The Emergence of the $\Delta U = 0$ Rule in Charm Physics

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Abstract

We discuss the implications of the recent discovery of CP violation in two-body SCS D decays by LHCb. We show that the result can be explained within the SM without the need for any large $SU(3)$ breaking effects. It further enables the determination of the imaginary part of the ratio of the $\Delta U = 0$ over $\Delta U = 1$ matrix elements in charm decays, which we find to be (0.65 ± 0.12). Within the standard model, the result proves the non-perturbative nature of the penguin contraction of tree operators in charm decays, similar to the known non-perturbative enhancement of $\Delta I = 1/2$ over $\Delta I = 3/2$ matrix elements in kaon decays, that is, the $\Delta I = 1/2$ rule. As a guideline for future measurements, we show how to completely solve the most general parametrization of the $D \to P^+P^-$ system.

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I. INTRODUCTION

In a recent spectacular result, LHCb discovered direct CP violation in charm decays at 5.3σ [1]. The new world average of the difference of CP asymmetries [2–13]

$$\Delta a_{CP}^{\rm dir} \equiv a_{CP}^{\rm dir}(D^0 \to K^+K^-) - a_{CP}^{\rm dir}(D^0 \to \pi^+\pi^-)\,, \tag{1}$$

where

$$a_{CP}^{\rm dir}(f) \equiv \frac{|\mathcal{A}(D^0 \to f)|^2 - |\mathcal{A}(\overline{D}^0 \to f)|^2}{|\mathcal{A}(D^0 \to f)|^2 + |\mathcal{A}(\overline{D}^0 \to f)|^2}\,, \tag{2}$$

and which is provided by the Heavy Flavor Averaging Group (HFLAV) [14], is given as [15]

$$\Delta a_{CP}^{\rm dir} = -0.00164 \pm 0.00028\,. \tag{3}$$

Our aim in this paper is to study the implications of this result. In particular, working within the Standard Model (SM) and using the known values of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements as input, we see how Eq. (3) can be employed in order to extract low energy QCD quantities, and learn from them about QCD.

The new measurement allows for the first time to determine the CKM-suppressed amplitude of singly-Cabibbo-suppressed (SCS) charm decays that contribute a weak phase difference relative to the CKM-leading part, which leads to a non-vanishing CP asymmetry. More specifically, $\Delta a_{CP}^{\rm dir}$ allows to determine the imaginary part of the $\Delta U = 0$ over $\Delta U = 1$ matrix elements.

As we show, the data suggest the emergence of a $\Delta U = 0$ rule, which has features that are similar to the known "$\Delta I = 1/2$ rule" in kaon physics. This rule is the observation that in $K \to \pi\pi$ the amplitude into a $I = 0$ final state is enhanced by a factor ~ 20 with respect to the one into a $I = 2$ final state [16–26]. This is explained by large non-perturbative rescattering effects. Analogous enhancements in charm decays have previously been discussed in Refs. [27–34]. For further recent theoretical work on charm CP violation see Refs. [35–51].

In Sec. II we review the completely general U-spin decomposition of the decays $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$ and $D^0 \to K^\pm\pi^\mp$. After that, in Sec. III we show how to completely determine all U-spin parameters from data. Our numerical results which are based on the current measurements are given in Sec. IV. In Sec. V we interpret these as the emergence of

a $\Delta U = 0$ rule, and in Sec. VI we compare it to the $\Delta I = 1/2$ rules in K, B and D decays. The different effect of $\Delta U = 0$ and $\Delta I = 1/2$ rules on the phenomenology of charm and kaon decays, respectively, is discussed in Sec. VII. In Sec. VIII we conclude.

II. MOST GENERAL AMPLITUDE DECOMPOSITION

The Hamiltonian of SCS decays can be written as the sum

$$\mathcal{H}_{\rm eff} \sim \Sigma(1,0) - \frac{\lambda_b}{2}(0,0)\,, \tag{4}$$

where $(i,j) = \mathcal{O}^{\Delta U=i}_{\Delta U_3=j}$, and the appearing combination of CKM matrix elements are

$$\Sigma \equiv \frac{V_{cs}^* V_{us} - V_{cd}^* V_{ud}}{2}\,, \qquad -\frac{\lambda_b}{2} \equiv -\frac{V_{cb}^* V_{ub}}{2} = \frac{V_{cs}^* V_{us} + V_{cd}^* V_{ud}}{2}\,, \tag{5}$$

where numerically, $|\Sigma| \gg |\lambda_b|$. The corresponding amplitudes have the structure

$$\mathcal{A} = \Sigma(A^s_\Sigma - A^d_\Sigma) - \frac{\lambda_b}{2} A_b\,, \tag{6}$$

where A^s_Σ, A^d_Σ and A_b contain only strong phases and we write also $A_\Sigma \equiv A^s_\Sigma - A^d_\Sigma$.

For the amplitudes we use the notation

$$\mathcal{A}(K\pi) \equiv \mathcal{A}(\overline{D}^0 \to K^+\pi^-)\,, \tag{7}$$

$$\mathcal{A}(\pi\pi) \equiv \mathcal{A}(\overline{D}^0 \to \pi^+\pi^-)\,, \tag{8}$$

$$\mathcal{A}(KK) \equiv \mathcal{A}(\overline{D}^0 \to K^+K^-)\,, \tag{9}$$

$$\mathcal{A}(\pi K) \equiv \mathcal{A}(\overline{D}^0 \to \pi^+K^-)\,. \tag{10}$$

The U-spin related quartet of charm meson decays into charged final states can then be written as [30, 37, 52]

$$\mathcal{A}(K\pi) = V_{cs}V_{ud}^*\left(t_0 - \frac{1}{2}t_1\right)\,, \tag{11}$$

$$\mathcal{A}(\pi\pi) = -\Sigma^*\left(t_0 + s_1 + \frac{1}{2}t_2\right) - \lambda_b^*\left(p_0 - \frac{1}{2}p_1\right)\,, \tag{12}$$

$$\mathcal{A}(KK) = \Sigma^*\left(t_0 - s_1 + \frac{1}{2}t_2\right) - \lambda_b^*\left(p_0 + \frac{1}{2}p_1\right)\,, \tag{13}$$

$$\mathcal{A}(\pi K) = V_{cd}V_{us}^*\left(t_0 + \frac{1}{2}t_1\right)\,. \tag{14}$$

The subscript of the parameters denotes the level of U-spin breaking at which they enter. We write $A(K\pi)$ and $A(\pi K)$ for the Cabibbo-favored (CF) and doubly Cabibbo-suppressed (DCS) amplitude without the CKM factors, respectively. We emphasize that the SM parametrization in Eqs. (11)–(14) is completely general and independent from U-spin considerations. For example, further same-sign contributions in the CF and DCS decays can be absorbed by a redefinition of t_0 and t_2, see Ref. [30]. The meaning as a U-spin expansion only comes into play if we assume a hierarchy for the parameters according to their subscript.

The letters used to denote the amplitudes should not be confused with any ideas about the diagrams that generate them. That is, the use of p_0 and t_0 is there since in some limit p_0 is dominated by penguin diagrams and t_0 by tree diagrams. Yet, this is not always the case, and thus it is important to keep in mind that all that we do know at this stage is that the above is a general reparametrization of the decay amplitudes, and that each amplitude arises at a given order in the U-spin expansion. In the topological interpretation of the appearing parameters, t_0 includes both tree and exchange diagrams, which are absorbed [52]. Moreover, s_1 contains the broken penguin and p_0 includes contributions from tree, exchange, penguin and penguin annihilation diagrams [30, 52].

We note that the U-spin parametrization is completely general when we assume no CPV in the CF and DCS decays, which is also the case to a very good approximation in the SM. Beyond the SM, there can be additional amplitude contributions to the $\overline{D}^0 \to K^+\pi^-$ and $\overline{D}^0 \to \pi^+K^-$ decays which come with a relative weak phase from CP violating new physics. We do not discuss this case any further here.

In terms of the above amplitudes, the branching ratios are given as

$$\begin{aligned}\mathcal{BR}(D \to P_1P_2) &= |\mathcal{A}|^2 \times \mathcal{P}(D, P_1, P_2)\,, \\ \mathcal{P}(D, P_1, P_2) &= \tau_D \times \frac{1}{16\pi m_D^3}\sqrt{(m_D^2 - (m_{P_1} - m_{P_2})^2)(m_D^2 - (m_{P_1} + m_{P_2})^2)}\,. \end{aligned} \tag{15}$$

The direct CP asymmetries are [29, 35, 53]

$$a_{CP}^{\rm dir} = \mathrm{Im}\left(\frac{\lambda_b}{\Sigma}\right)\mathrm{Im}\left(\frac{A_b}{A_\Sigma}\right)\,. \tag{16}$$

III. SOLVING THE COMPLETE U-SPIN SYSTEM

We discuss how to extract the U-spin parameters of Eqs. (11)–(14) from the observables. We are mainly interested in the ratios of parameters and less in their absolute sizes and therefore we consider only quantities normalized on t_0, that is

$$\tilde{t}_1 \equiv \frac{t_1}{t_0}\,, \qquad \tilde{t}_2 \equiv \frac{t_2}{t_0}\,, \qquad \tilde{s}_1 \equiv \frac{s_1}{t_0}\,, \qquad \tilde{p}_0 \equiv \frac{p_0}{t_0}\,, \qquad \tilde{p}_1 \equiv \frac{p_1}{t_0}\,. \tag{17}$$

We choose, without loss of generality, the tree amplitude t_0 to be real. The relative phase between $\mathcal{A}(K\pi)$ and $\mathcal{A}(\pi K)$ is physical and can be extracted in experimental measurements. However, the relative phases between $\mathcal{A}(\pi\pi)$, $\mathcal{A}(KK)$ and $\mathcal{A}(K\pi)$ are unphysical, i.e. not observable on principal grounds. This corresponds to two additional phase choices that can be made in the U-spin parametrization. Consequently, without loss of generality, we can also choose the two parameters $\tilde{s}_1$ and $\tilde{t}_2$ to be real. Altogether, that makes eight real parameters, that we want to extract, not counting the normalization t_0. Of these, four parameters are in the CKM-leading part of the amplitudes and four in the CKM-suppressed one. In the CP limit $\mathrm{Im}\lambda_b \to 0$ we can absorb $\tilde{p}_0$ and $\tilde{p}_1$ into $\tilde{t}_2$ and $\tilde{s}_1$ respectively, which makes four real parameters in that limit.

The eight parameters can be extracted from eight observables that can be used to completely determine them. Additional observables can then be used in order to overconstrain the system. We divide the eight observables that we use to determine the system into four categories:

(i) Branching ratio measurements (3 observables) [16]. They are used to calculate the squared matrix elements. We neglect the tiny effects of order $|\lambda_b/\Sigma|$ and we get

$$|A_\Sigma(KK)|^2 = \frac{\mathcal{B}(\overline{D}^0 \to K^+K^-))}{|\Sigma|^2\mathcal{P}(D^0,K^+,K^-)}\,, \tag{18}$$

$$|A_\Sigma(\pi\pi)|^2 = \frac{\mathcal{B}(\overline{D}^0 \to \pi^+\pi^-)}{|\Sigma|^2\mathcal{P}(D^0,\pi^+,\pi^-)}\,, \tag{19}$$

$$|A(K\pi)|^2 = \frac{\mathcal{B}(\overline{D}^0 \to K^+\pi^-)}{|V_{cs}V_{ud}^*|^2\mathcal{P}(D^0,K^+,\pi^-)}\,, \tag{20}$$

$$|A(\pi K)|^2 = \frac{\mathcal{B}(\overline{D}^0 \to K^-\pi^+)}{|V_{cd}V_{us}^*|^2\mathcal{P}(D^0,K^-,\pi^+)}\,. \tag{21}$$

We consider three ratios of combinations of the four branching ratios, which are

$$R_{K\pi} \equiv \frac{|A(K\pi)|^2 - |A(\pi K)|^2}{|A(K\pi)|^2 + |A(\pi K)|^2}\,, \tag{22}$$

$$R_{KK,\pi\pi} \equiv \frac{|A(KK)|^2 - |A(\pi\pi)|^2}{|A(KK)|^2 + |A(\pi\pi)|^2}\,, \tag{23}$$

$$R_{KK,\pi\pi,K\pi} \equiv \frac{|A(KK)|^2 + |A(\pi\pi)|^2 - |A(K\pi)|^2 - |A(\pi K)|^2}{|A(KK)|^2 + |A(\pi\pi)|^2 + |A(K\pi)|^2 + |A(\pi K)|^2}\,. \tag{24}$$

(*ii*) Strong phase which does not require CP violation (1 observable). The relative strong phase between CF and DCS decay modes

$$\delta_{K\pi} \equiv \arg\left(\frac{\mathcal{A}(\overline{D}^0 \to K^-\pi^+)}{\mathcal{A}(D^0 \to K^-\pi^+)}\right) = \arg\left(\frac{\mathcal{A}(D^0 \to K^+\pi^-)}{\mathcal{A}(D^0 \to K^-\pi^+)}\right) \tag{25}$$

can be obtained from time-dependent measurements [40, 54–63] or correlated $D^0\overline{D}^0$ decays [64–69] at a charm-τ factory.

(*iii*) Integrated direct CP asymmetries (2 observables). In particular we use [27–31, 33–42, 44–51]

$$\Delta a_{CP}^{\rm dir} \equiv a_{CP}^{\rm dir}(D^0 \to K^+K^-) - a_{CP}^{\rm dir}(D^0 \to \pi^+\pi^-)\,, \tag{26}$$

$$\Sigma a_{CP}^{\rm dir} \equiv a_{CP}^{\rm dir}(D^0 \to K^+K^-) + a_{CP}^{\rm dir}(D^0 \to \pi^+\pi^-)\,. \tag{27}$$

(*iv*) Strong phases that require CP violation (2 observables) [36, 40, 60, 62, 64–67]. These are the relative phases of the amplitudes of a $\overline{D}^0$ and D^0 going into one of the CP eigenstates. They are proportional to CPV effects and thus very hard to extract. In particular,

$$\delta_{KK} \equiv \arg\left(\frac{\mathcal{A}(\overline{D}^0 \to K^+K^-)}{\mathcal{A}(D^0 \to K^+K^-)}\right)\,, \qquad \delta_{\pi\pi} \equiv \arg\left(\frac{\mathcal{A}(\overline{D}^0 \to \pi^+\pi^-)}{\mathcal{A}(D^0 \to \pi^+\pi^-)}\right)\,. \tag{28}$$

These can be obtained from time-dependent measurements or measurements of correlated $D^0\overline{D}^0$ pairs.

In principle, using the above observables the system Eqs. (11)–(14) is exactly solvable as long as the data is very precise. In the CP limit the branching ratio measurements (*i*) and the strong phase (*ii*) are sufficient to determine $\tilde{t}_1$, $\tilde{t}_2$ and $\tilde{s}_1$, which are the complete set of independent parameters in this limit.

For our parameter extraction with current data, we expand the observables to first non-vanishing order in the U-spin expansion. We measure the power counting of that expansion with a generic parameter ε, which, for nominal U-spin breaking effects is expected to be

$\varepsilon \sim 25\%$. All of the explicit results that we give below have the nice feature that the parameters can be extracted from them up to relative corrections of order $\mathcal{O}(\varepsilon^2)$. Below it is understood that we neglect all effects of that order.

In terms of our parameters the ratios of branching ratios are given as

$$R_{K\pi} = -\mathrm{Re}(\tilde{t}_1)\,, \tag{29}$$

$$R_{KK,\pi\pi} = -2\tilde{s}_1\,, \tag{30}$$

$$R_{KK,\pi\pi,K\pi} = \frac{1}{2}\left(\tilde{s}_1^2 - \frac{1}{4}|\tilde{t}_1|^2 + \tilde{t}_2\right)\,. \tag{31}$$

By inserting the expressions for $R_{K\pi}$ and $R_{KK,\pi\pi}$ into Eq. (24) we can solve the above equations for the independent parameter combinations. The result up to $\mathcal{O}(\varepsilon^2)$ is

$$\mathrm{Re}(\tilde{t}_1) = -R_{K\pi}\,, \tag{32}$$

$$\tilde{s}_1 = -\frac{1}{2}R_{KK,\pi\pi}\,, \tag{33}$$

$$-\frac{1}{4}\left(\mathrm{Im}\,\tilde{t}_1\right)^2 + \tilde{t}_2 = 2R_{KK,\pi\pi,K\pi} - \frac{1}{4}R_{KK,\pi\pi}^2 + \frac{1}{4}R_{K\pi}^2\,. \tag{34}$$

We are then able to determine $\tilde{t}_1$ with Eq. (32) and the strong phase between the CF and DCS mode, see also Ref. [60],

$$\delta_{K\pi} = \arg\left(-\frac{1-\frac{1}{2}\tilde{t}_1}{1+\frac{1}{2}\tilde{t}_1}\right) = -\mathrm{Im}(\tilde{t}_1)\,, \tag{35}$$

where in the last step we neglect terms of relative order of ε^2.

After that we can determine $\tilde{s}_1$ and $\tilde{t}_2$ from Eqs. (33) and (34), respectively. The sum and difference of the integrated direct CP asymmetries can be used together with the phases δ_{KK} and $\delta_{\pi\pi}$ to determine $\tilde{p}_0$ and $\tilde{p}_1$. We have

$$\Delta a_{CP}^{\mathrm{dir}} = \mathrm{Im}\left(\frac{\lambda_b}{\Sigma}\right) \times 4\,\mathrm{Im}\left(\tilde{p}_0\right)\,, \tag{36}$$

and

$$\Sigma a_{CP}^{\mathrm{dir}} = 2\,\mathrm{Im}\left(\frac{\lambda_b}{\Sigma}\right) \times \left[2\,\mathrm{Im}(\tilde{p}_0)\tilde{s}_1 + \mathrm{Im}(\tilde{p}_1)\right]\,. \tag{37}$$

Note that also $\Delta a_{CP}^{\mathrm{dir}}$ and $\Sigma a_{CP}^{\mathrm{dir}}$ share the feature of corrections entering only at the relative order $\mathcal{O}(\varepsilon^2)$ compared to the leading result. The measurement of $\Delta a_{CP}^{\mathrm{dir}}$ is basically a direct measurement of $\mathrm{Im}\,\tilde{p}_0$,

$$\mathrm{Im}\,\tilde{p}_0 = \frac{1}{4\mathrm{Im}(\lambda_b/\Sigma)}\Delta a_{CP}^{\mathrm{dir}}\,. \tag{38}$$

The phases δ_{KK} and $\delta_{\pi\pi}$ give (see e.g. Ref. [36])

$$\mathrm{Re}\left(\frac{A_b(D^0\to K^+K^-)}{A_\Sigma(D^0\to K^+K^-)}\right)-\mathrm{Re}\left(\frac{A_b(D^0\to \pi^+\pi^-)}{A_\Sigma(D^0\to \pi^+\pi^-)}\right)=4\mathrm{Re}(\tilde{p}_0)\,, \tag{39}$$

and

$$\begin{aligned}\mathrm{Re}\left(\frac{A_b(D^0\to K^+K^-)}{A_\Sigma(D^0\to K^+K^-)}\right)+\mathrm{Re}\left(\frac{A_b(D^0\to \pi^+\pi^-)}{A_\Sigma(D^0\to \pi^+\pi^-)}\right)&=2\,\mathrm{Re}(2\tilde{p}_0\tilde{s}_1+\tilde{p}_1)\\ &=2\left[2\,\mathrm{Re}(\tilde{p}_0)\tilde{s}_1+\mathrm{Re}(\tilde{p}_1)\right]\,. \end{aligned}\tag{40}$$

As $\tilde{s}_1$ is already in principle determined from the other observables, this gives us then the full information on $\tilde{p}_0$ and $\tilde{p}_1$.

As the observables δ_{KK} and $\delta_{\pi\pi}$ are the hardest to measure, we are not providing here the explicit relation of Eq. (39) and Eq. (40) to these observables, acknowledging just that the corresponding parameter combinations can be determined from these in a straight forward way.

Taking everything into account, we conclude that the above system of eight observables for eight parameters can completely be solved. This is done where the values of the CKM elements are used as inputs. We emphasize that in principle with correlated double-tag measurements at a future charm-tau factory [64–66, 68–76] we could even overconstrain the system.

IV. NUMERICAL RESULTS

We use the formalism introduced in Sec. III now with the currently available measurements. As not all of the observables have yet been measured, we cannot determine all of the U-spin parameters. Yet, we use the ones that we do have data on to get useful information on some of them.

- Using Gaussian error propagation without taking into account correlations, from the branching ratio measurements [16]

$$\mathcal{BR}(D^0\to K^+K^-)=(3.97\pm0.07)\cdot10^{-3}\,, \tag{41}$$

$$\mathcal{BR}(D^0\to \pi^+\pi^-)=(1.407\pm0.025)\cdot10^{-3}\,, \tag{42}$$

$$\mathcal{BR}(D^0\to K^+\pi^-)=(1.366\pm0.028)\cdot10^{-4}\,, \tag{43}$$

$$\mathcal{BR}(D^0\to K^-\pi^+)=(3.89\pm0.04)\cdot10^{-2}\,, \tag{44}$$

we obtain the normalized combinations

$$R_{K\pi} = -0.11 \pm 0.01\,, \tag{45}$$

$$R_{KK,\pi\pi} = 0.534 \pm 0.009\,, \tag{46}$$

$$R_{KK,\pi\pi,K\pi} = 0.071 \pm 0.009\,. \tag{47}$$

- The strong phase between DCS and CF mode for the scenario of no CP violation in the DCS mode is [14]

$$\delta_{K\pi} = \left(8.6^{+9.1}_{-9.7}\right)^\circ\,. \tag{48}$$

- The world average of $\Delta a_{CP}^{\rm dir}$ is given in Eq. (3).

- The sum of CP asymmetries $\Sigma a_{CP}^{\rm dir}$ in which CP violation has not yet been observed. In order to get an estimate we use the HFLAV averages for the single measurements of the CP asymmetries [2–7, 11, 14]

$$A_{CP}(D^0 \to \pi^+\pi^-) = 0.0000 \pm 0.0015\,, \tag{49}$$

$$A_{CP}(D^0 \to K^+K^-) = -0.0016 \pm 0.0012\,, \tag{50}$$

  and subtract the contribution from indirect charm CP violation $a_{CP}^{\rm ind} = (0.028 \pm 0.026)\%$ [15]. We obtain

$$\begin{aligned}\Sigma a_{CP}^{\rm dir} &= A_{CP}(D^0 \to K^+K^-) + A_{CP}(D^0 \to \pi^+\pi^-) - 2a_{CP}^{\rm ind} \\ &= -0.002 \pm 0.002\,,\end{aligned} \tag{51}$$

  where we do not take into account correlations, which may be sizable.

- The phases δ_{KK} and $\delta_{\pi\pi}$ have not yet been measured, and we cannot get any indirect information about them.

From Eqs. (32)–(35) it follows that

$$\mathrm{Re}(\tilde{t}_1) = 0.109 \pm 0.011\,, \tag{52}$$

$$\mathrm{Im}(\tilde{t}_1) = -0.15^{+0.16}_{-0.17}\,, \tag{53}$$

$$\tilde{s}_1 = -0.2668 \pm 0.0045\,, \tag{54}$$

$$-\frac{1}{4}\left(\mathrm{Im}\tilde{t}_1\right)^2 + \mathrm{Re}(\tilde{t}_2) = 0.075 \pm 0.018\,. \tag{55}$$

Employing [16]

$$\mathrm{Im}\left(\frac{\lambda_b}{\Sigma}\right) = (-6.3 \pm 0.3) \cdot 10^{-4}\,, \tag{56}$$

and inserting the measurement of $\Delta a_{CP}^{\rm dir}$ into Eq. (38), we obtain

$$\mathrm{Im}\,\tilde{p}_0 = 0.65 \pm 0.12\,. \tag{57}$$

Using $\Sigma a_{CP}^{\rm dir}$ we get

$$2\mathrm{Im}(\tilde{p}_0)\tilde{s}_1 + \mathrm{Im}(\tilde{p}_1) = 1.7 \pm 1.6\,. \tag{58}$$

Few remarks are in order regarding the numerical values we obtained.

1. Among the five parameters defined in Eq. (17), $\tilde{p}_1$ is the least constrained parameter as we have basically no information about it. In order to learn more about it we need measurements of $\Sigma a_{CP}^{\rm dir}$ as well as of the phases δ_{KK} and $\delta_{\pi\pi}$.

2. The higher order U-spin breaking parameters are consistently smaller than the first order ones, and the second order ones are even smaller. This is what we expect assuming the U-spin expansion.

3. Eqs. (52)–(55) suggest that the $\mathrm{SU}(3)_F$ breaking of the tree amplitude $\tilde{t}_1$ is smaller than the broken penguin contained in $\tilde{s}_1$.

4. Using Eqs. (52)–(55) we can get a rough estimate for the $\mathcal{O}(\varepsilon^2)$ corrections that enter the expression for $\Delta a_{CP}^{\rm dir}$ in Eq. (36). The results on the broken penguin suggest that these corrections do not exceed a level of $\sim 10\%$. We cannot, however, determine these corrections completely without further knowledge on $\tilde{p}_1$.

V. THE $\Delta U = 0$ RULE

We now turn to discuss the implications of Eq. (57). We rewrite Eq. (36) as

$$\Delta a_{CP}^{\rm dir} = 4\,\mathrm{Im}\left(\frac{\lambda_b}{\Sigma}\right)|\tilde{p}_0|\sin(\delta_{\rm strong})\,, \tag{59}$$

with the unknown strong phase

$$\delta_{\rm strong} = \arg(\tilde{p}_0)\,. \tag{60}$$

Then the numerical result in Eq. (57) reads

$$|\tilde{p}_0| \sin(\delta_{\rm strong}) = 0.65 \pm 0.12\,. \tag{61}$$

Recall that in the group theoretical language the parameters t_0 and p_0 are the matrix elements of the $\Delta U = 1$ and $\Delta U = 0$ operators, respectively [51]. For the ratio of the matrix elements of these operators we employ now the following parametrization

$$\tilde{p}_0 = B + Ce^{i\delta}\,, \tag{62}$$

such that B is the short-distance (SD) ratio and the second term arises from long-distance (LD) effects. While the separation between SD and LD is not well-defined, what we have in mind here is that diagrams with a b quark in the loop are perturbative and those with quarks lighter than the charm are not.

In Eq. (73) of Sec. VI below we apply the same decomposition into a "no QCD" part and corrections to that also to the $\Delta I = 1/2$ rules in K, D and B decays to pions. It is instructive to compare all of these systems in the same language.

We first argue that in Eq. (62) to a very good approximation $B = 1$. This is basically the statement that perturbatively, the diagrams with intermediate b are tiny. More explicitly, in that case, that is when we neglect the SD b penguins, we have

$$Q^{\Delta U=1} \equiv \frac{Q^{\bar{s}s} - Q^{\bar{d}d}}{2}\,, \qquad Q^{\Delta U=0} \equiv \frac{Q^{\bar{s}s} + Q^{\bar{d}d}}{2}\,. \tag{63}$$

Setting $C = 0$ then corresponds to the statement that only $Q^{\bar{s}s}$ can produce K^+K^- and only $Q^{\bar{d}d}$ can produce $\pi^+\pi^-$. This implies that for $C = 0$

$$\left\langle K^+K^-\right| Q^{\bar{d}d} \left|D^0\right\rangle = \left\langle \pi^+\pi^-\right| Q^{\bar{s}s} \left|D^0\right\rangle = 0\,, \tag{64}$$

and

$$\left\langle K^+K^-\right| Q^{\bar{s}s} \left|D^0\right\rangle \neq 0\,, \qquad \left\langle \pi^+\pi^-\right| Q^{\bar{d}d} \left|D^0\right\rangle \neq 0\,. \tag{65}$$

We then see that $B = 1$ since

$$\frac{\langle K^+K^-| Q^{\Delta U=0} |D^0\rangle}{\langle K^+K^-| Q^{\Delta U=1} |D^0\rangle} = 1\,, \qquad \frac{\langle \pi^+\pi^-| Q^{\Delta U=0} |D^0\rangle}{\langle \pi^+\pi^-| Q^{\Delta U=1} |D^0\rangle} = -1\,. \tag{66}$$

We note that in the $\mathrm{SU}(3)_F$ limit we also have

$$\left\langle K^+K^-\right| Q^{\Delta U=1} \left|D^0\right\rangle = -\left\langle \pi^+\pi^-\right| Q^{\Delta U=1} \left|D^0\right\rangle\,, \tag{67}$$

$$\left\langle K^+K^-\right| Q^{\Delta U=0} \left|D^0\right\rangle = \left\langle \pi^+\pi^-\right| Q^{\Delta U=0} \left|D^0\right\rangle\,, \tag{68}$$

but this is not used to argue that $B = 1$.

We then argue that $\delta \sim \mathcal{O}(1)$. The reason is that non-perturbative effects involve on-shell particles, or in other words, rescattering, and such effects give rise to large strong phases to the LD effects independent of the magnitude of the LD amplitude.

In the case that $B = 1$, $\delta \sim \mathcal{O}(1)$ and using the fact that the CKM ratios are small we conclude that the CP asymmetry is roughly given by the CKM factor times C

$$\Delta a_{CP}^{\rm dir} = 4\,{\rm Im}\left(\frac{\lambda_b}{\Sigma}\right) \times C \times \sin\delta\,. \tag{69}$$

Now the question is: what is C? As at this time no method is available in order to calculate C with a well-defined theoretical uncertainty, we do not employ here a dynamical calculation in order to provide a SM prediction for C and $\Delta a_{CP}^{\rm dir}$. We rather show the different principal possibilities and how to interpret them in view of the current data. In order to do so we measure the order of magnitude of the QCD correction term C relative to the "no QCD" limit $\tilde{p}_0 = 1$. Relative to that limit, we differentiate between three cases

1. $C = \mathcal{O}(\alpha_s/\pi)$: Perturbative corrections to $\tilde{p}_0$.

2. $C = \mathcal{O}(1)$: Non-perturbative corrections that produce strong phases from rescattering but do not significantly change the magnitude of $\tilde{p}_0$.

3. $C \gg \mathcal{O}(1)$: Large non-perturbative effects with significant magnitude changes and strong phases from rescattering to $\tilde{p}_0$.

Note that category (2) and (3) are in principle not different, as they both include non-perturbative effects, which differ only in their size.

Some perturbative results concluded that $C = \mathcal{O}(\alpha_s/\pi)$, leading to $\Delta a_{CP}^{\rm dir} \sim 10^{-4}$ [40, 77]. Note that the value $\Delta a_{CP}^{\rm dir} = 1 \times 10^{-4}$, assuming $O(1)$ strong phase, would correspond numerically to $C \sim 0.04$. We conclude that if there is a good argument that C is of category (1), the measurement of $\Delta a_{CP}^{\rm dir}$ would be a sign of beyond the SM (BSM) physics, because it would indicate a relative $\mathcal{O}(10)$ enhancement.

If the value of $\Delta a_{CP}^{\rm dir}$ would have turned out as large as suggested by the central value of some (statistically unsignificant) earlier measurements [8, 9], we would clearly need category (3) in order to explain that, i.e. penguin diagrams that are enhanced in magnitude, see e.g. Refs. [30, 34, 44–48, 51]. Another example for category (3) is the $\Delta I = 1/2$ rule in the kaon sector which is further discussed in sections VI and VII.

The current data, Eq. (61), is consistent with category (2). In the SM picture, the measurement of $\Delta a_{CP}^{\rm dir}$ proves the non-perturbative nature of the $\Delta U = 0$ matrix elements with a mild enhancement from $\mathcal{O}(1)$ rescattering effects. This is the $\Delta U = 0$ rule for charm.

Note that the predictions for $\Delta a_{CP}^{\rm dir}$ of category (i) and (ii) differ by $\mathcal{O}(10)$, although category (ii) contains only an $\mathcal{O}(1)$ nonperturbative enhancement with respect to the "no QCD" limit $\tilde{p}_0 = 1$. We emphasize that a measure for a QCD enhancement is not necessarily its impact on an observable, but the amplitude level comparison with the absence of QCD effects.

We also mention that we do not need $\mathrm{SU}(3)_F$ breaking effects to explain the data. Yet, the observation of $|\tilde{s}_1| > |\tilde{t}_1|$ in Eqs. (52)–(54) provide additional supporting evidence that rescattering is significant. Though no proof of the $\Delta U = 0$ rule on its own, this matches its upshot and is indicative of the importance of rescattering effects also in the broken penguin which is contained in $\tilde{s}_1$.

With future data on the phases δ_{KK} and $\delta_{\pi\pi}$ we will be able to determine the strong phase δ of Eq. (62). In that way it will be possible to completely determine the characteristics of the emerging $\Delta U = 0$ rule.

VI. $\Delta I = 1/2$ RULES IN K, D AND B DECAYS

It is instructive to compare the $\Delta U = 0$ rule in charm with the $\Delta I = 1/2$ rule in kaon physics, and furthermore also to the corresponding ratios of isospin matrix elements of D and B decays. For a review of the $\Delta I = 1/2$ rule see e.g. Ref. [21].

In kaon physics we consider $K \to \pi\pi$ decays. Employing an isospin parametrization we have [21]

$$\begin{aligned}
\mathcal{A}(K^+ \to \pi^+\pi^0) &= \frac{3}{2} A_2^K e^{i\delta_2^K} \,, \\
\mathcal{A}(K^0 \to \pi^+\pi^-) &= A_0^K e^{i\delta_0^K} + \sqrt{\frac{1}{2}} A_2^K e^{i\delta_2^K} \,, \\
\mathcal{A}(K^0 \to \pi^0\pi^0) &= A_0^K e^{i\delta_0^K} - \sqrt{2} A_2^K e^{i\delta_2^K} \,.
\end{aligned} \tag{70}$$

Note that the strong phases of A_0^K and A_2^K are factored out, so that $A_{0,2}^K$ contain weak phases only. The data give

$$\left| \frac{A_0^K}{A_2^K} \right| \approx 22.35 \,, \qquad \delta_0^K - \delta_2^K = (47.5 \pm 0.9)^\circ \,, \tag{71}$$

see Ref. [21] and references therein for more details. $A_{0,2}^K$ have a small imaginary part stemming from the CKM matrix elements only. To a very good approximation the real parts $\mathrm{Re}(A_0^K)$ and $\mathrm{Re}(A_2^K)$ in the $\Delta I = 1/2$ rule depend only on the tree operators [25, 26]

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V-A}\,, \qquad Q_2 = (\bar{s}u)_{V-A}(\bar{u}d)_{V-A}\,. \tag{72}$$

The lattice results Refs. [22–24] show an emerging physical interpretation of the $\Delta I = 1/2$ rule, that is an approximate cancellation of two contributions in $\mathrm{Re}(A_2^K)$, which does not take place in $\mathrm{Re}(A_0^K)$. These two contributions are different color contractions of the same operator.

The isospin decompositions of $D \to \pi\pi$ and $B \to \pi\pi$ are completely analog to Eq. (70). To differentiate the charm and beauty isospin decompositions from the kaon one, we put the corresponding superscripts to the respective analog matrix elements. Leaving away the superscripts indicates generic formulas that are valid for all three meson systems.

In order to understand better the anatomy of the $\Delta I = 1/2$ rule we use again the form

$$\frac{A_0}{A_2} = B + Ce^{i\delta}\,, \tag{73}$$

analogously to Eq. (62) in Sec. V for the $\Delta U = 0$ rule. Here, B is again the contribution in the limit of "no QCD", and $Ce^{i\delta}$ contains the corrections to that limit. Now, as discussed in Refs. [21, 78], in the limit of no strong interactions only the Q_2 operator contributes in Eq. (73). Note that the operator Q_1 is only generated from QCD corrections. When we switch off QCD, the amplitude into neutral pions vanishes and we have for $K, D, B \to \pi\pi$ equally [21, 78]

$$B = \sqrt{2}\,. \tag{74}$$

This corresponds to the limit $\tilde{p}_0 = 1$ that we considered in Sec. V for the $\Delta U = 0$ rule. The exact numerical value in Eq. (74) of course depends on the convention used for the normalization of $A_{0,2}$ in the isospin decomposition Eq. (70), where we use the one present in the literature.

For the isospin decomposition of $D^+ \to \pi^+\pi^0$, $D^0 \to \pi^+\pi^-$ and $D^0 \to \pi^0\pi^0$, we simply combine the fit of Ref. [33] to get

$$\left|\frac{A_0^D}{A_2^D}\right| = 2.47 \pm 0.07\,, \qquad \delta_0^D - \delta_2^D = (\pm 93 \pm 3)^\circ\,. \tag{75}$$

Reproducing the $\Delta I = 1/2$ rule for charm Eq. (75) is an optimal future testing ground for emerging new interesting non-perturbative methods [42]. Very promising steps on a conceptual level are also taken by lattice QCD [79].

In K and D decays the contributions of penguin operators to A_0 is CKM-suppressed, i.e. to a good approximation A_0 is generated from tree operators only. In B decays the situation is more involved because there is no relative hierarchy between the relevant CKM matrix elements. However, one can separate tree and penguin contributions by including the measurements of CP asymmetries within a global fit, as done in Ref. [31]. From Fig. 3 therein we find for the ratio of matrix elements of tree operators that

$$\left|\frac{A_0^B}{A_2^B}\right| \sim \sqrt{2} \tag{76}$$

is well compatible with the data, the best fit point having $|A_0^B/A_2^B| = 1.5$. The fit result for the phase difference $\delta_0^B - \delta_2^B$ is not given in Ref. [31].

The emerging picture is: The $\Delta I = 1/2$ rule in B decays is compatible or close to the "no QCD" limit. The $\Delta I = 1/2$ rule in kaon physics clearly belongs to category (3) of Sec. V. Here, the non-perturbative rescattering affects not only the phases but also the magnitudes of the corresponding matrix elements. Finally, the $\Delta I = 1/2$ rule in charm decays is intermediate and shows an $\mathcal{O}(1)$ enhancement, similar to the $\Delta U = 0$ rule that we found in Sec. V.

We can understand these differences from the different mass scales that govern K, D and B decays. Rescattering effects are most important in K decays, less important but still significant in D decays, and small in B decays.

VII. PHENOMENOLOGY OF THE $\Delta U = 0$ VS. $\Delta I = 1/2$ RULE

An interesting difference between the $\Delta I = 1/2$ rule in kaon decays and the $\Delta U = 0$ rule in charm decays is their effect on the phenomenology. Large rescattering enhances the CP violation effects in D decays, but it reduces the effect in kaon decays. The reason for the difference lies in the fact that in kaon decays the SD decay generates only a $u\bar{u}$ final state, while in charm decays it generates to a very good approximation the same amount of $d\bar{d}$ and $s\bar{s}$ states.

We write the amplitudes very generally and up to a normalization factor as

$$\mathcal{A} = 1 + rae^{i(\phi+\delta)}\,, \tag{77}$$

such that r is real and depends on CKM matrix elements, a is real and corresponds to the ratio of the respective hadronic matrix elements, ϕ is a weak phase and δ is a strong phase. For kaons a is the ratio of matrix elements of the operators $Q^{\Delta I=1/2}$ over $Q^{\Delta I=3/2}$, while for charm it is the ratio of matrix elements of the operators $Q^{\Delta U=0}$ over $Q^{\Delta U=1}$.

We first consider the case where we neglect the third generation. In that limit for kaons we have the decomposition

$$\mathcal{A}_K = V_{us}V_{ud}^*(A_{1/2} + r_{CG}A_{3/2})\,, \tag{78}$$

where r_{CG} is the CG coefficient that can be read from Eq. (70). For charm we have

$$\mathcal{A}_D = V_{cs}V_{us}^* A_1. \tag{79}$$

That means that in the two-generational limit for kaons we have $r = 1$ and in charm $r = 0$. If we switch on the third generation we get small corrections to these values in each case: $r \ll 1$ for charm and $|r - 1| \ll 1$ for kaons. These effects come from the non-unitarity of the 2×2 CKM. For the kaon case there is an extra effect that stems from SD penguins that come with $V_{ts}V_{td}^*$. In both cases we have $\delta \sim \mathcal{O}(1)$ from non-perturbative rescattering, as well as $\phi \sim \mathcal{O}(1)$.

The general formula for direct CP asymmetry is given as [16]

$$A_{CP} = -\frac{2ra\sin(\delta)\sin(\phi)}{1 + (ra)^2 + 2ra\cos(\delta)\cos(\phi)} \approx \begin{cases} 2ra\sin(\delta)\sin(\phi) & \text{for } ra \ll 1, \\ 2(ra)^{-1}\sin(\delta)\sin(\phi) & \text{for } ra \gg 1. \end{cases} \tag{80}$$

Non-perturbative effects enhance a in both kaon and charm decays. This means the effect which is visible in the CP asymmetry is different depending on the value of r. For $ra \ll 1$ increasing a results in enhancement of the CP asymmetry, while for $ra \gg 1$ it is suppressed. These two cases correspond to the charm and kaon cases, respectively. It follows that the $\Delta I = 1/2$ rule in kaons reduces CP violating effects, while the $\Delta U = 0$ rule in charm enhances them.

VIII. CONCLUSIONS

From the recent determination of $\Delta a_{CP}^{\rm dir}$ we derive the ratio of $\Delta U = 0$ over $\Delta U = 1$ amplitudes as

$$|\tilde{p}_0| \sin(\delta_{\rm strong}) = 0.65 \pm 0.12\,. \tag{81}$$

In principle two options are possible in order to explain this result: In the perturbative picture beyond the SM (BSM) physics is necessary to explain Eq. (81). On the other hand, in the SM picture, we find that all that is required in order to explain the result is a mild non-perturbative enhancement due to rescattering effects. Therefore, it is hard to argue that BSM physics is required.

Our interpretation of the result is that the measurement of $\Delta a_{CP}^{\rm dir}$ provides a proof for the $\Delta U = 0$ rule in charm. The enhancement of the $\Delta U = 0$ amplitude is not as significant as the one present in the $\Delta I = 1/2$ rule for kaons. In the future, with more information on the strong phase of $\tilde{p}_0$ from time-dependent measurements or measurements of correlated $D^0\overline{D}^0$ decays, we will be able to completely determine the extent of the $\Delta U = 0$ rule.

Interpreting the result within the SM implies that we expect a moderate non-perturbative effect and nominal $SU(3)_F$ breaking. The former fact implies that we expect U-spin invariant strong phases to be $\mathcal{O}(1)$. The latter implies that we anticipate the yet to be determined $SU(3)_F$ breaking effects not to be large. Thus, there are two qualitative predictions we can make

$$\delta_{\rm strong} \sim \mathcal{O}(1), \qquad a_{CP}^{\rm dir}(D^0 \to K^+K^-) \approx -a_{CP}^{\rm dir}(D^0 \to \pi^+\pi^-)\,. \tag{82}$$

Verifying these predictions will make the SM interpretation of the data more solid.

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