ISBN: 978-81-941567-9-6
January 3-6, 2020

EXAMINING THE ROLE OF COVARIATIONAL REASONING IN DEVELOPING STUDENTS' UNDERSTANDING OF THE GREENHOUSE EFFECT

Debasmita Basu[1] and Nicole Panorkou[2]
Eugene Lang College of Liberal Arts, The New School[1], Montclair State University[2]
basud1@newschool.edu[1], panorkoun@montclair.edu[2]

Climate change is a pressing issue of the present age. Most of the important information about climate change in the news and public media is in the form of data and graphs, however students often focus on the shape of a graph, overlooking the covariational relationship between the represented quantities. Building on the framework of covariational reasoning, we designed two simulations in NetLogo in which students investigated the relationships between different covarying quantities underlying the phenomenon of the greenhouse effect. In this paper, we present the analysis of two cycles of whole-class design experiments in two sixth-grade classrooms. We discuss the development of students' covariational reasoning as they engaged with the simulations and how this type of reasoning helped them develop their critical thinking about the greenhouse effect.

PURPOSE OF THE STUDY

In the 1992 Earth Summit in Rio de Janeiro, the United Nations Framework Convention on Climate Change (UNFCCC) defined climate change as the "change of climate which is attributed directly or indirectly to human activity" (Kolbert, 2006, p. 153). Indeed, the increasing human population has maintained its dominance over the earth's ecosystem (Karl & Trenberth, 2003). From large-scale burning of fossil fuels (Dolman & Verhagen, 2003) to choosing hazardous household items (Black & Cherrier, 2010), human activities have enhanced the global greenhouse gas emission rate by .5 to 1% every year (Karl & Trenberth, 2003). If this trend of greenhouse gas emission continues, then global temperature might rise between 2 to 5-degrees Celsius in next few decades (Boyes, Chuckran, & Stanisstreet, 1993), which in turn would melt polar ice caps and rise the sea level. To restrain the pace of the existing climatic disruption, though the governments and different organizations have taken a number of initiatives, such as "The Paris Agreement", research shows that introduction of climatic issues in school curriculum would develop within students an awareness about the climate (Shepardson, Niyogi, Choi, and Charusombat, 2009). Mathematics education inarguably plays a significant role in the process of educating students about the complex yet pressing issues related to climate (Barwell, 2013). Consequently, in this study, we aimed to explore the power of mathematical reasoning for developing students' understanding of the greenhouse effect, a major cause behind climate change and help them identify their contribution to the problem.

CLIMATE CHANGE AND MATHEMATICS LITERACY

Climate change and mathematics are closely related. Mathematics serves as an essential tool in every phase of describing, predicting, and communicating the effects of climate change. Governments and policymakers develop laws and policies around environmental conservation, largely based on the predictions made by mathematical models of climate Barwell (2013). Acknowledging the role of mathematics for addressing climate change, Abtahi, Gotze, Steffensen, Hauge, and Barwell (2017) questioned the "ethical and moral responsibilities" (p. 2) of mathematics educators to educate their students about climate. They argued, if teachers assume their ethical responsibilities and incorporate climate change into their mathematics instruction, then that would facilitate students' ability to identify the role of mathematics in climate change and prepare future decision makers to affect change for the betterment of the climate.

THEORETICAL FRAMEWORK

According to Barwell (2013), mathematics literacy is essential for students to interpret data and graphs about the greenhouse effect, as available in news and public media. Consistent with that argument, this study focused on students' covariational reasoning as a fundamental concept for interpreting graphs (Moore, Paoletti, Stevens & Hobson (2016). Covariational reasoning involves coordinating two quantities as the values of those quantities change (Confrey & Smith, 1995). A student reasons covariationally when she envisions two quantities varying simultaneously (Thompson & Carlson, 2017). For instance, a students' articulation, as the amount of carbon-dioxide increases, the air temperature increases simultaneously, illustrates her covariational reasoning. While investigating students' covariational reasoning, Carlson, Jacobs, Coe, Larsen, and Hsu (2002) developed a framework describing five mental actions that characterizes students' covariational reasoning when engaged in graphical activities. According to Carlson et al. (2002), students exhibit first mental action (MA1) when they focus on the coordination of two quantities (For example, carbon-dioxide changes, air temperature changes). Under MA2, students focus on the direction of change of two quantities and reason, as the amount of carbon-dioxide increases, air temperature increases. Mental Action 3 (MA3) involves the coordination between the amount of change in one quantity due to change in the other quantity. For example, students identify, as the value of carbon-dioxide increases by 100 units, air temperature increases by 5 degree Celsius. Students exhibit MA4 and MA5 if they can coordinate the average and instantaneous rate of change of one quantity with respect to change in the other quantity. We used this framework of mental actions to engineer learning opportunities for students to reason covariationally and study how this type of reasoning may create scope for students in advancing their understanding of the greenhouse effect. More specifically, we explored: How may students' covariational reasoning help them develop an awareness about the causes and consequences of the greenhouse effect?

METHOD

The primary methodology of this study is whole-class design experiment (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003). This highly interventionist method was chosen to engineer particular forms of covariational reasoning and examine the impact of those forms on developing students' understanding of the greenhouse effect (Basu & Panorkou, 2019). More specifically, we undertook the following three primary objectives: a) develop dynamic mathematical activities and implement in middle school classrooms; b) study students' thinking as they engage with the activities and observe the progression of their covariational reasoning; and c) examine the role of covariational reasoning in developing students' understanding of the greenhouse effect. We made a humble conjecture that the dynamic activities would provide students an exploratory space to engage in covariational reasoning, which in turn would help students develop their understanding of the causes and consequences of the greenhouse effect. The activities went through two iterations of implementation and revision (Cobb et al., 2003) to ensure an extent of generalizability.

The Role of Technology in Task Design

Akgun (2013) stated that technology nurtures within students an affinity for STEM literacy and make learning more meaningful and efficient through active participation and social interaction. Indeed, when students are introduced to a dynamic interactive environment through technology, they are motivated to play and tinker with the different features of the interface and engage in learning of mathematical concepts through observation and self-exploration (Resnick, 2014). Prior research on covariational reasoning shows that technology helps students envision the change in quantities as well as to reverse change, which is not always practical with physical manipulations (Castillo-Garsow, Johnson, & Moore, 2013). Consistently, in this study, we used NetLogo (Wilensky, 1999), a multi-agent programmable modeling environment, to develop three simulations on the greenhouse effect. We hypothesized that the dynamic environment of NetLogo, its animated outputs, and the result plots would help students understand the dynamics of the interaction between the different quantities included in the simulations (Basu & Panorkou, 2019). This paper specifically focuses on two simulations, the Climate Change and the Carbon Calculator. We hoped that the simulations would engage students in covariational reasoning and help them understand the causes and consequences of the greenhouse effect. The simulations were accompanied with a set of tasks and questions that we anticipated would provide students explicit and implicit prompts to engage them in critical thinking and shape their cognition (Boaler & Brodie, 2004).

Simulation 1: Climate Change

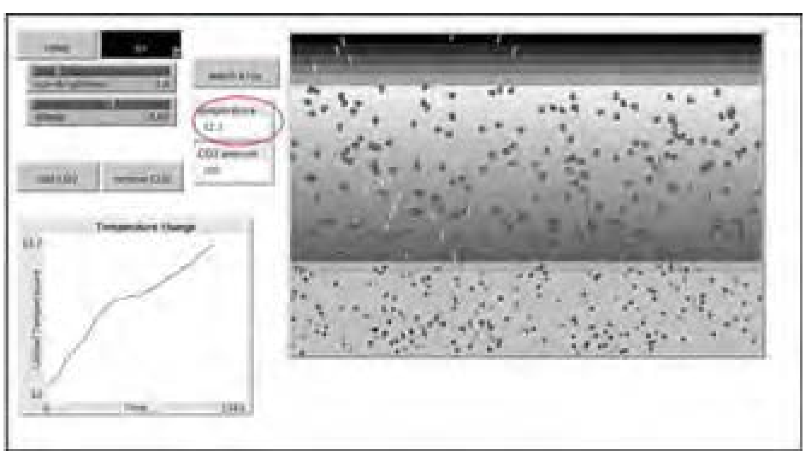


Figure 1: Climate Change Simulation

The first simulation of this study, the Climate Change (Figure 1), is adapted from NetLogo (https://ccl.northwestern.edu/netlogo/models/ClimateChange). This dynamic simulation is a model of the heat energy flow in the earth which provides users a space to explore how two environmental factors, the albedo of the earth and the amount of carbon-dioxide might impact global temperature. Users can move the albedo slider from left to right to change its value between zero and one and observe its impact on air temperature. The simulation also contains Add CO_2 and Remove CO_2 buttons, which the users can use to increase and decrease the amount of carbon-dioxide molecules (represented by green dots in Figure 1) and investigate how the value of global temperature changes along with it. Users can read the value of global temperature as recorded in a temperature monitor on the upper left side of the simulation (highlighted by red color); or they can observe a time-series graph on the lower left side of the simulation representing the change in global temperature with respect to time.

Simulation 2: Carbon Calculator

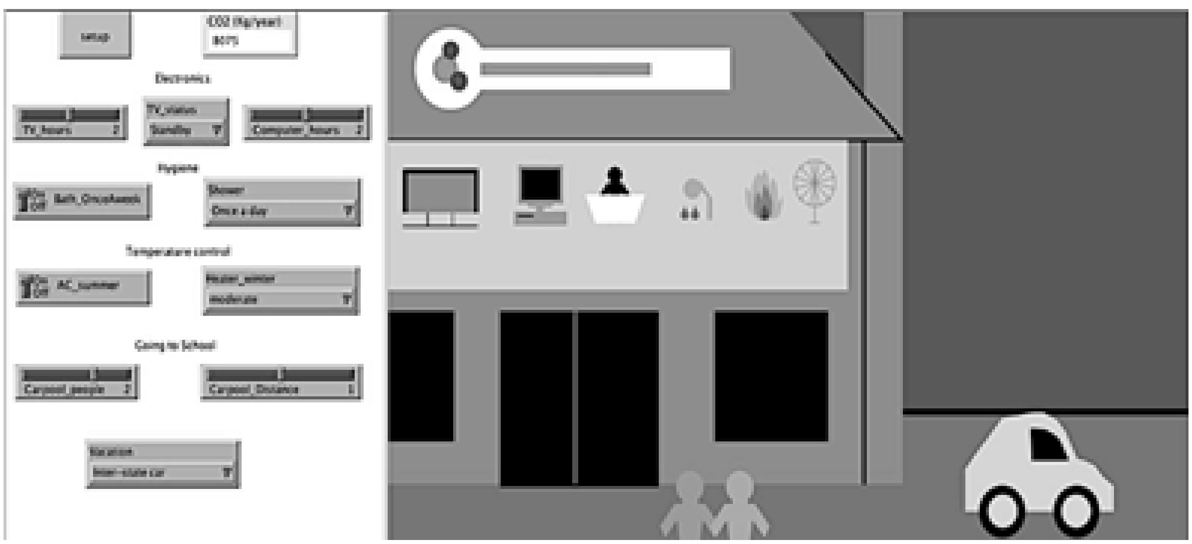


Figure 2: Carbon Calculator

Research suggests that if individuals identify various sources of carbon-dioxide and can estimate their contributions to the issue, then that would lead them to change their own behavior and work towards mitigation of the problem (Padgett, Steinemann, Clarke, & Vandenbergh, 2008). With a similar goal in mind, we designed the Carbon Calculator simulation (Figure 2). The simulation contained several activities, such as watching TV, playing video games, and using air conditioners, that we assumed would be familiar to the students, and are some of the factors responsible for enhanced carbon-dioxide concentration in the atmosphere. The simulation allowed the students to manipulate the values of these factors and observe the impact of the change on annual carbon-dioxide discharge. For example, students could drag the TV_hours slide to the right and left to increase and decrease the total number of TV_hours between zero and four and check the corresponding value of carbon-dioxide in the CO_2 (Kg/year) output box. Likewise, the simulation contained a drop-down menu for shower, that allowed the students to choose the number of times they take a shower in a week and calculate the annual amount of carbon-dioxide released.

Participants and Research Settings

We collaborated with two sixth-grade teachers, Doug and Chelsea (pseudonyms) from North-Eastern region of the United States. We implemented the dynamic activities in their classrooms through two cycles of design experiment. The first cycle took place in Doug's science classroom containing 27 students, and the second cycle took place in Chelsea's mathematics classroom containing 17 students. While Doug and Chelsea conducted the whole-class instruction, a member of the research team interacted with a small group of students to "create a small-scale version of the learning ecology so that it can be studied in depth and detail" (Cobb et al., 2003, p. 9). All the sessions were audio- and video-recorded, and students' written artifacts were collected as a complementary data source. We conducted two stages of data analysis, ongoing data analysis (Cobb, Stephan, McClain, & Gravemeijer, 2001) and retrospective analysis (Cobb et al., 2003). The ongoing analysis, conducted at the end of every session, informed us about students' covariational reasoning and helped us revise the tasks accordingly. Retrospective analysis, on the other hand, guided us to develop more robust theories about students' covariational reasoning that we anticipate might have formed as a result of their interaction with the simulations.

RESULTS

From the retrospective analysis we identified students' reasoning about three sets of quantities a) carbon-dioxide and air temperature b) TV_hours and carbon-dioxide, and c) carpool people and carbon-dioxide. In the following sub-sections, we discuss the forms of covariational reasoning students exhibited as they expressed these relationships and discuss how these forms of reasoning helped them develop an awareness about the greenhouse effect.

Relationship Between Carbon-dioxide and Air Temperature

The session began with students exploring the Climate Change simulation. Students focused on the covariational relationship between carbon-dioxide and air temperature. As students explored the relationship between the two quantities, we asked the them, "what will happen if I increase carbon-dioxide?" In response Nia said, "it increases the temperature." Aiming to examine how Nia identified the increase of temperature, we asked her, "how do you know it is increasing?" Nia pointed to the increasing time graph (Figure 1, lower left corner) in the simulation and identified that if carbon-dioxide increases, temperature gets higher.

Next, students engaged in a graphing activity. We anticipated that the graph might prompt students to focus on the numerical values of the two quantities, thus allowing them to recognize the amount of change and rate of change of air temperature with the change in carbon-dioxide (MA3, MA4). Students plotted the carbon-dioxide and air temperature ordered pairs and graphed the relationship between the two quantities. When we asked them to explain the graph, Ani measured the 'space' between two consecutive values of air temperature in the graph and argued, "this one from here (interval B) has more space than this one from here (interval A). This one has more space in between of them (interval B)." (Figure 3). Referring to the air temperature intervals he further added, "here from here, like 3 fingers and from here to here like 4 fingers. So, it has more space here than here."

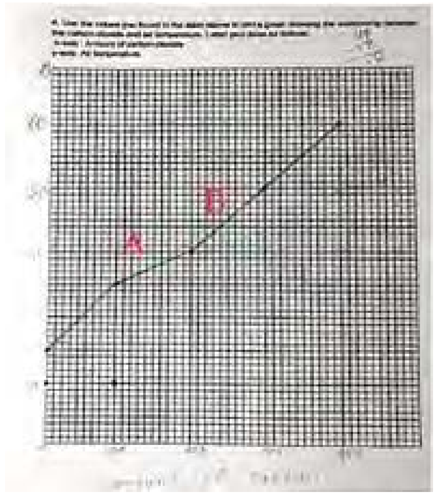


Figure 3: Ani's graph representing the relationship between carbon-dioxide and air temperature

By measuring the space between the various intervals, Ani seemed to focus on the change of value of air temperature in each interval of carbon-dioxide ([0-100], [100-200], and so on). Ani's response indicates that the graphing activity provided him a space to correlate the amount of change of air temperature with change in the value of carbon-dioxide, a type of reasoning aligned to Carlson et al.'s (2002) MA3.

Relationship Between TV-hours and Carbon-dioxide Amount

Next, students worked on the Carbon Calculator simulation. They manipulated the number of hours they watch TV between one and four, observed the corresponding values of carbon-dioxide, and recorded the ordered pairs in the TV hours and CO_2 amount table.

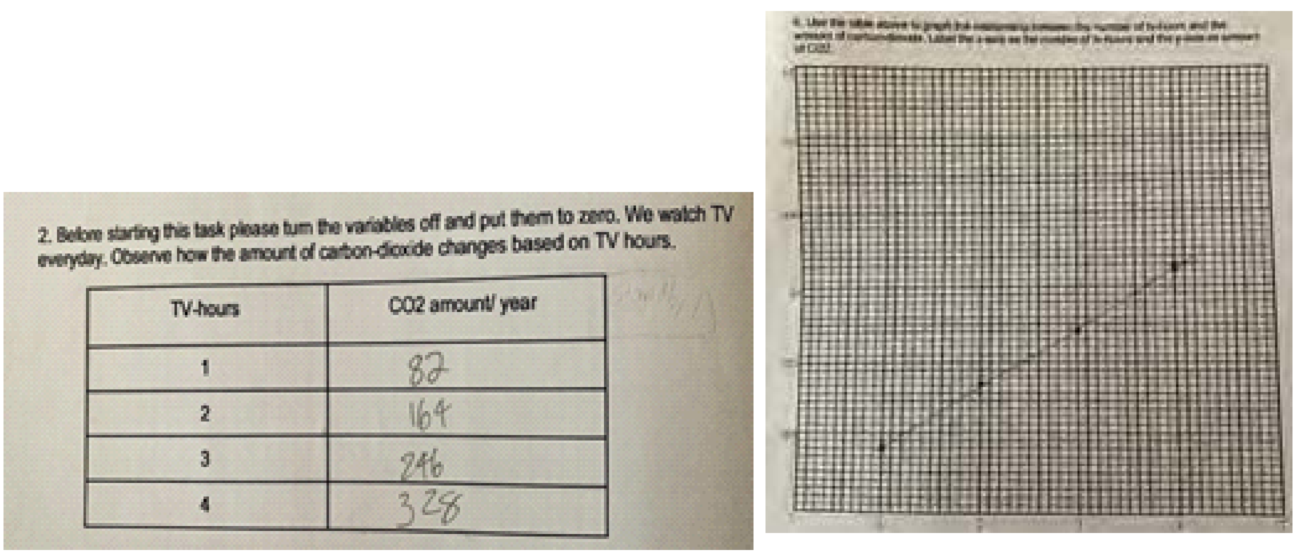


Figure 4: Amber's graph expressing the relationship between TV_hours and carbon-dioxide

Students engaged in a graphing activity where they plotted the TV hours versus amount of carbon-dioxide ordered pairs and graphed the covariational relationship between the two quantities. When we asked the students to reflect on the relationship between the two quantities, Amber said that when the TV hours doubles, the amount of carbon-dioxide also doubles. When we asked Amber to explain her response of doubling carbon-dioxide, she added, "You just keep adding depending on the hours of usage of TV." Being unable to understand if Amber was thinking across the two quantities or coordinating the change of one quantity with the change in another, we prompted Amber to explain her answer. She replied,

Amber : *Per hour it is 82. The amount of....*

Interviewer : *Carbon-dioxide?*

Amber : *Yeah. And if you multiply 82 times 2, the total is 164 which is 2 hours. So, for just every hour you just keep adding 82.*

Amber's statement, "So, for just every hour you just keep adding 82" indicates that Amber used the table to focus on the amount of change of carbon-dioxide for every hour change of TV usage and incremented the carbon-dioxide amount by 82 for each unit increment of TV hours. The above discussion suggests Amber's MA3 reasoning.

Relationship Between Carpooling and Carbon-dioxide Amount

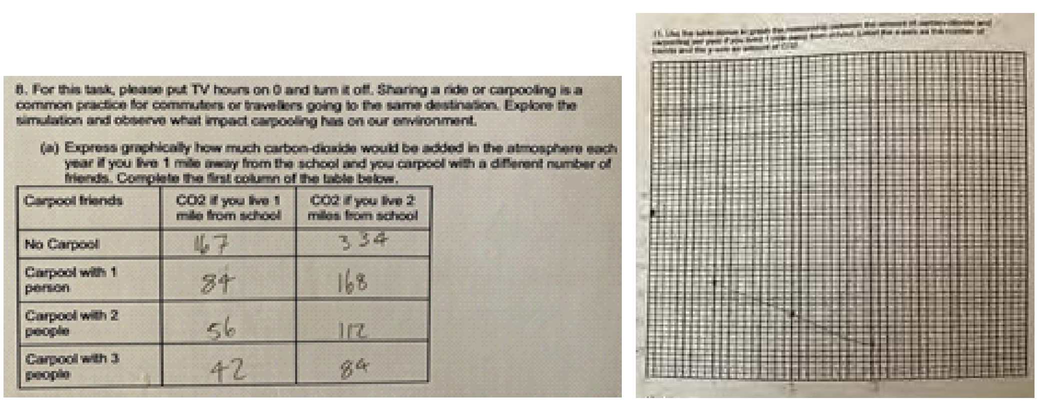
Another activity in the Carbon Calculator simulation asked the students to explore the covariational relationship between the number of people carpooling and the amount of carbon-dioxide being released. Like the TV_hour activity, here as well students modified the number of people carpooling by dragging the carpool_people slider and recorded the corresponding values of carbon-dioxide in a carpooling versus CO_2 table (Figure 5). Next, students plotted the ordered pairs and graph the relationship between the two quantities. However, before drawing the graph, we asked them to predict the nature of the graph. The following excerpt illustrates our conversation with Amber on this regard.

Interviewer : *Before plotting can you give me some idea how the graph will look like?*

Amber : *It will start going down, decreasing.*

Interviewer : *Why?*

Amber : *Because since you are carpooling, the more people you carpool, the less cars you use. So, that means the less carbon-dioxide you are using.*



8. For this task, please put TV hours on 0 and turn it off. Sharing a ride or carpooling is a common practice for commuters or travellers going to the same destination. Explore the simulation and observe what impact carpooling has on our environment.

(a) Express graphically how much carbon-dioxide would be added in the atmosphere each year if you live 1 mile away from the school and you carpool with a different number of friends. Complete the first column of the table below.

Carpool friends	CO2 if you live 1 mile from school	CO2 if you live 2 miles from school
No Carpool	167	334
Carpool with 1 person	84	168
Carpool with 2 people	56	112
Carpool with 3 people	42	84

Figure 5: Amber's graph expressing the relationship between carpooling and carbon-dioxide

Amber focused on the directional relationship between the two quantities and identified the greater number of people carpool, the lesser would be the number of cars, and as a result, the reduced would be the amount of emitted carbon-dioxide (MA2).

The activities stimulated a discussion in the classroom regarding the causes and consequences of carbon-dioxide emission. Gio said, "I did not think even if the TV is plugged in or the video game or X-box is plugged in it still releases CO2." When we asked the students to propose some strategies to lower atmospheric carbon-dioxide emission, Ani suggested that all students need to "talk to their parents not to use cars so much." He recommended: "stop driving, more walking, using bicycles." Echoing Ani, Gio suggested using "public transport" to reduce the carbon-dioxide concentration in the atmosphere. From the students' response it seemed to us that the activities acted as a pre-cursor towards developing their sense of agency towards mitigating carbon-dioxide emission by adjusting their daily practices.

DISCUSSION

Our research has revealed that dynamic mathematical activities allowed students to engage in different levels of covariational reasoning and identify the relationship between quantities expressed graphically (for example, as carbon-dioxide increases, temperature increases), and interpret those relationship in light of the context of the greenhouse effect (excess carbon-dioxide enhances air temperature). Observed trends suggest that covariational reasoning, as illustrated by the students' excerpts, bridged the mathematical and scientific aspects of the greenhouse effect and helped students develop an integrated understanding of the phenomenon. We expect that this article will leave mathematics and science teachers with afterthoughts regarding their roles and responsibilities in empowering students mathematically and scientifically and helping future citizens to become more sensitive towards their environment.

ACKNOWLEDGEMENTS

This research is supported by a grant from the National Science Foundation under grant #1742125. We thank Dr. Michelle Zhu and Dr. Pankaj Lal for sharing their expertise and insight in the study.

REFERENCES

Akgun, O. E. (2013). Technology in STEM project-based learning. In *STEM Project-Based Learning* (pp. 65-75). Brill Sense.

Abtahi, Y., Gotze, P., Steffensen, L., Hauge, K. H., & Barwell, R. (2017). Teaching climate change in mathematics classrooms: An ethical responsibility'. *Philosophy of Mathematics Education Journal*, (32).

Barwell, R. (2013). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education*, 15(1), 1-16.

Basu, D., & Panorkou, N. (2019). Integrating Covariational Reasoning and Technology into the Teaching and

Learning of the Greenhouse Effect. *Journal of Mathematics Education*, 12(1), 6-23.

Black, I. R., & Cherrier, H. (2010). Anti consumption as part of living a sustainable lifestyle: daily practices, contextual motivations and subjective values. *Journal of Consumer Behaviour*, 9(6), 437-453.

Boaler, J., & Brodie, K. (2004, October). The importance, nature, and impact of teacher questions. In *Proceedings of the twenty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 774-782).

Boyes, E., Chuckran, D., & Stanisstreet, M. (1993). How do high school students perceive global climatic change: What are its manifestations? What are its origins? What corrective action can be taken? *Journal of Science Education and Technology*, 2(4), 541-557.

Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. (2002). Applying covariational reasoning while modeling dynamic events: A framework and a study. *Journal for Research in Mathematics Education*, 33(5), 352-378.

Castillo-Garsow, C., Johnson, H. L., & Moore, K. C. (2013). Chunky and smooth images of change. *For the Learning of Mathematics*, 33(3), 31-37.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The journal of the Learning Sciences*, 10(1-2), 113-163.

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational researcher*, 32(1), 9-13.

Confrey, J., & Smith, E. (1995). Splitting, covariation, and their role in the development of exponential functions. *Journal for research in mathematics education*, 26(1): 66-86.

Dolman, A. J., & Verhagen, A. (2003). Land use and global environmental change. In Dolman A.J., Verhagen A., Rovers C.A. (eds) Global environmental change and land use (pp. 3-13). Springer, Dordrecht.

Karl, T. R., & Trenberth, K. E. (2003). Modern global climate change. *science*, 302(5651), 1719-1723.

Kolbert, E. (2006). *The Darkening Sea*. The New Yorker, 20, 66-75.

Moore, K. C., Paoletti, T., Stevens, I. E., & Hobson, N. L. F. (2016). Graphing habits:" I just don't like that. In *Proceedings of the 19th Meeting of the MAA Special Interest Group on Research in Undergraduate Mathematics Education*. Pittsburgh, PA: RUME.

Padgett, J. P., Steinemann, A. C., Clarke, J. H., & Vandenbergh, M. P. (2008). A comparison

of carbon calculators. *Environmental impact assessment review*, 28(2-3), 106-115.

Resnick, M. (2014, August). Give P's a chance: Projects, peers, passion, play. In Constructionismand creativity: *Proceedings of the Third International Constructionism Conference. Austrian Computer Society*, Vienna (pp. 13-20).

Shepardson, D. P., Niyogi, D., Choi, S., & Charusombat, U. (2009). Seventh grade students' conceptions of global warming and climate change. *Environmental Education Research*, 15(5), 549-570.

Thompson, P. W., & Carlson, M. P. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Eds.) *Compendium for research in mathematics education*, 421-456, Reston, VA: NCTM.

Wilensky, U. (1999). NetLogo. (http://ccl.northwestern.edu/netlogo/). *Center for Connected Learning and Computer-Based Modeling*, Northwestern University, IL.