Emergence of Maximal Symmetry

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An emergent global symmetry of the composite sector (called maximal symmetry) can soften the ultraviolet behavior of the Higgs potential and also significantly modify its structure. We explain the conditions for the emergence of maximal symmetry as well as its main consequences. We present two simple implementations and generalize both to N-site as well as full warped extra dimensional models. The gauge symmetry of these models enforces the emergence of maximal symmetry. The corresponding Higgs potentials have unique properties: one case minimizes the tuning while the other allows heavy top partners evading direct LHC bounds.

I. INTRODUCTION

The discovery of Higgs boson has been a milestone for particle physics [1, 2]. However, the potential for such an elementary scalar particle is generically sensitive to physics at extremely high scales, rendering the Higgs potential unstable to quantum corrections. One can impose additional symmetries to eliminate this ultraviolet (UV) sensitivity. Besides supersymmetry [3] or discrete symmetry like twin parity [4] or trigonometric parity [5], one widely considered possibility is a spontaneously broken (approximate) global symmetry, with the Higgs identified as one of the pseudo-Nambu-Goldstone bosons (pNGB) of this symmetry breaking [6–8] (for reviews see [9, 10]). Particular implementations of this global symmetry breaking can forbid the UV divergences of the Higgs potential. Some of the leading ideas along this direction are collective symmetry breaking and little Higgs [11] models, dimensional deconstruction [12, 13], warped extra dimensions [14–16], and the Weinberg sum rule for a composite Higgs [17].

Recently a new concept has been proposed as an alternative to these methods mentioned above; the UV divergences of the Higgs potential from the top sector are absent because of "maximal symmetry" [18]. The structure of the low energy effective Lagrangian differs from the generic case: maximal symmetry forbids Higgs corrections for the effective kinetic terms of the top quark which source the UV divergence and are often the leading sources for the quadratic term in the Higgs potential. Although maximal symmetry is simple, elegant and can have many model building applications, the exact nature of maximal symmetry and its emergence in the low-energy effective action have remained somewhat mysterious. One would like to understand what exactly the origin of this symmetry is and how to systematically realize it in the low energy effective Lagrangian by integrating out the heavy composite fields or some bulk KK modes in extra dimensions.

In this paper, we show that the origin of maximal symmetry is actually very simple: it is simply an enhanced global symmetry of the composite sector! Generically composite Higgs models are based on a coset G/H corresponding to the $G \to H$ symmetry breaking pattern and the composite sector only has an H symmetry. Whenever H is enhanced to G we will obtain a maximally symmetric model. After explaining the basic principles behind maximal symmetry we demonstrate them by constructing the simplest two site models. We show that maximal symmetry can be easily enforced by the gauge symmetries of the model, indicating that maximal symmetry is secretly a remnant of some of the gauge symmetries broken at higher energies. Next we generalize this simple models to the N-sites. In these models the fermions at the intermediate sites automatically have a global G symmetry, and the key point of realizing maximal symmetry is to preserve a global symmetry G or G' other than H for the fermions at the last site. Using this observation, we also build simple warped extra dimensional models realizing the two implementations of maximal symmetry. Note that the second implementation ("minimal maximal symmetry") is a brand new setup that has never been discussed before. We finally briefly discuss the structure of the Higgs potential and find that a light Higgs can be obtained without light top partners in the model with minimal maximal symmetry.

II. MAXIMAL SYMMETRY

Maximal symmetry is the enhancement of the spontaneously broken G/H symmetry back to the full G in some sector of the composites. Generically composites form representations of the unbroken group H, and the original G symmetry is not implemented in the composite sector. In the most general situation the the composites don't even have to fill out a complete G representation. There may however be sectors of composite fields (for example the fermionic top partners) where the composites themselves still form a complete G multiplet, hence the composite sector may have an emergent enhanced

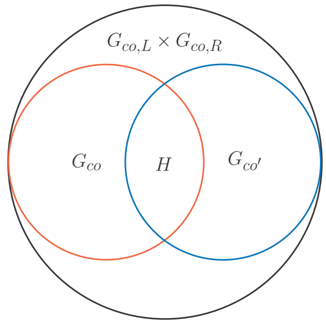

FIG. 1: Sketch of the pattern of symmetries leading to maximal symmetry in the composite sector.

G global symmetry. This enhanced global symmetry is maximal symmetry, which can play an important role in the structure of the induced Higgs potential.

For concreteness we will consider the $G = SO(5)$ and $H = SO(4)$ symmetry breaking pattern corresponding to the minimal choice that incorporates a custodial symmetry for the SM. The Higgs field is contained in the non-linear sigma field U which transforms as [19, 20]

$$U \to gUh^\dagger \tag{1}$$

where $g \in G$ is an element of the linearly realized full G symmetry while $h \in H$ is the non-linearly realized shift-symmetry. Thus the U field can also be interpreted as the field connecting $SO(5)$ symmetry of the elementary sector with the spontaneously broken $SO(5)$ symmetry of the composite sector: it transforms under an $SO(5)_{el} \times SO(5)_{co}$ symmetry, where the composite sector breaks $SO(5)_{co}$ to $SO(4)$. The special case of maximal symmetry is when some remnant of the full $SO(5)_{el} \times SO(5)_{co}$ is left over providing additional protection for the Higgs potential and softening its UV behavior. If either $SO(5)_{el}$ or $SO(5)_{co}$ is unbroken there will simply be no Higgs potential: either of these symmetries is sufficient to fully protect the pNGB's from acquiring any potential. We thus need to break both $SO(5)$'s but in such a manner that some remnant bigger than $SO(4)$ is left over. There are two simple options emerging, depending on the embedding of the SM fermions into the global symmetries.

1. Both the left handed top doublet q_L and the right handed top t_R are embedded into $SO(5)_{el}$. This is the standard assumption, corresponding to the SM fermions being mainly elementary. The embedding of these fields into incomplete $SO(5)_{el}$ multiplets breaks the elementary symmetry, but does not say anything about the structure of the composite sector. The enhancement of the global symmetries will depend entirely on the structure of the composite fields. To achieve our goal we need to preserve an $SO(5)$ symmetry that does not coincide with the original $SO(5)_{co}$. The original proposal of maximal symmetry is exactly that: an $SO(5)_{co'}$ symmetry that appears in the composite sector, where the $SO(5)_{co'}$ is not identical to $SO(5)_{co}$, see Fig.1.

2. The second option is when q_L is embedded in the elementary sector, but t_R in the composite sector. Since t_R is an $SU(2)_L$ singlet this can be easily achieved by simply making t_R a singlet under $SO(4)_{co}$. In this case already the embedding of q_L and t_R will have the right symmetry breaking pattern of $SO(5)_{el} \times SO(5)_{co}$ to ensure that a Higgs potential will be generated. If the remaining composites maintain any form of $SO(5)$ symmetry (which now could also coincide with the original $SO(5)_{co}$) a softening of the UV behavior of the Higgs potential is expected. We call this new possibility the minimal realization of maximal symmetry.

III. IMPLICATIONS OF MAXIMAL SYMMETRY

We have argued that maximal symmetry is an emergent accidental symmetry in the composite sector. Next we show how such a symmetry will potentially soften the UV behavior of the Higgs potential. Consider first the case considered in [18] when both q_L and t_R are embedded in the elementary sector. For concreteness we assume that they are both in fundamentals of $SO(5)_{el}$ transforming as $\Psi_{q_L} \to g\Psi_{q_L}$ and $\Psi_{t_R} \to g\Psi_{t_R}$. We also assume that the coset G/H is a so-called "symmetric space" (which means that there exists a Higgs-parity operator V). In this case one can always construct the linearly realized pNGB matrix $\Sigma' = UVU^\dagger = U^2V$, which transforms linearly under the full set of global symmetries $\Sigma' \to g\Sigma' g^\dagger$. By integrating out the composite sector one can always find the effective Lagrangian for the elementary fields. Since the composite sector is fully integrated out, its effect will show up only via some form factors, and the effective Lagrangian can be constrained by only considering the elementary symmetries. This will restrict the form to be [18]

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \bar{\Psi}_{q_L}\not{p}(\Pi_0^L(p) + \Pi_1^L(p)\Sigma')\Psi_{q_L} - \bar{\Psi}_{q_L}M_1^t(p)\Sigma'\Psi_{t_R} \\ &+ \bar{\Psi}_{t_R}\not{p}(\Pi_0^R(p) + \Pi_1^R(p)\Sigma')\Psi_{t_R} + h.c.\ ,\end{aligned} \tag{2}$$

where the form factors $\Pi_{0,1}^{L/R}(p)$ and M_1^t encode the effects of the composite sector. A global symmetry in the composite sector will manifest itself in some special relation among the form factors, in most cases some of the form factors are simply vanishing due to the symmetry. Most interesting is the example where $\Pi_1^{L,R} = 0$ due to the symmetry in the composite sector, the example considered in [18], while $M_1^t \neq 0$. Note that $\Pi_1^{L,R} = 0$ automatically ensures the finiteness of the Higgs potential. How can one use composite global symmetries to forbid Π_1 but allow M_1^t (which is necessary to generate any Higgs potential)? The method used in [18] was to introduce an $SO(5)_{co,L} \times SO(5)_{co,R}$ chiral global symmetry in the composite sector and ensure that the composite mass term breaks this chiral global symmetry to $SO(5)_{co'}$

defined by $g_{co,L} V g_{co,R}^\dagger = V$ for $g_{co,L} \in SO(5)_{co,L}$ and $g_{co,R} \in SO(5)_{co,R}$. One can then trace back the action of the composite symmetries in the Lagrangian (2) by noting that the dressed elementary fields $U^\dagger \Psi_{q_L,t_R}$ transform under the composite symmetries. For the chirally enhanced composite global symmetries $U^\dagger \Psi_{q_L,t_R} \to g_{co,L,R} U^\dagger \Psi_{q_L,t_R}$. The $\bar{\Psi}_{q_L} \not{p} \Sigma' \Psi_{q_L}$ can be rewritten in terms of the dressed fields as $(\bar{\Psi}_{q_L} U) V (U^\dagger \Psi_{q_L})$ and is not invariant under the $SO(5)_{co'}$ symmetry. The structure of the M_1^t term however exactly coincides with the symmetry breaking pattern of the chiral composite symmetries $\bar{\Psi}_{q_L} \Sigma' \Psi_{t_R} = (\bar{\Psi}_{q_L} U) V (U^\dagger \Psi_{t_R})$ which is invariant for transformations obeying $g_{co,L} V g_{co,R}^\dagger = V$.

Let us now consider the second possibility, not discussed so far in the literature, which we will refer to as the "minimal maximal symmetry". In this case q_L is still embedded in the elementary sector, however t_R is now assumed to be transforming under the global symmetries of the composite sector. Note that this does not necessarily imply that t_R is a composite itself. Since t_R is an $SU(2)_L$ singlet, it can for example easily mix with a singlet from the composite sector *without* being dressed by the U field. Such a mixing would imply that t_R will be transforming under the composite global symmetries rather than the elementary ones. For simplicity we will again assume that q_L and t_R are embedded into fundamental representations of $SO(5)_{el}$ and $SO(5)_{co}$ respectively, transforming as $\Psi_{q_L} \to g_{el} \Psi_{q_L}$ and $\Psi_{t_R} \to g_{co} \Psi_{t_R}$ with $g_{el} \in SO(5)_{el}$ and $g_{co} \in SO(5)_{co}$. Since the composite sector must be $H \equiv SO(4) \subset SO(5)_{co}$ invariant, Ψ_{t_R} should be a full H representation to keep H unbroken. After integrating out the composite sector the general form of the effective Lagrangian invariant under the $SO(5)_{el}$ global symmetry can be written as

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \bar{\Psi}_{q_L} \not{p} (\Pi_0^L(p) + \Pi_1^L(p) \Sigma') \Psi_{q_L} + \bar{\Psi}_{t_R} \not{p} \Pi_0^R(p) \Psi_{t_R} \\ &+ \bar{\Psi}_{q_L} M_1^t(p) U \Psi_{t_R} + h.c. \end{aligned} \quad (3)$$

differing slightly from (2): the form factor Π_1^R is automatically vanishing, while the M_1^t mass term has a U insertion connecting the elementary and composite sectors, rather than the Σ'. Following the discussion in the first case, if any $SO(5)$ subgroup of the chirally enhanced composite global symmetries, defined as $U^\dagger \Psi_{q_L} \to g_{co,L} U^\dagger \Psi_{q_L}$ and $\Psi_{t_R} \to g_{co,R} \Psi_{t_R}$, is unbroken, the form factor Π_1^L will again be forbidden. However the M_1^t term is automatically invariant under this symmetry and will be allowed. Note that in some sense this scenario is even more powerful than the traditional implementation of maximal symmetry. Usually one needs to choose an alignment for the composite mass terms to point exactly in the $SO(5)_{co'}$ direction, one which will also leave the M_1^t term in the effective action invariant. For the minimal maximal symmetry however *any* $SO(5)$ subgroup of the chiral global symmetries is sufficient - the modified M_1^t term will always be left invariant. However the embedding of the q_L and t_R into Ψ_{q_L} and Ψ_{t_R} will now explicitly break both the elementary and the composite global symmetries, and a Higgs potential will be generated.

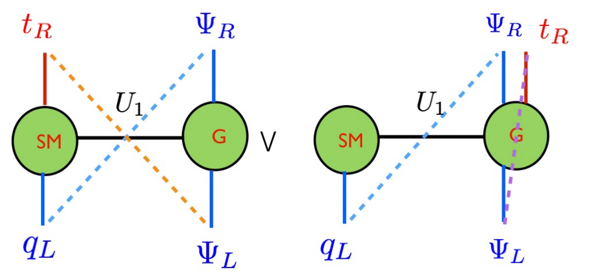

FIG. 2: Moose diagrams for the two-site models. Left: ordinary maximal symmetry; Right: minimal maximal symmetry.

IV. SIMPLEST MODEL FOR FINITE EWSB: 2-SITE MODEL WITH MAXIMAL SYMMETRY

In this section we present the simplest concrete examples of models with maximal symmetry. These also represent the simplest realistic finite EWSB models. One of the main takeaways from these models is that gauge symmetry can be used to enforce the relations needed for the appearance of maximal symmetry, and no special tuning or coincidence of parameters is needed to achieve the maximally symmetric limit. We present the two-site models corresponding to both implementations of maximal symmetry explained in detail in our general discussion above. Later we will show how to generalize them to N sites as well as to full extra dimensional constructions.

A. The Minimal Model for a Maximally Symmetric Composite Higgs

The two-site model can realize maximal symmetry in an extremely simple way. The appearance of the maximal symmetry will be a consequence of the gauge symmetries of the model. For concreteness, we take a two-site model realizing the coset $SO(5)/SO(4)$ [16] as an example. We will have a global $SO(5)$ symmetry at both sites, hence the full global symmetry of the moose is $SO(5)_1 \times SO(5)_2$. The link field U_1 connecting these two sites is in the bi-fundamental representation of the global symmetry, breaking it to the diagonal subgroup $SO(5)_V$. We gauge the $SU(2)_L \times U(1)_Y$ subgroup of $SO(5)_1$ at the first site, which will correspond to the usual EW symmetry, while at the second site the entire $SO(5)_2$ is fully gauged, as shown in the left panel in Fig. 2.[29] This gauged $SO(5)_2$ will be at the heart of the appearance of the maximal symmetry. In order for the moose to realize the $SO(5)/SO(4)$ coset, the gauge symmetry at the last site should be broken to $SO(4)$. To achieve this we will use a VEV in the symmetric representation of $SO(5)_2$. The choice of a symmetric VEV is the main difference in the construction of the gauge sector of our 2-site model compared to the traditional deconstructed models of minimal composite Higgs where the breaking is usually achieved via the 5-dimensional vector representation of $SO(5)$. The reason for the choice of a symmetric VEV is that this will be coinciding with the Higgs parity operator $V = \mathrm{diag}(1,1,1,1,-1)$, allowing the possible appearance of maximal symmetry. The linear pNGB field Σ corresponding to this breaking can be parametrized as $\Sigma = U'VU'^{\dagger}$, where U' is non-linear sigma field of coset space $SO(5)_2/SO(4)$. The $SO(5)_1$ and $SO(5)_2$ global symmetries (below the breaking scale of the gauged $SO(5)_2$) can be identified with the $SO(5)_{el}$ and $SO(5)_{co}$ symmetries of the general discussion in the Section II. The breaking of the gauge symmetries will eat some of the pNGB's such that in the end we are left with a single set of pNGBs corresponding to the $SO(5)/SO(4)$ coset. These uneaten NGBs can be described by the linear sigma field $\Sigma' = UVU^{\dagger}$ with $U = U_1U'$.

For the fermion sector of the model we introduce the LH top doublet and the RH top singlet at the first site, while at the second site we introduce a complete Dirac fermion Ψ in the fundamental representation of the entire $SO(5)_2$ - this is required by the fact that we have gauged $SO(5)_2$. This is the point where the gauging of the global symmetry will require that the composite fermions fill out a complete $SO(5)_2$ multiplet, a prerequisite for the emergence of maximal symmetry. This fermion multiplet will have an enhanced $SO(5)_{2L} \times SO(5)_{2R}$ chiral global symmetry in the limit when it is massless, and its bare Dirac mass breaks this chiral global symmetry to $SO(5)_2$. On the other hand its Yukawa coupling to Σ breaks it to $SO(5)_{2'}$: a differently oriented $SO(5)$ subgroup of the chiral global symmetries which keeps the VEV V invariant $g_L V g_R^{\dagger} = V$. As discussed in the previous section, the appearance of such a global $SO(5)$ symmetry differing from the Higgs shift symmetry is necessary to obtain a maximal symmetric model. We can see that $SO(5)_{2'}$ is the only candidate for such a symmetry. However the Dirac mass for Ψ would break this symmetry, hence we have to assume that Ψ has no bare mass but obtains a mass only from the $SO(5)/SO(4)$ breaking. This is a technically natural assumption that could also be enforced by a discrete Z_2 or Z_3 symmetry. The SM fermions t_L, t_R, b_L should be embedded in the fundamental representation of $SO(5)_1$ so that they can mix with Ψ:

$$\Psi_{q_L} = \frac{1}{\sqrt{2}} \begin{pmatrix} ib_L \\ b_L \\ it_L \\ -t_L \\ 0 \end{pmatrix} \quad \Psi_{t_R} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ t_R \end{pmatrix}. \tag{4}$$

The most general interactions for these fermions invariant under the gauge symmetries and with a vanishing bare mass are given by

$$\begin{aligned} \mathcal{L}_f &= \bar{q}_L i\not{D} q_L + \bar{\Psi} i\not{D}\Psi + \bar{t}_R i\not{D} t_R \\ &- \epsilon_L \bar{\Psi}_{q_L} U_1 \Psi_R - M\bar{\Psi}_L \Sigma \Psi_R - \epsilon_R \bar{\Psi}_L U_1^{\dagger} \Psi_{t_R} + h.c. \end{aligned} \tag{5}$$

The global symmetry of the composite fermions in the absence of the Yukawa terms is enlarged to a chiral global $SO(5)_{coL} \times SO(5)_{coR}$. However the Yukawa coupling of Ψ to Σ breaks this enlarged chiral global symmetry to the maximal symmetry $SO(5)_{co'}$ which keeps the VEV V invariant $g_L V g_R^{\dagger} = V$. The appearance of this enhanced global symmetry in the fermion sector will ensure the vanishing of the $\Pi_1^{L,R}$ form factors in the effective Lagrangian in Eq. (2) . This can also be explicitly seen via the following analysis. If we turn off any one of the three Yukawa couplings $\epsilon_{L,R}$ or M, the Higgs shift symmetry is restored. Thus the Higgs potential must be proportional to the product of these three couplings, which indicates that the only dependence on the Higgs field will be via the top Yukawa coupling, as expected in models with maximal symmetry. Since power counting tells us that the leading contribution to the Higgs potential should be proportional to the square of the effective top Yukawa coupling, the Higgs potential will be finite at one-loop for this minimal model.

Note that eventually this model can be easily promoted to full extra dimensional theories by identifying the first site with a UV brane and the second site with an IR brane.

B. 2-site Model with Minimal Maximal Symmetry

The two site model can also easily realize the minimal implementation of maximal symmetry described in Sec. II. For this we can choose the same basic model with two sites and $SO(5)_1 \times SO(5)_2$ broken to $SO(5)_V$, and the $SU(2)_L \times U(1)_Y$ gauged on the first site. The main difference will be that the singlet top t_R will be introduced at the second site $SO(5)_2$ as a gauge singlet, as shown in the right panel in Fig.2. In addition the $SO(5)_2/SO(4)$ breaking will this time be via a VEV $\mathcal{V} = (0,0,0,0,1)$ in the vector 5 of $SO(5)_2$ with its corresponding sigma field $\mathcal{H}' = U'\mathcal{V}$. The uneaten NGBs are still in the coset $SO(5)_1/SO(4)$ which can again be described by $\mathcal{H} = U\mathcal{V}$ and $U = U_1U'$. The bare mass term of Dirac fermion Ψ can be introduced to breaks its chiral symmetry to $SO(5)_2$ as maximal symmetry. The most general Lagrangian then is

$$\begin{aligned} \mathcal{L}_f &= \bar{q}_L i\not{D} q_L + \bar{\Psi} i\not{D}\Psi + \bar{t}_R i\not{D} t_R \\ &- \epsilon_L \bar{\Psi}_{q_L} U_1 \Psi_R - M\bar{\Psi}_L \Psi_R - \epsilon_R \bar{\Psi}_L \mathcal{H}' t_R + h.c. \end{aligned} \tag{6}$$

Following the same analysis, the Higgs potential is still dependent on the product of mixing Yukawa couplings and Dirac mass. Again we can see that because there is a $SO(5)$ global symmetry in Ψ sector the effective kinetic terms of SM field are independent on Higgs field.

V. N-SITE MODEL FOR MAXIMAL SYMMETRY

In the previous section we explained how maximal symmetry can be realized in a simple two site model and how it can be used for a realistic model of EWSB. Here we generalize this to an N-site model with maximal symmetry whose topology is just a 1 dimensional interval. This will also allow us to find a simple extra dimensional realization of the model in the continuum limit. We also explain how the N-site model with maximal symmetry differs from the usual deconstructed composite Higgs models (CHMs) [21, 22].

The N-site model can be obtained from the two site model presented above by inserting $N-2$ intermediate sites, which corresponds to the bulk in the extra dimensional theory. The global symmetry G at each intermediate site is fully gauged, corresponding to the bulk gauge symmetry. This gauge symmetry guarantees that a Dirac fermion at this site will have a global G symmetry, which is the diagonal subgroup of its enhanced chiral symmetry broken by a bare mass term. Thus one can obtain a maximally symmetric N-site model as long as the last site leaves this global G symmetry unbroken. For the last site we can just use the same structures as we used for the two site models presented above.

We will briefly present the explicit form of the N-site model corresponding to the $SO(5)/SO(4)$ minimal composite Higgs model (MCHM) [16], which is a simply direct generalization of the 2-site model. First we discuss the case of minimal maximal symmetry with a chiral Ψ_{Q_L} on the first site where only the $SU(2)\times U(1)_Y$ subgroup of $SO(5)$ is gauged, while the chiral singlet t_R is on the last N^{th} site, where the $SO(5)$ gauge symmetry is broken to $SO(4)$ by the scalar $\mathcal{H}'$. The bulk of the moose is obtained by introducing the Dirac fermions $\Psi_i, i=2,\dots,N$ in the fundamental representation of $SO(5)$ living at the i^{th} site and with masses M_i. These will be the composite top partners forming a KK tower of top partners. We also introduce the the link fields U_i, $i=1,\dots,N-1$ which will connect the neighboring fermions. Similar to the case of the 2-site model we find that the intermediate fermions interacting with the SM top sector preserve the global G symmetry. The most general Lagrangian realizing maximal symmetry is then given by

$$\begin{aligned}\mathcal{L}_f &= \bar{q}_L i\not{D} q_L + \sum_{i=2}^{N}\bar{\Psi}_i i\not{D}_i\Psi_i + \bar{t}_R i\not{D} t_R \\ &- \epsilon_1\bar{\Psi}_{q_L}U_1\Psi_{2R} - \sum_{j=2}^{N-1}\epsilon_j\bar{\Psi}_{jL}U_j\Psi_{j+1R} \\ &- \sum_{i=2}^{N}M_i\bar{\Psi}_{iL}\Psi_{iR} - \epsilon_N\bar{\Psi}_{NL}\mathcal{H}'t_R + h.c. \end{aligned} \qquad (7)$$

where $D_i\Psi_i = \partial_\mu\Psi_i - i\rho_i\Psi_i$ and ρ_i is the gauge boson at the **i**th site. An illustration of this model can be seen in Fig. 3. The uneaten Goldstone bosons corresponding to the $SO(5)/SO(4)$ coset is contained in the product operator $U=\prod_{i=1}^{N-1}U_iU'$ which in the continuum limit clearly corresponds to the Wilson line connecting the endpoints of the $5D$ interval. U transforms under the $SO(5)_{L_1}\times SO(5)_{R_N}$ global symmetry as

$$U\to g_{L_1}Ug^\dagger_{R_N}, \qquad (8)$$

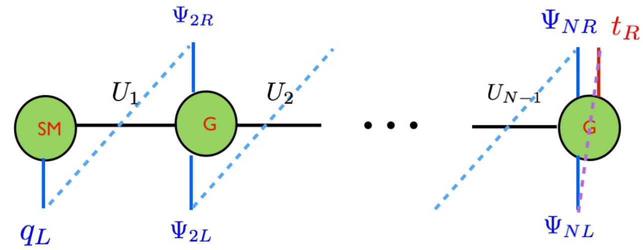

FIG. 3: Moose diagrams for the N-site model with minimal maximal symmetry.

which corresponds to the enlarged $SO(5)_{el}\times SO(5)_{co}$ shift symmetry of our general discussion. This enlarged global symmetry is explicitly broken by the embedding of q_L into Ψ_{q_L} and t_R into $\Psi_{t_R}\equiv\mathcal{V}t_R$. As explained before, such global symmetry breaking pattern will forbid the Higgs dependent Π_1 form factors in the kinetic terms but allow the M_1 form factor responsible for the top Yukawa coupling, and will generate the finite Higgs potential.

We can see the emergence of maximal symmetry and the specific form of the effective action in Eq. (2) in more detail by simply integrating out the Ψ_i Dirac fermions site by site (corresponding to integrating out the bulk in the continuum limit). Since the bulk Dirac fermion at each site has an unbroken global $SO(5)_{V_i}$ symmetry we can always redefine Ψ_i

$$\Psi_{iL}\to\prod_{j=1}^{i}U_j\Psi_{iL} \quad \Psi_{iR}\to\prod_{j=1}^{i}U_j\Psi_{iR} \quad 2\le i\le N\,. \quad (9)$$

In this redefined basis the kinetic term of the SM fermions and the Ψ_i mass terms will be independent of the NGBs. After integrating out all the bulk fermions Ψ_i, the NGBs only remain in the link between Ψ_{q_L} and $\Psi_{t_R}\equiv\mathcal{V}t_R$ as $\Psi_{q_L}U\Psi_{t_R}$. Therefore we find that the effective Lagrangian for Ψ_{q_L} and Ψ_{t_R} is invariant under $SO(5)_{L_1}\times SO(5)_{R_N}$ with the form exactly as predicted in Eq. (3)

$$\begin{aligned}\mathcal{L}_f &= \bar{\Psi}_{q_L}\not{p}\Pi_0^L(p)\Psi_{q_L} + \bar{\Psi}_{t_R}\not{p}\Pi_0^R(p)\Psi_{t_R} \\ &- \Big(M_1(p)\bar{\Psi}_{q_L}U\Psi_{t_R} + h.c.\Big). \end{aligned} \qquad (10)$$

We can see that the terms for Ψ_{q_L} and Ψ_{t_R} have the $SO(5)_{L_1}$ and $SO(5)_{R_N}$ global symmetry and the mass term breaks it into the diagonal part giving the minimal implementation of maximal symmetry.

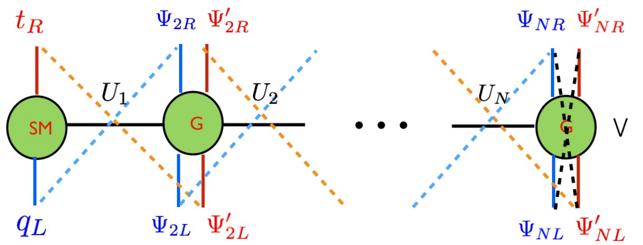

FIG. 4: Moose diagrams for the N-site model with ordinary maximal symmetry.

Next we consider the N-site model where both the top doublet and singlet live at the first site, similar to the two site model discussed in section IV A. In this case there will be a separate bulk fermion Ψ coupling to q_L and another Ψ' coupling to t_R, both in the **5** representation of $SO(5)$. In the continuum limit the top doublet and singlet will be the zero modes of the bulk fermions Ψ and Ψ'. We are adding the second bulk fermion Ψ' (unlike in the corresponding 2-site model) to obtain a theory that has a simple extra dimensional interpretation. To realize maximal symmetry, the $SO(5)\times SO(5)'$ global symmetry of these two bulk fermions will be broken to $SO(5)_{V'}$ by the Yukawa coupling to the scalar Σ mixing Ψ and Ψ' at the last site. The general Lagrangian of this N-site model is of the form

$$\begin{aligned}
\mathcal{L}_f &= \bar{q}_L i\not{D} q_L + \bar{t}_R i\not{D} t_R + \sum_{i=2}^{N}(\bar{\Psi}_i i\not{D}_i \Psi_i)\\
&- \epsilon_1 \bar{\Psi}_{q_L} U_1 \Psi_{2R} - \sum_{j=2}^{N-1} \epsilon_j \bar{\Psi}_{jL} U_j \Psi_{j+1R} - M_i \sum_{i=2}^{N} \bar{\Psi}_{iL}\Psi_{iR}\\
&- \epsilon'_1 \bar{\Psi}_{t_R} U_1 \Psi'_{2L} - \sum_{j=2}^{N-1} \epsilon'_j \bar{\Psi}'_{jL} U_j \Psi'_{j+1R} - M'_i \sum_{i=2}^{N} \bar{\Psi}'_{iL}\Psi'_{iR}\\
&- M(\bar{\Psi}_{NL}\Sigma\Psi'_{NR} + \bar{\Psi}'_{NL}\Sigma\Psi_{NR}) + h.c. \qquad (11)
\end{aligned}$$

Note that due to the doubling of the bulk fermions all gauge invariant mass terms can be non-vanishing. In this model, the global symmetry of the fermions at each intermediate site is $SO(5)\times SO(5)'$, while at the last site we have the $SO(5)_{V'}$. This symmetry breaking pattern is exactly what is needed to realize maximal symmetry. We can now explicitly find the low-energy effective Lagrangian for this model. First we redefine the phases of Ψ'_i and Ψ_i as in Eq. (9) and then re-do the integrating out of the bulk fermions site-by-site. We again obtain the effective Lagrangian invariant under $SO(5)_{V'}$

$$\begin{aligned}
\mathcal{L}_f &= \bar{\Psi}_{q_L}\not{p}\Pi_0^L(p)\Psi_{q_L} + \bar{\Psi}_{t_R}\not{p}\Pi_0^R(p)\Psi_{t_R}\\
&- M_1(p)\bar{\Psi}_{q_L} U V U^\dagger \Psi_{t_R} \qquad (12)
\end{aligned}$$

confirming our expectation that maximal symmetry of the composite sector really forbids the Higgs dependent kinetic terms. The key difference between the maximally symmetric N-site model and the ordinary deconstructed composite Higgs model is the unbroken global symmetry in the bulk fermion sector at the last site (in the ordinary moose, only the $SO(4)$ global symmetry is unbroken at the last site while here we have $SO(5)_{V'}$). More discussion about the ordinary deconstruction can be found in App. B.

VI. MAXIMAL SYMMETRY FROM EXTRA DIMENSIONS

The N-site model presented above can be directly generalized to a 5D theory warped in AdS space with a metric [14]

$$ds^2 = a(z)^2(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) \equiv g_{MN}dx^M dx^N, \quad (13)$$

where the coordinate z along the fifth dimension ranges $R < z < R'$ and the warp factor $a(z) = \frac{R}{z}$ corresponds to an S^1/Z_2 orbifold with R the AdS curvature. We will present the warped extra dimensional implementation of both the original maximally symmetric model as well as the new minimal maximal symmetry.

A. $5+5$ bulk fermions - ordinary maximal symmetry

We first show how to realize the original maximal symmetry in AdS space. From the construction of the N-site model we learn that for the 5D model using the $SO(5)/SO(4)$ MCHM coset [16] we will need to introduce two separate bulk fermions Ψ_1 and Ψ_2 in the fundamental representation of the $SO(5)$ bulk gauge symmetry. The zero modes of the Ψ_1 and Ψ_2 will be identified with the top doublet and singlet. To preserve an unbroken $SO(5)$ global symmetry at the IR brane, their boundary conditions (BCs) at the IR brane should be $SO(5)$ invariant. Hence the BCs will be

$$\begin{aligned}
\Psi_1 &= \begin{bmatrix} (\mathbf{2},\mathbf{2})^{\mathbf{q_1}}_{\mathbf{L}} = \begin{bmatrix} q'_{1L}(-+) \\ q_{1L}(++) \end{bmatrix} \\ (\mathbf{1},\mathbf{1})^{\mathbf{q_1}}_{\mathbf{L}} = T_L(-+) \end{bmatrix},\\
\Psi_2 &= \begin{bmatrix} (\mathbf{2},\mathbf{2})^{\mathbf{u}}_{\mathbf{R}}(-+) \\ (\mathbf{1},\mathbf{1})^{\mathbf{u}}_{\mathbf{R}} = t_R(++) \end{bmatrix}, \qquad (14)
\end{aligned}$$

where we decomposed the 5 dimensional vector representation of $SO(5)$ as $(2,2)+(1,1)$ under the unbroken $SO(4) \sim SU(2)_L \times SU(2)_R$ subgroup, $\pm$ denote Neumann or Dirichlet BCs, and for Ψ_1 we only displayed the BCs for the left handed χ fields of the Dirac fermion (while for Ψ_2 the BCs are for the right handed fields). The fields with opposite chiralities will have opposite BCs. If these two bulk fermions do not mix with each

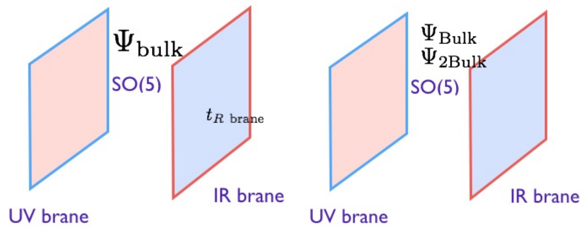

FIG. 5: Extra dimensional setup.

other (ie. both have their respective bulk masses $m_{1,2}$ but no bulk mixing mass) their global symmetry at the IR brane will be $G_L \times G_R$:

$$\Psi_1 \to g_L \Psi_1 \quad \Psi_2 \to g_R \Psi_2 \quad g_{L,R} \in G_{L,R}. \tag{15}$$

Since we chose the BCs of the fermions at the IR brane to be $SO(5)$ invariant, the global $SO(5)_L \times SO(5)_R$ symmetry (which contains the Wilson line shift symmetry) will ensure that the bulk gauge interactions do not contribute to a potential for the Wilson line A_5. In order to break the A_5 shift symmetry while still preserving the global $SO(5)_{co'}$ symmetry we introduce the brane localized mass term mixing the two bulk fermions twisted by the Higgs parity V:

$$S_{mix} = \frac{1}{g_5^2} \int d^4x \sqrt{-g_{ind}}\, \tilde{m} (\bar{\Psi}_{1L} V \Psi_{2R} + h.c.), \tag{16}$$

where g_{ind} is the induced metric at the IR brane. Due to the maximal symmetry, the Wilson line can be removed from the fermion sector if we turn off the interactions of either Ψ_1 or Ψ_2. Hence (as suggested by the N-site model) one needs the KK modes of the bulk fermions to connect the top doublet and top singlet with the Wilson line, while the effective kinetic terms of the SM fermions will be independent of the Wilson line. As expected, the Higgs potential can only depend on the effective top Yukawa coupling.

Now we can explicitly calculate the holographic effective Lagrangian at the UV brane to verify our conclusions. We will choose $\chi_L = \Psi_{1L}(x, R)$ and $\psi_R = \Psi_{2R}(x, R)$ as the holographic fields. Imposing the bulk equations of motion one can show that the entire holographic effective action becomes a boundary term

$$S_4 = \frac{1}{2g_5^2} \int \left[d^4x \sqrt{-g_{ind}} (\bar{\Psi}_{1L} \Psi_{1R} - \bar{\Psi}_{2L} \Psi_{2R} + h.c.) \right]_R^{R'}, \tag{17}$$

Note that the sign of the holographic action changes depending on whether the LH or the RH field is taken as the basic holographic field.

The addition of (16) will modify the IR boundary conditions of the fermions at $z = R'$ to

$$\Psi_{1R} = \tilde{m} V \Psi_{2R} \quad \Psi_{2L} = -\tilde{m} V \Psi_{1L}. \tag{18}$$

Plugging the classical solutions of the bulk equations satisfying the BCs in Eq. (18), the holographic Lagrangian will be

$$\begin{aligned} \mathcal{L}_H &= \bar{\chi}_L \not{p} \Pi^L(\tilde{m}) \chi_L - \bar{\psi}_R \not{p} \Pi^R(\tilde{m}) \psi_R \\ &+ M^{LR} (\bar{\chi}_L V \psi_R + \bar{\psi}_R V \chi_L), \end{aligned} \tag{19}$$

where the form factors $\Pi^{L,R}$ and M^{LR} are explicitly shown in Eq. (A7) in App. A.

The presence of the non-vanishing Wilson line can be incorporated by a bulk rotation of the fields such that the Wilson line disappears from the bulk equations of motion, however will reappear in the BCs. The effect of this rotation will be the dressing of the holographic fields with the Wilson line via the expression [9]

$$\begin{aligned} \chi_L, \psi_R &\to U(R, R') \chi'_L, \psi'_R, \\ U(R, R') &= \mathrm{Exp}\Big(i \frac{-\sqrt{2} \pi^{\hat{a}} T^{\hat{a}}}{f} \Big), \end{aligned} \tag{20}$$

where $f = \frac{2\sqrt{R}}{g_5 R'}$, $\pi^{\hat{a}}$ are the A_5 zero modes, and $T^{\hat{a}}$ is the generator in $SO(5)/SO(4)$. Finally the effective Lagrangian takes the form

$$\begin{aligned} \mathcal{L}_{eff} &= \bar{\chi}'_L \not{p} \Pi^L(\tilde{m}) \chi'_L - \bar{\psi}'_R \not{p} \Pi^R(\tilde{m}) \psi'_R \\ &+ M^{LR} (\bar{\chi}'_L \Sigma \psi'_R + h.c.) \end{aligned} \tag{21}$$

where $\Sigma = U^\dagger V U = U^{\dagger 2} V$ and

$$\chi'_L = \frac{1}{\sqrt{2}} \begin{pmatrix} i b_L \\ b_L \\ i t_L \\ -t_L \\ 0 \end{pmatrix} \qquad \psi'_R = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ t_R \end{pmatrix}. \tag{22}$$

We again see that due to the $SO(5)_{co'}$ symmetry of the bulk sector, the effective kinetic terms are independent of Σ. Only the effective Yukawa coupling can depend on the NGBs, which is $SO(5)_{co'}$ invariant. While the global symmetry is broken to $SO(4)$ by the Σ VEV, the presence of maximal symmetry in the top sector will again soften the divergences consistent with our expectations from the N-site model.

B. 5 + 1 case: minimal maximal symmetry

Finally we show the extra dimensional implementation of minimal maximal symmetry. Based on the structure of the N-site model corresponding to this scenario we expect that the SM top doublet would still be a zero mode of the bulk fermion Ψ_1 in Eq. (14). However the top singlet t_R should be a bulk singlet localized on the IR brane. The simplest choice is to add t_R simply as a brane localized chiral Weyl fermion, though we can really think of it as the limit of a very heavy bulk fermion whose zero mode is sharply localized on the IR brane.

To realize minimal maximal symmetry, the bulk fermion at the IR brane should respect the $SO(5)_L$ global symmetry so its BC should be the same as the one in the previous case. Hence as in the previous analysis, the bulk fermion does not break the Wilson line shift symmetry. To break the shift symmetry and produce the top Yukawa coupling, the singlet t_R should interact with the Wilson line via the mixing with the bulk fermion. Since the mixing breaks the $SO(5)_L$ global symmetry to $SO(4)$ on the IR brane, the induced potential for the Wilson line must be proportional to the mixing parameter. Thus the $SO(5)_L$ global symmetry of the bulk fermion implies that the A_5 potential only depends on the effective top Yukawa coupling. Since the gauge symmetry at the IR brane is reduced to $SO(4)$, the $SO(4)$ singlet component of Ψ_1 can mix with the brane localized t_R:

$$S_{IR} = \frac{1}{g_5^2}\int d^4x\sqrt{-g_{ind}}\Big(\tilde{m}\bar{T}_L t_R + h.c. + \bar{t}_R \not{D} t_R\Big) \tag{23}$$

where $\not{D} = e_a^\mu \Gamma^a(\partial_\mu - iA_\mu)$. We can again choose the left-handed Ψ_{1L} as the holographic field $\chi_L = \Psi_L^I(x, R)$. Again the holographic action will be a boundary term

$$S_4 = \frac{1}{2g_5^2}\int d^4x\sqrt{-g_{ind}}\Big[\bar{\Psi}_{1L}\Psi_{1R} + h.c.\Big]_R^{R'}, \tag{24}$$

The boundary terms on the IR brane will modify the BCs to be

$$T_R(x, R') = \tilde{m} t_R \tag{25}$$

Using this BC we can again obtain the classical solution in the bulk (for details see App. A). Following the same procedure as in the previous section (and normalizing the t_R kinematic term by rescaling $t_R \to t_R(\frac{R'}{R})^{\frac{3}{2}}$, we can get the holographic effective Lagrangian

$$\mathcal{L}_H = \bar{\chi}_L \not{p}\Pi_L(p)\chi_L + \bar{t}_R \not{p} t_R + (M(p)\bar{\chi}_L t_R(p) + h.c.)\,, \tag{26}$$

where the form factors Π_L and M are shown in detail in (A9). Rotating the holographic field will restore the dependence on the NGBs fields, and turning off the component vanishing at the UV boundary,

$$\chi_{L,R} \to U(R, R')\chi'_L \quad \chi'_L = \frac{1}{\sqrt{2}}\begin{pmatrix} ib_L \\ b_L \\ it_L \\ -t_L \\ 0 \end{pmatrix}, \tag{27}$$

we obtain the effective Lagrangian for the top sector

$$\begin{aligned}\mathcal{L}_{eff} &= \bar{\chi}'_L \not{p}\Pi_L(p)\chi'_L + \bar{t}_R \not{p} t_R \\ &+ (M(p)\bar{\chi}'_L \mathcal{H} t_R + h.c.)\end{aligned} \tag{28}$$

where $\mathcal{H} = U^\dagger \mathcal{V}$ with $\mathcal{V} = (0,0,0,0,1)$. As discussed before, the interval between the top doublet and singlet is $SO(5)$ invariant so their effective kinetic terms should be independent of the NGBs to preserve the global $SO(5)$, leading to the minimal maximal symmetry.

VII. COMMENTS ON THE GAUGE SECTOR AND THE HIGGS POTENTIAL

Maximal symmetry is the unbroken global symmetry G in the (composite/bulk) fermion sector. However, in the gauge sector, there can not be such an unbroken global symmetry because the global symmetry of the bulk gauge sector is the same as the gauge symmetry, which eventually must be broken to H. This statement can also be verified by inspecting the gauge boson effective Lagrangian which can be parametrized as

$$\mathcal{L} = \frac{P_t^{\mu\nu}}{2}\Big(\Pi_0(p)\text{Tr}[A_\mu A_\nu] + \Pi_1(p)\text{Tr}[\Sigma A_\mu \Sigma A_\nu]\Big) \tag{29}$$

The effects of the global symmetry in the composite sector can be traced by the dressed operator $U^\dagger A_\mu U$. If there was a global symmetry G in the composite gauge sector then the dressed operator would transform as $U^\dagger A_\mu U \to g U^\dagger A_\mu U g^\dagger$. We can see that while the Π_0 kinetic term is G invariant (but independent of the NGB's), however the Π_1 term breaks G to H due to the VEV of the Sigma field. If Π_1 is vanishing there would be no gauge contribution to the Higgs potential. As soon as Π_1 is non-zero there can be no unbroken global symmetry larger than H. This proves that maximal symmetry can not be implemented in the composite gauge sector. However the Higgs potential from the gauge sector can be still finite if the breaking of the global symmetries in the gauge sector is sufficiently collective (which is usually the case in deconstructed models with more than two sites).

Finally we explain the utility of maximal symmetry in achieving a phenomenologically viable Higgs potential. It is usually parametrized using the variable $s_h \equiv \sin\frac{\langle h\rangle}{f} \ll 1$, and expanded to leading order as

$$V(h) = -(\gamma_f - \gamma_g)s_h^2 + \beta_f s_h^4 \tag{30}$$

where γ_f, β_f are the contributions from the top sector and β_g is the contribution from the gauge sector. If $\gamma_f - \gamma_g$ and β_f are positive, the Higgs will acquire a VEV, $\xi \equiv s_h^2 = (\gamma_f - \gamma_g)/(2\beta_f)$. To achieve a small ξ, the coefficient of the s_h^2 term has to be suppressed via cancellations. The tuning measuring this cancellation is around

$$\Delta \approx \frac{1}{\xi}\frac{\gamma_f}{\beta_f}. \tag{31}$$

In a generic CHMs without any mechanism for softening the UV behavior of the Higgs potential, the leading contribution to γ_f is usually from the effective top kinetic terms and is quadratically divergent, while the leading contribution to β_f is log divergent, also originating from the top kinetic terms but at sub-leading order in y_t. Since these two terms have a different degree of divergence, the tuning in Eq. (31) will usually be very large and the Higgs very heavy. In composite Higgs models based on deconstruction or Holographic Higgs models the Higgs potential is usually finite. However γ_f and β_f are still from the leading and sub-leading contributions of the top effective kinetic terms so they will have a different dependence on the top Yukawa coupling y_t: $\gamma_f \sim \mathcal{O}(y_t)$ and $\beta_f \sim \mathcal{O}(y_t^2)$. For a viable Higgs potential one first needs to tune γ_f to be the same order as β_f and then tune it to be $\xi\beta_f$ to get small ξ, which results in a double tuning. In this model the tuning is around $\Delta \sim g_f^2/\xi$ for a light Higgs, where $g_f \equiv M_f/f$ and M_f is the top partner mass. Note however that in these models the Higgs potential could be completely generated by the top sector.

In the CHM with maximal symmetry, the Higgs potential from the top sector originates entirely from the top Yukawa coupling. This contribution is finite, and there is no freedom to impose a cancellation purely within γ_f to get a small ξ. The only way to achieve the suppression of the s_h^2 term is to use the contribution from the gauge sector γ_g to cancel the contribution from the top sector γ_f. This is the main source of tuning in the maximally symmetric model. In the original maximally symmetric model [18] these two terms are of the same order in the top Yukawa coupling thus the tuning is minimal, $\Delta \sim 1/\xi$.

We want to emphasize that in all of the CHMs discussed so far in this section, the Higgs mass explicitly depends on the top partner mass, implying that the top partner mass must be light, around $g_f \approx 1$, to obtain a 125 GeV Higgs. However for the case of minimal maximal symmetry this situation changes. Due to the different choice of embeddings the effective top Yukawa term is proportional to s_h (vs. proportional to s_{2h} in ordinary maximal symmetry), as a result of which $\gamma_f \sim \mathcal{O}(y_t^2)$ and $\beta_f \sim \mathcal{O}(y_t^4)$. Hence in this scenario we still end up with a double tuning for small Higgs VEVs, similar to the ordinary CHM. However β_f is at $\mathcal{O}(y_t^4)$ and can be parametrized as

$$\beta_f \approx c\frac{M_f^4}{(4\pi)^2}(\frac{y_t}{g_f})^4, \tag{32}$$

where c is a numerical constant. We can see that β_f is not sensitive to the top partner mass, which leads to an expression for the Higgs mass of the form $m_h^2 = 8\beta_f\xi/f^2 \sim c/(2\pi^2)y_t^4v^2$. We find the Higgs is always somewhat too light, $m_h \approx 100$ GeV for $M_f \approx 10f$. However the Higgs mass can be easily enhanced while the tuning still significantly suppressed if an independent Higgs quartic coupling can be produced through some mechanism [23, 24]. For this case we will end up with a simple model where one only needs a reasonably small (smaller than for the traditional MCHM) tuning to get a small ξ, while at the same time the Higgs mass is not sensitive to the top partner mass avoiding the LHC direct detection constraints. For example fixing $\xi = 0.1$ as well as the Higgs mass $m_h = 125$ GeV we find that in this model the tuning is around $\Delta \approx 40$ if the lightest top partner is heavier than 2 TeV. On the other hand for the traditional composite Higgs models one needs a tuning larger than 100 to keep the lightest top partner heavier than 2 TeV [25]. In these models the masses of the Higgs and the top partners are strongly correlated and it is very difficult to obtain heavy top partners while keeping Higgs light. For these traditional models, one needs a careful tuning to obtain small ξ and a separate tuning to keep the Higgs light, while for the model with minimal maximal symmetry the only tuning needed is to obtain small ξ.

VIII. CONCLUSIONS AND OUTLOOK

The composite sector of composite Higgs models may have emergent (accidental) global symmetries. This maximal symmetry will have far-reaching consequences for the structure of the Higgs potential. It can forbid the Higgs dependence of the effective top kinetic terms thereby softening its UV behavior. The particular forms of the maximally symmetric Higgs potential may also minimize the tuning needed or allow heavy top partners. We have explained the general conditions for the emergence of maximal symmetry as well as its major consequences.

We have shown two major options for implementing maximal symmetry depending on the embeddings of the top doublet and singlet into the global symmetry. We find that the essence of the emergence of maximal symmetry in a deconstructed or extra dimensional theory is to preserve a global $SO(5)$ symmetry of the top partners at the last site/IR brane. We also showed that the structure of the gauge symmetries can enforce the emergence of maximal symmetry in the top sector. In the ordinary maximal symmetric model, the Higgs potential parameters γ_f and β_f are almost equal so the tuning is always minimal. However in the new model with minimal maximal symmetry, the double tuning will still be present because γ_f and β_f are order $\mathcal{O}(y_t^2)$ and $\mathcal{O}(y_t^4)$ in the top Yukawa coupling. The critical aspect of this new implementation of maximal symmetry is that the Higgs mass is not sensitive to the top partner mass so a light Higgs can be obtained without light top partners.

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Appendix A: Calculation of the Holographic Form Factors

In AdS background, the equation of motion for the bulk fermion field in mixed momentum-coordinate space (p,z) are [26]

$$(\partial_z + 2\frac{\partial_z a(z)}{a(z)} \pm a(z)m)\Psi_{L,R} = \pm \not{p}\Psi_{R,L} \quad \text{(A1)}$$

If we parametrize the bulk fermion field as

$$\Psi_{L,R}(p,z) = f_{L,R}(p,z)\Psi_{L,R}(p). \quad \text{(A2)}$$

with $\not{p}\Psi_L(p) = p\Psi_R(p)$, we obtain the following equation

$$(\partial_z + 2\frac{\partial_z a(z)}{a(z)} \pm a(z)m) f_L(p,z) = \pm p f_R(p,z) \quad \text{(A3)}$$

Rewriting the equation of motion for $f_{L,R}$ as

$$\Big(\partial_z^2 + p^2 - \frac{4}{z}\partial_z + \frac{6}{z^2} \mp \frac{mR}{z^2} - \frac{(mR)^2}{z^2}\Big) f_{L,R} = 0 \quad \text{(A4)}$$

we obtain the solutions

$$\begin{aligned} f_L(p,z) &= z^{5/2}\left[J_\alpha(pz)Y_\beta(pR') - J_\beta(pR')Y_\alpha(pz)\right] \\ f_R(p,z) &= z^{5/2}\left[J_{\alpha-1}(pz)Y_\beta(pR') - J_\beta(pR')Y_{\alpha-1}(pz)\right], \end{aligned} \quad \text{(A5)}$$

where $\alpha \equiv mR + 1/2$ and β is still undetermined. To determine β, the IR BC is needed, which is different for the $5+5$ and $5+1$ cases.

For our convenience, we define the following propagators

$$\begin{aligned} G^{++}(\alpha_i) &\equiv f_L(p,R)|_{\beta=\alpha_i} \quad & G^{-+}(\alpha_i) &\equiv f_R(p,R)|_{\beta=\alpha_i}. \\ G^{+-}(\alpha_i) &\equiv f_L(p,R)|_{\beta=\alpha_i-1} \quad & G^{--}(\alpha_i) &\equiv f_R(p,R)|_{\beta=\alpha_i-1}, \end{aligned} \quad \text{(A6)}$$

with $\alpha_i \equiv m_i R + 1/2$.

For $\mathbf{5}+\mathbf{5}$ case, the form factor is [27, 28]

$$\begin{aligned} \Pi^L &= \frac{1}{p}\frac{G^{--}(\alpha_1)G^{-+}(\alpha_2) - \tilde{m}^2 G^{-+}(\alpha_1)G^{--}(\alpha_2)}{G^{+-}(\alpha_1)G^{-+}(\alpha_2) + \tilde{m}^2 G^{++}(\alpha_1)G^{--}(\alpha_2)} \\ \Pi^R &= \frac{1}{p}\frac{G^{+-}(\alpha_1)G^{++}(\alpha_2) - \tilde{m}^2 G^{++}(\alpha_1)G^{+-}(\alpha_2)}{G^{+-}(\alpha_1)G^{-+}(\alpha_2) + \tilde{m}^2 G^{++}(\alpha_1)G^{--}(\alpha_2)} \\ M^{LR} &= \frac{\tilde{m}}{2}\frac{G^{-+}(\alpha_1)G^{+-}(\alpha_1) + G^{++}(\alpha_1)G^{--}(\alpha_1)}{G^{-+}(\alpha_1)G^{+-}(\alpha_2) + \tilde{m}^2 G^{++}(\alpha_1)G^{--}(\alpha_2)} \\ &+ \frac{\tilde{m}}{2}\frac{G^{-+}(\alpha_2)G^{+-}(\alpha_2) + G^{++}(\alpha_2)G^{--}(\alpha_2)}{G^{+-}(\alpha_1)G^{-+}(\alpha_2) + \tilde{m}^2 G^{++}(\alpha_1)G^{--}(\alpha_2)}. \end{aligned} \quad \text{(A7)}$$

For the $\mathbf{5}+\mathbf{1}$ case, we have the solutions for the right-handed field at the UV brane [27]

$$\begin{aligned} T_R(p,L_0) &= \frac{G^{--}(\alpha_1)}{G^{+-}(\alpha_1)}\frac{\not{p}}{p}\chi_L^5 + \frac{\tilde{G}^{+-}(\alpha_1)}{G^{+-}(\alpha_1)}\tilde{m}t_R(p), \\ \Psi_{1R}^a(p,L_0) &= \frac{G^{--}(\alpha_1)}{G^{+-}(\alpha_1)}\frac{\not{p}}{p}\chi_L^a, \end{aligned} \quad \text{(A8)}$$

with

$$\tilde{G}^{+-}(\alpha) = z^{5/2}\left[J_\alpha(pR)Y_{\alpha-1}(pR) - J_{\alpha-1}(pR)Y_\alpha(pR)\right]$$

and index $a = 1,\cdots,4$, and thus the form factors read

$$\Pi_L = \frac{1}{p}\frac{G^{--}(\alpha_1)}{G^{+-}(\alpha_1)}, \quad M = \tilde{m}\frac{\tilde{G}^{+-}(\alpha_1)}{G^{+-}(\alpha_1)}. \quad \text{(A9)}$$

Appendix B: Traditional composite Higgs model in 4D moose

The essential difference between a traditional deconstructed MCHM and a model with maximal symmetry is whether the global $SO(5)$ symmetry of the bulk fermions remains unbroken at the last site. In the traditional approach only the $SO(4)$ subgroup of $SO(5)$ at last site is gauged. The $SO(5)_L \times SO(5)_R$ global chiral symmetry of the top partners is broken to the $SO(4)$ gauge symmetry at the last site. We will explain that this will lead to the Higgs dependence of top effective kinetic terms. To simplify the discussion we only focus on the interactions of the top doublet and its partners, which is given by

$$\begin{aligned} \mathcal{L}_f &= \bar{q}_L i\not{D} q_L + \sum_{i=2}^{N-1}(\bar{\Psi}_i i\not{D}_i \Psi_i) \\ &- \epsilon_1 \bar{\Psi}_{q_L} U_1 \Psi_{2R} - \sum_{j=2}^{N-2}\epsilon_j \bar{\Psi}_{jL} U_j \Psi_{j+1R} - \sum_{i=2}^{N-1} M_i \bar{\Psi}_{iL}\Psi_{iR} \\ &- \epsilon_{N-1}\bar{\Psi}_{N-1L} U_{N-1}(\Psi_{NR}^4 + \Psi_{NR}^1) \\ &- M_4 \bar{\Psi}_{NR}^4 \Psi_{NL}^4 - M_1 \bar{\Psi}_{NR}^1 \Psi_{NL}^1 + h.c., \end{aligned} \quad \text{(B1)}$$

where $D_i = \partial - i g_{\rho_i}\rho_i$ and ρ_i and g_{ρ_i} are the $SO(5)$ gauge bosons and gauge coupling at the i^{th} site. Since the gauge symmetry at the last site is $SO(4)$, the $\Psi_N^{4,1}$ are forming an $SO(4)$ quadruplet and singlet. All the information of the $SO(5)/SO(4)$ breaking is encoded in the mass terms $M_4 \neq M_1$. In the extra dimensional case the corresponding statement is that the IR boundary condition of the bulk fermion corresponding to the top doublet is only $SO(4)$ invariant (while their Yukawa couplings are assumed to be global $SO(5)$ invariant). Since the intermediate fields respect the global $SO(5)$ symmetry, integrating them out (as in the main text) the contributions to the top kinetic terms remain independent of the Higgs and only the effective Yukawa couplings of the top doublet Ψ_{q_L} to $\Psi_N^{4,1}$ can depend on the non-linear sigma field $U \equiv \prod_{i=1}^{N-1} U_i$ containing the Higgs (which corresponds to the Wilson line in the extra dimensional case).

Since the fermion masses at the last site only respect $SO(4)$, which breaks the Higgs shift symmetry, after integrating out the Dirac fermions $\Psi_N^{4,1}$ at the last site the Ψ_{q_L} effective kinetic term will in general pick up a dependence on the linearly realized pNGB field Σ. This reflects that the global symmetry of the doublet top partners is only $SO(4)$. We will find a similar conclusion in the extra dimensional warped MCHM: since the IR BCs of the bulk fermions corresponding to the top doublet and singlet only respects a global $SO(4)$ the effective top kinetic terms in general will depend on the Wilson line.

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[29] The gauging of the $SO(5)_2$ is the main difference compared to the ordinary deconstruction model discussed in appendix B.