Editors' Suggestion

$T\bar{T}$ flows and (2, 2) supersymmetry

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(Received 6 November 2019; published 7 January 2020)

We construct a solvable deformation of two-dimensional theories with (2, 2) supersymmetry using an irrelevant operator which is a bilinear in the supercurrents. This supercurrent-squared operator is manifestly supersymmetric and equivalent to $T\bar{T}$ after using conservation laws. As illustrative examples, we deform theories involving a single (2, 2) chiral superfield. We show that the deformed free theory is on-shell equivalent to the (2, 2) Nambu-Goto action. At the classical level, models with a superpotential exhibit more surprising behavior: the deformed theory exhibits poles in the physical potential which modify the vacuum structure. This suggests that irrelevant deformations of $T\bar{T}$ type might also affect infrared physics.

DOI: 10.1103/PhysRevD.101.026008

I. INTRODUCTION

Understanding the space of quantum field theories (QFTs) is a fascinating question. A typical approach to this question is to start with a particularly tractable model, such as a free, conformal, or exactly solvable theory, and deform it infinitesimally by adding an integrated local operator. An infinitesimal *relevant* deformation generates a renormalization group flow. The resulting theory will differ in the infrared from the original undeformed theory. When the original theory is conformal, there might exist exactly *marginal* deformations which preserve the conformal symmetry for finite values of the deformation parameters; the space of marginal parameters defines the moduli space of the conformal field theory. Finally, if the deforming operator is *irrelevant*, the ultraviolet properties of the theory change, and it is usually difficult to understand this change in terms of any kind of flow. This case is the most difficult to understand because, in essence, the definition of the theory changes.

Irrelevant deformations of two-dimensional Poincaré-invariant QFTs generated by the determinant of the stress-energy tensor, $\det(T) = T_{00}T_{11} - T_{01}T_{10}$, are special. These $T\bar{T}$ deformations define a flow along which certain properties of the deformed theory can be computed exactly [1]. Most important is the energy spectrum [2,3]. However, in many cases the classical action can also be determined in closed form along the flow [3,4]. This prompted the study of $T\bar{T}$ deformations of integrable theories [3,5,6], as well as of more general theories [7–11], with a number of applications to (effective) string theory [12–14], to two-dimensional gravity [15–18], and to the AdS_3/CFT_2 correspondence [19–28].[1]

One of the first examples studied was the deformation of a theory of free bosons, which resulted in the Nambu-Goto action [3].[2] Interestingly, the $T\bar{T}$ deformed action for a scalar theory with an arbitrary potential can also be exactly constructed, at least classically. Imposing the $T\bar{T}$ flow equation for the Lagrangian

$$\frac{d}{d\lambda}\mathcal{L}_\lambda = \det(T[\mathcal{L}_\lambda]), \tag{1.1}$$

where the stress-energy tensor $T_{\mu\nu}[\mathcal{L}_\lambda]$ is computed in the deformed theory itself, and setting the initial condition

$$\mathcal{L}_0 = \frac{1}{2}\partial_{++}\phi\partial_{--}\phi + V(\phi), \tag{1.2}$$

[1]More general "$T\bar{J}$" deformations, which break Lorentz invariance, have also been considered [29–35].

[2]This can also be seen by studying the world-sheet S matrix of strings in flat space [12,13]. For a theory of free bosons and fermions, one instead finds the Green-Schwarz action in the light-cone gauge [36]. See also Refs. [23,37] for a discussion of the relation between light-cone gauge-fixed strings and $T\bar{T}$ deformations.

gives [3,4]

$$\mathcal{L}_\lambda = \frac{1}{2\lambda}\frac{1-2\lambda V(\phi)}{1-\lambda V(\phi)}\left[-1+\sqrt{1+2\lambda\frac{(\partial_{++}\phi\partial_{--}\phi+2V(\phi))(1-\lambda V(\phi))}{(1-2\lambda V(\phi))^2}}\right]. \tag{1.3}$$

This Lagrangian is fairly involved. It is interesting to consider the potential energy at zero momentum, which means discarding all interaction terms which involve derivatives. This captures the potential for slowly varying fields,

$$\mathcal{L}_\lambda = \frac{1}{2}\partial_{++}\phi\partial_{--}\phi + \frac{V(\phi)}{1-\lambda V(\phi)} + \cdots, \tag{1.4}$$

where the ellipses denote interaction terms involving derivatives. Although this is just a classical result, the form of the deformed potential is striking: if we start from a regular potential, we will generically develop poles for sufficiently large $|\lambda|$. These poles are invisible in perturbation theory in the flow parameter λ. Were we able to trust this result at the quantum level, this would point to a dramatic modification of the theory. Namely, an *irrelevant* deformation would end up changing the *infrared* structure of the theory, resulting in a kind of "IR/UV mixing."

It is generally not possible to draw firm conclusions about the quantum properties of a theory by studying its classical potential. A truly quantum analysis would certainly be preferable. Unfortunately, our current understanding of the $T\bar{T}$ deformation at the quantum level is far from complete. When the theory is studied in infinite volume, the deformation can be defined by postulating that the S matrix only change by a Castillejo-Dalitz-Dyson (CDD) factor [3]. This CDD factor, however, spoils the analytic properties of the scattering matrix at large values of the deformation parameter, taking us away from the framework of local QFT. In finite volume, on the other hand, a flow equation for the energy spectrum follows from (1.1) [2].[3] However, generically along the flow some energy levels will become complex; this phenomenon is not completely understood. Which physical observables make sense in the deformed theory is also currently rather mysterious. All in all, a rigorous exploration of possible IR/UV mixing requires a deeper understanding of the quantum properties of the deformed theory.

One instance where a *classical* analysis of the potential might allow us to draw more reliable conclusions about the quantum theory is for models with extended supersymmetry. As long as there is sufficient supersymmetry for the potential to be partly controlled by a holomorphic quantity, there will be partial protection from perturbative (and sometimes nonperturbative) quantum effects. Models with $\mathcal{N}=(2,2)$ supersymmetry in two dimensions are precisely of this type, provided that the $T\bar{T}$ deformation is compatible with manifest $\mathcal{N}=(2,2)$ supersymmetry.

Recently it was shown that the $T\bar{T}$ flow preserves manifest $\mathcal{N}=(0,1)$, $\mathcal{N}=(1,1)$ [36,38], and $\mathcal{N}=(0,2)$ supersymmetry [39]. Specifically, we can view the flow as generated by the supersymmetric descendant of a composite operator; this composite operator is built from a bilinear in supercurrents. This construction both ensures supersymmetry along the flow and is sufficient to reproduce, and indeed slightly generalizes Zamolodchikov's argument for the well definedness and solvability of $T\bar{T}$ [1]. Moreover, for some simple supersymmetric actions it was possible to explicitly construct the deformed Lagrangian in superspace, gaining some insight on the resulting theory [36,38,39].

The main aim of this paper is to repeat this analysis in the $\mathcal{N}=(2,2)$ case and find a manifestly $\mathcal{N}=(2,2)$ supersymmetric version of the $T\bar{T}$ flow. The case of $\mathcal{N}=(2,2)$ is particularly interesting for at least two reasons: first, it is the most heavily studied class of two-dimensional supersymmetric theories because of applications to string compactifications. Second, these models are closely connected to the dimensional reduction of $\mathcal{N}=1$ theories in four dimensions. Understanding more about the structure of the $\mathcal{N}=(2,2)$ theory might shed light on how to generalize $T\bar{T}$ to higher dimensions; see [38,40–42] for discussions of such higher-dimensional generalizations. We plan to report on results along this direction in [43].

In this work, we will establish the appearance of a singularity in the physical potential, such as the one appearing in Eq. (1.4), in a manifestly $\mathcal{N}=(2,2)$ form—where, as usual, the role of $V(\phi)$ will be played by $|W'(\phi)|^2$ with $W(\phi)$ the holomorphic superpotential. Our results on the $\mathcal{N}=(2,2)$ version of $T\bar{T}$ provide a stepping stone toward a fully quantum analysis of the vacuum structure of nonconformal $T\bar{T}$-deformed theories, which we plan to explore in the future.

The paper is structured as follows: in Sec. II we review the structure of the $\mathcal{N}=(2,2)$ supercurrent multiplets which we need to construct the supersymmetric deformation. In Sec. III we construct the supercurrent-squared operator $\mathcal{T}\bar{\mathcal{T}}$ as a bilinear in the supercurrents and discuss its well definedness. Finally, in Sec. IV we construct the deformed action for a few examples of $\mathcal{N}=(2,2)$ theories. In particular, we focus on theories involving a single chiral multiplet with an action determined by an arbitrary Kähler potential, as well as models with a superpotential. In Appendixes A, B, and C, we collect assorted results used in the main body of the text.

[3]For integrable theories, such a flow equation may also be derived from the CDD deformation using the thermodynamic Bethe ansatz [3].

II. $D = 2\mathcal{N} = (2,2)$ SUPERCURRENT MULTIPLETS

Our manifestly supersymmetric modification of $T\bar{T}$ is built from bilinears in fields of the supercurrent multiplet. In this section we review the structure of such multiplets in $D = 2$, $\mathcal{N} = (2, 2)$ theories.

A. Conventions

We work in two-dimensional $\mathcal{N} = (2, 2)$ superspace with a Lorentzian signature; see [44] for a classic reference on the subject. Our four anticommuting coordinates are written $\theta^{\pm}$ and $\bar{\theta}^{\pm}$, and we will collectively denote the superspace coordinates by $\zeta^M = (x^\mu, \theta^{\pm}, \bar{\theta}^{\pm})$. To more easily interpret expressions involving both vector and spinor quantities, we change to light-cone coordinates using the bispinor conventions

$$x^{\pm\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^1), \tag{2.1}$$

and we define the corresponding partial derivatives

$$\partial_{\pm\pm} = \frac{1}{\sqrt{2}}(\partial_0 \pm \partial_1), \tag{2.2}$$

so that $\partial_{\pm\pm}x^{\pm\pm} = 1$ and $\partial_{\pm\pm}x^{\mp\mp} = 0$.

Spinors in two dimensions carry a single index which is raised or lowered as follows:

$$\psi^+ = -\psi_-, \qquad \psi^- = \psi_+. \tag{2.3}$$

We write all vector indices as pairs of spinor indices. This allows us to nicely compare terms in equations involving combinations of spinor, vector, spinor-vector, and tensor quantities. Using this notation, for example, the supercurrent has components $S_{+++}, S_{---}, S_{+--}$, and S_{-++}, which we can immediately identify as a spinor-vector because it has three indices. Similarly, the stress-energy tensor carries two vector indices which are repackaged into four bispinor indices $T_{++++}, T_{----}, T_{++--} = T_{--++}$.

The supercovariant derivatives, collectively denoted by $D_A = (\partial_a, D_{\pm}, \bar{D}_{\pm})$, are defined by

$$D_{\pm} = \frac{\partial}{\partial\theta^{\pm}} - \frac{i}{2}\bar{\theta}^{\pm}\partial_{\pm\pm}, \qquad \bar{D}_{\pm} = -\frac{\partial}{\partial\bar{\theta}^{\pm}} + \frac{i}{2}\theta^{\pm}\partial_{\pm\pm}, \tag{2.4}$$

and satisfy

$$\{D_{\pm}, \bar{D}_{\pm}\} = i\partial_{\pm\pm}, \tag{2.5}$$

with all other (anti)commutators vanishing.

The supersymmetry transformations for an $\mathcal{N} = (2, 2)$ superfield $\mathcal{F}(\zeta) = \mathcal{F}(x^{\pm\pm}, \theta^{\pm}, \bar{\theta}^{\pm})$ are given by

$$\delta_Q\mathcal{F} := i\epsilon^+\mathcal{Q}_+\mathcal{F} + i\epsilon^-\mathcal{Q}_-\mathcal{F} - i\bar{\epsilon}^+\bar{\mathcal{Q}}_+\mathcal{F} - i\bar{\epsilon}^-\bar{\mathcal{Q}}_-\mathcal{F}, \tag{2.6}$$

where on superfields the supercharges are represented by the following differential operators:

$$\mathcal{Q}_{\pm} = \frac{\partial}{\partial\theta^{\pm}} + \frac{i}{2}\bar{\theta}^{\pm}\partial_{\pm\pm}, \qquad \bar{\mathcal{Q}}_{\pm} = -\frac{\partial}{\partial\bar{\theta}^{\pm}} - \frac{i}{2}\theta^{\pm}\partial_{\pm\pm}, \tag{2.7}$$

satisfying

$$\{\mathcal{Q}_{\pm}, \bar{\mathcal{Q}}_{\pm}\} = -i\partial_{\pm\pm}, \tag{2.8}$$

and commuting with the covariant derivatives D_A.

B. The $\mathcal{S}$ multiplet

For Lorentz invariant supersymmetric theories, there is an essentially unique supermultiplet which contains the stress-energy tensor $T_{\mu\nu}$, the supercurrent $S_{\mu\alpha}$, and no other operators with spin larger than one, under the assumption that the multiplet, though in general reducible, cannot be separated into decoupled supersymmetry multiplets; namely that it is indecomposable [45]. This $\mathcal{S}$-multiplet can be defined in any theory with $D = 2$, $\mathcal{N} = (2, 2)$ supersymmetry. By "essentially unique," we mean that the $\mathcal{S}$-multiplet is unique up to improvement terms which preserve the superspace constraint equations.

For two-dimensional theories with (2, 2) supersymmetry, the $\mathcal{S}$-multiplet consists of superfields $\mathcal{S}_{\pm\pm}$, $\chi_{\pm}$, and $\mathcal{Y}_{\pm}$ which satisfy the constraints:

$$\bar{D}_{\pm}\mathcal{S}_{\mp\mp} = \pm(\chi_{\mp} + \mathcal{Y}_{\mp}), \tag{2.9a}$$

$$\bar{D}_{\pm}\chi_{\pm} = 0, \qquad \bar{D}_{\pm}\chi_{\mp} = \pm C^{(\pm)}, \qquad D_+\chi_- - \bar{D}_-\bar{\chi}_+ = k, \tag{2.9b}$$

$$D_{\pm}\mathcal{Y}_{\pm} = 0, \quad \bar{D}_{\pm}\mathcal{Y}_{\mp} = \mp C^{(\pm)}, \quad D_+\mathcal{Y}_- + D_-\mathcal{Y}_+ = k'. \tag{2.9c}$$

Here k and k' are real constants and $C^{(\pm)}$ is a complex constant. The $\mathcal{S}$-multiplet contains $8 + 8$ independent real component operators and the constants $k, k', C^{(\pm)}$ [45]. The expansion in components of $\mathcal{S}_{\pm\pm}, \chi_{\pm}$, and $\mathcal{Y}_{\pm}$ are given for convenience in Appendix A.

Among the various component fields it is important to single out the complex supersymmetry current $S_{\alpha\mu}$ and the energy-momentum tensor $T_{\mu\nu}$. The complex supersymmetry current, associated with $S_{+\pm\pm}$ and $S_{-\pm\pm}$, is conserved: $\partial^\mu S_{\alpha\mu} = 0$. The energy-momentum tensor, associated with $T_{\pm\pm\pm\pm}$ and $T_{++--} = T_{--++}$, is real, conserved ($\partial^\mu T_{\mu\nu} = 0$), and symmetric ($T_{\mu\nu} = T_{\nu\mu}$). In light-cone notation the conservation equations are given by

$$\partial_{++}S_{+--}(x) = -\partial_{--}S_{+++}(x), \tag{2.10a}$$

$$\partial_{++}\bar{S}_{+--}(x) = -\partial_{--}\bar{S}_{+++}(x), \tag{2.10b}$$

$$\partial_{++}T_{----}(x) = -\partial_{--}\Theta(x), \tag{2.10c}$$

$$\partial_{++}\Theta(x) = -\partial_{--}T_{++++}(x), \tag{2.10d}$$

where we have defined as usual

$$\Theta(x) \coloneqq T_{++--}(x) = T_{--++}(x). \tag{2.11}$$

To conclude this subsection, let us describe the ambiguity in the form of the $\mathcal{S}$ multiplet which is parametrized by a choice of improvement terms. If $\mathcal{U}$ is a real superfield, we are free to modify the $\mathcal{S}$-multiplet superfields as follows:

$$\mathcal{S}_{\pm\pm} \to \mathcal{S}_{\pm\pm} + [D_\pm, \bar{D}_\pm]\mathcal{U}, \tag{2.12a}$$

$$\chi_\pm \to \chi_\pm - \bar{D}_+\bar{D}_-D_\pm\mathcal{U}, \tag{2.12b}$$

$$\mathcal{Y}_\pm \to \mathcal{Y}_\pm - D_\pm\bar{D}_+\bar{D}_-\mathcal{U}, \tag{2.12c}$$

which keeps invariant the conservation equations (2.9). In general, the $\mathcal{S}$ multiplet is a reducible representation of supersymmetry, and some of its component can consistently be set to zero by a choice of improvement. The reduced Ferrara-Zumino supercurrent multiplet, which plays a central role in our paper, is described next.

C. The Ferrara-Zumino multiplet and old-minimal supergravity

If there exists a well-defined superfield $\mathcal{U}$ such that $\chi_\pm = \bar{D}_+\bar{D}_-D_\pm\mathcal{U}$, then we may use the transformation (2.12) to set $\chi_\pm = 0$ in the $\mathcal{S}$ multiplet. If in addition $k = C^{(\pm)} = 0$, then the fields $\mathcal{S}_{\pm\pm}$ and $\mathcal{Y}_\pm$ satisfy the defining equations for the Ferrara-Zumino (FZ) multiplet. In this case, it is conventional to rename the field $\mathcal{S}_{\pm\pm}$ to $\mathcal{J}_{\pm\pm}$ and write these defining equations as

$$\bar{D}_\pm\mathcal{J}_{\mp\mp} = \pm\mathcal{Y}_\mp, \tag{2.13a}$$

$$D_\pm\mathcal{Y}_\pm = 0, \tag{2.13b}$$

$$\bar{D}_\pm\mathcal{Y}_\mp = 0, \tag{2.13c}$$

$$D_+\mathcal{Y}_- + D_-\mathcal{Y}_+ = k'. \tag{2.13d}$$

The superfield $\mathcal{J}_{\pm\pm}$ in the FZ multiplet turns out to be associated with the axial $U(1)_A$ R-symmetry current and satisfies the conservation equation

$$\partial_{--}\mathcal{J}_{++} - \partial_{++}\mathcal{J}_{--} = 0. \tag{2.14}$$

This multiplet, which has $4+4$ real components, is the dimensionally reduced version of the $D=4$ $\mathcal{N}=1$ FZ multiplet [46]; see Appendix A for more details. All of the models we consider in Sec. IV have the property that $\chi_\pm$ can be improved to zero; that is, they all have a well-defined FZ multiplet.

Just as the bosonic Hilbert stress tensor $T_{\mu\nu}$ represents the response function of the Lagrangian to a linearized perturbation $h_{\mu\nu}$ of the metric, the supercurrent multiplets correspond to linearized couplings to supergravity.[4]

Different formulations of off-shell supergravity couple to different supercurrent multiplets. If a theory has a well-defined FZ multiplet, as is the case for all the examples found in Sec. IV, then the theory can be consistently coupled to the old-minimal supergravity prepotentials $H^{\pm\pm}$ and σ. The nomenclature "old-minimal" is again inherited from $D=4$, $\mathcal{N}=1$ supergravity; see [48,49] for pedagogical reviews and references. Here $H^{\pm\pm}$ is the conformal supergravity prepotential—the analogue of the traceless part of the metric—and σ is a chiral conformal compensator.

We refer the reader to [50–54] and references therein for an exhaustive description of $D=2$, $\mathcal{N}=(2,2)$ off-shell supergravity in superspace, which we will use in our analysis; see also Appendix B. For the scope of this work, it will be enough to know the structure of linearized old-minimal supergravity. For instance, at the linearized level the gauge symmetry of the supergravity prepotentials $H^{\pm\pm}, \sigma$, and $\bar{\sigma}$, can be parametrized as follows:

$$\delta H^{++} = \frac{i}{2}(\bar{D}_-L^+ - D_-\bar{L}^+), \tag{2.15a}$$

$$\delta H^{--} = \frac{i}{2}(\bar{D}_+L^- - D_+\bar{L}^-), \tag{2.15b}$$

$$\delta\sigma = -\frac{i}{2}\bar{D}_+\bar{D}_-(D_+L^+ - D_-L^-), \tag{2.15c}$$

$$\delta\bar{\sigma} = -\frac{i}{2}D_-D_+(\bar{D}_+\bar{L}^+ - \bar{D}_-\bar{L}^-), \tag{2.15d}$$

in terms of unconstrained spinor superfields $L^\pm$ and their complex conjugates.

The conservation law (2.13) for the FZ multiplet can be derived by using the previous gauge transformations. The linearized supergravity couplings for a given model are written as[5]

[4]Rather than coupling to supergravity, one could define the supercurrent multiplets using a superspace Noether procedure, as is done for $D=4$ theories with $\mathcal{N}=1$ supersymmetry in [47]. This was the approach followed for (1, 1) supersymmetry in [38].

[5]We use the notation $d^2\theta \coloneqq d\theta^-d\theta^+$, $d^2\bar{\theta} \coloneqq d\bar{\theta}^+d\bar{\theta}^-$, and $d^4\theta \coloneqq d^2\theta d^2\bar{\theta}$.

$$\mathcal{L}_{\text{linear}} = \int d^4\theta (H^{++}\mathcal{J}_{++} + H^{--}\mathcal{J}_{--}) - \int d^2\theta \sigma \mathcal{V} - \int d^2\bar{\theta}\,\bar{\sigma}\bar{\mathcal{V}}, \tag{2.16}$$

with $\mathcal{V}$ a chiral superfield and $\bar{\mathcal{V}}$ its complex conjugate. Assuming the matter superfields satisfy their equations of motion, the change in the Lagrangian (2.16) under the gauge transformation (2.15) is

$$\begin{aligned}\delta\mathcal{L}_{\text{linear}} &= \int d^4\theta (\delta H^{++}\mathcal{J}_{++} + \delta H^{--}\mathcal{J}_{--}) - \int d^2\theta \delta\sigma \mathcal{V} - \int d^2\bar{\theta}\delta\bar{\sigma}\bar{\mathcal{V}} \\ &= \frac{i}{2}\int d^4\theta \{(\bar{D}_- L^+ - D_- \bar{L}^+)\mathcal{J}_{++} + (\bar{D}_+ L^- - D_+ \bar{L}^-)\mathcal{J}_{--} - (D_+ L^+ - D_- L^-)\mathcal{V} \\ &\quad - (\bar{D}_+ \bar{L}^+ - \bar{D}_- \bar{L}^-)\bar{\mathcal{V}}\} \\ &= \frac{i}{2}\int d^4\theta \{L^+(\bar{D}_-\mathcal{J}_{++} + D_+\mathcal{V}) + L^-(\bar{D}_+\mathcal{J}_{--} - D_-\mathcal{V}) + \text{c.c.}\},\end{aligned} \tag{2.17}$$

where we have integrated by parts. Demanding that the variation vanishes for any gauge parameter $L^\pm$ gives

$$\bar{D}_-\mathcal{J}_{++} + D_+\mathcal{V} = 0, \qquad \bar{D}_+\mathcal{J}_{--} - D_-\mathcal{V} = 0. \tag{2.18}$$

This matches the constraints (2.13) for the FZ multiplet if we identify

$$\mathcal{Y}_\pm = D_\pm \mathcal{V} \tag{2.19}$$

and set $k' = 0$.

As we will soon see, studying $T\bar{T}$ deformations requires consideration of a composite operator constructed out of the square of the supercurrent multiplet. Hence to solve the $T\bar{T}$ flow equations we need to be able to calculate the supercurrent multiplet explicitly. The coupling to supergravity provides a straightforward prescription for computing the FZ multiplet for matter models that can be coupled to old-minimal supergravity.[6] In particular, for a given $\mathcal{N} = (2,2)$ matter theory we will

(1) Begin with an undeformed superspace Lagrangian $\mathcal{L}$ in flat $\mathcal{N} = (2,2)$ superspace.
(2) Minimally couple $\mathcal{L}$ to the supergravity superfield prepotentials $H^{\pm\pm}$, σ, and $\bar{\sigma}$.
(3) Extract the superfields $\mathcal{J}^{\pm\pm}$, $\mathcal{V}$, and $\bar{\mathcal{V}}$ which couple linearly to $H^{\pm\pm}$, σ, and $\bar{\sigma}$, respectively, in the D and F terms of (2.16).

Thanks to the analysis given above, the superfields $\mathcal{J}^{\pm\pm}$, $\mathcal{V}$, and $\bar{\mathcal{V}}$ will automatically satisfy the FZ-multiplet constraints (2.18). A detailed description of the computation of the FZ multiplet for the models relevant for our paper is given in Appendix B.

[6]Though we will not need it in our paper, it is worth mentioning that the nonminimal supergravity results of [50–54] allow the computation of the supercurrent multiplet for more general classes of models.

III. THE $T\bar{T}$ OPERATOR AND $\mathcal{N} = (2,2)$ SUPERSYMMETRY

After having reviewed in the previous section the structure of the $\mathcal{S}$ multiplet, we are ready to describe $\mathcal{N} = (2,2)$ $T\bar{T}$ deformations.

A. The $\mathcal{T}\bar{\mathcal{T}}$ operator

Given a $D = 2$, $\mathcal{N} = (2,2)$ supersymmetric theory with an $\mathcal{S}$ multiplet, we define the supercurrent-squared deformation of this theory, denoted $\mathcal{T}\bar{\mathcal{T}}$ in analogy with $T\bar{T}$, by the flow equation

$$\partial_\lambda \mathcal{L} = -\frac{1}{8}\mathcal{T}\bar{\mathcal{T}}, \tag{3.1}$$

where $\mathcal{T}\bar{\mathcal{T}}$ is constructed from current bilinears with

$$\mathcal{T}\bar{\mathcal{T}} \equiv -\int d^4\theta \mathcal{S}_{++}\mathcal{S}_{--} - \left(\int d\theta^- d\theta^+ \chi_+\chi_- + \int d\bar{\theta}^- d\theta^+ \bar{\mathcal{Y}}_+\mathcal{Y}_- + \text{c.c.}\right), \tag{3.2}$$

and where the factor of $\frac{1}{8}$ is chosen for later convenience. This deformation generalizes the results we recently obtained for $D = 2$ theories possessing $\mathcal{N} = (0,1)$, $\mathcal{N} = (1,1)$, and $\mathcal{N} = (0,2)$ supersymmetry [36,38,39] to theories with $\mathcal{N} = (2,2)$ supersymmetry.

Let us recall the form of the $T\bar{T}$ composite operator [1], which we denote

$$T\bar{T}(x) = T_{++++}(x)T_{----}(x) - [\Theta(x)]^2. \tag{3.3}$$

An important property of the $\mathcal{N} = (0,1)$, $\mathcal{N} = (1,1)$, and $\mathcal{N} = (0,2)$ cases is that the $T\bar{T}$ operator turns out to be the bottom component of a long supersymmetric multiplet. This is true up to both total vector derivatives (∂_{++} and ∂_{--}) and terms that vanish upon using the supercurrent conservation equations (Ward identities). For this reason, in the supersymmetric cases studied previously, the original $T\bar{T}$ deformation of [1] is manifestly supersymmetric and equivalent to the deformations constructed in terms of the full superspace integrals of primary supercurrent-squared composite operators [36,38,39].

Remarkably, despite the much more involved structure of the (2, 2) $\mathcal{S}$ multiplet compared to theories with fewer supersymmetries, it is possible to prove that the following relation holds:

$$\mathcal{T}\bar{\mathcal{T}}(x) = T\bar{T}(x) + \text{EOM's} + \partial_{++}(\cdots) + \partial_{--}(\cdots). \tag{3.4}$$

In (3.4), we use EOM's to denote terms that are identically zero when Eqs. (2.9) are used. Showing (3.4) requires using (A1)–(A3), along with several cancellations, integration by parts, and the use of the (2, 2) $\mathcal{S}$-multiplet conservation equations (2.9).

In fact, the specific combination of current superfields given in (3.2) was chosen precisely for (3.4) to hold. The combination (3.4) is also singled out by being invariant under the improvement transformation (2.12). The important implication of (3.4) is that the $T\bar{T}$ deformation for an $\mathcal{N} = (2, 2)$ supersymmetric quantum field theory is manifestly supersymmetric and equivalent to the $\mathcal{T}\bar{\mathcal{T}}$ deformation of Eq. (3.2).

Note that in the $\mathcal{N} = (2, 2)$ case the deformation we have introduced in (3.2) is conceptually different from the cases with less supersymmetry. Specifically, the deformation is not given by the descendant of a *single* composite superfield. On the other hand, suppose the $\mathcal{S}$ multiplet is such that $C^{(\pm)} = k = k' = 0$, and it is possible to improve the superfields $\chi_\pm$ and $\mathcal{Y}_\pm$ to a case where

$$\mathcal{Y}_\pm = D_\pm \mathcal{V}, \qquad \bar{\mathcal{Y}}_\pm = \bar{D}_\pm \bar{\mathcal{V}}, \tag{3.5a}$$

$$\chi_+ = i\bar{D}_+\bar{\mathcal{B}}, \qquad \chi_- = i\bar{D}_-\mathcal{B}, \qquad \bar{\chi}_+ = -iD_+\mathcal{B}, \qquad \bar{\chi}_- = -iD_-\bar{\mathcal{B}}, \tag{3.5b}$$

with $\mathcal{V}$ chiral and $\mathcal{B}$ twisted chiral:

$$\bar{D}_\pm \mathcal{V} = 0, \qquad D_\pm \bar{\mathcal{V}} = 0, \tag{3.6a}$$

$$\bar{D}_+\mathcal{B} = D_-\mathcal{B} = 0, \qquad D_+\bar{\mathcal{B}} = \bar{D}_-\bar{\mathcal{B}} = 0. \tag{3.6b}$$

In this case (3.2) simplifies to

$$\begin{aligned}\mathcal{T}\bar{\mathcal{T}} &= -\int d^4\theta \mathcal{S}_{++}\mathcal{S}_{--} + \left(\int d\theta^- d\theta^+ \bar{D}_+\bar{\mathcal{B}}\bar{D}_-\bar{\mathcal{B}} \right. \\ &\quad \left. - \int d\bar{\theta}^- d\theta^+ \bar{D}_+\bar{\mathcal{V}} D_-\mathcal{V} + \text{c.c.}\right) \\ &= -\int d^4\theta(\mathcal{S}_{++}\mathcal{S}_{--} - 2\mathcal{B}\bar{\mathcal{B}} - 2\mathcal{V}\bar{\mathcal{V}}),\end{aligned} \tag{3.7}$$

and we see that, up to EOM's, $T\bar{T}(x)$ is the bottom component of a long supersymmetric multiplet. In this situation, once we define the composite superfield

$$\mathcal{O}(\zeta) \coloneqq -\mathcal{S}_{++}(\zeta)\mathcal{S}_{--}(\zeta) + 2\mathcal{B}(\zeta)\bar{\mathcal{B}}(\zeta) + 2\mathcal{V}(\zeta)\bar{\mathcal{V}}(\zeta), \tag{3.8}$$

Eq. (3.4) turns into the equivalent result[7]

$$\begin{aligned}\int d^4\theta \mathcal{O}(\zeta) &= D_-D_+\bar{D}_+\bar{D}_-\mathcal{O}(\zeta)|_{\theta=0} \\ &= T\bar{T}(x) + \text{EOM's} + \partial_{++}(\cdots) + \partial_{--}(\cdots),\end{aligned} \tag{3.9}$$

stating that the D term of the operator $\mathcal{O}(\zeta)$ is equivalent to the standard $T\bar{T}(x)$ operator.

For a matter theory that can be coupled to old-minimal supergravity, leading to the FZ multiplet described by (2.18), the operator $\mathcal{O}(\zeta)$ further simplifies thanks to the fact that the twisted-(anti)chiral operators $\mathcal{B}$ and $\bar{\mathcal{B}}$ disappear. For these cases, the $T\bar{T}$ flow turns into the following equation:

$$\partial_\lambda \mathcal{L} = \frac{1}{8}\int d^4\theta(\mathcal{J}_{++}\mathcal{J}_{--} - 2\mathcal{V}\bar{\mathcal{V}}). \tag{3.10}$$

This will be our starting point in analyzing $\mathcal{N} = (2, 2)$ deformed models in Sec. IV.

B. Point splitting and well definedness

The $T\bar{T}(x)$ operator (3.3) is quite magical because it is a well-defined irrelevant composite local operator, free of short distance divergences [1]. In fact, this property generalizes to the larger class of operators

$$[A_s(x)A'_{s'}(x) - B_{s+2}(x)B_{s'-2}(x)], \tag{3.11}$$

where (A_s, B_{s+2}) and $(A'_{s'}, B'_{s'-2})$ are two pairs of conserved currents with spins s and s'. The operator $T\bar{T}(x)$ is a particular example with $s = s' = 0$. As proven in [2], these composite operators of "Smirnov-Zamolodchikov'–type have a well-defined point splitting which is free of short-distance divergences. In the case of $\mathcal{N} = (0, 1)$ and $\mathcal{N} = (1, 1)$ supersymmetric $T\bar{T}$ deformations, the entire supermultiplet whose bottom component is $T\bar{T}(x)$ is composed of well-defined Smirnov-Zamolodchikov–type operators [36,38]. In the $\mathcal{N} = (0, 2)$ case, the primary[8] operator whose bottom component is $T\bar{T}(x)$ is not of Smirnov-Zamolodchikov–type. Nevertheless, also in this case it was recently shown that, thanks to supersymmetry, the whole multiplet is well defined [39].

[7]In the subsequent discussion by $\theta = 0$ we will always mean $\theta^\pm = \bar{\theta}^\pm = 0$.

[8]We denote as primary operator the top component of a supersymmetric multiplet even when the theory is not superconformal.

In the $\mathcal{N}=(2,2)$ case it is clear that the situation is more complicated than any of the cases mentioned above. First, in the general situation, according to (3.2), the $\mathcal{T}\bar{\mathcal{T}}$ deformation is a linear combination of a D term together with chiral and twisted-chiral F term contributions. Though the F terms might be protected by standard perturbative nonrenormalization theorems [see, for example, [48,49] for the $D=4$, $\mathcal{N}=1$ case which dimensionally reduces to $D=2$, $\mathcal{N}=(2,2)$], the D term associated with the $\mathcal{S}_{++}\mathcal{S}_{--}$ operator has no clear reason to be protected in general from short-distance divergences in point-splitting regularization, and hence has no obvious reason to be well defined. This indicates that there might be a clash between supersymmetry and a point-splitting procedure in the general setting.

We will not attempt to analyze this issue in full generality in the current paper; instead our aim is to describe a subclass of models for which the $\mathcal{T}\bar{\mathcal{T}}$ deformation turns out to be well defined. A natural restriction to impose is that the $\mathcal{S}$ multiplet is constrained by (3.5) and the $\mathcal{T}\bar{\mathcal{T}}$ deformation is therefore described by the D term (3.7). By trivially extending the arguments used in [39] for the $\mathcal{N}=(0,2)$ case, it is not difficult to show that these restrictions are sufficient to imply that the multiplet described by the $\mathcal{N}=(2,2)$ primary operator $\mathcal{O}(\zeta)$ of (3.8) is indeed well defined despite not being of Smirnov-Zamolodchikov–type. As in the $\mathcal{N}=(0,2)$, unbroken $\mathcal{N}=(2,2)$ supersymmetry turns out to be the reason for this to happen.

Let us quickly explain how this works for the FZ multiplet and the deformation (3.10), which are the main players in our paper. Note, however, that the same argument extends to more general cases where both chiral and twisted-chiral current superfields, $\chi_\pm$ and $\mathcal{Y}_\pm$, satisfying (3.5) are turned on. We also refer to [39] for details that we will skip in the following discussion, which are trivial extensions from the (0, 2) to the (2, 2) case.

A first indication of the well definedness of the multiplet associated with $\mathcal{O}(\zeta)$ comes by looking at the vacuum expectation value of its lowest component. Define the primary composite operator

$$O(x) \coloneqq -j_{--}(x)j_{++}(x) + 2v(x)\bar{v}(x) = \mathcal{O}(\zeta)|_{\theta=0} \tag{3.12}$$

and its point-split version

$$O(x,x') \coloneqq -j_{--}(x)j_{++}(x') + v(x)\bar{v}(x') + \bar{v}(x)v(x'), \tag{3.13}$$

where

$$j_{\pm\pm}(x) \coloneqq \mathcal{J}_{\pm\pm}(\zeta)|_{\theta=0}, \qquad v(x) \coloneqq \mathcal{V}(\zeta)|_{\theta=0}, \qquad \bar{v}(x) \coloneqq \bar{\mathcal{V}}(\zeta)|_{\theta=0}. \tag{3.14}$$

Note that Eq. (2.18) implies the following relation among the component operators:

$$[\bar{Q}_\pm, j_{\mp\mp}(x)] = \pm[Q_\mp, v(x)], \qquad [Q_\pm, j_{\mp\mp}(x)] = \pm[\bar{Q}_\mp, \bar{v}(x)], \tag{3.15}$$

with $Q_\pm$ and $\bar{Q}_\pm$ denoting the $\mathcal{N}=(2,2)$ supercharges.[9] By then using $\partial_{\pm\pm} = i\{Q_\pm, \bar{Q}_\pm\}$, $\{Q_+, Q_-\} = \{\bar{Q}_+, \bar{Q}_-\} = 0$, $[\bar{Q}_\pm, v(x)] = [Q_\pm, \bar{v}(x)] = 0$, super-Jacobi identities, together with the conservation equations (3.15), and the assumption that the vacuum is invariant under supersymmetry, it is straightforward to show that the vacuum expectation value (VEV) of $O(x,x')$ satisfies

$$\begin{aligned}
\partial_{++}\langle j_{--}(x)j_{++}(x')\rangle &= i\langle[\{Q_+, \bar{Q}_+\}, j_{--}(x)]j_{++}(x')\rangle \\
&= i\langle\{Q_+, [Q_-, v(x)]\}j_{++}(x') + \{\bar{Q}_+, [\bar{Q}_-, \bar{v}(x)]\}j_{++}(x')\rangle \\
&= -i\langle[Q_+, v(x)][Q_-, j_{++}(x')] + [\bar{Q}_+, \bar{v}(x)][\bar{Q}_-, j_{++}(x')]\rangle \\
&= i\langle[Q_+, v(x)][\bar{Q}_+, \bar{v}(x')] + [\bar{Q}_+, \bar{v}(x)][Q_+, v(x')]\rangle \\
&= \langle[i\{\bar{Q}_+, Q_+\}, v(x)\}\bar{v}(x') + [i\{Q_+, \bar{Q}_+\}, v(x)\}\bar{v}(x')\rangle \\
&= \partial_{++}\langle v(x)\bar{v}(x') + v(x)\bar{v}(x')\rangle,
\end{aligned} \tag{3.16}$$

and, after performing a similar calculation for $\langle\partial_{--}j_{--}(x)\times j_{++}(x')\rangle = -\langle j_{--}(x)\partial'_{--}j_{++}(x')\rangle$, it is clear that the relation

$$\partial_{\pm\pm}\langle O(x,x')\rangle = 0 \tag{3.17}$$

holds. Therefore, $\langle O(x,x')\rangle$ is independent of the positions and free of short distance divergences. It is worth noting that similar to the argument showing that the two point function of two chiral or twisted-chiral operators is independent of the positions x and x', the previous analysis for $\langle O(x,x')\rangle$ necessarily relies on unbroken $\mathcal{N}=(2,2)$ supersymmetry.

The argument given above can be generalized to a statement about operators in superspace in complete analogy to the $\mathcal{N}=(0,2)$ case of [39]. Let us investigate the short distance singularities in the bosonic coordinates by defining a point-split version of the $\mathcal{N}=(2,2)$ primary $\mathcal{T}\bar{\mathcal{T}}$ operator,

$$\mathcal{O}(x,x',\theta) \coloneqq -\mathcal{J}_{--}(x,\theta)\mathcal{J}_{++}(x',\theta) + \mathcal{V}(x,\theta)\bar{\mathcal{V}}(x',\theta) + \bar{\mathcal{V}}(x,\theta)\mathcal{V}(x',\theta). \tag{3.18}$$

We want to show that the preceding bilocal superfield is free of short distance divergences in the limit $x \to x'$. A straightforward calculation shows that

[9] Given an operator $F(x)$ defined as the $\theta=0$ component of the superfield $\mathcal{F}(\zeta)$, $F(x) \coloneqq \mathcal{F}(\zeta)|_{\theta=0}$, then its supersymmetry transformations are such that $[Q_\pm, F(x)\} = \mathcal{Q}_\pm\mathcal{F}(\zeta)|_{\theta=0} = D_\pm\mathcal{F}(\zeta)|_{\theta=0}$ and $[\bar{Q}_\pm, F(x)\} = \bar{\mathcal{Q}}_\pm\mathcal{F}(\zeta)|_{\theta=0} = \bar{D}_\pm\mathcal{F}(\zeta)|_{\theta=0}$.

$$\begin{aligned}\partial_{++}\mathcal{O}(x,x',\theta) = -\{&iD_+\mathcal{V}(\zeta)[D'_-\mathcal{J}_{++}(\zeta') + \bar{D}'_+\bar{\mathcal{V}}(\zeta')] + i\bar{D}_+\bar{\mathcal{V}}(\zeta)[\bar{D}'_-\mathcal{J}_{++}(\zeta') + D'_+\mathcal{V}(\zeta')]\\ &+ i(\mathcal{Q}_+ + \mathcal{Q}'_+)[(\bar{D}_+\bar{\mathcal{V}}(\zeta))\mathcal{V}(\zeta')] + i(\bar{\mathcal{Q}}_+ + \bar{\mathcal{Q}}'_+)[(D_+\mathcal{V}(\zeta))\bar{\mathcal{V}}(\zeta')]\\ &+ i(\mathcal{Q}_- + \mathcal{Q}'_-)[(D_+\mathcal{V}(\zeta))\mathcal{J}_{++}(\zeta')] + i(\bar{\mathcal{Q}}_- + \bar{\mathcal{Q}}'_-)[(\bar{D}_+\bar{\mathcal{V}}(\zeta))\mathcal{J}_{++}(\zeta')]\\ &+ (\partial_{++} + \partial'_{++})[\bar{\theta}^+(\bar{D}_+\bar{\mathcal{V}}(\zeta))\mathcal{V}(\zeta') + \bar{\theta}^-(D_+\mathcal{V}(\zeta))\mathcal{J}_{++}(\zeta')]\\ &- (\partial_{++} + \partial'_{++})[\theta^+(D_+\mathcal{V}(\zeta))\bar{\mathcal{V}}(\zeta') + \theta^-(\bar{D}_+\bar{\mathcal{V}}(\zeta))\mathcal{J}_{++}(\zeta')]\}|_{\theta=\theta'}. \end{aligned} \tag{3.19}$$

Note that the first line in the preceding expression is zero because of the FZ conservation equations (2.18), which hold up to contact terms in correlation functions. The other lines are either total vector derivatives or supersymmetry transformations of bilocal operators. A similar equation holds for $\partial_{--}\mathcal{O}(x,x',\theta)$ showing that the operator $\mathcal{O}(x,x',\theta)$ satisfies

$$\partial_{\pm\pm}\mathcal{O}(x,x',\theta) = 0 + \text{EOM's} + [P,\ldots] + [Q,\ldots], \tag{3.20}$$

where $[P,\ldots]$ and $[Q,\ldots]$ schematically indicate a translation and supersymmetry transformation of some bilocal superfield operator.[10] To conclude, by employing an operator product expansion (OPE) argument completely analogous to the one originally given by Zamolodchikov in [1] and extended to the $\mathcal{N}=(0,2)$ supersymmetric case in [39], one can show that Eq. (3.20) implies

$$\mathcal{O}(x,x',\theta) = \mathcal{O}(\zeta) + \text{derivative terms}. \tag{3.21}$$

Here "derivative terms" indicate superspace covariant derivatives $D_A = (\partial_{\pm\pm}, D_\pm, \bar{D}_\pm)$ acting on local superfield operators while $\mathcal{O}(\zeta)$ arises from the regular, nonderivative part of the OPE of $\mathcal{O}(x,x',\theta)$. As a result the integrated operator

$$\begin{aligned} S_{\mathcal{O}} &= \int d^2x d^4\theta \lim_{\varepsilon\to 0}\mathcal{O}(x,x+\varepsilon,\theta)\\ &= \int d^2x d^4\theta :\mathcal{O}(x,x,\theta):, \end{aligned} \tag{3.22}$$

which can be considered as a definition of the integrated $\mathcal{T}\bar{\mathcal{T}}(x)$ operator,[11] is free of short distance divergences and is well defined in complete analogy to the nonsupersymmetric case [1] and the $\mathcal{N}=(0,1)$, $\mathcal{N}=(1,1)$, and $\mathcal{N}=(0,2)$ cases [36,38,39].

[10] See Appendix A of [39] for the relation between the operators $(\mathcal{Q}_\pm + \mathcal{Q}'_\pm)$, $(\bar{\mathcal{Q}}_\pm + \bar{\mathcal{Q}}'_\pm)$ and the generators of supersymmetry transformations on bilocal superfields such as $\mathcal{O}(x,x',\theta)$. The extension of that analysis from $\mathcal{N}=(0,2)$ to $\mathcal{N}=(2,2)$ is straightforward.

[11] Note that, consistently, one can show that

$$\begin{aligned}\{\mathcal{Q}_+,[\bar{\mathcal{Q}}_+,\{\mathcal{Q}_-,[\bar{\mathcal{Q}}_-,O(x,x')]\}]\} = &\,T_{----}(x)T_{++++}(x')\\ &- \Theta(x)\Theta(x') + \text{EOM's}\\ &+ [P,\ldots], \end{aligned} \tag{3.23}$$

implying that the descendant of the point-split primary operator $O(x)$ is equivalent, up to Ward identities and total vector derivatives $(\partial_{\pm\pm})$, to the point-split version of the descendant $T\bar{T}(x)$ operator.

IV. DEFORMED (2, 2) MODELS

In this section, we will apply our supercurrent-squared deformation (3.10) to a few examples of $\mathcal{N}=(2,2)$ supersymmetric theories for a chiral multiplet Φ. The superfield Φ can be written in components as

$$\begin{aligned}\Phi = &\,\phi + \theta^+\psi_+ + \theta^-\psi_- + \theta^+\theta^- F - i\theta^+\bar{\theta}^+\partial_{++}\phi\\ &- i\theta^-\bar{\theta}^-\partial_{--}\phi - i\theta^+\theta^-\bar{\theta}^-\partial_{--}\psi_+ - i\theta^-\theta^+\bar{\theta}^+\partial_{++}\psi_-\\ &- \theta^+\theta^-\bar{\theta}^-\bar{\theta}^+\partial_{++}\partial_{--}\phi, \end{aligned} \tag{4.1}$$

where ϕ is a complex scalar field, $\psi_\pm$ are Dirac fermions, and F is a complex auxiliary field. The multiplet Φ satisfies the chirality constraint $\bar{D}_\pm\Phi = 0$.

We denote the physical Lagrangian by $\mathcal{L}$ and the superspace D term Lagrangian by $\mathcal{A}$, so that

$$S = \int d^2x\mathcal{L} = \int d^2x d^4\theta\mathcal{A}. \tag{4.2}$$

A broad class of two-derivative theories for a chiral superfield can be described by superspace Lagrangians of the form

$$\mathcal{L} = \int d^4\theta K(\Phi,\bar{\Phi}) + \int d^2\theta W(\Phi) + \int d^2\bar{\theta}\,\bar{W}(\bar{\Phi}), \tag{4.3}$$

where $K(\Phi,\bar{\Phi})$ is a real function called the Kähler potential and $W(\Phi)$ is a holomorphic function called the superpotential. These are $\mathcal{N}=(2,2)$ Landau-Ginzburg models. In order for the kinetic terms of the component fields of Φ to have the correct sign, we will assume that $K_{\Phi\bar{\Phi}} = \frac{\partial^2 K}{\partial\Phi\partial\bar{\Phi}}$ is positive.

Although we will not expand on this point in detail, all the results found in this section can be derived almost identically for the case of a generic model of a single scalar twisted-chiral superfield $\mathcal{Y}$, $\bar{D}_+\mathcal{Y} = D_-\mathcal{Y} = 0$, and its conjugate. This is not surprising since theories containing only chiral superfields are physically equivalent to theories

formulated in terms of twisted-chiral superfields; see, for example, [50–54] for a discussion of this equivalence in models with global and local supersymmetry. There are also many more involved (2, 2) theories that one might also want to study involving chiral, twisted-chiral and semi-chiral superfields; see, for example, Ref. [55] for a recent discussion and references. For this analysis, we have chosen to consider only models based on a single chiral multiplet.

A. Kähler potential

First we will set the superpotential W to zero and begin with an undeformed superspace Lagrangian of the form

$$\mathcal{L} = \int d^4\theta K(\Phi, \bar{\Phi}) \tag{4.4}$$

for some Kähler potential K. To leading order around this undeformed theory, the FZ supercurrents are

$$\mathcal{J}_{\pm\pm} = 2K_{\Phi\bar{\Phi}} D_\pm \Phi \bar{D}_\pm \bar{\Phi}, \tag{4.5a}$$

$$\mathcal{V} = 0, \tag{4.5b}$$

where $K_\Phi = \frac{\partial K}{\partial \Phi}$, $K_{\Phi\bar{\Phi}} = \frac{\partial^2 K}{\partial \Phi \partial \bar{\Phi}}$, etc. Therefore, at first order the supercurrent-squared deformation driven by $\mathcal{O} = (-\mathcal{S}_{++}\mathcal{S}_{--} + 2\mathcal{V}\bar{\mathcal{V}})$ will source a four-fermion contribution in the D term, giving

$$\mathcal{L}^{(1)} = \mathcal{L}^{(0)} + \frac{1}{2}\lambda K^2_{\Phi\bar{\Phi}} D_+\Phi\bar{D}_+\bar{\Phi}D_-\Phi\bar{D}_-\bar{\Phi}. \tag{4.6}$$

Next, we would like to find the all-orders solution for the deformed theory. We make the ansatz that, at finite deformation parameter λ, the Lagrangian takes the form

$$\mathcal{L}_\lambda = \int d^4\theta \{K(\Phi, \bar{\Phi}) + f(\lambda, x, \bar{x}, y) K^2_{\Phi\bar{\Phi}} D_+\Phi\bar{D}_+\bar{\Phi}D_-\Phi\bar{D}_-\bar{\Phi}\}, \tag{4.7}$$

where we define the combinations

$$x = K_{\Phi\bar{\Phi}}\partial_{++}\Phi\partial_{--}\bar{\Phi}, \qquad y = K_{\Phi\bar{\Phi}}(D_+D_-\Phi)(\bar{D}_+\bar{D}_-\bar{\Phi}). \tag{4.8}$$

Using the results in Appendix B, one finds that the superfields $\mathcal{J}_{\pm\pm}$ and $\mathcal{V}$ appearing in our supercurrent-squared deformation, computed for the Lagrangian (4.7), are given by

$$\begin{aligned}
\mathcal{J}_{++} = {} & 2K_{\Phi\bar{\Phi}} D_+\Phi\bar{D}_+\bar{\Phi}\left[1 + f(x + \bar{x} - 3y) + x\frac{\partial f}{\partial x}(\bar{x} - y) + \bar{x}\frac{\partial f}{\partial \bar{x}}(x - y) + y\frac{\partial f}{\partial y}(x + \bar{x} - 2y)\right] \\
& + 2K^2_{\Phi\bar{\Phi}} D_-\Phi\bar{D}_-\bar{\Phi}\partial_{++}\Phi\partial_{++}\bar{\Phi}\left[-f - x\frac{\partial f}{\partial x} - \bar{x}\frac{\partial f}{\partial \bar{x}} + y\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial \bar{x}}\right)\right] \\
& - 2iK^2_{\Phi\bar{\Phi}} D_+\Phi D_-\Phi\partial_{++}\bar{\Phi}\bar{D}_+\bar{D}_-\bar{\Phi}\left[-f + (x - \bar{x})\frac{\partial f}{\partial \bar{x}} + (x - y)\frac{\partial f}{\partial y}\right] \\
& - 2iK^2_{\Phi\bar{\Phi}} \bar{D}_+\bar{\Phi}\bar{D}_-\bar{\Phi}\partial_{++}\Phi D_+D_-\Phi\left[f + (x - \bar{x})\frac{\partial f}{\partial x} + (y - \bar{x})\frac{\partial f}{\partial y}\right],
\end{aligned} \tag{4.9}$$

$$\begin{aligned}
\mathcal{J}_{--} = {} & 2K_{\Phi\bar{\Phi}} D_-\Phi\bar{D}_-\bar{\Phi}\left[1 + f(x + \bar{x} - 3y) + x\frac{\partial f}{\partial x}(\bar{x} - y) + \bar{x}\frac{\partial f}{\partial \bar{x}}(x - y) + y\frac{\partial f}{\partial y}(x + \bar{x} - 2y)\right] \\
& + 2K^2_{\Phi\bar{\Phi}} D_+\Phi\bar{D}_+\bar{\Phi}\partial_{--}\Phi\partial_{--}\bar{\Phi}\left[-f - x\frac{\partial f}{\partial x} - \bar{x}\frac{\partial f}{\partial \bar{x}} + y\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial \bar{x}}\right)\right] \\
& - 2iK^2_{\Phi\bar{\Phi}} D_+\Phi D_-\Phi\partial_{--}\bar{\Phi}\bar{D}_+\bar{D}_-\bar{\Phi}\left[-f + (\bar{x} - x)\frac{\partial f}{\partial x} + (\bar{x} - y)\frac{\partial f}{\partial y}\right] \\
& - 2iK^2_{\Phi\bar{\Phi}} \bar{D}_+\bar{\Phi}\bar{D}_-\bar{\Phi}\partial_{--}\Phi D_+D_-\Phi\left[f + (\bar{x} - x)\frac{\partial f}{\partial \bar{x}} + (y - x)\frac{\partial f}{\partial y}\right],
\end{aligned} \tag{4.10}$$

and

$$\begin{aligned}
\mathcal{V} = {} & 2K^2_{\Phi\bar{\Phi}}\left(f + y\frac{\partial f}{\partial y} + x\frac{\partial f}{\partial x} + \bar{x}\frac{\partial f}{\partial \bar{x}}\right)[-i\partial_{++}\bar{\Phi}(D_+D_-\Phi)D_-\Phi\bar{D}_-\bar{\Phi} + \partial_{++}\bar{\Phi}\partial_{--}\bar{\Phi}D_-\Phi D_+\Phi \\
& - \bar{D}_-\bar{\Phi}\bar{D}_+\bar{\Phi}(D_+D_-\Phi)^2 - i\partial_{--}\bar{\Phi}(D_+D_-\Phi)D_+\Phi\bar{D}_+\bar{\Phi}].
\end{aligned} \tag{4.11}$$

The supercurrent-squared flow then induces a differential equation for the superspace Lagrangian $\mathcal{A}_\lambda$ (where, again, $\mathcal{L}_\lambda = \int d^4\theta \mathcal{A}_\lambda$) given by

$$\frac{d}{d\lambda}\mathcal{A}_\lambda = -\frac{1}{8}\mathcal{O} = \frac{1}{8}(\mathcal{J}_{++}\mathcal{J}_{--} - 2\mathcal{V}\bar{\mathcal{V}}). \tag{4.12}$$

Given our ansatz (4.7), we see that

$$\frac{d\mathcal{A}_\lambda}{d\lambda} = \frac{df}{d\lambda}K^2_{\Phi\bar{\Phi}}D_+\Phi\bar{D}_+\bar{\Phi}D_-\Phi\bar{D}_-\bar{\Phi}. \tag{4.13}$$

On the other hand, plugging our expressions (4.9), (4.10), and (4.11) for the supercurrents into the right-hand side of (4.12) also gives a result proportional to $K^2_{\Phi\bar{\Phi}}D_+\Phi\bar{D}_+\bar{\Phi}D_-\Phi\bar{D}_-\bar{\Phi}$. Equating the coefficients, we find a differential equation for f:

$$\begin{aligned}\frac{df}{d\lambda} = \frac{1}{2}\Bigg\{&-\bar{x}y\left[f + (\bar{x}-x)\frac{\partial f}{\partial \bar{x}} + (y-x)\frac{\partial f}{\partial y}\right]^2 - xy\left[f + (x-\bar{x})\frac{\partial f}{\partial x} + (y-\bar{x})\frac{\partial f}{\partial y}\right]^2 \\ &+ 2(x-y)(y-\bar{x})\left[f + y\frac{\partial f}{\partial y} + \bar{x}\frac{\partial f}{\partial \bar{x}} + x\frac{\partial f}{\partial x}\right]^2 + x\bar{x}\left[f + (\bar{x}-y)\frac{\partial f}{\partial \bar{x}} + (x-y)\frac{\partial f}{\partial x}\right]^2 \\ &+ \left[1 + (x+\bar{x}-3y)f + (x+\bar{x}-2y)y\frac{\partial f}{\partial y} + \bar{x}(x-y)\frac{\partial f}{\partial \bar{x}} + x(\bar{x}-y)\frac{\partial f}{\partial x}\right]^2\Bigg\}.\end{aligned} \tag{4.14}$$

In particular, this shows that our ansatz (4.7) for the finite-λ superspace action is consistent: the supercurrent-squared deformation closes on an action of this form. It could have been otherwise: the flow equation might have sourced additional terms proportional, say, to two-fermion combinations $D_+\Phi\bar{D}_+\bar{\Phi}$, or required dependence on other dimensionless variables such as $\lambda(D_+D_-\Phi)^2$, but these complications do not arise in the case where the undeformed theory only has a Kähler potential.

On dimensional grounds, f must be proportional to λ times a function of the dimensionless combinations λx and λy. Thus, although the differential equation for f determined by (4.14) is complicated, one can solve order by order in λ. The solution to $\mathcal{O}(\lambda^3)$ is

$$\begin{aligned}f(\lambda, x, \bar{x}, y) = &\frac{\lambda}{2} + \lambda^2\left(\frac{x+\bar{x}}{4} - \frac{3}{4}y\right) \\ &+ \lambda^3\left(\frac{x^2+\bar{x}^2+3x\bar{x}}{8} + \frac{37}{24}y^2 - \frac{25}{24}(x+\bar{x})y\right) \\ &+ \cdots.\end{aligned} \tag{4.15}$$

We were unable to find a closed-form expression for f to all orders in λ. However, the differential equation simplifies dramatically when we impose the equations of motion for the theory, and in this case one can write down an exact formula. This is similar to the $T\bar{T}$ flow of the free action for a real $\mathcal{N} = (1,1)$ scalar multiplet that was analyzed in [36,38].

We claim that, on-shell, one may drop any terms where $y \sim (D_+D_-\Phi)(\bar{D}_+\bar{D}_-\bar{\Phi})$ multiplies the four-fermion term $|D\Phi|^4 \equiv D_+\Phi\bar{D}_+\bar{\Phi}D_-\Phi\bar{D}_-\bar{\Phi}$. This is shown explicitly in Appendix C and follows directly from the superspace equation of motion and nilpotency of the fermionic terms $D_\pm\Phi$ and $\bar{D}_\pm\bar{\Phi}$. It is also an intuitive statement associated with the fact that for these models, on-shell, $\mathcal{N} = (2,2)$ supersymmetry is not broken. In fact, note that the superfields $(D_+D_-\Phi)$ and $(\bar{D}_+\bar{D}_-\bar{\Phi})$ have as their lowest components the auxiliary fields F and $\bar{F}$. If supersymmetry is not broken, the VEV of F has to be zero, $\langle F\rangle = 0$, which implies that the auxiliary field F is on-shell at least quadratic in fermions and, more precisely, can be proven to be at least linear in $\psi_\pm = D_\pm\Phi|_{\theta=0}$ and $\bar{\psi}_\pm = \bar{D}_\pm\bar{\Phi}|_{\theta=0}$. From this argument it follows that on-shell $(D_+D_-\Phi)$ is at least linear in $D_\pm\Phi$ and $\bar{D}_\pm\bar{\Phi}$, and then the two conditions $(D_+D_-\Phi)|D\Phi|^4 = 0$ and $y|D\Phi|^4 = 0$ follow.

After removing from (4.14) the y-dependent terms which vanish on-shell, we find a simpler differential equation for the function f,

$$\begin{aligned}\frac{df}{d\lambda} = \frac{1}{2}\Bigg\{&-x\bar{x}\left[f + x\frac{\partial f}{\partial x} + \bar{x}\frac{\partial f}{\partial \bar{x}}\right]^2 \\ &+ \left[1 + (x+\bar{x})f + x\bar{x}\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial \bar{x}}\right)\right]^2\Bigg\},\end{aligned} \tag{4.16}$$

whose solution is

$$\begin{aligned}&f(\lambda, x, \bar{x}, y = 0) \\ &= \frac{\lambda}{1 - \frac{\lambda}{2}(x+\bar{x}) + \sqrt{1 - \lambda(x+\bar{x}) + \frac{\lambda^2}{4}(x-\bar{x})^2}}.\end{aligned} \tag{4.17}$$

Thus we have shown that the supercurrent-squared deformed Lagrangian at finite λ is equivalent on-shell to the following superspace Lagrangian:

$$\mathcal{L}_\lambda = \int d^4\theta \Bigg(K(\Phi,\bar{\Phi}) + \frac{\lambda K^2_{\Phi\bar{\Phi}} D_+\Phi \bar{D}_+\bar{\Phi} D_-\Phi \bar{D}_-\bar{\Phi}}{1-\frac{1}{2}\lambda K^2_{\Phi\bar{\Phi}} A + \sqrt{1-\lambda K^2_{\Phi\bar{\Phi}} A + \frac{1}{4}\lambda^2 K^4_{\Phi\bar{\Phi}} B^2}} \Bigg), \tag{4.18}$$

where

$$A = \partial_{++}\Phi\partial_{--}\bar{\Phi} + \partial_{++}\bar{\Phi}\partial_{--}\Phi,$$
$$B = \partial_{++}\Phi\partial_{--}\bar{\Phi} - \partial_{++}\bar{\Phi}\partial_{--}\Phi. \tag{4.19}$$

When $K(\Phi,\bar{\Phi}) = \bar{\Phi}\Phi$, it is simple to show that this model represents an $\mathcal{N} = (2,2)$ off-shell supersymmetric extension of the $D = 4$ Nambu-Goto string in an appropriate gauge—often referred to as a static gauge in the presence of a B field, though it can be more naturally described as a uniform light-cone gauge [56,57] (see Refs. [23,37] for a discussion of this point). In particular, by setting various component fields to zero and performing the superspace integrals, one can show that (4.18) matches the expected answer for $T\bar{T}$ deformations in previously known nonsupersymmetric cases. For instance, setting the fermions to zero and integrating out the auxiliary fields F and $\bar{F}$ gives the $T\bar{T}$ deformation of the complex free boson ϕ, whose Lagrangian is

$$\mathcal{L}_{\lambda,\mathrm{bos}} = \frac{\sqrt{1+2\lambda a+\lambda^2 b^2}-1}{4\lambda} = \frac{a}{4} - \lambda\frac{\partial_{++}\phi\partial_{--}\phi\partial_{++}\bar{\phi}\partial_{--}\bar{\phi}}{1+\lambda a+\sqrt{1+2\lambda a+\lambda^2 b^2}}, \tag{4.20}$$

where

$$a = \partial_{++}\phi\partial_{--}\bar{\phi} + \partial_{++}\bar{\phi}\partial_{--}\phi,$$
$$b = \partial_{++}\phi\partial_{--}\bar{\phi} - \partial_{++}\bar{\phi}\partial_{--}\phi. \tag{4.21}$$

The Lagrangian (4.20) indeed describes the $D = 4$ light-cone gauge-fixed Nambu-Goto string model.

Alternatively, setting all the bosons to zero in (4.18) can be shown to give the $T\bar{T}$ deformation of a complex free fermion. These calculations are similar to those in the case of the (0, 2) supercurrent-squared action, which are presented in [39]. In fact, it can even be easily shown that an $\mathcal{N} = (0,2)$ truncation of (4.18) gives precisely the $T\bar{T}$ deformation of a free $\mathcal{N} = (0,2)$ chiral multiplet that was derived in [39].

It is worth highlighting that, unlike the $\mathcal{N} = (2,2)$ case, an off-shell (0, 2) chiral scalar multiplet contains only physical degrees of freedom and no auxiliary fields. Interestingly, related to this fact, it turns out that (up to integration by parts and total derivatives) the $\mathcal{N} = (0,2)$ off-shell supersymmetric extension of the $D = 4$ Nambu-Goto string action in the light-cone gauge is unique and precisely matches the off-shell $T\bar{T}$ deformation of a free $\mathcal{N} = (0,2)$ chiral multiplet action [39].

In the $\mathcal{N} = (2,2)$ case, because of the presence of the auxiliary field F in the chiral multiplet Φ, there are an infinite set of inequivalent $\mathcal{N} = (2,2)$ off-shell extensions of the Lagrangian (4.20) that are all equivalent on-shell. A representative of these equivalent actions is described by (4.18) when $K(\Phi,\bar{\Phi}) = \bar{\Phi}\Phi$.

The nonuniqueness of dynamical systems described by actions of the form (4.18) can also be understood by noticing that, for example, it is possible to perform a class of redefinitions that leaves the action (4.18) invariant on-shell. As a (very particular) example, note that we are free to perform a shift of the form

$$D_+\bar{D}_-(\bar{D}_+\bar{\Phi}D_-\Phi) \to D_+\bar{D}_-(\bar{D}_+\bar{\Phi}D_-\Phi) + a(D_+D_-\Phi + \bar{D}_+\bar{D}_-\bar{\Phi})^2 \tag{4.22}$$

for any real number a. In terms of A and B, Eq. (4.22) implements the shifts

$$A \to A + a((D_+D_-\Phi)^2 + 2y + (\bar{D}_+\bar{D}_-\bar{\Phi})^2), \qquad B \to B \tag{4.23}$$

in (4.18). The resulting Lagrangian would enjoy the same on-shell simplifications described in Appendix C and would turn out to be on-shell equivalent to the Lagrangian (4.18). In this infinite set of on-shell equivalent actions, a particular choice would represent an exact solution of the $T\bar{T}$ flow equation (4.12)–(4.14), whose leading terms in a λ series expansion are given in (4.15). Another representative in this on-shell equivalence class is the simplified model described by (4.18).

These types of redefinition and on-shell equivalentness are not a surprise, nor really new. In fact, they are of the same nature as redefinitions that have been studied in detail in [58] (see also [59] for a description of these types of "trivial symmetries") in the context of $D = 4, \mathcal{N} = 1$ chiral and linear superfield models possessing a nonlinearly realized additional supersymmetry [58,60]. As in (4.23), the field redefinition in this context does not affect the dynamics of the physical fields—it basically corresponds only to an arbitrariness in the definition of the auxiliary fields that always appear quadratically in the action and then are set to zero (up to fermion terms that will not contribute due to nilpotency in the action) on-shell. Although here we only focused on discussing the on-shell ambiguity of the solution of the $\mathcal{N} = (2,2)$ $T\bar{T}$ flow, we expect that the exact solution of the flow equations with y nonzero (4.12)–(4.14) can be found by a field redefinition of the kind we made in the action (4.18).

It is also interesting to note that similar freedoms and field redefinitions are also described in the construction of $D = 4$, $\mathcal{N} = 1$ supersymmetric Born-Infeld actions; see,

for example, Ref. [61]. In fact, as will be analyzed in more detail elsewhere [43], it can be shown that the Lagrangian (4.18) is structurally of the type described by Bagger and Galperin for the $D = 4$, $\mathcal{N} = 1$ supersymmetric Born-Infeld action [61]. The equivalence can be formally shown by identifying $W_+ = \bar{D}_+\bar{\Phi}$, $W_- = D_-\Phi$, $W^2 = \bar{D}_+\bar{\Phi}D_-\Phi$, and $D^\alpha W_\alpha = D_+D_-\Phi + \bar{D}_-\bar{D}_+\bar{\Phi}$ to match their conventions. As a consequence, we can show that our solution for the $T\bar{T}$ flow possesses a second nonlinearly realized $\mathcal{N} = (2,2)$ supersymmetry, besides the (2, 2) supersymmetry which is made manifest by the superspace construction. This property is analyzed in detail in [43]. We note that the presence of a second supersymmetry is analogous to what happens in the $\mathcal{N} = (0,2)$ case [39].

B. Adding a superpotential

Now suppose we begin with an undeformed theory that has a superpotential $W(\Phi)$,

$$\mathcal{L}^{(0)} = \int d^4\theta K(\Phi,\bar{\Phi}) + \left(\int d^2\theta W(\Phi)\right) + \left(\int d^2\bar{\theta}\,\bar{W}(\bar{\Phi})\right). \tag{4.24}$$

As shown in Appendix B, the superpotential F term gives a contribution $\delta\mathcal{V} = 2W(\Phi)$ to the field $\mathcal{V}$ which appears in supercurrent squared. To leading order in the deformation parameter, the Lagrangian takes the form

$$\begin{aligned}\mathcal{L}^{(0)} &\to \mathcal{L}^{(0)} + \mathcal{L}^{(1)} \\ &= \mathcal{L}^{(0)} + \lambda\int d^4\theta \\ &\quad\times\left(\frac{1}{2}K^2_{\Phi\bar{\Phi}}D_+\Phi\bar{D}_+\bar{\Phi}D_-\Phi\bar{D}_-\bar{\Phi} + W(\Phi)\bar{W}(\bar{\Phi})\right).\end{aligned} \tag{4.25}$$

In addition to the four-fermion term which we saw in Sec. IV A, we see that the deformation modifies the Kähler potential, adding a term proportional to $|W(\Phi)|^2$.

Next consider the second order term in λ. For convenience, we use the combination $|D\Phi|^4 = D_+\Phi\bar{D}_+\bar{\Phi}D_-\Phi\bar{D}_-\bar{\Phi}$, which is the four-fermion combination that appeared at first order. Then

$$\begin{aligned}\mathcal{L}^{(2)} = \frac{\lambda^2}{4}\int d^4\theta(&x + \bar{x} - 3y - 2|W'(\Phi)|^2 \\ &+ WD_-D_+ + \bar{W}\bar{D}_+\bar{D}_-)|D\Phi|^4.\end{aligned} \tag{4.26}$$

The new terms involving supercovariant derivatives of $|D\Phi|^4$ will generate contributions with two fermions in the D term.

As we continue perturbing to higher orders, the form of the superspace Lagrangian becomes more complicated. It is no longer true that the supercurrent-squared flow closes on a simple ansatz with one undetermined function, as it did in the case with only a Kähler potential. Indeed, the finite-λ deformed superspace Lagrangian in the case with a superpotential will depend not only on the variables x, $\bar{x}$, and y as in Sec. IV A but also, for example, on combinations such as $\partial_{++}\Phi\bar{D}_+\bar{D}_-\bar{\Phi}$, which can appear multiplying the two-fermion term $D_-\Phi\bar{D}_-\bar{\Phi}$ in the superspace Lagrangian. To find the full solution, one would need to determine several functions contributing to the D term—one multiplying the four-fermion term $|D\Phi|^4$ as in the Kähler case; one for the deformed Kähler potential which may now depend on x, y, and other combinations; and four functions multiplying the two-fermion terms $D_+\Phi D_-\Phi$, $D_+\Phi\bar{D}_-\bar{\Phi}$, etc. Each function can depend on several dimensionless combinations.

In the presence of a superpotential, the situation might further be complicated by the fact that supersymmetry can be spontaneously broken. This would make it impossible, for example, to use on-shell simplifications such as $y|D\Phi|^4 = 0$ that we employed in Sec. IV A, where supersymmetry is never spontaneously broken.

It should be clear that the case with a superpotential is significantly more involved and rich than just a pure Kähler potential. In this case, we have not attempted to find a solution of the $T\bar{T}$ flow equation in closed form. However, it is evident from the form of supercurrent-squared Eq. (4.12)—which is always written as a D term integral of current bilinears—that this deformation will only affect the D term and not the $\mathcal{N} = (2,2)$ superpotential W appearing in the chiral integral. Therefore the superpotential, besides being protected from perturbative quantum corrections, is also protected from corrections along the supercurrent-squared flow.

C. The physical classical potential

In view of the difficulty of finding the all-orders deformed superspace action for a theory with a superpotential, we now consider the simpler problem of finding the local-potential approximation (or zero-momentum potential) for the bosonic complex scalar ϕ contained in the superfield Φ. We stress that our analysis here is purely classical, and we will make a couple of comments about possible quantum effects later in this section. For simplicity, we will also restrict to the case in which the Kähler potential is flat, $K(\Phi,\bar{\Phi}) = \bar{\Phi}\Phi$. By "zero-momentum potential" we mean the physical potential $V(\phi)$ which appears in the Lagrangian after performing the superspace integral in the deformed theory and then setting $\partial_{\pm\pm}\phi = 0$. For instance, consider the undeformed Lagrangian

$$\mathcal{L}^{(0)} = \int d^4\theta\bar{\Phi}\Phi + \int d^2\theta W(\Phi) + \int d^2\bar{\theta}\,\bar{W}(\bar{\Phi}). \tag{4.27}$$

When we ignore all terms involving derivatives and the fermions $\psi_\pm$, the only contributions to the physical Lagrangian (after performing the superspace integral) come

from an $|F|^2$ term from the kinetic term, plus the term $W(\Phi) = W(\phi) + W'(\phi)\theta^+\theta^- F$. This gives us the zero-momentum, zero-fermion component action

$$S = \int d^2x(|F|^2 + W'(\phi)F + \bar{W}'(\bar{\phi})\bar{F}). \tag{4.28}$$

We may integrate out the auxiliary field F using its equation of motion $\bar{F} = -W'(\phi)$, which yields

$$S = \int d^2x(-|W'(\phi)|^2), \tag{4.29}$$

so the zero-momentum potential for ϕ is $V = |W'(\phi)|^2$, as expected. Note that the previous potential might have extrema that break $\mathcal{N} = (2, 2)$ supersymmetry while supersymmetric vacua will always set $\langle F\rangle = \langle W'(\phi)\rangle = 0$. We will assume supersymmetry of the undeformed theory not to be spontaneously broken in our discussion.

Now suppose we deform by the supercurrent-squared operator to second order in λ, which gives the superspace expression (4.26). If we again perform the superspace integral and discard any terms involving derivatives or fermions, we now find the physical Lagrangian

$$\begin{aligned}\mathcal{L}|_{\partial_{\pm\pm}\phi=0} = |F|^2 + FW' + \bar{F}\bar{W}' + \lambda\left(\frac{1}{2}|F|^4 - |F|^2|W'|^2\right) \\ + \frac{1}{4}\lambda^2|F|^4(W'F + \bar{W}'\bar{F}) \\ - \frac{1}{2}\lambda^2|W'|^2|F|^4 + \frac{3}{4}\lambda^2|F|^6.\end{aligned} \tag{4.30}$$

Remarkably, the equations of motion for the auxiliary F in (4.30) admit the solution $F = -\bar{W}'(\bar{\phi})$, $\bar{F} = -W'(\phi)$, which is the same as the unperturbed solution. This, for instance, implies that if we start from a supersymmetric vacua in the undeformed theory, we will remain supersymmetric along the $T\bar{T}$ flow. On the one hand, this is not a surprise considering that we know the $T\bar{T}$ flow preserves the structure of the spectrum, and in particular should leave a zero-energy supersymmetric vacuum unperturbed. On the other hand, it is a reassuring check to see this property explicitly appearing in our analysis.

Returning to (4.30) and integrating out the auxiliary fields gives

$$\mathcal{L}|_{\partial_{\pm\pm}\phi=0} = -|W'(\phi)|^2 - \frac{1}{2}\lambda|W'(\phi)|^4 - \frac{1}{4}\lambda^2|W'(\phi)|^6. \tag{4.31}$$

These are the leading terms in the geometric series $\frac{-|W'|^2}{1-\frac{1}{2}\lambda|W'|^2}$. In fact, up to conventions for the scaling of λ, one could have predicted this outcome from the form of the supercurrent-squared operator and the known results for $T\bar{T}$ deformations of a bosonic theory with a potential [3]. We know that, up to terms which vanish on-shell, the effect of adding supercurrent squared to the physical Lagrangian is to deform by the usual $T\bar{T}$ operator. However, in the zero-momentum sector, we see that the $T\bar{T}$ deformation reduces to deforming by the square of the potential:

$$T\bar{T}|_{\partial_{\pm\pm}\phi=0} = \mathcal{L}^2|_{\partial_{\pm\pm}\phi=0} = V^2. \tag{4.32}$$

Therefore, it is easy to solve for the deformed potential if we deform a physical Lagrangian $\mathcal{L} = f(\lambda, \partial_{\pm\pm}\phi) + V(\lambda, \phi)$ by $T\bar{T}$, since the flow equation for the potential term is simply

$$\partial_\lambda\mathcal{L} = \frac{\partial V}{\partial\lambda} = V^2, \tag{4.33}$$

which admits the solution

$$V(\lambda, \phi) = \frac{V(0, \phi)}{1 - \lambda V(0, \phi)}. \tag{4.34}$$

We can apply this result to the Lagrangian (4.28), treating the entire expression involving the auxiliary field F as a potential (since it is independent of derivatives). The deformed theory has a zero-momentum piece which is therefore equivalent to

$$S(\lambda)|_{\partial_{\pm\pm}\phi=0} = \int d^2x\frac{(|F|^2 + W'(\phi)F + \bar{W}'(\bar{\phi})\bar{F})}{1 - \lambda(|F|^2 + W'(\phi)F + \bar{W}'(\bar{\phi})\bar{F})}, \tag{4.35}$$

at least on-shell. Integrating out the auxiliary now gives

$$S(\lambda)|_{\partial_{\pm\pm}\phi=0} = \int d^2x\frac{-|W'(\phi)|^2}{1 - \lambda|W'(\phi)|^2} \tag{4.36}$$

as the deformed physical potential. This matches the first few terms of (4.31), up to a convention-dependent factor of $\frac{1}{2}$ in the scaling of λ.

Now one might ask what superspace Lagrangian would yield the physical action (4.36) after performing the $d\theta$ integrals. One candidate is

$$\begin{aligned}\mathcal{L}(\lambda)|_{\partial_{\pm\pm}\phi=0} \sim \int d^4\theta(\bar{\Phi}\Phi - \lambda|W(\Phi)|^2) + \int d^2\theta W(\Phi) \\ + \int d^2\bar{\theta}\,\bar{W}(\bar{\Phi}),\end{aligned} \tag{4.37}$$

where $\sim$ means "this superspace Lagrangian gives an equivalent zero-momentum physical potential for the boson ϕ on-shell."

It is important to note that (4.37) is *not* the true solution for the deformed superspace Lagrangian using supercurrent squared. The genuine solution involves a four-fermion

term, all possible two-fermion terms, and more complicated dependence on the variable $y = \lambda(D_+D_-\Phi)(\bar{D}_+\bar{D}_-\bar{\Phi})$ in the zero-fermion term. However, if one were to perform the superspace integral in the true solution and then integrate out the auxiliary field F using its equation of motion, one would obtain the same zero-momentum potential for ϕ as we find by performing the superspace integral in (4.37) and integrating out F.

The form (4.37) is interesting because it shows that the effect of supercurrent-squared on the physical potential for ϕ can be interpreted as a change in the Kähler metric, which for this Lagrangian is

$$K_{\Phi\bar{\Phi}} = 1 - \lambda|W'(\Phi)|^2. \tag{4.38}$$

When one performs the superspace integrals in (4.37), the result is

$$\mathcal{L}|_{\partial_{\pm\pm}\phi=0} = K_{\Phi\bar{\Phi}}|F|^2 + W'(\phi)F + \bar{W}'(\bar{\phi})\bar{F}, \tag{4.39}$$

which admits the solution $F = -\frac{\bar{W}'(\bar{\phi})}{K_{\Phi\bar{\Phi}}}$. Substituting this solution gives

$$\mathcal{L}|_{\partial_{\pm\pm}\phi=0} = \frac{-|W'(\phi)|^2}{K_{\Phi\bar{\Phi}}} = \frac{-|W'(\phi)|^2}{1-\lambda|W'(\phi)|^2}, \tag{4.40}$$

which agrees with (4.36).

As already mentioned, supersymmetric vacua of the original, undeformed, theory are associated with critical points of the superpotential $W(\phi)$. Any vacuum of the undeformed theory will persist in the deformed theory: near a point where $W'(\phi) = 0$, we see that the physical potential $V(\phi) = \frac{|W'|^2}{1-\lambda|W'|^2}$ also vanishes (away from the pole $|W'|^2 = \frac{1}{\lambda}$, the deformed potential is a monotonically increasing function of $|W'|^2$). Further, the auxiliary field F does not acquire a vacuum expectation value because $F = -\bar{W}'(\bar{\phi})$ remains a solution to its equations of motion in the deformed theory. Once more, this indicates that supersymmetry is unbroken along the whole $T\bar{T}$ flow if it is in the undeformed theory.

However, this classical analysis suggests that the soliton spectrum of the theory has changed dramatically at any finite deformation parameter λ. There are now generically poles in the physical potential $V(\phi)$ at points where $|W'|^2 = \frac{1}{\lambda}$ which might separate distinct supersymmetric vacua of the theory. For instance, if the original theory had a double-well superpotential with two critical points ϕ_1, ϕ_2 where $W'(\phi_i) = 0$, then this undeformed theory supports Bogomol'nyi-Prasad-Sommerfield (BPS) soliton solutions which interpolate between these two vacua. But if the superpotential W reaches a value of order $\frac{1}{\lambda}$ at some point between ϕ_1 and ϕ_2, then this soliton solution appears naively forbidden in the deformed theory because it requires crossing an infinite potential barrier. Another way of seeing this is by considering the effective Kähler potential (4.38), which would change sign at some point between the two supersymmetric vacua in the deformed theory and thus give rise to a negative-definite Kähler metric.

Our discussion has been purely classical. As we emphasized in the Introduction, a fully quantum analysis of this problem is desirable, though subtle because of the nonlocal nature of the $T\bar{T}$ deformation. The advantage of performing such an analysis in models with extended supersymmetry is that holomorphy and associated nonrenormalization theorems provide control over the form of any possible quantum corrections. For example, the superpotential for the models studied in this work is not renormalized perturbatively along the flow. It would be interesting to examine the structure of perturbative quantum corrections along the lines of [62], but in superspace with manifest supersymmetry. It should be possible to absorb any quantum corrections visible in perturbation theory by a change in the D-term Kähler potential, meaning that at least the structure of the supersymmetric vacua would be preserved.

ACKNOWLEDGMENTS

We thank Marco Baggio for discussions and collaboration in the initial stages of this project. C. F. acknowledges support from the University of Chicago's divisional MS-PSD program. C. C., C. F., and S. S. are supported in part by NSF Grant No. PHY1720480. This work is partially supported through a research grant of the Swiss National Science Foundation, as well as by the NCCR SwissMAP, funded by the Swiss National Science Foundation. The work of A. S. was also supported by ETH Career Seed Grant No. SEED-23 19-1. The work of G. T.-M. is supported by the Albert Einstein Center for Fundamental Physics, University of Bern, and by the Australian Research Council (ARC) Future Fellowship FT180100353. We also thank the organizers and participants of the workshop on "*$T\bar{T}$ and Other Solvable Deformations of Quantum Field Theories*" for providing a stimulating atmosphere, and the Simons Center for Geometry and Physics for hospitality and partial support during the final stages of this work. We thank the participants of the workshop "New frontiers of integrable deformations" at Villa Garbald in Castasegna for stimulating discussions.

APPENDIX A: THE $\mathcal{S}$ MULTIPLET IN COMPONENTS

In this appendix we provide the component expansion of the superfields of the $\mathcal{S}$ multiplet introduced in Sec. II B. The results presented below are equivalent to the results first obtained in [45] up to differences in notation.

The constraints (2.9) are solved in terms of component fields by

$$\begin{aligned}\mathcal{S}_{\pm\pm} = {} & j_{\pm\pm} - i\theta^{\pm}S_{\pm\pm\pm} - i\theta^{\mp}(S_{\mp\pm\pm} \mp 2\sqrt{2}i\bar{\rho}_{\pm}) - i\bar{\theta}^{\pm}\bar{S}_{\pm\pm\pm} \\ & - i\bar{\theta}^{\mp}(\bar{S}_{\mp\pm\pm} \pm 2\sqrt{2}i\rho_{\pm}) - \theta^{\pm}\bar{\theta}^{\pm}T_{\pm\pm\pm\pm} + \theta^{\mp}\bar{\theta}^{\mp}\left(A \mp \frac{k+k'}{2}\right) \\ & + i\theta^{+}\theta^{-}\bar{Y}_{\pm\pm} + i\bar{\theta}^{+}\bar{\theta}^{-}Y_{\pm\pm} \pm i\theta^{+}\bar{\theta}^{-}\bar{G}_{\pm\pm} \mp i\theta^{-}\bar{\theta}^{+}G_{\pm\pm} \\ & \mp \frac{1}{2}\theta^{+}\theta^{-}\bar{\theta}^{\pm}\partial_{\pm\pm}S_{\mp\pm\pm} \mp \frac{1}{2}\theta^{+}\theta^{-}\bar{\theta}^{\mp}\partial_{\pm\pm}(S_{\pm\mp\mp} \pm 2\sqrt{2}i\bar{\rho}_{\mp}) \\ & \mp \frac{1}{2}\bar{\theta}^{+}\bar{\theta}^{-}\theta^{\pm}\partial_{\pm\pm}\bar{S}_{\mp\pm\pm} \mp \frac{1}{2}\bar{\theta}^{+}\bar{\theta}^{-}\theta^{\mp}\partial_{\pm\pm}(\bar{S}_{\pm\mp\mp} \mp 2\sqrt{2}i\rho_{\mp}) + \frac{1}{4}\theta^{+}\theta^{-}\bar{\theta}^{+}\bar{\theta}^{-}\partial_{\pm\pm}^{2}j_{\mp\mp}. \end{aligned} \tag{A1}$$

Let us introduce the usual useful combinations: $y^{\pm\pm} = x^{\pm\pm} - \frac{i}{2}\theta^{\pm}\bar{\theta}^{\pm}$ and $\tilde{y}^{\pm\pm} = x^{\pm\pm} \mp \frac{i}{2}\theta^{\pm}\bar{\theta}^{\pm}$. The chiral superfields $\chi_{\pm}$ are

$$\chi_{+} = -i\lambda_{+}(y) - i\theta^{+}\bar{G}_{++}(y) + \theta^{-}\left(E(y) + \frac{k}{2}\right) + \bar{\theta}^{-}C^{(-)} + \theta^{+}\theta^{-}\partial_{++}\bar{\lambda}_{-}(y), \tag{A2a}$$

$$\chi_{-} = -i\lambda_{-}(y) - \theta^{+}\left(\bar{E}(y) - \frac{k}{2}\right) + i\theta^{-}G_{--}(y) - \bar{\theta}^{+}C^{(+)} - \theta^{+}\theta^{-}\partial_{--}\bar{\lambda}_{+}(y), \tag{A2b}$$

$$\lambda_{\pm} = \pm\bar{S}_{\mp\pm\pm} + \sqrt{2}i\rho_{\pm}, \tag{A2c}$$

$$E = \frac{1}{2}(\Theta - A) + \frac{i}{4}(\partial_{++}j_{--} - \partial_{--}j_{++}), \tag{A2d}$$

$$0 = \partial_{++}G_{--} - \partial_{--}G_{++}, \tag{A2e}$$

and the twisted-(anti)chiral superfields $\mathcal{Y}_{\pm}$ are given by

$$\mathcal{Y}_{+} = \sqrt{2}\rho_{+}(\tilde{y}) + \theta^{-}\left(F(\tilde{y}) + \frac{k'}{2}\right) - i\bar{\theta}^{+}Y_{++}(\tilde{y}) - \bar{\theta}^{-}C^{(-)} + \sqrt{2}i\theta^{-}\bar{\theta}^{+}\partial_{++}\rho_{-}(\tilde{y}), \tag{A3a}$$

$$\mathcal{Y}_{-} = \sqrt{2}\rho_{-}(\tilde{y}) - \theta^{+}\left(F(\tilde{y}) - \frac{k'}{2}\right) + \bar{\theta}^{+}C^{(+)} - i\bar{\theta}^{-}Y_{--}(\tilde{y}) + \sqrt{2}i\theta^{+}\bar{\theta}^{-}\partial_{--}\rho_{+}(\tilde{y}), \tag{A3b}$$

$$F = -\frac{1}{2}(\Theta + A) - \frac{i}{4}(\partial_{++}j_{--} + \partial_{--}j_{++}), \tag{A3c}$$

$$0 = \partial_{++}Y_{--} - \partial_{--}Y_{++}. \tag{A3d}$$

For the FZ multiplet defined by the constraints (2.18), the $\mathcal{S}$-multiplet reduces to a set of $4+4$ real independent component fields described by the $j_{\pm\pm}$ $U(1)_A$ axial conserved R-symmetry current ($\partial_{++}j_{--} - \partial_{--}j_{++} = 0$). In addition, there is a complex scalar field $v(x)$ [see Eq. (3.14)], together with the independent supersymmetry current and energy momentum tensor:

$$S_{\pm\pm\pm}(x) \coloneqq iD_{\pm}\mathcal{J}_{\pm\pm}(\zeta)|_{\theta=0}, \tag{A4a}$$

$$\bar{S}_{\pm\pm\pm}(x) \coloneqq -i\bar{D}_{\pm}\mathcal{J}_{\pm\pm}(\zeta)|_{\theta=0}, \tag{A4b}$$

$$S_{\mp\pm\pm}(x) \coloneqq -iD_{\mp}\mathcal{J}_{\pm\pm}(\zeta)|_{\theta=0} = \pm i\bar{D}_{\pm}\bar{\mathcal{V}}(\zeta)|_{\theta=0}, \tag{A4c}$$

$$\bar{S}_{\mp\pm\pm}(x) \coloneqq i\bar{D}_{\mp}\mathcal{J}_{\pm\pm}(\zeta)|_{\theta=0} = \mp iD_{\pm}\mathcal{V}(\zeta)|_{\theta=0}, \tag{A4d}$$

$$T_{\pm\pm\pm\pm}(x) \coloneqq \frac{1}{2}[D_{\pm}, \bar{D}_{\pm}]\mathcal{J}_{\pm\pm}(\zeta)|_{\theta=0}, \tag{A4e}$$

$$\Theta(x) \coloneqq -\frac{1}{2}[D_+, \bar{D}_+]\mathcal{J}_{--}(\zeta)|_{\theta=0} = -\frac{1}{2}[D_-, \bar{D}_-]\mathcal{J}_{++}(\zeta)|_{\theta=0}$$
$$= -\frac{1}{2}D_+D_-\mathcal{V}(\zeta)|_{\theta=0} + \frac{1}{2}\bar{D}_+\bar{D}_-\bar{\mathcal{V}}(\zeta)|_{\theta=0}. \tag{A4f}$$

For the FZ multiplet, the following relation holds:

$$\begin{aligned}\mathcal{J}_{\pm\pm} = {} & j_{\pm\pm} - i\theta^{\pm}S_{\pm\pm\pm} - i\bar{\theta}^{\pm}\bar{S}_{\pm\pm\pm} + i\theta^{\mp}S_{\mp\pm\pm} + i\bar{\theta}^{\mp}\bar{S}_{\mp\pm\pm} \\ & - \theta^{\pm}\bar{\theta}^{\pm}T_{\pm\pm\pm\pm} + \theta^{\mp}\bar{\theta}^{\mp}\Theta + i\theta^{+}\theta^{-}\partial_{\pm\pm}\bar{v} + i\bar{\theta}^{+}\bar{\theta}^{-}\partial_{\pm\pm}v \\ & \mp \frac{1}{2}\theta^{+}\theta^{-}\bar{\theta}^{\pm}\partial_{\pm\pm}S_{\mp\pm\pm} \pm \frac{1}{2}\theta^{+}\theta^{-}\bar{\theta}^{\mp}\partial_{\pm\pm}S_{\pm\mp\mp} \\ & \mp \frac{1}{2}\bar{\theta}^{+}\bar{\theta}^{-}\theta^{\pm}\partial_{\pm\pm}\bar{S}_{\mp\pm\pm} \pm \frac{1}{2}\bar{\theta}^{+}\bar{\theta}^{-}\theta^{\mp}\partial_{\pm\pm}\bar{S}_{\pm\mp\mp} + \frac{1}{4}\theta^{+}\theta^{-}\bar{\theta}^{+}\bar{\theta}^{-}\partial_{\pm\pm}^2 j_{\mp\mp}.\end{aligned} \tag{A5}$$

Moreover, the chiral superfields $\chi_\pm$ are set to zero and the twisted-(anti)chiral superfields $\mathcal{Y}_\pm = D_\pm\mathcal{V}$ are given by

$$\mathcal{Y}_+ = i\bar{S}_{-++}(\tilde{y}) + \theta^-G(\tilde{y}) - i\bar{\theta}^+\partial_{++}v(\tilde{y}) + \theta^-\bar{\theta}^+\partial_{++}\bar{S}_{+--}(\tilde{y}), \tag{A6a}$$

$$\mathcal{Y}_- = -i\bar{S}_{+--}(\tilde{y}) - \theta^+G(\tilde{y}) - i\bar{\theta}^-\partial_{--}v(\tilde{y}) + \theta^+\bar{\theta}^-\partial_{++}\bar{S}_{---}(\tilde{y}), \tag{A6b}$$

$$G = -\Theta - \frac{i}{2}\partial_{++}j_{--}. \tag{A6c}$$

APPENDIX B: DETAILS OF THE SUPERCURRENT MULTIPLET CALCULATION

In this appendix, we compute the fields $\mathcal{J}_{\pm\pm}$ and σ appearing in the FZ multiplet for Lagrangians of a chiral superfield Φ with the general form

$$\mathcal{L}_0 = \left(\int d^4\theta\mathcal{A}(\Phi, D_\pm\Phi, D_+D_-\Phi, \partial_{\pm\pm}\Phi, \text{c.c.})\right) + \left(\int d^2\theta W(\Phi)\right) + \left(\int d^2\bar{\theta}\,\bar{W}(\bar{\Phi})\right), \tag{B1}$$

where "c.c." indicates dependence on the conjugates $\bar{\Phi}, \bar{D}_\pm\bar{\Phi}, \bar{D}_+\bar{D}_-\bar{\Phi}$, and $\partial_{\pm\pm}\bar{\Phi}$. To do this, we will minimally couple the theory to supergravity using the old-minimal supergravity formulation and extract the currents which couple to the metric superfield $H^{\pm\pm}$ and the chiral compensator σ. The minimal coupling prescription involves promoting $\mathcal{L}_0$ to[12]

$$\mathcal{L}_0 \to \mathcal{L}_{\text{SUGRA}} = \left(\int d^4\theta E^{-1}\mathcal{A}(\mathbf{\Phi}, \nabla_\pm\mathbf{\Phi}, \nabla_+\nabla_-\mathbf{\Phi}, \nabla_{\pm\pm}\mathbf{\Phi}, \text{c.c.})\right)$$
$$+ \left(\int d^2\theta\mathcal{E}^{-1}W(\mathbf{\Phi})\right) + \left(\int d^2\bar{\theta}\bar{\mathcal{E}}^{-1}\bar{W}(\bar{\mathbf{\Phi}})\right). \tag{B2}$$

Here $\nabla_\pm$ is the derivative which is covariant with respect to the full local supergravity gauge group, E^{-1} is the full superspace measure, $\mathcal{E}^{-1}$ is the chiral measure, and $\mathbf{\Phi}$ is the covariantly chiral version of the chiral superfield Φ—that is, $\bar{\nabla}_\pm\mathbf{\Phi} = 0$, whereas $\bar{D}_\pm\Phi = 0$.

Expressions for these supercovariant derivatives and measures have been worked out in a series of papers [50–54] from which we will import the results that we need for our analysis. To leading order in H^m, the linearized inverse superdeterminant of the supervielbein is

$$E^{-1} = 1 - [\bar{D}_+, D_+]H^{++} - [\bar{D}_-, D_-]H^{--}, \tag{B3}$$

while the chiral measure is given by

$$\mathcal{E}^{-1} = e^{-2\sigma}(1 \cdot e^{iH^m\tilde{\partial}_m}) = 1 - 2\sigma + i(\partial_m H^m) + \cdots, \tag{B4}$$

[12]Conforming to notation of [50–54], in this section we will sometimes use the index notations $\alpha = +, -$ and $Zm = ++, --$.

where the ellipses are terms of higher order in H^m and σ. The covariantly chiral superfield Φ is related to the ordinary chiral superfield Φ by

$$\Phi = e^{iH^m\partial_m}\Phi = \Phi + i(H^{++}\partial_{++} + H^{--}\partial_{--})\Phi + \mathcal{O}(H^2). \tag{B5}$$

The spinor supercovariant derivatives $\nabla_\pm$ are

$$\nabla_\alpha = E_\alpha + \Omega_\alpha M + \Gamma_\alpha \bar{M} + \Sigma_\alpha N, \tag{B6}$$

where M and N are linear combinations of the Lorentz, $U(1)_V$, and $U(1)_A$ generators which act on spinors as

$$[M, \psi_\pm] = \pm\frac{1}{2}\psi_\pm, \qquad [M, \bar{\psi}_\pm] = 0, \tag{B7a}$$

$$[\bar{M}, \bar{\psi}_\pm] = \pm\frac{1}{2}\bar{\psi}_\pm, \qquad [\bar{M}, \psi] = 0, \tag{B7b}$$

$$[N, \psi_\pm] = -\frac{i}{2}\psi_\pm, \qquad [N, \bar{\psi}_\pm] = +\frac{i}{2}\bar{\psi}_\pm. \tag{B7c}$$

The spinor inverse of the supervielbein $E_\alpha = E_\alpha{}^M\partial_M$, and the structure group connections Ω_α, Γ_α, and Σ_α can be expressed to linear order in terms of the metric superfield $H^{\pm\pm}$ and an unconstrained complex scalar compensator S. In the case of old-minimal supergravity, the unconstrained superfield S is related to the chiral compensator σ by

$$S = \sigma - \frac{i}{2}\partial_m H^m - \frac{1}{2}[\bar{D}_+, D_+]H^{++} - \frac{1}{2}[\bar{D}_-, D_-]H^{--} \tag{B8}$$

to linear order. In the following analysis we will first obtain expressions for the supercovariant derivatives in terms of $S = S(H^m, \sigma)$, and we use (B8) to give them in terms of H^m and σ.

The spinorial inverse of the supervielbein is given at first order in the prepotentials by

$$E_\pm = (1 + \bar{S})D_\pm + i(D_\pm H^m)\partial_m - 2(\bar{D}_\mp D_\pm H^{\mp\mp})D_\mp, \tag{B9}$$

together with their complex conjugates. Meanwhile, the connections Ω_α, Γ_α, and Σ_α can be written to leading order as

$$\Gamma_\pm = \pm 2D_\pm(S + \bar{S}) \mp 2D_\mp\bar{D}_\mp D_\pm H^{\mp\mp}, \tag{B10a}$$

$$\Sigma_\pm = -2iD_\pm\bar{S} + 2iD_\mp\bar{D}_\mp D_\pm H^{\mp\mp}, \tag{B10b}$$

$$\Omega_\pm = \mp 2D_\mp\bar{D}_\mp D_\pm H^{\mp\mp}. \tag{B10c}$$

Using (B6), the vielbeins (B9), and the expression (B5) for $\mathbf{\Phi}$, we find the supercovariant derivatives

$$\nabla_\pm\mathbf{\Phi} = (1 + \bar{S})D_\pm\Phi + 2i(D_\pm H^m)\partial_m\Phi + iH^m(D_\pm\partial_m\Phi) - 2(\bar{D}_\mp D_\pm H^{\mp\mp})D_\mp\Phi, \tag{B11a}$$

$$\bar{\nabla}_\pm\bar{\mathbf{\Phi}} = (1 + S)\bar{D}_\pm\bar{\Phi} - 2i(\bar{D}_\pm H^m)\partial_m\bar{\Phi} - iH^m(\bar{D}_\pm\partial_m\bar{\Phi}) - 2(\bar{D}_\pm D_\mp H^{\mp\mp})\bar{D}_\mp\bar{\Phi}. \tag{B11b}$$

To compute the second supercovariant derivatives acting on $\mathbf{\Phi}$ and $\bar{\mathbf{\Phi}}$, we must include the contributions from Ω_α, Γ_α, Σ_α, and their conjugates. One finds

$$\begin{aligned}\bar{\nabla}_+\nabla_+\mathbf{\Phi} = {} & i(1 + S + \bar{S})\partial_{++}\Phi - 2(\bar{D}_+\bar{D}_-D_+H^{--})D_-\Phi + 2i(\bar{D}_+D_+H^m)\partial_m\Phi \\ & - H^m\partial_{++}\partial_m\Phi + 2(\bar{D}_+(S + \bar{S}) + \bar{D}_-D_-\bar{D}_+H^{--})D_+\Phi,\end{aligned} \tag{B12a}$$

$$\begin{aligned}\nabla_+\nabla_-\mathbf{\Phi} = {} & (1 + 2\bar{S})D_+D_-\Phi + 2i(D_+D_-H^m)\partial_m\Phi - 2i(D_-H^m)D_+\partial_m\Phi \\ & + 2i(D_+H^m)D_-\partial_m\Phi + iH^mD_+D_-\partial_m\Phi - 2(D_+\bar{D}_+D_-H^{++})D_+\Phi \\ & + 2(D_-\bar{D}_-D_+H^{--})D_-\Phi,\end{aligned} \tag{B12b}$$

$$\begin{aligned}\bar{\nabla}_-\nabla_-\mathbf{\Phi} = {} & i(1 + S + \bar{S})\partial_{--}\Phi - 2(\bar{D}_-\bar{D}_+D_-H^{++})D_+\Phi + 2i(\bar{D}_-D_-H^m)\partial_m\Phi \\ & - H^m\partial_{--}\partial_m\Phi + 2(\bar{D}_-(S + \bar{S}) + \bar{D}_+D_+\bar{D}_-H^{++})D_-\Phi,\end{aligned} \tag{B12c}$$

together with their complex conjugates. Armed with these expressions, we can linearize the supergravity couplings in (B2). First let us consider the contribution from the D term. We would like to extract the terms proportional to $H^{\pm\pm}$ and σ in

$$\begin{aligned}\mathcal{L} &= \int d^4\theta E^{-1}\mathcal{A}(\mathbf{\Phi}, \nabla_\pm\mathbf{\Phi}, \nabla_+\nabla_-\mathbf{\Phi}, \nabla_{\pm\pm}\mathbf{\Phi}, \text{c.c.}),\\ &\sim \int d^4\theta\Big(H^{\alpha\dot\alpha}[D_\alpha, \bar D_{\dot\alpha}]\mathcal{A} + i\frac{\partial\mathcal{A}}{\partial\Phi}H^m\partial_m\Phi + (\nabla_\alpha\Phi - D_\alpha\Phi)\frac{\partial\mathcal{A}}{\partial\nabla_\alpha\Phi}\\ &\quad + \frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}(\nabla_+\nabla_-\mathbf{\Phi} - D_+D_-\Phi) + \frac{\partial\mathcal{A}}{\partial\nabla_m\Phi}(\nabla_m\Phi - \partial_m\Phi) + \text{c.c.}\Big),\end{aligned} \tag{B13}$$

where $\nabla_{\pm\pm} = -i\{\nabla_\pm, \bar\nabla_\pm\}$. Doing so, we see that the currents which couple to $H^{\pm\pm}$ are

$$\begin{aligned}\mathcal{J}_{++} &= [D_+, \bar D_+]\Big[\frac{1}{2}\mathcal{A} - \frac{1}{2}\frac{\partial\mathcal{A}}{\partial\nabla_-\Phi}D_-\Phi - \frac{1}{2}\frac{\partial\mathcal{A}}{\partial\nabla_+\Phi}D_+\Phi + \frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_+D_-\Phi + \frac{\partial\mathcal{A}}{\partial\nabla_{++}\Phi}\partial_{++}\Phi\\ &\quad + 2i\bar D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{--}\Phi}D_-\Phi\Big) + 2i\bar D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{++}\Phi}D_+\Phi\Big) + \frac{\partial\mathcal{A}}{\partial\nabla_{--}\Phi}\partial_{--}\Phi\Big]\\ &\quad + i\Big[\frac{\partial\mathcal{A}}{\partial\Phi}\partial_{++}\Phi + \frac{1}{2}\partial_{++}\Big(\frac{\partial\mathcal{A}}{\partial\nabla_-\Phi}D_-\Phi\Big) - \partial_{++}\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_+D_-\Phi\Big) - \frac{\partial\mathcal{A}}{\partial\nabla_-\Phi}D_-\partial_{++}\Phi\\ &\quad - 2D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_-\Phi}\partial_{++}\Phi\Big) + \frac{1}{2}\partial_{++}\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\Phi}D_+\Phi\Big) - \frac{\partial\mathcal{A}}{\partial\nabla_+\Phi}D_+\partial_{++}\Phi - 2D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\Phi}\partial_{++}\Phi\Big)\\ &\quad - 2D_-D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}\partial_{++}\Phi\Big) + 2D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_+\partial_{++}\Phi\Big)\\ &\quad - 2D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_-\partial_{++}\Phi\Big) + \frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_+D_-\partial_{++}\Phi + 2iD_+\bar D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{++}\Phi}\partial_{++}\Phi\Big)\\ &\quad + 2iD_-\bar D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{--}\Phi}\partial_{++}\Phi\Big) + \frac{\partial\mathcal{A}}{\partial\nabla_{++}\Phi}\partial_{++}^2\Phi + \frac{\partial\mathcal{A}}{\partial\nabla_{--}\Phi}\partial_{--}\partial_{++}\Phi\Big]\\ &\quad + 2\Big[-D_-\bar D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_-\Phi}D_+\Phi\Big) - D_-\bar D_+D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_+\Phi\Big)\\ &\quad + iD_-\bar D_+\bar D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{--}\Phi}D_+\Phi\Big) - i\bar D_-D_+\bar D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{--}\Phi}D_-\Phi\Big)\Big] + \text{c.c.}\end{aligned} \tag{B14}$$

and

$$\begin{aligned}\mathcal{J}_{--} &= [D_-, \bar D_-]\Big[\frac{1}{2}\mathcal{A} - \frac{1}{2}\frac{\partial\mathcal{A}}{\partial\nabla_-\Phi}D_-\Phi - \frac{1}{2}\frac{\partial\mathcal{A}}{\partial\nabla_+\Phi}D_+\Phi + \frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_+D_-\Phi + \frac{\partial\mathcal{A}}{\partial\nabla_{++}\Phi}\partial_{++}\Phi\\ &\quad + 2i\bar D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{--}\Phi}D_-\Phi\Big) + 2i\bar D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{++}\Phi}D_+\Phi\Big) + \frac{\partial\mathcal{A}}{\partial\nabla_{--}\Phi}\partial_{--}\Phi\Big]\\ &\quad + i\Big[\frac{\partial\mathcal{A}}{\partial\Phi}\partial_{--}\Phi + \frac{1}{2}\partial_{--}\Big(\frac{\partial\mathcal{A}}{\partial\nabla_-\Phi}D_-\Phi\Big) - \partial_{--}\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_+D_-\Phi\Big) - \frac{\partial\mathcal{A}}{\partial\nabla_-\Phi}D_-\partial_{--}\Phi\\ &\quad - 2D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_-\Phi}\partial_{--}\Phi\Big) + \frac{1}{2}\partial_{--}\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\Phi}D_+\Phi\Big) - \frac{\partial\mathcal{A}}{\partial\nabla_+\Phi}D_+\partial_{--}\Phi - 2D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\Phi}\partial_{--}\Phi\Big)\\ &\quad - 2D_-D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}\partial_{--}\Phi\Big) + 2D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_+\partial_{--}\Phi\Big) - 2D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_-\partial_{--}\Phi\Big)\\ &\quad + \frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_+D_-\partial_{--}\Phi + 2iD_+\bar D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{++}\Phi}\partial_{--}\Phi\Big) + 2iD_-\bar D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{--}\Phi}\partial_{--}\Phi\Big)\\ &\quad + \frac{\partial\mathcal{A}}{\partial\nabla_{++}\Phi}\partial_{++}\partial_{--}\Phi + \frac{\partial\mathcal{A}}{\partial\nabla_{--}\Phi}\partial_{--}^2\Phi\Big] + 2\Big[-D_+\bar D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\Phi}D_-\Phi\Big) + D_+\bar D_-D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_-\Phi\Big)\\ &\quad + iD_+\bar D_-\bar D_+\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{++}\Phi}D_-\Phi\Big) - i\bar D_+D_-\bar D_-\Big(\frac{\partial\mathcal{A}}{\partial\nabla_{++}\Phi}D_+\Phi\Big)\Big] + \text{c.c.},\end{aligned} \tag{B15}$$

where +c.c. means to add the complex conjugates of *all* preceding terms (including the real quantity $\frac{1}{2}[D_\pm, \bar{D}_\pm]\mathcal{A}$ for which the complex conjugate merely removes the factor of $\frac{1}{2}$).

The field $\mathcal{V}$ which appears in our deformation (3.10) receives two contributions, one from the D term coupling which depends only on $\mathcal{A}$ and one from the F term coupling which depends only on the superpotential W. Adding them, we find

$$\begin{aligned}\mathcal{V} = \bar{D}_+\bar{D}_-\Bigg[&-\frac{\partial\mathcal{A}}{\partial\nabla_\alpha\Phi}D_\alpha\Phi + 2\frac{\partial\mathcal{A}}{\partial\nabla_+\nabla_-\Phi}D_+D_-\Phi \\ &+ \frac{\partial\mathcal{A}}{\partial\nabla_m\Phi}\partial_m\Phi + \frac{\partial\mathcal{A}}{\partial\nabla_m\bar{\Phi}}\partial_m\bar{\Phi} \\ &+ 2i\bar{D}_+\left(\frac{\partial\mathcal{A}}{\partial\nabla_{++}\Phi}D_+\Phi\right) + 2iD_+\left(\frac{\partial\mathcal{A}}{\partial\nabla_{++}\bar{\Phi}}\bar{D}_+\bar{\Phi}\right) \\ &+ 2i\bar{D}_-\left(\frac{\partial\mathcal{A}}{\partial\nabla_{--}\Phi}D_-\Phi\right) + 2iD_-\left(\frac{\partial\mathcal{A}}{\partial\nabla_{--}\bar{\Phi}}\bar{D}_-\bar{\Phi}\right)\Bigg] \\ &+ 2W(\Phi). \end{aligned} \tag{B16}$$

APPENDIX C: SIMPLIFYING THE DEFORMATION ON-SHELL

In this appendix, we prove the claim that one can drop all terms which involve products of $(D_+D_-\Phi)$ or $(\bar{D}_+\bar{D}_-\bar{\Phi})$ and the four-fermion term $|D\Phi|^4 = D_+\Phi\bar{D}_+\bar{\Phi}D_-\Phi\bar{D}_-\bar{\Phi}$ when the equations of motion are satisfied.

To see this for the models we consider, it suffices to consider a superspace Lagrangian of the form

$$\begin{aligned}\mathcal{L} &= \int d^4\theta\mathcal{A}(\Phi, D_\pm\Phi, D_+D_-\Phi, \partial_{\pm\pm}\Phi, \text{c.c.}) \\ &= \int d^4\theta(K(\Phi,\bar{\Phi}) + f(x,\bar{x},y)|D\Phi|^4), \end{aligned} \tag{C1}$$

which has the superspace equation of motion

$$\begin{aligned}\bar{D}_+\bar{D}_-K_\Phi = \bar{D}_+\bar{D}_-\Bigg\{&D_\alpha\left[\frac{\partial(f|D\Phi|^4)}{\partial D_\alpha\Phi}\right] \\ &- D_+D_-\left[\frac{\partial(f|D\Phi|^4)}{\partial D_+D_-\Phi}\right] - \partial_m\left[\frac{\partial(f|D\Phi|^4)}{\partial(\partial_m\Phi)}\right]\Bigg\} \end{aligned} \tag{C2}$$

for Φ, and the conjugate equation of motion for $\bar{\Phi}$. If we multiply (C2) on both sides by the four-fermion term $|D\Phi|^4 = D_+\Phi\bar{D}_+\bar{\Phi}D_-\Phi\bar{D}_-\bar{\Phi}$, then any term containing $(D_\pm\Phi)$ and $(\bar{D}_\pm\bar{\Phi})$ fermions in (C2) will vanish by nilpotency. On the left, the only surviving term is $K_{\Phi\bar{\Phi}}\bar{D}_+\bar{D}_-\bar{\Phi}$, while on the right we get contributions from the first and second terms:

$$\begin{aligned}K_{\Phi\bar{\Phi}}(\bar{D}_+\bar{D}_-\bar{\Phi})|D\Phi|^4 = (\bar{D}_+\bar{D}_-\bar{\Phi})|D\Phi|^4 \\ \times\left\{\lambda\bar{D}_+\bar{D}_-\left[\frac{\partial f}{\partial y}(\partial_{--}\bar{\Phi})(\partial_{++}\bar{\Phi})\right]\right. \\ \left. - \left(\frac{x+\bar{x}}{\lambda}\right)f\right\}. \end{aligned} \tag{C3}$$

On collecting terms, the previous equation turns into

$$\begin{aligned}(\bar{D}_+\bar{D}_-\bar{\Phi})|D\Phi|^4\Bigg\{&K_{\Phi\bar{\Phi}} + \left(\frac{x+\bar{x}}{\lambda}\right)f \\ &- \lambda\bar{D}_+\bar{D}_-\left[\frac{\partial f}{\partial y}(\partial_{--}\bar{\Phi})(\partial_{++}\bar{\Phi})\right]\Bigg\} = 0. \end{aligned} \tag{C4}$$

The parentheses multiplying $(\bar{D}_+\bar{D}_-\bar{\Phi})|D\Phi|^4$ in the previous expression do not vanish in general, at least for λ small enough. Then for (C4) to be satisfied, the equation

$$(\bar{D}_+\bar{D}_-\bar{\Phi})|D\Phi|^4 = 0 \tag{C5}$$

has to hold when the equations of motion are satisfied. This justifies our claim in Sec. IV A that we may drop all terms involving the product $y|D\Phi|^4$ in the deformation, assuming we restrict to on-shell configurations.

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