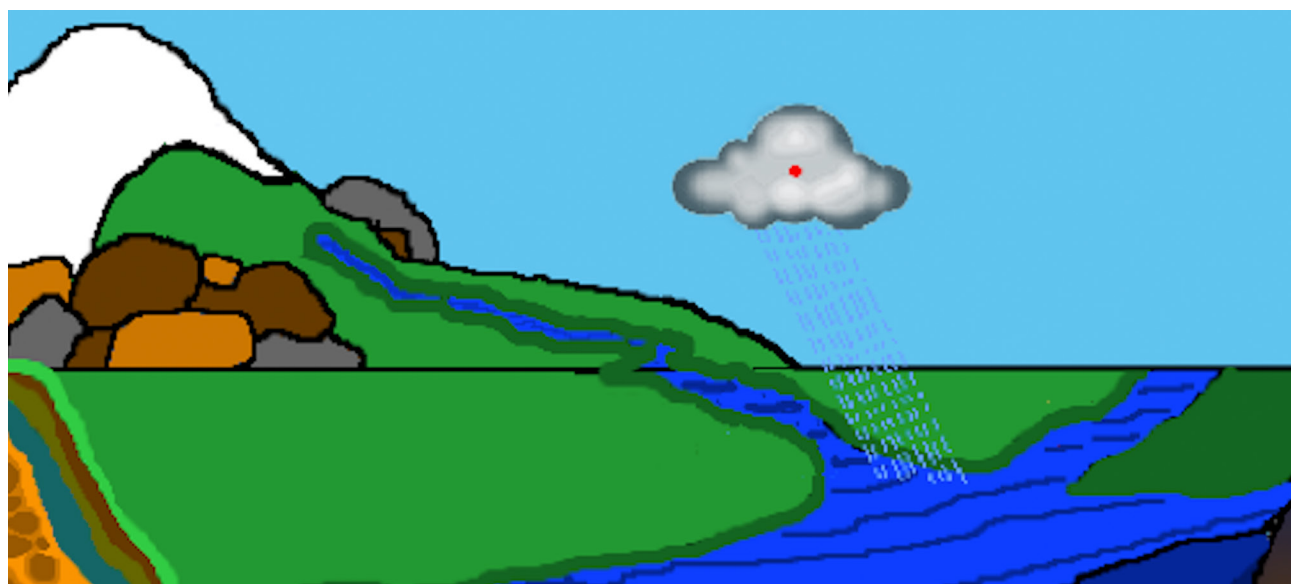




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Using Scratch Programming to Explore Coordinates

A Scratch task is designed and implemented for teaching and learning coordinates in an engaging, dynamic manner.

Erell Germia and Nicole Panorkou

Integrating coding in the mathematics curricula is a worldwide trend that aims to help students develop 21st-century skills by visualizing abstract concepts, exploring real-world applications of concepts, and developing problem-solving skills. Recent developments of programming languages, such as Scratch (<http://scratch.mit.edu>), allow young students to program interactive projects using a drag-and-drop and snapping blocks system even without any previous

programming experience (Maloney et al. 2010; Resnick et al. 2010).

This development of easy-to-use programming languages opens up a whole new opportunity for educators to study how these programs can be used for learning math. However, prior research on using programming tasks in mathematics classrooms has documented the difficulty of achieving a balance between the instructional goals of the teacher and the goals of students' self-directed activity

(Hoyles and Noss 1992). This conflict of goals has been deemed the “play paradox” (Hoyles and Noss 1992, p. 47): The ways students explore and solve a problem may not lead to the mathematics content the teacher desires. Aiming to address this challenge, Benton and colleagues (2016) developed the “5Es” framework of five constructs (Explore, Explain, Envisage, Exchange, and bridgeE) to provide guidance on the pedagogical strategies that can be used by teachers to successfully implement such Scratch tasks. In this article, we describe the 5Es framework by presenting how a teacher used it to implement a Scratch programming task and engage sixth-grade students in productive practices for learning about coordinates.

WHY USE SCRATCH TO TEACH COORDINATES?

Developing students’ understanding of coordinates is important because it relates to other areas of mathematics, such as the study of maps, the interpretation and construction of graphs, and even the graphing of functions in later years of schooling (Sarama et al. 2003; Somerville and Bryant 1985). However, students experience several difficulties when working with coordinates (Jones and Mooney 2003). Examples include students reversing the coordinates into (y, x) , treating x and y as separate entities, failing to understand that a horizontal or vertical distance should be zero when an object is located along the x -axis or the y -axis, not considering the negative sign as implying a direction, and treating negative coordinates the same way as the positive coordinates (Sarama et al. 2003). Research also shows that students think of coordinates as “dots” that are drawn on lines; therefore, they are uncomfortable in plotting points between grid lines; for example, plotting $(3, 5)$ when the x -axis uses increments of two (Sarama et al. 2003).

The above difficulties exist because students do not consider coordinates as quantities that have a specific *direction* and *magnitude* (a specific distance) from the origin. Students already have knowledge about measuring distances and a spatial understanding for navigational

directions “upward/downward” and “left/right” to signify vertical and horizontal directionality, respectively, (Clements and Sarama 2014) that we as educators can use to help them construct such meanings. Research on students’ understanding of translation as a “sliding” motion found that by engaging in tasks in a dynamic animation environment, students were able to associate the use of a minus sign in the coordinates with the direction of “go back,” “left,” or “down” that reverses the positive direction of “right” or “upward” (Panorkou and Maloney 2015). The same study also showed that by manipulating the x and y coordinates of the objects to animate them to move to another location, students were able to reason about the change in location in terms of *magnitude* (a specific distance) and *direction* (Panorkou and Maloney 2015). In other words, they mentally structured the coordinate plane as a two-dimensional (2D) space constructed by infinite imaginary number lines (Clements and Sarama 2014).

The Panorkou and Maloney (2015) study illustrated the potential of animation technology for introducing coordinates as describing a dynamic motion of an object from one location to another. By experimenting with different coordinate values and using the feedback of the animation software, students engaged in a *debugging* process (Papert 1980) in which they had to search for what they had done wrong and try to find a way to fix it. This process helped them develop their previous conceptions about coordinates. Scratch programming has these capabilities. It not only provides students with dynamic animation experiences for coordinate explorations but also demands syntactic correctness, and this can help students engage in a *debugging* process to understand the purpose and *utility* (Ainley, Pratt, and Hansen 2006) of ordered pairs.

THE SCRATCH RAIN CLOUD TASK

We designed the Rain Cloud task for use after students have completed a module on the Water Cycle, aiming to

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relate it to the content they have been exploring in science. The task was implemented in a sixth-grade classroom. Students had a basic knowledge of coordinates on the first quadrant from their mathematics classes. Because they did not have any prior experience with Scratch, first we introduced some basic elements of the Scratch interface (see figure 1). Students became familiar with the blocks in the *blocks palette*. The blocks contain programming syntax and are shaped into puzzle pieces that can be dragged to the *scripts area* and snapped together vertically to form a script. Then students

Fig. 1



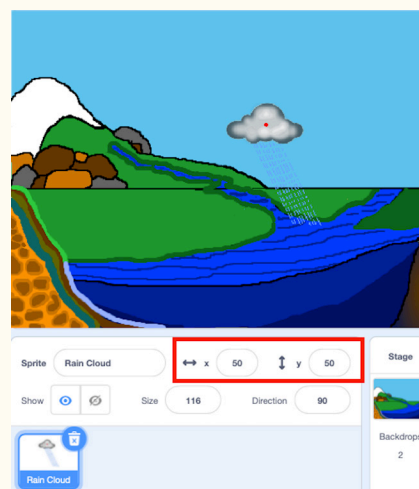
Basic elements of the Scratch interface familiarized students with the blocks palette, programming syntax, and script.

explored the “sprites” in Scratch, which are the objects that perform the programmed actions on the stage pane.

In Scratch, the x - and y -coordinates of the sprite, in this case the Rain Cloud, are shown below the stage (see figure 2). As an introductory activity (see Task A in the appendix, available as an online supplemental file), we asked students to observe how these coordinates change as they move the cloud sprite in different positions on the stage and then to discuss their observations with their partner.

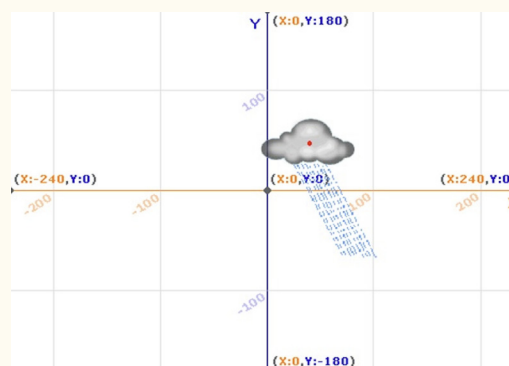
Next we designed the program in such a way that when students press the spacebar on their computer, the stage pane changes into a coordinate grid, as figure 3 shows. According to Clements and Sarama (2014), if the grid can be turned on and off the screen, it can help students create a mental image of the coordinates system. For the second activity (see Task B in the appendix), we asked students to use the grid to describe how they understand the meaning of x and y values, including the negative values.

Fig. 2



Students observed the changing of coordinates and then discussed their observations with a partner.

Fig. 3



Turning the coordinate grid on and off can help students create a mental image of the coordinates system.

Then students were introduced to the “go to ($_,_$)” and “glide_secs to ($_,_$)” blocks. We asked them to use the “go to ($_,_$)” block to change the location of the sprite and use the “glide_secs to ($_,_$)” block to move the sprite to various positions on the grid in a number of seconds (see Task C in the appendix). The first block introduces the idea of change of coordinates as jumps of the sprite from one location to another, whereas the second block visibly illustrates this change as a smooth dynamic motion from one location to another (see the appendix for the task.)

Finally students were asked to use what they had learned about coordinates to move the rain cloud to the

top of the mountain (see Task D in the appendix). This task (see figure 1) presents to students a messy simulation that they must fix. In the simulation, a sprite in the form of a rain cloud needs to travel to the mountain, but the code is “broken.” This fixing challenge is a simple debugging process in which students should identify and resolve the defect in the code of the project. This purpose creates the necessity for learners to use their thinking about coordinates to complete the task. At the end of the exploration, we asked students to reflect on their learning and connect what they had been doing in Scratch with the mathematics of coordinates (see Task E in the appendix).

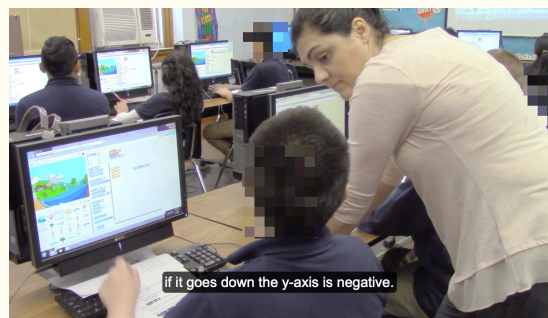
IMPLEMENTING THE RAIN CLOUD TASK

In this section, we describe how the teacher in our study used the 5Es framework to successfully implement the Rain Cloud task, and we discuss how sixth-grade students engaged in productive practices for learning about coordinates.

Explore

The first construct, Explore, involves supporting students’ activity to take control of their own learning and explore different opportunities and constraints in the tool and task, investigate ideas, and debug errors (Benton et al. 2016). Students were asked to change the values in the “go to (__,_)” block to make the cloud move to different places on the screen. To do that, they explored positive and negative values for the coordinates to examine the direction of the cloud. Video 1 shows the discussion between the teacher and James.

Video 1 James in the Explore Construct



 Watch the full video online.

Similar to James in video 1, students considered the negative sign in the coordinates as implying a direction, avoiding the difficulty reported by Sarama and colleagues (2003). In the specific episode, James is using the terms x -axis and y -axis to actually mean x - and y -coordinates, probably because he was coordinating the two axes to find their intersection point. By experimenting with different coordinate values, he was able to coordinate the x and y values to distinguish between locations in the four quadrants (see figure 4). During this experimentation, the teacher walked around and prompted students to explain what they noticed. Asking such open-ended questions as “What have you noticed so far?” allows students to reflect on their explorations and talk about what they *themselves* found fascinating in this process.

Explain

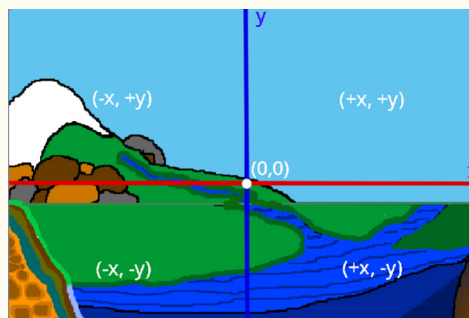
In the second construct, Explain, the teacher asks students to explain what they have learned in relation to the task and articulate the reasons behind their

Fig. 4

(a)

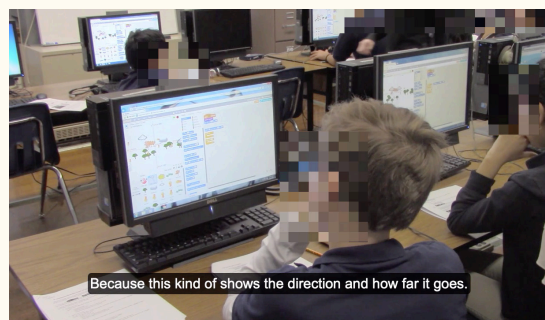


(b)



Experimenting with different coordinate values allowed James (a) to reason about the nature of coordinates and then (b) to distinguish between locations in the four quadrants.

Video 2 Jason Explaining His Choice



[Watch the full video online.](#)

approach. In video 2, Jason was asked to explain his choice for the values of the coordinates after he had explored different options. Again, the student uses the term *axis* instead of *coordinate*.

Video 2 shows that Jason was able to construct a meaning of coordinates as direction and magnitude (“this shows the direction and how far it goes”). Next the teacher prompted students to think about the difference in positive and negative values:

Teacher: So that (50, 0), where is it [the sprite] going to go?

Harry: The (50, 0) is going to go to the right 50 steps.

Teacher: OK. And then the (–50, 0)?

Harry: It goes to the left 50 steps.

Similar to Jason, Harry showed that he was able to define (50, 0) as specifying a distance (“go to the right”) and a magnitude (“50 steps”). He also understood that

Video 3 Harry Explaining a Location



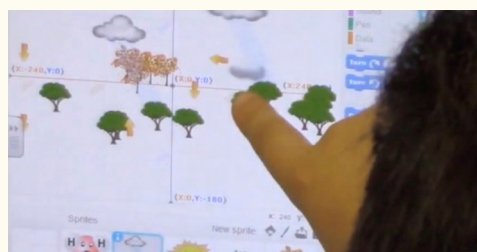
[Watch the full video online.](#)

the vertical distance should be zero and that he could plot points between grid lines (the grid given was in increments of 100; see figure 3), avoiding the difficulties reported by Sarama and her colleagues (2003). In explaining the difference between (50, 0) and (–50, 0), Harry also recognized the negative sign to imply a change in direction compared to the positive sign. Video 3 presents the rest of the discussion with Harry, during which he was asked to place his sprite in the location (240, 180) on the grid.

Next the teacher prompted students to explain the difference between the “Go to” and “Glide to” tools (see video 4 for a discussion between the teacher and Carol). Carol was able to distinguish between the “go to (–,–)” block as changing the location of the sprite as teleporting and the “glide_secs to (–,–)” block as illustrating visually the movement of the sprite from one location to another. This latter form of reasoning might be

Fig. 5

(a)

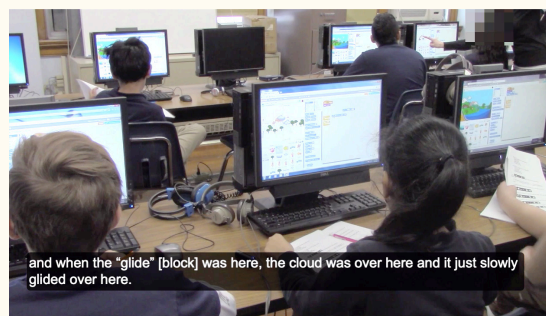


(b)



Harry shows where (a) the coordinates (50,0) and (b) the coordinates (–50,0) are located.

Video 4 Carol Making a Distinction



 Watch the full video online.

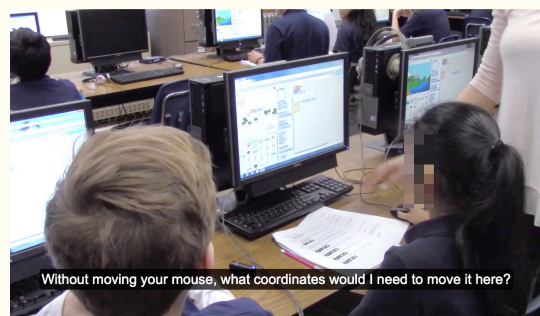
useful in making the connection between coordinates and translations (Panorkou and Maloney 2015). During these explorations, we found that asking students to explain is a powerful tool that can help increase their level of understanding and intensify their motivation to work further on their learning as they reflect on their thinking while speaking. Moreover, they can clarify their ideas with the teacher or their peers, and the teacher can use this for formative assessment. Asking open-ended questions, such as those below, allows students to consider their own ideas and thoughts and conveys the message that what a student thinks matters in the learning process.

- What have you noticed so far?
- What did you use?
- Why did you think that would work?
- How is this different from this?

Envisage

In the third construct, Envisage, students are prompted to envision the output of their task before acting it out. This process helps students to mentally imagine the output on the basis of their prior construction of knowledge and then refine and develop this knowledge through this experience. In video 5, Carol was asked to predict which coordinates she would need to move the cloud to a position pointed to by the teacher. Carol was able to mentally imagine—without acting it out—the coordinates that would move the sprite. This shows that she structured the grid as a 2D space constructed by mental number lines (Clements and Sarama 2014). The episode shows that the teacher's role is crucial in this

Video 5 Carol Envisioning a Process



 Watch the full video online.

development: by asking students to make conjectures, act them out, and reflect on their thinking. This “envision” process helps the teacher to get a glimpse of students’ understanding at various points of the lesson.

Exchange

In the fourth construct, Exchange, students are encouraged to exchange their ideas by collaborating and sharing what they have learned or noticed while working on the task. Although all students had their own computer, the teacher asked them to collaborate in groups of two to solve the task. Below we present an exchange between Carol and Jason.

Carol: I got an idea; it might not work. [*Typing and looking amazed after checking the output, then turning to Jason*].

Jason: [*Looking at Carol's screen*]

Carol: Look, I did minus one [*typing and turning to Jason for approval*].

Jason: The glitch? Look, this is positive 240 or whatever. And then I'm going to put negative to this one.

Carol: I remember when you were playing around with it.

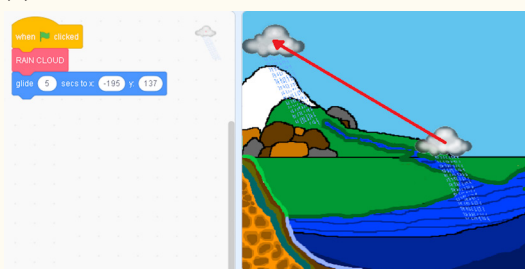
As Carol observed how Jason played with the blocks, she was encouraged to try other ideas. Although she was not confident that her idea would work, she tried, shared, and waited for feedback from Jason to solve the task. While working together to help resolve disagreements and answer questions of one another, students develop their understanding of ideas. This social sharing process is one of the benefits of undertaking constructionist activities in computational environments

(Bruckman 1998; Kafai and Resnick 1996; Papert 1980). The teacher can support this process by encouraging students to work together to share their thinking.

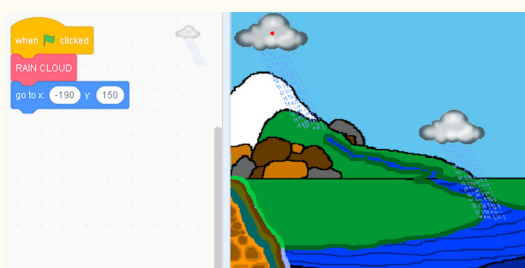
Also, at the end of the exploration, the teacher shared different solutions on the screen (see figure 6 for two of those possible solutions) and engaged students in a whole-class discussion about the similarities and differences of each solution. Task D was designed to support this exchange of ideas because more than one correct solution exists for moving the rain cloud to the mountain. During this discussion, students discussed the difference in using the “go to (x, y)” or “glide_secs to (x, y)” blocks and also a range of x and y values that moved the rain cloud above the mountain.

Fig. 6

(a)



(b)



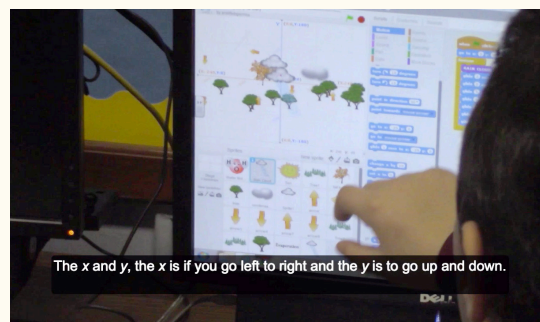
This shows two different solutions of the Rain Cloud task, using (a) the “glide_secs to (x, y)” block and using (b) the “go to (x, y)” block.

BridgE

In the final construct, bridgE, students are prompted to combine their ideas formed in the previous constructs and describe those into a more meaningful way of understanding. In video 6, the teacher initiated a discussion about the mathematics of the blocks.

Similar to the episode in video 6, through such targeted questioning as “What do all of these mean?” or

Video 6 Bridging Ideas Together



[Watch the full video online.](#)

“How does this relate to what we have been doing in class?” the teacher can help students make connections between “Scratch time” and ideas in mathematics, other disciplines, or even their everyday lives.

CONCLUDING REMARKS

In this article, we discussed how a teacher used the 5Es framework to implement a Scratch programming task and engage sixth-grade students in productive practices for learning about coordinates. By engaging in these practices while working with the Rain Cloud task, students were able to avoid some of the difficulties about coordinates that previous studies have presented. Specifically, students were able to define coordinates as illustrating both a direction and a magnitude. They reasoned about the negative sign as indicating a direction opposite that of the positive values and used this knowledge to locate sprites in all four quadrants. By experimenting with the “go to” and “glide to” blocks, they were also able to reason about the purpose and utility of coordinates both as a change in location and as a movement from one location to another. We consider these generalizations to be foundational for developing more robust understandings of graphing and transformations in later years of schooling.

The 5Es framework can be used as a guide for implementing other Scratch tasks in our mathematics classrooms while achieving a balance between our instructional goals and the goals of students’ self-directed activity. In particular, it is important that we offer opportunities for students to explore the environment on their own, ask them questions about their

observations, and encourage them to articulate their thinking in words. To help students generalize, asking them to predict the outcomes of specific tasks before performing them is valuable. Finally, by sharing their strategies with other students and exchanging views on the task, students can develop their thinking further. Note that these constructs are considered

unordered (Benton et al. 2016), and although we described each construct separately, we are not dismissing the phenomena that some of these constructs intertwine and can be observed at the same time. The role of the teacher is crucial in the 5Es implementation, especially for helping students make connections to the mathematics embedded in the task. [—](#)

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