Improving the Efficiency of Maxwell's Equations FDTD Modeling for Space Weather Applications by Scaling the Speed of Light

Yisong Zhang
Electrical and Computer Engineering
University of Utah
Salt Lake City, Utah, USA

Jamesina J. Simpson
Electrical and Computer Engineering
University of Utah
Salt Lake City, Utah, USA

Daniel Welling
Electrical and Computer Engineering
University of Texas at Arlington
Arlington, Texas, USA

Michael Liemohn
Electrical and Computer Engineering
University of Michigan
Ann Arbor, Michigan, USA

***Abstract*—Space weather can affect the Earth over time spans of hours and days. However, time-stepping increments for FDTD models are typically on the order of a fraction of a second. This paper introduces a means of increasing the time stepping increment's upper limit by artificially slowing down the speed of light. Numerically slowing down the speed of light is achieved by appropriately modifying the permittivity, permeability, and conductivity values in the model. Proof-of-concept results are provided to show that the method works well for homogeneous media.**

***Index Terms*—FDTD, Earth-ionosphere waveguide, electromagnetic propagation, space weather.**

I. Introduction

Geomagnetic disturbances caused by solar storms have the potential to create large-scale geomagnetically induced currents in long conductors at the Earth's surface. These may disrupt the operation of electric power grids and cause blackouts. For example, energized particles and magnetic fields ejected by Coronal Mass Ejections (CMEs) in the direction of Earth may disturb the Earth's magnetosphere and generate a geomagnetic storm.

The travel time of the energized particles and magnetic fields between the Sun and the Earth is 1-2 days. The resulting geomagnetic storm on Earth (including the initial, main, and recover phases) can continue for more than a day. Finite-difference time-domain (FDTD) simulations of the near-Earth environment during geomagnetic storms should be as efficient as possible to make it possible to run the simulations throughout the entire time span of the storm and predict the potential impact caused by the CME.

To ensure numerical stability of an FDTD model, based on a Von Neumann stability analysis the time step increment is restricted by the spatial grid resolution and the electromagnetic wave propagation velocity. Specifically, the time stepping increment may be increased by either decreasing the spatial grid resolution or by reducing the propagation velocity of the waves throughout the grid. For the application of near-Earth space weather hazards, a relatively fine spatial grid resolution is desired so that the details of the lithosphere, topography, and ionosphere may be included. As a result, in this work, the possibility of reducing the speed of light is investigated to improve the simulation efficiency.

Motivation for this study is as follows: Relative to the time-step increment of a typical Earth-ionosphere waveguide model (on the order of microseconds), the overlying disturbed ionospheric currents during a geomagnetic storm are typically considered to change very slowly (on the order of a second or longer). As a result, slowing down the wave speed from the speed of light by a factor of, say, ten, is not expected to negatively impact the observations at the Earth's surface. Electromagnetic waves traveling from an altitude of 110 km in the ionosphere would reach the ground in 3 milliseconds instead of 0.3 milliseconds.

Section II provides a description of the proposed method, including how the permittivity, permeability, conductivity, and lithosphere layer thicknesses should be scaled in order to yield identical results at the Earth's surface between a regular FDTD code using the speed of light and an FDTD code using a scaled speed of light. We have found that using a scaled speed of light FDTD code, the computation time of the FDTD model may be reduced by a factor of at least six to ten.

II. Methods

A. Scale the speed of light

Based on the Von Neumann stability analysis and the stability analysis in [1], the time step is restricted by the propagation velocity and spatial resolution.

$$\Delta t \leq \frac{1}{c\sqrt{\left(\frac{1}{\Delta x}\right)^2+\left(\frac{1}{\Delta y}\right)^2+\left(\frac{1}{\Delta z}\right)^2}} \tag{1}$$

where c is the speed of light. Multiplying both permittivity and permeability in free space by a scaling factor, sf, as follows

$$\mu_{new} = sf * \mu_0 \quad ; \quad \varepsilon_{new} = sf * \varepsilon_0 \tag{2}$$

where $sf \geq 1$ is defined as the scaling factor, the speed of light would be slowed down by factor of sf. Then, the upper bound of the time step given by (1) is increased by factor of sf.

B. Scale the electrical conductivity

Besides the permittivity and the permeability, the electrical conductivity, σ, must be considered in this FDTD model. In order to maintain the same reflection coefficient between two different media (air and ground, or different layers of ground), the electrical conductivity must be scaled in the same manner as the permittivity and permeability in (2), i.e.

$$\sigma_{new} = sf * \sigma \tag{3}$$

C. Scale the grid size in lossy media

After scaling the parameters as shown in (2) and (3), the new attenuation and the phase constants become

$$\alpha_{new} = sf * \alpha \quad ; \quad \beta_{new} = sf * \beta \tag{4}$$

The scaling of the phase constant indicates a reduced the propagation velocity [1], which is what desired. However, the scaling of the attenuation factor changes how quickly the electromagnetic wave is attenuated in a conductive medium. In order to maintain the same attenuation rate when the attenuation factor is increased, the thickness of the conductive layer should be sf times thinner to correspondingly reduce the total attenuation.

Note that the scaled propagation velocity and the scaled grid size lead to zero time delay in lossy media. Since there is no attenuation in the free space, scaling grid size is not necessary.

III. RESULTS

A one-dimensional (1D) FDTD model is used to generate proof-of-concept results [2]. The two components in the 1-D FDTD model are Ez and Hy. Fig. 1 shows a result of electromagnetic wave propagation in free space for a regular FDTD grid and for the scaled speed-of-light FDTD model. Fig. 1 demonstrates a time delay between the two waveforms, which matches the different expected propagation velocities in free space, However, the amplitude of the wave is consistent, which is most important. Fig. 2 shows a wave propagating in a homogenous lossy medium with conductivity of 0.015 S/m. Note that for the scaled speed-of-light code, the thickness of the layer is reduced. In Fig. 2, the amplitudes of the waves are about the same. Also, no time delay is observed in this case because, as explained in Section IIC, no time delay is expected because of the thickness of the layer is reduced.

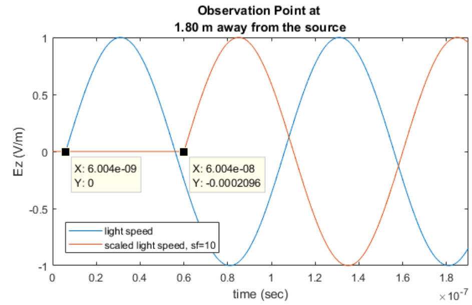

Fig. 1. The observed electric field at 1.8 m away from the source in free space. The source in this case is a sinusoidal hard source (E_z) with frequency of 10 MHz and magnitude of 1 V/m. The blue line is the observation of the model with the speed of light and the red line is the observation with scaled speed of light ($sf = 10$).

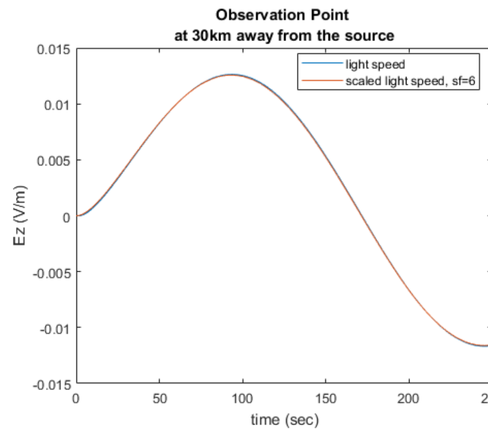

Fig. 2. The observed electric field at 30km (or 5km in the scaled grid) away from the source in a lossy medium with conductivity $\sigma = 0.015$ S/m. The source in this case is a sinusoidal hard source (E_z) with frequency of 0.003 Hz and magnitude of 1 V/m. The blue line is the observation of the model with the speed of light and the red line is the observation with scaled speed of light ($sf = 6$).

IV. CONCLUSION

This paper demonstrates that the scaling method works well for homogeneous media whether the conductivity is zero or non-zero. As part of future work, wave propagation in multi-layered media will be tested. In addition, the effects of the scaled speed of light on boundary conditions, such as perfectly matched layers, and surface impedance boundary condition will be investigated.

ACKNOWLEDGEMENT

This material is based upon work supported by the National Science Foundation under Grant No. 1662318.

REFERENCES

[1] A. Taflove and S. C. Hagness, Computational Electrodynamics: The Finite-Difference Time-Domain Method, 3rd ed. Norwood, MA, USA: Artech House, 2005.

[2] Computational and Information Systems Laboratory. 2019. Cheyenne: HPE/SGI ICE XA System (University Community Computing). Boulder, CO: National Center for Atmospheric Research. doi:10.5065/D6RX99HX.