Examining Students' Quantitative Reasoning in a Virtual Ecosystem Simulation of the Water Cycle

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Abstract: In this study, we designed and tested an instructional module to help students develop their quantitative reasoning while examining the scientific phenomenon of the water cycle. The results from a design experiment in a sixth-grade classroom showed that students exhibited reasoning that integrated science and mathematics and avoided the typical naïve conceptions about the water cycle. Specifically, students engaged in covariational and multivariational reasoning as they coordinated the simultaneous change of the quantities involved in the water cycle. They also exhibited transitive reasoning in terms of those quantities and relational reasoning connecting the water cycle to other science content. These forms of reasoning illustrate the power of mathematical reasoning for helping students construct an advanced understanding of the water cycle phenomenon. The findings also show that the context of science helped students to construct advanced forms of mathematical reasoning about quantities, illustrating the reciprocal relationship between math and science learning.

Math and science integration: The case of the water cycle learning

Mathematics and science integration efforts illustrate the common practices between math and science and how these commonalities serve as a platform to help students experience meaningful and relevant interdisciplinary learning (e.g., National Research Council [NRC], 2013). A seamless integration of mathematics into science topics can help students develop high levels of understanding about the science phenomena (Roschelle et al., 2007). Although integrated mathematics and science activities have been developed, these activities only encourage students to solve science problems using their existing mathematical knowledge without exhibiting any new forms of mathematical reasoning during the process (Tytler, Williams, Hobbs & Anderson, 2019). In other words, we have limited information about how integrated activities can be designed that would develop students' reasoning in ways that illustrate a *reciprocal relationship* (English, 2016) between mathematics and science. A reciprocal relationship would involve developing a deeper understanding of the science phenomenon because of the integration with mathematics while also advancing their mathematical reasoning because of the integration with science.

To examine the design of activities that help illustrate such reciprocal relationships between math and science, we designed an instructional module that integrates the mathematics of quantitative reasoning and the science of the water cycle. We chose to focus on the water cycle because it is one of the first, most abstract, and complex scientific phenomena introduced to students (Vinisha & Ramadas, 2013). To understand the complex system of the water cycle, research suggests that its learning must be integrated with other educational contexts in order for students to develop a coherent understanding (Vinisha & Ramadas, 2013). In contrast to this view, the water cycle has been exclusively taught in science classes (Barrutia, Ruíz-González, Villarroel, & Díez, 2019). In the current science curricula, the water cycle is commonly taught using illustrations from science textbooks, where students are asked to describe the water cycle through drawings (Vinisha & Ramadas, 2013). Although the use of textbook illustrations or drawings has been found to help students develop a basic understanding about the water cycle (Cardak, 2009), research shows that students also develop many naïve conceptions about the phenomenon. For example, some students believe that evaporation does not happen in cold temperatures (Ben-zvi-Assarf & Orion, 2005), or that the amount of water vapor in the air remains unchanged (Cardak, 2009). Moreover, the learning of the water cycle involves students understanding how water goes through a cyclical movement (Ben-zvi-Assarf & Orion, 2005); however, studies show that students do not have this cyclic notion (Barrutia et al., 2019). Instead, they usually over-simplify the cycle, believing that water only travels to the atmosphere from the earth through evaporation and returns to the earth through condensation (Cardak, 2009). To develop the cyclic notion of the water cycle and develop these naïve conceptions to more sophisticated understandings, students must be able to coordinate its different components into a coherent unitary system (Kali, Orion, & Eylon, 2003). The next section describes the theoretical framework that we used to design such experiences for students.

Theoretical framework

Across the mathematics and science education literature, the development of students' reasoning is often an isolated process and exclusive to each discipline. To bridge the two disciplines, we first examined their similarities. We found that studying change is predominant in both mathematics and science concepts; thus, we examined its potential for developing students' integrated reasoning in the two disciplines. In science phenomena, we usually have a single variable changing, or two variables changing simultaneously. Reasoning about these variables typically focuses on cause-and-effect relationships where a student identifies the factors that influence a phenomenon, but this form of reasoning does not explain how these factors interact with each other. In other words, reasoning about the cause-and-effect relationships, such as identifying that humidity is a factor influencing precipitation, is different from reasoning about their quantities and how these quantities covary, such as reasoning that as the humidity increases, the precipitation increases.

To investigate students' reasoning about the quantities embedded in the water cycle, we followed the work of Thompson and colleagues on quantitative reasoning. Quantitative reasoning is a mental process of analyzing a phenomenon, translating it into a system of quantities, and reasoning about the quantities and the relationships between them (Smith III & Thompson, 2008). Hence, by taking a quantitative reasoning perspective, understanding the water cycle includes conceiving its measurable quantities such as air temperature, lake temperature, mountain temperature, land temperature, and humidity, and constructing relationships between those and the processes of the water cycle, such as precipitation, evaporation, infiltration, and runoff.

When envisioning two quantities changing as those quantities change simultaneously, an individual is engaging in what research refers to as *covariational reasoning* (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Confrey & Smith, 1994). Reasoning covariationally does not necessarily involve reasoning using numerical values. In fact, research has shown that students can identify and reason about quantities using their magnitudes and not numerical values. For example, when coordinating quantities, students can attend to the direction of their change (Carlson et al., 2002) or to the intensity of change in quantities (Johnson, 2012). In coordinating the direction of change, students may reason about the *direct relationships* between quantities when quantities change in the same direction or the *inverse relationships* when quantities change in opposite directions. For example, a student may reason that as air temperature increases, the amount of water that evaporates also increases, illustrating the direct relationship between air temperature and the amount of evaporation as both are increasing. Also, a student may reason that as infiltration decreases, the amount of runoff increases, illustrating an inverse relationship between infiltration and runoff. In attending to the intensity of change in quantities, a student may reason that if the lake temperature is higher or lower, the evaporation is faster or slower, respectively.

Although quantitative and covariational reasoning has been significantly studied and found to be an important form of reasoning about quantities, most of these investigations use these forms of reasoning to frame their investigation of a mathematical concept, such as rate-of-change (Johnson, 2012), trigonometric relationships (Moore, 2012), or linear relationships and quadratic relationships (Ellis, 2011). As Thompson and Carlson (2017) pointed out, the great majority of these studies do not contribute directly to defining the covariation construct. In a previous study using quantitative reasoning in the learning of gravity (Basu et al., 2020), we found that sixth-grade students were able to coordinate both the mass and the distance when making judgments about gravity, showing that the science context helped them reason multivariationally about quantities. In mathematics education, multivariational reasoning has been studied as a form of reasoning in advanced mathematics (Kuster & Jones, 2019). Consequently, we started with the conjecture that integrating quantitative reasoning with science will not only help students with the science concept but will also give us an insight into sophisticated mathematical forms of reasoning about quantities that are possible through this integration. Specifically, we aimed to explore:

a) How may students reason about quantities in the context of the water cycle?

b) How may students' quantitative reasoning help them develop an understanding of the water cycle?

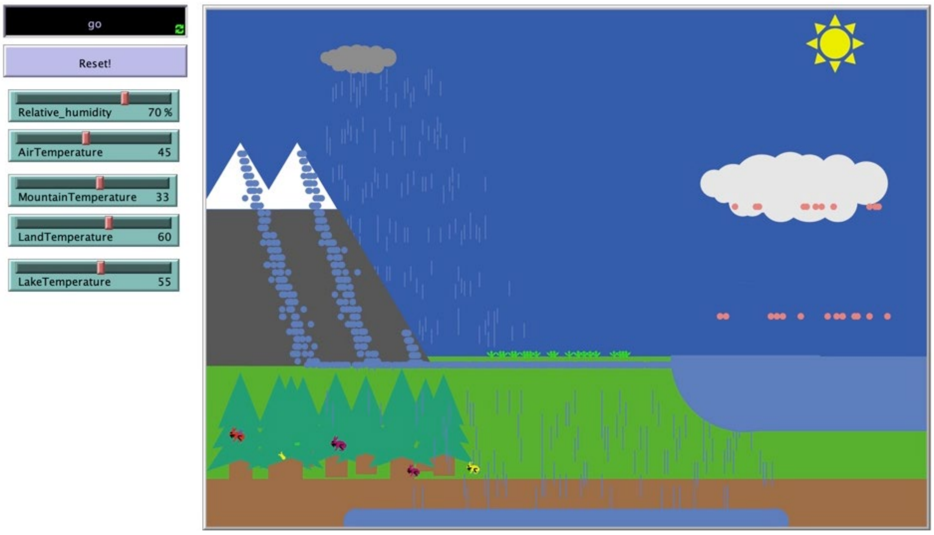
Method

We conducted a whole-class design experiment (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003) with sixth-grade students to engineer and study particular forms of quantitative and scientific reasoning in the context of the water cycle and explore the means that supported these forms of reasoning. This paper reports on the first iteration of our design, aiming to examine the integration of quantitative reasoning in the learning of the water cycle.

Tools and tasks design

For our design, we first considered that constructing dynamic models of the water cycle may help students progress past some of their naïve conceptions about the phenomenon. To provide students with opportunities for a dynamic exploration of the phenomenon, we developed the Water Cycle simulation (Figure 1) on NetLogo

(Wilensky, 1999), a multi-agent modeling environment. To help students visualize how the variables in water cycle change and interplay affecting the ways in which water molecules move around the virtual ecosystem, parametric values were assigned to relative humidity and temperatures of air, mountain, land, and lake (Zhu et al., 2018). In terms of relative humidity, the user can change the relative humidity in the virtual ecosystem by dragging the relative humidity slider between values of 0% and 100%. In terms of temperature, the user can modify the values for air, mountain, land, and lake temperature by dragging their sliders between values of 0 and 100 degrees Fahrenheit. As these values change, the students can observe the magnitude of the amount of water molecules in each process is also changing. For instance, as students increase or decrease the percentage of relative humidity, they can observe that the magnitude of red dots representing water molecules decreases or increases, respectively. In some processes, the state or production rate of water molecules also changes when temperatures are modified at a range of values. For example, when the air temperature slider is set below 32 degrees Fahrenheit, the precipitation turns into sleet from liquid rain. Similarly, when the lake temperature is above 32 degrees Fahrenheit, the evaporation slows down. The “Reset!” and “go” buttons can be used to reset the simulation between experiments so that the user can observe the changes that result from dragging only one particular slider at a time.



Relative_humidity 70 %
AirTemperature 45
MountainTemperature 33
LandTemperature 60
LakeTemperature 55

Figure 1. The Water Cycle simulation.

First, we asked the students to freely explore the simulation. Then, we engaged them in two sets of tasks that prompted them to explore covariational relationships between the quantities. In the first task, the students were asked to manipulate the humidity and the various temperatures one at a time and explore any changes in the processes of evaporation, precipitation, runoff, and infiltration in the simulation. We also intentionally asked them to move particular sliders at specific values and respond to questions such as “What do you observe?” and “What relationships have you found?” In the second task, students were asked to identify which of the variables influence a specific water cycle process and under what conditions of other variables the water molecules in a process would increase or decrease. For instance, “To make more rain, what would you do to the air temperature and humidity?” We conjectured that by experimenting with the Water Cycle simulation, changing variables, and receiving instant feedback from the environment, students would identify and reason covariationally about the quantities presented in the virtual ecosystem.

Participants, research context, and analysis

The whole-class design experiment took place in a sixth-grade classroom in a public elementary school in the northeastern United States. We focused on sixth grade because students at this grade are expected to reason about relationships between varying quantities (Common Core State Standards for Mathematics 6.EE.9 and 6.RP.A.3) (NGA & CCSSO, 2010) and are expected to learn about water and its continuous cycle between different spheres of the earth (Earth and Space Science 2.C for Middle School) (NRC, 2013). The design experiment consisted of five 45-minute sessions over five days. Each session was video-recorded. While the teachers led the whole class instruction of the module, we observed and interviewed three pairs of students so that we could study these students’ learning in depth as a microscale of “learning ecology” (Cobb et al., 2003, p. 9). These students were described by their teacher as being active participants in class discussions. We collected students’ work and transcribed our interviews with the three pairs.

At the end of the experiment, we conducted a retrospective analysis (Cobb et al., 2003) of the data. We focused on students' reasoning about quantities and their relationships as the unit of our analysis. First, we worked chronologically through the videos and transcriptions and identified students' episodes that illustrated forms of quantitative reasoning according to our theoretical framework mentioned above. We were also open to any new forms of reasoning that students might exhibit that may not have been documented in previous studies. Next, we identified the qualitatively different ways that students expressed quantitative reasoning and examined the reproducible patterns that the students exhibited when engaging with specific forms of activities (research question *a*). We categorized the qualitatively different ways of reasoning about quantities in two sections, namely a) coordinating the change of two quantities as they change in relation to each other, and b) coordinating the change of three quantities as they change in relation to each other. In the following section, we describe these categories and we also discuss how these forms of reasoning contributed to students' understanding of the water cycle phenomenon (research question *b*).

Findings

In this section, we provide examples from student episodes to discuss how students reasoned quantitatively and scientifically in the context of the water cycle and describe the ways in which students' activity within the tools, tasks, and questioning supported their forms of reasoning.

Coordinating the change in two quantities as they change in relation to each other

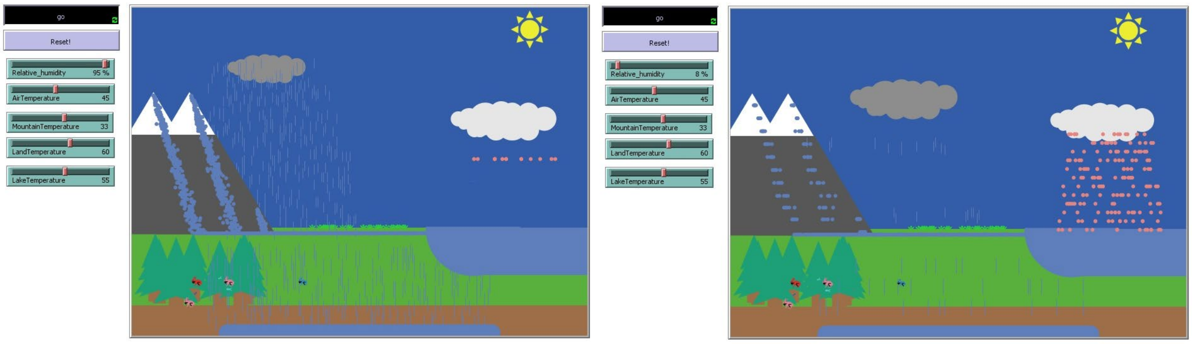
At the beginning of the experiment, students were asked to manipulate the humidity and the various temperatures one at a time and explore any changes in the process of evaporation. As they increased and decreased the percentage of relative humidity, students observed that the amount of evaporation decreases and increases, respectively (Figure 2). For instance, Chloe [Pair A] noticed that evaporation "increases when the humidity is low." Similar to Chloe, students used covariational reasoning to describe how the two quantities, humidity and evaporation, change at the same time. They were also able to describe the direction of change in the quantities and point to their inverse relationship. When we asked them why this happens, Justin [Pair A] explained, "when there is more humidity, there is more air, there's more water vapor in the air. It's kind of preventing it from adding another layer, affecting how evaporation goes." Similarly, Von [Pair B] explains this in the excerpt below:

Researcher: If relative humidity is high, then, why do you think evaporation is low?

Von: Because there's already enough water vapor in the air. So that, when it evaporates, it makes them less.

Researcher: Why is there more evaporation when there is less relative humidity?

Von: Because there's less water vapor, so the evaporation [pointing at evaporation], they, fills it up.



Relative_humidity 95 %
AirTemperature 45
MountainTemperature 33
LandTemperature 60
LakeTemperature 55

Relative_humidity 8 %
AirTemperature 45
MountainTemperature 33
LandTemperature 60
LakeTemperature 55

Figure 2. Amount of evaporation when relative humidity is increased (left) or decreased (right).

Next, as students manipulated the lake temperature slider, they observed that the amount of evaporation also changes (Figure 3). For example, Chloe observed that when she increased the lake temperature, "it makes it [evaporation] grow," illustrating that she noticed the direct relationship between lake temperature and evaporation. Justin noticed that "the higher the lake temperature, the faster the molecules evaporate. The lower the temperatures, let's say at zero, the molecules start to a very low timing." There are a few things we would like to discuss about Justin's reasoning. First, his reasoning about the rate of change of evaporation as "faster" or "a very low timing" is similar to what Johnson (2012) describes as the coordination of intensity of change in a quantity by systematically changing one quantity. It is important to emphasize here the role of simulation in the development of Justin's reasoning as it was designed to illustrate the rate of change of evaporation based on the lake temperature. Also, this reasoning that evaporation happens when the temperature is low but at "a very (s)low timing" shows that, by manipulating the temperature in low and high temperatures in the simulation, he avoided the misconception that evaporation does not occur at low temperatures (Ben-zvi-Assarf & Orion, 2005). Finally, his reasoning that when the relative humidity is higher, there is "more water vapor in the air," and it is forming "layers," shows that, by observing the change in the amount of water molecules in the simulation, he avoided the common belief that the amount of water vapor in the air remains the same (Cardak, 2009).

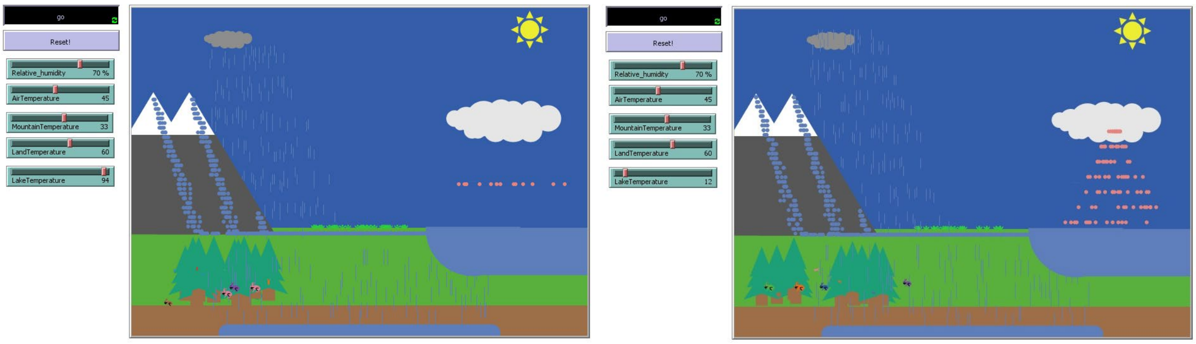


Figure 3. Rate of evaporation when the lake temperature is increased (left) or decreased (right).

In exploring the relationship between lake temperature and evaporation further, Von [Pair B] changed the lake temperature to 100 degrees and noticed that "when it gets hot enough, it turns into a gas, water vapor." Drake [Pair B] added that "the lake temperature, if the higher it is, they [pointing at the water molecules] spread out more, the lower it is, and if they [water molecules] stick together more [dragging the slider of lake temperature to the left]." Aside from illustrating their reasoning about the relationship of the direction of change in temperature with the rate of change in evaporation in the simulation, both Drake and Von emphasized the molecular structure of water during evaporation. We interpret their reasoning of "they spread out more" and "turns into a gas, water vapor" as an attempt to integrate scientific ideas about the impact of heat absorption on the state of matter, that when the temperature is high, the hydrogen bonds between water molecules break and the water molecules move freely. Drake was also able to combine this reasoning with the intensity of change in the quantity of evaporation:

Researcher:	Now, if your lake temperature is at 15 [degrees Fahrenheit], what happens?
Drake:	[Dragging the lake temperature to 15]. Slow [pointing at the evaporating water molecules]. The evaporation is slower.
Researcher:	What will happen if I make it 75 [degrees Fahrenheit]?
Drake:	[Dragging the lake temperature to 75 and pointing on the cloud] It's faster but they spread apart.

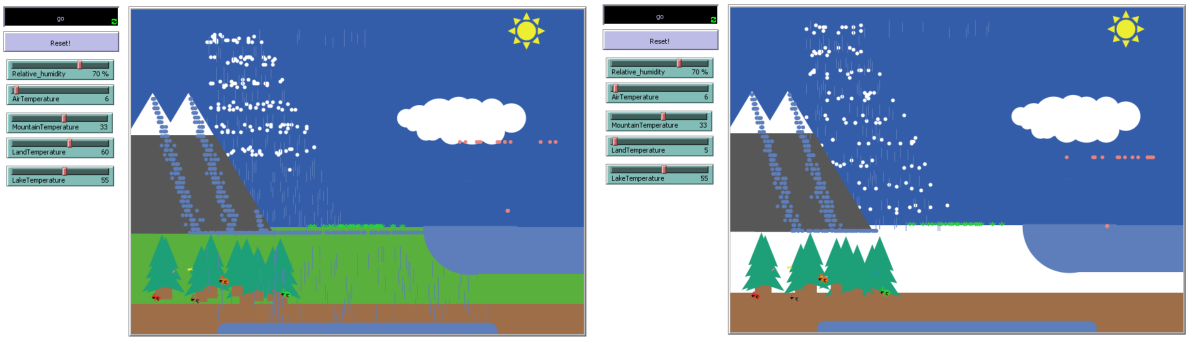
In the episode above, Drake [Pair B] shows that he was able to envision the "faster" or "slower" change in evaporation with the change in the molecular structure of water ("they spread apart") while coordinating the change in lake temperature.

Although in this paper we focus only on evaporation to show how students coordinated two quantities, in the tasks that followed students continued to coordinate the direction of change in humidity and temperature as those influence the variation in precipitation, runoff, or infiltration. For instance, Ray [Pair C] reasoned that "there will be more rain if the relative humidity was higher, and lower rain, lower amount of rain if the relative humidity is lower." Through these tasks, students were able to coordinate the change in two quantities in multiple processes and reason covariationally in that way without having the need to assign any values to those quantities.

Coordinating the change in three quantities as they change in relation to each other

All pairs were able to reason covariationally about the direction of change of two quantities as they explored the variables involved in evaporation. During those tasks, we also noticed that some students began to reason about changes in three quantities simultaneously. For example, when we asked, "How do changes in lake temperature and relative humidity change evaporation?" Diego [Pair C] stated, "The lower the lake temperature, the lower the rate of evaporation, and the lower the amount of water vapor that goes into the air." In his reasoning, Diego exhibited that he considered the changes in three quantities, namely lake temperature, rate of evaporation, and amount of water vapor, illustrating in that way a form of *multivariational reasoning*. He also attended to the direct relationship between the three quantities changing. Similar to other students before, he recognized that evaporation occurs even at low temperatures, avoiding the common naïve conception that it does not, and elaborated that as a result, there will be a low amount of water molecules in the form of vapor in the air.

Indeed, the water cycle is a complex system with several components that interplay. To further engage students with more complex quantitative relationships of the water cycle, we prompted them to reason about three quantities simultaneously. For example, we asked, "Manipulate only the air temperature and the land temperature to make the precipitation be in the form of snow. What conditions will release more snow?" Chloe and Justin [Pair A] immediately argued that "We need both of them to be cold." When we asked them to explain their reasoning, Chloe dragged the air temperature slider to the left (lower values) and she showed that the cloud started to release sleet, a mixture of rain and snow (Figure 4, left), describing this as "if you just move for air temperature, it only snows a little bit." Then, she changed the land temperature by moving its slider to the left and showed that the cloud released more snow which accumulated on the ground and stated "But if you put it with a land temperature, it starts to accumulate in the ground, and it produces more" snow (Figure 4, right). Chloe shows that she transitioned from coordinating two quantities, air temperature and snow, to adding a third quantity into this relationship that would make even more snow. As a result, she constructed a quantitative relationship that included three quantities changing simultaneously, illustrating a form of multivariational reasoning.



Relative_humidity 70 %
AirTemperature 6
MountainTemperature 33
LandTemperature 60
LakeTemperature 55

Relative_humidity 70 %
AirTemperature 6
MountainTemperature 33
LandTemperature 5
LakeTemperature 55

Figure 4. Amount of snow when only the air temperature is low (left) and when both the land temperature and the air temperature are low (right).

Next, we explored students' reasoning about coordinating quantities using open-ended generic questions, such as "What influences runoff?" without prompting them to focus on specific variables. For example, Justin [Pair A] stated, "if there's a very low humidity, it doesn't rain as much, and the rain will not get to the mountains. So, if there's zero humidity, then, there's no rain. So, it's like, there's not much runoff from the mountains." Similar to the other students, Justin reasoned multivariationally by reasoning about the direction of change in humidity and coordinating this change with the change in the quantity of precipitation in the form of rain while also coordinating the change in the quantity of runoff. What is also noteworthy in Justin's statement is that he was able to generalize that "if there's zero humidity, then, there's no rain" showing that he recognizes that if one of the quantities in the quantitative relationship is zero, then the other quantity will be zero.

In his statement above, Justin began making connections between two processes, precipitation and runoff. Similarly, students began coordinating the changes in three water cycle processes, namely evaporation, precipitation, and runoff. For instance, Ray [Pair C] stated, "If the rate of evaporation is higher, there could be higher rate of precipitation. If there's a higher rate of precipitation, there could be more runoff. So, the higher rate of evaporation, there can be more runoff." We consider Ray's reasoning to be another form of multivariational reasoning as he exhibited a *transitive relationship* between three quantities that are changing simultaneously. Specifically, he argued that because evaporation (quantity *a*) is related to precipitation (quantity *b*), and precipitation (quantity *b*) is related to runoff (quantity *c)*, then evaporation (quantity *a*) is related to runoff (quantity *c*). We would argue that reasoning about the transitive relationships of quantities can be useful in understanding the synthesis of multiple components of a complex scientific concept. Although Ray recognized a transitive relationship between these three quantities, it is also apparent in his language that he was cautious when stating such a relationship since other components in the water cycle can interplay, causing inverse changes to any of the three variables. His language "there *could* be more runoff" shows this caution.

Finally, we noticed that some students illustrated evidence of coordinating changes of more than three variables. For example, Chloe [Pair A] reasoned, “So, for this model…gravity makes it rain. But when there’s more rain, there would be more runoff. Over here [mountain], less than 32 [mountain temperature], when it’s snowing, it doesn’t affect runoff because it doesn’t, the mountain is freezing it, and the land is also freezing it because it’s cold.” Chloe’s reasoning shows that she recognizes the water cycle to be a complex system with multiple components. She exhibited a sophisticated coordination of the changing mountain temperature and land temperature and their relationships, including the role of gravity in precipitation and runoff in the water cycle (Ben-zvi-Assarf & Orion, 2005). We consider Chloe’s reference to gravity as illustrating a form of *relational reasoning* as it expresses a connection between the science of the water cycle with what she has learned about other science concepts. We define this form of reasoning as *relational* in the sense of Van de Walle et al. (2014), a form of reasoning that illustrates the connections of an idea with other ideas involving mathematics and science.

Conclusions

This study investigated students’ quantitative reasoning as they explored the simultaneous change of quantities in the water cycle. In exploring the Water Cycle simulation on NetLogo, students examined some varying quantities that influence the water cycle processes. As students interacted with and received feedback from the simulation, they were able to identify the quantities involved and reason about those quantities and their relationships. Our findings show that students reasoned covariationally about the direction of change of two quantities as these quantities change simultaneously in the Water Cycle simulation. The carefully designed models of those quantities in the simulation led them to also reason about the changes in the intensity (Johnson, 2012) and the structure of those quantities. By experimenting with multiple variables at the same time to explain the phenomenon of the water cycle, students illustrated more sophisticated forms of reasoning such as multivariational reasoning (Kuster & Jones, 2019), transitive reasoning, and relational reasoning. We consider these types of reasoning to show evidence of students’ understanding of how the various quantities interplay in the water cycle (Ben-zvi-Assarf & Orion, 2005) and their synthesis of relationships of the water cycle components (Kali et al., 2003). By examining and reasoning about quantities and the relationships between these quantities, students developed an advanced understanding about the water cycle processes while avoiding the common naïve conceptions found in previous studies (Ben-zvi-Assarf & Orion, 2005; Cardak, 2009).

Students were able to identify and reason about the quantities of evaporation, precipitation, and runoff by observing the change in their magnitudes (amount and intensity of molecules, amount of rain/snow) as presented in the simulation and not numerical values. The design of the simulation included numeric values on each temperature slider and a percentage value on the humidity slider because we considered those quantities to be more abstract. However, we saw that students did not use these values in their reasoning, but instead, they reasoned non-numerically (e.g., the higher the temperature, the faster the evaporation). Therefore, in future iterations of our study, it will be interesting to explore whether we can represent those quantities as magnitudes as well. Additionally, it will be noteworthy to develop a new version of the simulation that has values in all quantities in order to examine how students could reason numerically about all of the relationships of magnitudes that they constructed in this version.

Our findings also provide evidence that reasoning quantitatively was a powerful way for developing a rigorous understanding of the complex phenomenon of the water cycle. In turn, the complexity of the water cycle assisted students in accessing advanced forms of quantitative reasoning. In other words, our findings illustrate that integrating quantitative reasoning in the learning of science phenomena has the potential to promote the reciprocal relationship (English, 2016) between science and mathematics. In contrast to exhibiting mathematics reasoning or science reasoning as isolated learning constructs bounded by disconnected disciplines (Tytler et al., 2019), our students’ reasoning illustrates the reciprocal relationship between the two disciplines. Consequently, we suggest that the design of our tools embedded in the Water Cycle simulation, the tasks in the instructional module, and the researcher-teacher questioning can be used as a framework for designing other modules that integrate science phenomena and quantitative reasoning.

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Acknowledgments

This research was supported by the National Science Foundation under grant number 1742125. The reviews expressed do not necessarily reflect official positions of the Foundation. The researchers would also like to thank Sowmith Etikyala, Michelle Zhu, and Pankaj Lal for the development of the simulation and Debasmita Basu for assisting in the data collection of the study.