first new particle discovered. In contrast, along this model line, none of the singlinos and singlet scalars are ever much lighter than the massive vector boson. For all values of the ratio $\mu_\Phi/m_{1/2}$, a gluino or a sgluon is the lightest of the non-MSSM states. Of course, the above results hold for a very specific set of assumptions about the RG boundary conditions and vectorlike supermultiplet content, but I have checked that they are qualitatively typical at least for a (certainly non-exhaustive) variety of modifications of the above assumptions.

V. COMMENTS ON COLLIDER PHENOMENOLOGY

The collider phenomenology of colorons, Dirac and mixed Majorana/Dirac gluinos, and sgluons has already been the subject of many papers, see refs. [9, 12–18, 21, 22, 65, 66], and [67–73], and [74–79], respectively. A detailed discussion of the LHC phenomenology is beyond the scope of the present paper, but a few brief comments are in order, with emphasis on qualitative issues where the model described above differs from the situation encountered in previous studies based on pure Dirac gluinos from supersoft and and hybrid models with an $N = 2$ gauge sector. In this section,

the particle mass eigenstates beyond those of the MSSM[†] will be denoted:

$$X = \text{color octet massive vector} \tag{5.1}$$

$$\tilde{g}_j = \text{color octet gluinos, } (j = 1, 2, 3, 4) \tag{5.2}$$

$$\tilde{\chi}_j = \text{color singlet fermion singlinos, } (j = 1, 2) \tag{5.3}$$

$$S_j = \text{color octet real scalar sgluons, } (j = 1, 2, 3) \tag{5.4}$$

$$\varphi_j = \text{color singlet real scalars, } (j = 1, 2, 3, 4), \tag{5.5}$$

where the ordering is in increasing mass. In the example model of the previous section, the lightest of these is either $\tilde{g}_1$ or S_1. If R-parity is conserved, then the bosons have even R-parity and the fermions have odd R-parity.

A stringent experimental constraint comes from the fact that the color octet vectors X (colorons) have tree-level couplings to ordinary quarks, and so can be detected in dijet events at hadron colliders. They have a partial width $\Gamma_X = \frac{g_A^4}{24\pi(g_A^2+g_B^2)} M_X$ to each flavor of quark-antiquark pair, but can also in principle have loop-induced decays to gluon pairs. In particular, they can be produced singly as dijet resonances via $q\overline{q} \to X \to q\overline{q}$, resulting in the most recent LHC bound of $M_X > 6.6$ TeV assuming $g_A = g_B$ [80–85]. However, in the context of the present paper the bounds will be somewhat weaker if $g_A < g_B$, as in the example model of the previous section. The experimental limits also assume that the di-jet decays of X dominate. If kinematically allowed, they could also in principle decay to squark-antisquark $\tilde{q}\tilde{q}^*$ or gluino pairs $\tilde{g}_j\tilde{g}_k$ or sgluon pairs S_jS_k. They could even decay to $S_j\varphi_k$ (for a related study see [22]) or $\tilde{\chi}\tilde{g}$, although these are kinematically forbidden throughout most of the example model line of the previous section.

The gluinos $\tilde{g}_j$ will be pair-produced in gluon-gluon and quark-antiquark fusion, as is familiar from standard supersymmetry. Just as in the MSSM, they can always decay to quark-squark final states if kinematically allowed, and in the alternative through virtual squarks to $q\overline{q}\tilde{N}$ or $q\overline{q}'\tilde{C}$ where $\tilde{N}$ and $\tilde{C}$ are ordinary neutralinos and charginos. If kinematically allowed, they can also decay in a variety of 2-body modes at tree-level, to $\tilde{g}_kX$ or $\tilde{\chi}_kX$ or $\tilde{g}_kS$ or $\tilde{\chi}_kS$ or $\tilde{g}_k\varphi_l$ (if $\tilde{g}_k$ is a lighter gluino). The couplings $S\tilde{g}\tilde{g}$ and $S\tilde{g}\tilde{\chi}$ and $\varphi\tilde{g}\tilde{g}$ needed for the last three decays arise from both supersymmetric gauge interactions (scalar-fermion-gaugino) and the $y, \overline{y}$ Yukawa couplings. As in the MSSM, the final states of pair-produced gluino decays will always lead to at least four jets plus missing transverse energy signatures, sometimes with leptons from chargino or neutralino decays, and often with bottom jets from the kinematic enhancement of lighter bottom and top squarks in the cascade decays. As noted in the previous section, one of the gluinos is likely to have an enhanced coupling to quark-squark pairs compared to the MSSM, unlike the case in models with pure or mostly Dirac gluinos. However, the gluino with enhanced couplings may not be the lightest gluino $\tilde{g}_1$. In the example model of the previous section, when $\tilde{g}_1$ is not gaugino-like and has essentially no couplings to quark-squark pairs, it is accompanied by a much lighter sgluon.

The sgluons S can also be pair-produced in gluon-gluon and quark-antiquark fusion, but can also be singly produced due to 1-loop effective couplings. The diagrams leading to an effective Sgg vertex are shown in Figure 5.1. Here, I note a difference compared to the sgluon models previously

[†] One of the $\tilde{g}_j$ corresponds to the MSSM gluino. The vectorlike quarks and leptons introduced for their renormalization group running contributions in subsection IV A will not be discussed; assume they are heavier.

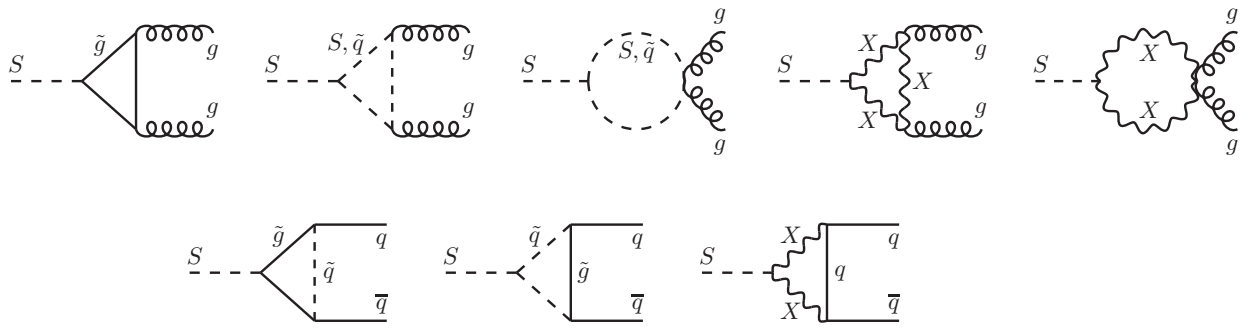

FIG. 5.1: Feynman diagrams leading to effective sgluon-gluon-gluon (top row) and sgluon-quark-antiquark (bottom row) couplings, which provide for single production and two-body decays of sgluons.

analyzed in refs. [74, 75]. In those cases, the gluino-loop contribution to the Sgg vertex (and, more generally, the 1-loop gluino-induced effective couplings of S to any number of gluons) was found to vanish, because the $S_j\tilde{g}_k\tilde{g}_l$ vertex in the 1-loop diagram was proportional to f^{abc}, which then requires $k \neq l$, causing the effective Sgg coupling to vanish, since the gluon couples only diagonally to gluino mass eigenstates due to the unbroken QCD gauge invariance. However, in the model considered in the present paper, there are also $S_j\tilde{g}_k\tilde{g}_k$ couplings proportional to the symmetric factor d^{abc}, both from the gauge couplings g_A and g_B as can be seen from eq. (2.7) and from the Yukawa couplings y and $\overline{y}$ as seen in eq. (2.8). This coupling does not vanish when inserted in the first of the loop diagrams in Figure 5.1, although there is a gluino mixing factor suppression. There is also a contribution from sgluons in the loop, in addition to the ordinary squarks, as seen in the second and third diagrams of Figure 5.1. Another difference from the models analyzed in refs. [74, 75] is that massive vector loops can contribute to the effective Sgg vertex, as shown in the last two diagrams in the first row of eq. 5.1. The SXX vertex appearing here is proportional to d^{abc}; it vanishes for the pseudo-scalar sgluon if CP is conserved. These effects mean that the loop-induced Sgg vertices can be significant, and single production of S due to gluon fusion can be larger than considered previously.

There are also loop-induced contributions to the $Sq\overline{q}$ vertex, as shown in the second row of Figure 5.1, although this effective coupling is helicity-suppressed by the corresponding quark mass, as in [74, 75]. At tree-level the sgluons could also decay to MSSM squark-antisquark pairs if kinematically allowed, through the coupling inherited from the D-term contribution to the scalar potential. Other two-body decays that can occur at tree-level, if kinematically allowed, are $\tilde{g}\tilde{g}$, and $\tilde{\chi}\tilde{g}$, and XS, and $X\varphi$, and φS, and SS.

The lightest sgluon mass eigenstate can therefore be produced in gluon fusion and decay to gg (or to $t\overline{t}$), leading to a dijet signature for which LHC searches [80–85] exist. However, signal/background interference effects can be very large [86] for heavy scalar di-gluon resonances, so that if the di-gluon production and decay dominate, the resonance may manifest as a dip/peak or step-function invariant mass distribution rather than a pure resonance peak. These interference effects have not been included in the experimental limits, which could be quite significantly modified if they were taken into account.

The singlet scalars φ can likewise have loop-induced couplings to gluon-gluon and quark-antiquark. They can therefore also be singly produced at the LHC, and would decay to jet pairs,

where the same comments just made about dijet searches apply. If kinematically allowed, they could also decay to $\tilde{g}\tilde{g}$, $\tilde{\chi}\tilde{\chi}$, $\varphi\varphi$, SS, or XS final states.

The singlino fermions $\tilde{\chi}$ from the $\Phi, \overline{\Phi}$ supermultiplets are a new feature of the model considered here. They have tree-level couplings $\tilde{\chi}\tilde{g}S$ and $\tilde{\chi}\tilde{g}X$ (proportional to gauge couplings) and $\tilde{\chi}\tilde{\chi}\varphi$ (proportional to Yukawa couplings y and $\overline{y}$), which allows for them to decay to other odd R-parity final states, with decay chains that will eventually terminate in the MSSM lightest supersymmetric particle. They can always decay in this way, through off-shell intermediate states if necessary, so they are not stable unless $\tilde{\chi}_1$ is the lightest supersymmetric particle. However, they cannot be singly produced due to their R-parity, and cannot even be pair-produced at tree-level at colliders due to the lack of couplings to gluons or quarks. Therefore it seems unlikely that they could be part of a discovery, unless through the cascade decays of the other states mentioned above. This also seems quite unlikely due to kinematics, at least for the mass spectra along the sample model line considered in the previous section.

VI. OUTLOOK

If supersymmetric particles exist at a multi-TeV scale, as suggested by the tension between the big hierarchy problem and the 125 GeV Higgs scalar boson mass, it is sensible to consider extensions of the minimal supersymmetric framework, including even radical ones that would not be viable at lower mass scales. In this paper, I have considered the possibility that the color gauge group is extended to $SU(3)\times SU(3)$. This symmetry breaking pattern was shown to be easy to attain in the case of the most general renormalizable and softly broken potential that can be constructed using the minimal field content with the necessary order parameters. Indeed, other possible remnant groups of the symmetry breaking were found to be highly disfavored. The model predicts several new color octets of spin 1, 1/2, and 0, and new spin 1/2 and spin 0 singlets, all with masses that are presumably at multi-TeV scales. In an example model framework motivated by gauge coupling unification with an infrared quasi-fixed point for the Yukawa couplings, gaugino-mass-dominated supersymmetry breaking leads to weakly interacting superpartners that are relatively light, and still could be discovered at the LHC. The phenomenology of these models at future colliders (including a high-energy LHC) will involve multiple gluino and sgluon states, in addition to a coloron vector boson, all of which could be lighter than the ordinary squarks. The lightest gluino could have either enhanced or highly suppressed couplings to quarks and squarks. The phenomenology can differ from that found in previous studies of Dirac and mixed gluinos and sgluons that occur in supersoft and models with an $N = 2$ gauge sector. Although not explored here, it should also be possible to realize the same gauge symmetry breaking pattern by introducing new singlet or octet chiral superfields. It is also possible to enlarge the gauge group that breaks down to $SU(3)_C$ in various ways.

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