

# Experimental Model Updating of a Full-Scale Concrete Frame Structure

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## ABSTRACT

In structural analysis, it is common practice to construct a finite element (FE) model of an as-built structure using nominal material properties and idealized boundary conditions. However, behaviors of the FE model generally differ from the as-built structure in the field. To minimize the differences, selected parameters of the FE model can be updated using experimental measurements from the as-built structure. This paper investigates the FE model updating of a full-scale concrete frame structure with over a thousand degrees-of-freedom. Given experimental measurements obtained during a shaker test, frequency-domain modal properties of the concrete structure are identified. A non-convex optimization problem is then formulated to update parameter values of the FE model by minimizing the difference between the experimentally identified modal properties and those generated from the FE model. The selected optimization variables include concrete elastic moduli of the columns, beams and slabs. Upon model updating, the modal properties of the FE model can match better with the experimentally identified modal properties.

**KEYWORDS:** *model updating, modal analysis, full-scale concrete frame*

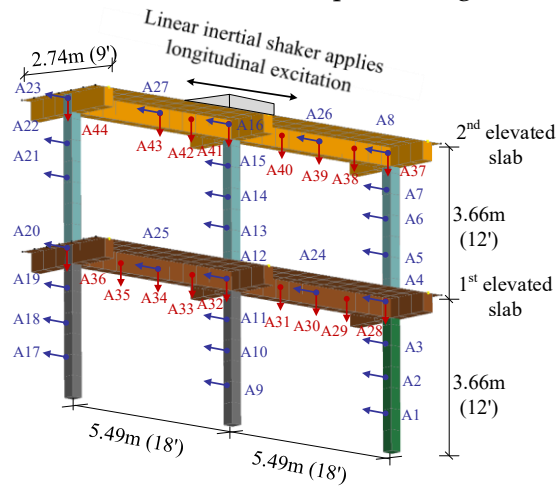
## INTRODUCTION

To a certain degree, as-built civil structure always behaves differently from its corresponding finite element (FE) model. The reason can be attributed to both model idealizations and nominal values of material properties. This paper investigates a frequency domain approach to update the material parameter values of an FE model by minimizing the difference between the experimentally identified modal properties and those of the FE model. The test structure is a full-scale concrete frame excited by a hydraulic shaker. Acceleration measurements of the structure are used to extract modal properties based on the Numerical Algorithms for Subspace State Space System Identification (N4SID). Given the experimentally identified modal properties, the

material parameters of the FE model are updated by solving a non-convex optimization problem through multiple starting points.

## TEST STRUCTURE

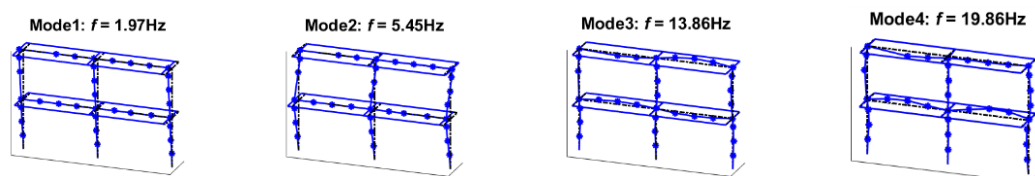
A full-scale reinforced concrete frame is used as the test structure in this study. The story height of the reinforced concrete frame is 3.66 meters, the column spacing is 5.49 meters and the width of the two elevated slabs is 2.74 meters (Figure 1). When constructing the frame, concrete pouring was conducted in five stages, indicated by five different colors shown in Figure 1. During testing, the frame was excited in the in-plane longitudinal direction by a hydraulic linear inertia shaker mounted at the center of the second elevated slab. With a scaled El Centro record, the inertia force of the moving mass on the shaker excited the structure dynamically. Structural responses under shaker excitation were measured using accelerometers (Kinematics EpiSensor ES-T and ES-U) instrumented at the middle and quarter locations of columns and longitudinal beams of the frame (see Figure 1 and [1]).



**Figure 1.** Accelerometer instrumentation of a full-scale test frame

## MODAL ANALYSIS AND FE MODEL UPDATING

A total of 27 in-plane longitudinal and 17 vertical acceleration channels are used to perform modal analysis and FE model updating. Using the experimental measurements when the maximum displacement of the shaker mass is scaled to 25.4 mm (1 inch), modal properties of the concrete frame are obtained using N4SID (Figure 2). The first two modes mainly consist of in-plane longitudinal movement of columns. On the contrary, higher modes are mainly characterized by vertical movement of beams.



**Figure 2.** Experimentally identified modes under shaker excitation

An FE model of the concrete frame is built using SAP2000. The initial FE model utilizes nominal material properties of the concrete, obtained from cylinder tests for five concrete pours. To update the model, the five concrete moduli of the FE model (corresponding to

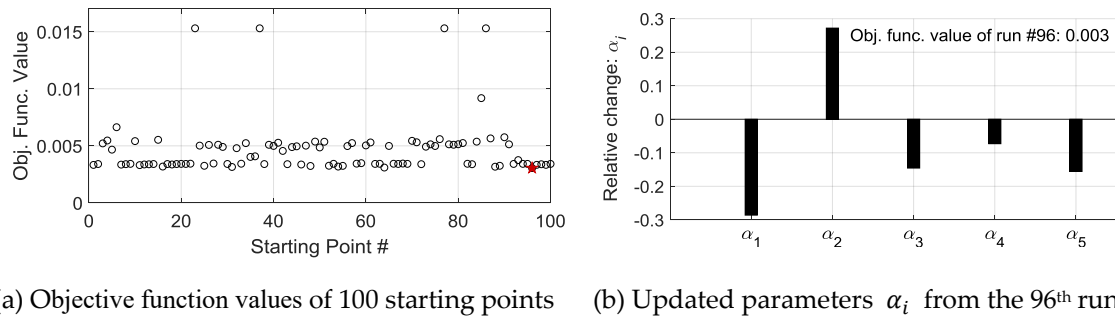
the five pours) are selected for updating. The structural stiffness matrix is thus parameterized on the five concrete moduli as  $\mathbf{K}(\boldsymbol{\alpha}) = \mathbf{K}_0 + \sum_{i=1}^{n_\alpha} \alpha_i \mathbf{K}_i$ . Here  $\boldsymbol{\alpha} \in \mathbb{R}^{n_\alpha}$  is a vector representing the relative changes of elastic moduli from nominal values and treated as the updating variables ( $n_\alpha = 5$ );  $\mathbf{K}_0$  is the initial stiffness matrix before model updating and using nominal concrete moduli; and  $\mathbf{K}_i$  is a constant stiffness matrix contributed by structural members from one pour and corresponding to one  $\alpha_i$ .

An optimization problem is formulated as follows to minimize the modal property differences between the FE model and experiments. Similarities in mode shapes between the FE model and experimental results are quantified using modal assurance criterion (MAC). For the  $i$ -th mode, the criterion is defined as  $\text{MAC}_i = \left( (\boldsymbol{\Psi}_i^{\text{EXP},m})^T \boldsymbol{\Psi}_i^{\text{FE},m} \right)^2 / \left( \|\boldsymbol{\Psi}_i^{\text{EXP},m}\|_2^2 \|\boldsymbol{\Psi}_i^{\text{FE},m}\|_2^2 \right)$ , where  $\boldsymbol{\Psi}_i^{\text{EXP},m}$  denotes the experimentally identified mode shape vector and  $\boldsymbol{\Psi}_i^{\text{FE},m}$  denotes the simulated mode shape vector at measured DOFs (from the FE model).

$$\underset{\boldsymbol{\alpha}}{\text{minimize}} \quad \sum_{i=1}^{n_{\text{modes}}} \left\{ \left( \frac{\lambda_i^{\text{EXP}} - \lambda_i(\boldsymbol{\alpha})}{\lambda_i^{\text{EXP}}} \cdot w_{\lambda_i} \right)^2 + \left( \frac{1 - \sqrt{\text{MAC}_i}}{\sqrt{\text{MAC}_i}} \cdot w_{\text{MAC}_i} \right)^2 \right\} \quad (1a)$$

$$\text{subject to} \quad \mathbf{L}_\alpha \leq \boldsymbol{\alpha} \leq \mathbf{U}_\alpha \quad (1b)$$

Here  $\mathbf{L}_\alpha$  and  $\mathbf{U}_\alpha$  denote the lower and upper bounds of the updating variable  $\boldsymbol{\alpha}$ ;  $n_{\text{modes}}$  denotes the number of modes used for updating;  $\lambda_i$  denotes the  $i$ -th eigenvalue of FE model obtained by solving the generalized eigenvalue problem between the stiffness matrix  $\mathbf{K}(\boldsymbol{\alpha})$  and mass matrix  $\mathbf{M}$ ;  $\lambda_i^{\text{EXP}}$  denotes the experimentally identified  $i$ -th eigenvalue;  $w_{\lambda_i}$  and  $w_{\text{MAC}_i}$  denote the weights of the eigenvalues and MAC values of the  $i$ -th mode, respectively. Note here the objective function is an oracle formulation of updating variable  $\boldsymbol{\alpha}$ , which results in a nonconvex optimization problem. An open-source MATLAB package for structural model updating (SMU) is used to solve the optimization problem with the trust-region-reflective algorithm [2]. The upper and lower bounds of  $\boldsymbol{\alpha}$  are set as 0.3 and -0.3. In this example, the weights are set the same to all four modes as  $w_{\lambda_i} = 1, 1, 1, 1$ , for  $i = 1, \dots, 4$ . The weights for MAC values are set as  $w_{\text{MAC}_i} = w_{\lambda_i}$ . Starting from 100 randomized points of  $\boldsymbol{\alpha} \in [\mathbf{L}_\alpha, \mathbf{U}_\alpha]$ , optimization searches are performed. Figure 3(a) plots the objective function values of the 100 runs, among which the 96<sup>th</sup> run (marked as a star) finishes as the smallest. Correspondingly, Figure 3(b) shows the optimal/updated values of  $\boldsymbol{\alpha}$  from the 96<sup>th</sup> run.



**Figure 3.** FE model updating results from 100 starting points

Finally, for both the initial model and the updated model, Table 1 summarizes the relative errors in resonance frequencies, defined as  $e \triangleq (f_i^{\text{FE}} - f_i^{\text{EXP}})/f_i^{\text{EXP}}$ , and the MAC values. Overall, a much better match in resonance frequencies of the 2<sup>nd</sup> to 4<sup>th</sup> modes is obtained, with a relatively small sacrifice in MAC values and a slight increase in the relative errors of the first mode.

**Table 1.** Comparison of modal properties before and after FE model updating

Mode	$f_i^{\text{EXP}}$ (Hz)	$f_i^{\text{FE,init}}$ (Hz)	$e^{\text{init}}$	$\text{MAC}^{\text{init}}$	$f_i^{\text{FE,updt}}$ (Hz)	$e^{\text{updt}}$	$\text{MAC}^{\text{updt}}$
1 <sup>st</sup>	1.972	1.964	-0.40%	0.999	1.952	-1.02%	0.997
2 <sup>nd</sup>	5.453	5.631	3.27%	0.990	5.499	0.84%	0.990
3 <sup>rd</sup>	13.861	14.957	7.91%	0.959	14.025	1.18%	0.936
4 <sup>th</sup>	19.864	20.612	3.76%	0.974	19.670	-0.98%	0.969

## CONCLUSIONS

In this study, material properties of a full-scale concrete frame model are updated using experimental measurements during a shaker test. A non-convex optimization problem is formulated to minimize the differences between the experimentally identified modal properties of the as-built frame and those of the initial FE model. The resonance frequencies of the updated FE model match better with the experimental modal analysis results than the initial model.

## REFERENCES

- [1] Dong, X., Liu, X., Wright, T., Wang, Y. and DesRoches, R. (2016) Validation of wireless sensing technology densely instrumented on a full-scale concrete frame structure. *Proceedings of International Conference on Smart Infrastructure and Construction (ICSIC)*, Cambridge, United Kingdom.
- [2] Wang, Y., Dong, X., Li, D. and Otsuki, Y. SMU: MATLAB Package for Structural Model Updating, version 1.1. (2019). <https://github.com/ywang-structures/Structural-Model-Updating>.