

$SU(3)_F$ Analysis for Beauty Baryon Decays

Avital Dery,^{*} Mitrajyoti Ghosh,[†] Yuval Grossman,[‡] and Stefan Schacht[§]

Department of Physics, LEPP, Cornell University, Ithaca, NY 14853, USA

Abstract

We perform a general $SU(3)_F$ analysis of $b \rightarrow c\bar{c}s(d)$ decays of members of the beauty baryon antitriplet to a member of the light baryon octet and a singlet. Under several reasonable assumptions we found $|\mathcal{A}(\Xi_b^0 \rightarrow \Lambda S)/\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 S)| \approx 1/\sqrt{6} |V_{cb}^* V_{cd}/(V_{cb}^* V_{cs})|$ and $|\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 S)/\mathcal{A}(\Lambda_b \rightarrow \Lambda S)| \sim 0.02$. These two relations have been recently probed by LHCb for the case of $S = J/\psi$. The former agrees with the measurement, while for the latter our prediction lies close to the upper bound set by LHCb.

arXiv:2001.05397v2 [hep-ph] 2 Apr 2020

^{*} avital.dery@cornell.edu

[†] mg2338@cornell.edu

[‡] yg73@cornell.edu

[§] ss3843@cornell.edu

I. INTRODUCTION

A tremendous amount of b -baryons is produced at the LHC [1]. This allows for angular analyses of Λ_b decays at LHCb [2] and ATLAS [3] and has led to evidence of CP violation in Λ_b decays [4]. It is now feasible to scrutinize rare or suppressed b -baryon decays: Recent results include the first observation of $\Lambda_b \rightarrow \Lambda \gamma$ [5] and the analysis of the isospin suppressed $\Lambda_b \rightarrow \Sigma^0 J/\psi$ decay and the Cabibbo-suppressed decay $\Xi_b^0 \rightarrow \Lambda J/\psi$ [6].

These increasingly precise measurements of baryon decays motivate us to perform an $SU(3)_F$ analysis of $b \rightarrow c\bar{c}q$ (with $q = s, d$) decays of the heavy b -baryon $\bar{\mathbf{3}}$ to the light baryon $\mathbf{8}$ and an $SU(3)_F$ singlet, $\bar{\mathbf{3}}_b \rightarrow \mathbf{8}_b \otimes \mathbf{1}$. From the perspective of $SU(3)_F$ it makes no difference if the singlet, which we denote as S , is a J/ψ or any final state particle that does not carry any $SU(3)_F$ flavor, for example, a photon or a lepton pair.

We start our analysis using two separate assumptions: (1) We work in the $SU(3)_F$ limit and (2) we treat the Λ and Σ^0 as isospin eigenstates. We emphasize that these assumptions are not connected to each other. We later relax these assumptions and take into account corrections to the $SU(3)_F$ limit as well as deviations of the mass eigenstates of Λ and Σ^0 from their isospin eigenstates.

At leading order the decays $\bar{\mathbf{3}}_b \rightarrow \mathbf{8}_b \otimes \mathbf{1}$ are mediated by tree-level $b \rightarrow c\bar{c}q$ transitions. These correspond to a $\mathbf{3}$ operator. In full generality however, we have to take into account additional contributions from loops that generate effective $b \rightarrow t\bar{t}q$ and $b \rightarrow u\bar{u}q$ transitions. The contribution from $b \rightarrow t\bar{t}q$ can be neglected as it is a penguin and therefore suppressed and it gives only another $\mathbf{3}$ under $SU(3)_F$. In contrast, the up quarks in $b \rightarrow u\bar{u}q$ can induce intermediate on-shell states leading to nontrivial effects from rescattering. Specifically, the $b \rightarrow u\bar{u}q$ transition has a more complicated isospin and $SU(3)_F$ structure and induces the higher $SU(3)_F$ representations $\bar{\mathbf{6}}$ and $\mathbf{15}$. Therefore, as higher $SU(3)_F$ representations stem from rescattering, in the literature it is often assumed that these are suppressed.

Our strategy is to start with a very general model-independent viewpoint and then introduce additional assumptions step by step. While we mainly concentrate in this paper on the case where $S = J/\psi$, the general nature of our results make it possible to apply them also to radiative and semileptonic decays.

CKM-leading $SU(3)_F$ limit Clebsch-Gordan coefficients for $\bar{\mathbf{3}}_b \rightarrow \mathbf{8}_b \otimes \mathbf{1}$ in $b \rightarrow s$ transitions have been presented in Refs. [7–9]. In Refs. [8, 10–12] hadronic models based on QCD

factorization have been utilized, and in Refs. [9, 13] a covariant confined quark model has been applied. An $SU(3)_F$ analysis of b -baryon antitriplet decays to the light baryon octet and the η_1 singlet can be found in Ref. [14].

Further applications of $SU(3)_F$ to b baryon decays can be found in Refs. [15–20]. Works on b baryon decays beyond their $SU(3)_F$ treatment are given in Refs. [21–25]. Applications of $SU(3)_F$ methods on non- b baryon decays can be found in Refs. [26–37]. Further literature on baryon decays is given in Refs. [38–40]. Discussions of baryonic form factors can be found in Refs. [41–50].

We present our $SU(3)_F$ analysis including isospin and $SU(3)_F$ breaking in Sec. II. After that we estimate in Sec. III the effect of Σ^0 – Λ mixing in Λ_b decays, which is in general scale- and process-dependent, i.e. non-universal. We compare with recent experimental results in Sec. IV and conclude in Sec. V.

II. $SU(3)_F$ ANALYSIS

A. General $SU(3)_F$ Decomposition

The $b \rightarrow c\bar{c}q$ (with $q = s, d$) decays of Λ_b , Ξ_b^- and Ξ_b^0 , which form the heavy baryon $\bar{\mathbf{3}}$, into a singlet S (e.g. $S = J/\psi, \gamma, l^+l^-, \dots$) and a member of the light baryon $\mathbf{8}$, share a common set of reduced $SU(3)_F$ matrix elements after the application of the Wigner-Eckart theorem. These decays are specifically:

- $b \rightarrow sc\bar{c}$ transitions:

$$\Lambda_b \rightarrow \Lambda S, \quad \Lambda_b \rightarrow \Sigma^0 S, \quad \Xi_b^0 \rightarrow \Xi^0 S, \quad \Xi_b^- \rightarrow \Xi^- S. \quad (1)$$

- $b \rightarrow dc\bar{c}$ transitions:

$$\Xi_b^0 \rightarrow \Lambda S, \quad \Xi_b^0 \rightarrow \Sigma^0 S, \quad \Lambda_b \rightarrow n S, \quad \Xi_b^- \rightarrow \Sigma^- S. \quad (2)$$

Note that there are two additional allowed decays $\Lambda_b \rightarrow \Xi^0 J/\psi$ and $\Xi_b^0 \rightarrow n J/\psi$ which are however highly suppressed by two insertions of weak effective operators, so we do not consider them in our study here. The $SU(3)_F$ quantum numbers and masses are given in Table I. In this section we discuss the $SU(3)_F$ limit, $SU(3)_F$ -breaking effects are treated in Sec. II E.

We can write the $SU(3)_F$ structure of the relevant $b \rightarrow s$ and $b \rightarrow d$ Hamiltonians as [51]

$$\begin{aligned}\mathcal{H}^{b \rightarrow s} &= \lambda_{cs}(\bar{c}b)(\bar{s}c) + \lambda_{us}(\bar{u}b)(\bar{s}u) + \lambda_{ts}(\bar{t}b)(\bar{s}t) \\ &= \lambda_{cs}(\mathbf{3})_{0,0,-\frac{2}{3}}^c + \lambda_{us} \left((\mathbf{3})_{0,0,-\frac{2}{3}}^u + (\bar{\mathbf{6}})_{1,0,-\frac{2}{3}}^u + \sqrt{6}(\mathbf{15})_{1,0,-\frac{2}{3}}^u + \sqrt{3}(\mathbf{15})_{0,0,-\frac{2}{3}}^u \right),\end{aligned}\quad (3)$$

$$\begin{aligned}\mathcal{H}^{b \rightarrow d} &= \lambda_{cd}(\bar{c}b)(\bar{d}c) + \lambda_{ud}(\bar{u}b)(\bar{d}u) + \lambda_{td}(\bar{t}b)(\bar{d}t) \\ &= \lambda_{cd}(\mathbf{3})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}}^c + \lambda_{ud} \left((\mathbf{3})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}}^u - (\bar{\mathbf{6}})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}}^u + \sqrt{8}(\mathbf{15})_{\frac{3}{2},-\frac{1}{2},\frac{1}{3}}^u + (\mathbf{15})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}}^u \right).\end{aligned}\quad (4)$$

See also Refs. [52] and [53] for the application of these Hamiltonians to $B \rightarrow J/\psi K$ and $B \rightarrow DD$, respectively. The notation for the subindices are such that $(\mathbf{N})_{I,I_3,Y}$ refers to the irreducible representation $\mathbf{N}$ of $SU(3)_F$ using the quantum numbers of strong isospin I , I_3 and strong hypercharge Y . In the standard basis of the Gell-Mann matrices I_3 and Y correspond to the eigenvalues of λ_3 and λ_8 , respectively. We further use the notation

$$\lambda_{cs} \equiv V_{cb}^* V_{cs} \sim \lambda^2, \quad \lambda_{us} \equiv V_{ub}^* V_{us} \sim \lambda^4, \quad \lambda_{ts} \equiv V_{tb}^* V_{ts} \sim \lambda^2, \quad (5)$$

$$\lambda_{cd} \equiv V_{cb}^* V_{cd} \sim \lambda^3, \quad \lambda_{ud} \equiv V_{ub}^* V_{ud} \sim \lambda^3, \quad \lambda_{td} \equiv V_{tb}^* V_{td} \sim \lambda^3, \quad (6)$$

for the CKM matrix element combinations, where we indicate the hierarchies using the Wolfenstein parameter λ .

Note that in Eqs. (3) and (4) it is understood that $SU(3)_F$ operators in front of different CKM matrix elements have to be differentiated as they stem from different underlying operators. For instance, even if the two triplets generate linearly dependent Clebsch-Gordan coefficients, the respective matrix elements themselves are independent. They can, for example, have a relative strong phase.

We write the reduced $SU(3)_F$ limit matrix elements as A_q^k , where k is the respective $SU(3)_F$ representation in the Hamiltonian and q denotes the operator it stems from. The initial state is always a $|\mathbf{3}\rangle$ and the final state is always a $|\mathbf{8}\rangle$, so that we are left with four reduced matrix elements in the $SU(3)_F$ limit:

$$A_c^3, \quad A_u^3, \quad A_u^{\bar{6}}, \quad A_u^{15}. \quad (7)$$

The $SU(3)_F$ limit decomposition is given in Table II. The CKM-leading part of the $b \rightarrow s$ transitions agrees with Refs. [7–9]. The Clebsch-Gordan coefficients are obtained using Refs. [54–56]. The normalization of the amplitudes is such that

$$\mathcal{B}(B_1 \rightarrow B_2 S) = |\mathcal{A}(B_1 \rightarrow B_2 S)|^2 \times \mathcal{P}(B_1, B_2, S), \quad (8)$$

with the two-body decay phase space factors

$$\mathcal{P}(B_1, B_2, S) \equiv \frac{\tau_{B_1}}{16\pi m_{B_1}^3} \sqrt{(m_{B_1}^2 - (m_{B_2} - m_S)^2)(m_{B_1}^2 - (m_{B_2} + m_S)^2)}. \quad (9)$$

Note that in cases where the $\text{SU}(3)_F$ singlet S is a multibody state, e.g. $S = l^+l^-$, we imply the appropriate phase space integration in Eq. (8). Note further, that we still work in the $\text{SU}(3)_F$ limit of the decay amplitudes. Eq. (9) only accounts for the trivial $\text{SU}(3)_F$ breaking from phase space effects. Additional $\text{SU}(3)_F$ breaking contributions are discussed in Sec. II E. Therein, we estimate $\text{SU}(3)_F$ breaking effects to be of order 20%. Note that the amplitudes in Eq. (8) have a mass dimension, but we always care about ratios, so we can think about them as dimensionless quantities. Note that phase space effects are of order 3% and thus they are well within the errors and could or could not be taken into account. For a model-dependent way to estimate these effects one can, for example, employ form factor results in Refs. [9, 57].

The reduced $\text{SU}(3)_F$ matrix elements can in principle be matched on a color suppressed tree diagram C , an exchange diagram E and penguin diagrams P_q with quark q running in the loop. As examples we show the topological diagrams for $\Lambda_b \rightarrow \Lambda J/\psi$ and $\Lambda_b \rightarrow \Sigma^0 J/\psi$ in Fig. 1. In the following, however, we only perform the group theory treatment.

The combined matrix of Clebsch-Gordan coefficients of $b \rightarrow s$ and $b \rightarrow d$ decays in Table II has matrix rank four, i.e., there are four sum rules, which read

$$-\sqrt{\frac{3}{2}}\mathcal{A}(\Lambda_b \rightarrow \Lambda S) + \frac{1}{\sqrt{2}}\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 S) + \mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 S) = 0, \quad (\text{SU}(3)_F \text{ sum rule}) \quad (10)$$

$$\sqrt{\frac{3}{2}}\mathcal{A}(\Xi_b^0 \rightarrow \Lambda S) - \frac{1}{\sqrt{2}}\mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 S) + \mathcal{A}(\Lambda_b \rightarrow n S) = 0, \quad (\text{SU}(3)_F \text{ sum rule}) \quad (11)$$

$$-\sqrt{2}\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 S) \frac{\lambda_{ud}}{\lambda_{us}} + \sqrt{6}\mathcal{A}(\Xi_b^0 \rightarrow \Lambda S) + \mathcal{A}(\Lambda_b \rightarrow n S) = 0, \quad (\text{SU}(3)_F \text{ sum rule}) \quad (12)$$

$$\begin{aligned} & \sqrt{\frac{3}{2}}\mathcal{A}(\Lambda_b \rightarrow \Lambda S) \frac{\lambda_{ud}}{\lambda_{us}} - \frac{3}{\sqrt{2}}\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 S) \frac{\lambda_{ud}}{\lambda_{us}} \\ & - \mathcal{A}(\Xi_b^- \rightarrow \Xi^- S) \frac{\lambda_{ud}}{\lambda_{us}} + \sqrt{6}\mathcal{A}(\Xi_b^0 \rightarrow \Lambda S) + \mathcal{A}(\Xi_b^- \rightarrow \Sigma^- S) = 0, \quad (\text{SU}(3)_F \text{ sum rule}) \quad (13) \end{aligned}$$

all of which are $\text{SU}(3)_F$ sum rules, and there is no isospin sum rule. Note that there are two sum rules which mix $b \rightarrow s$ and $b \rightarrow d$ decays and two which do not. These sum rules are valid in the $\text{SU}(3)_F$ limit irrespective of the power counting of the CKM matrix elements, assumptions on the reduced matrix elements, or the particular $\text{SU}(3)_F$ singlet S , i.e. they are completely generic.

B. Assumptions on CKM Hierarchy and Rescattering

We now make some assumptions, which are not completely generic, i.e. their validity can for example depend on the particular considered $SU(3)_F$ singlet S , e.g. if $S = J/\psi$ or $S = \gamma$.

We first neglect the CKM-suppressed amplitude in $b \rightarrow s$ decays, that is we set $\lambda_{us}/\lambda_{cs} \rightarrow 0$. In the isospin and $SU(3)_F$ limit for $b \rightarrow s$ decays we have then only one contributing reduced matrix element:

$$\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 S) = 0, \quad (\text{isospin sum rule}) \quad (14)$$

$$\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 S) = \mathcal{A}(\Xi_b^- \rightarrow \Xi^- S), \quad (\text{isospin sum rule}) \quad (15)$$

$$\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 S) = \sqrt{\frac{3}{2}} \mathcal{A}(\Lambda_b \rightarrow \Lambda S). \quad (SU(3)_F \text{ sum rule}) \quad (16)$$

We now move to make another assumption and that is to also neglect the λ_{ud} terms for the $b \rightarrow d$ transitions. Despite the formal power counting Eq. (6), that is $|\lambda_{ud}| \simeq |\lambda_{cd}|$, numerically we actually have [58]

$$\left| \frac{\lambda_{ud}}{\lambda_{cd}} \right| \approx 0.38. \quad (17)$$

Moreover, it is plausible that $A_u^{\bar{6}}$ and A_u^{15} are suppressed because they result from light quarks stemming from $b \rightarrow u\bar{u}s(d)$ which induce intermediate on-shell states that rescatter into $c\bar{c}$, see also Refs. [59–63]. Under the assumption that these terms are more or equally suppressed as $SU(3)_F$ -breaking effects we have many more relations. All seven non-zero decays we considered in Table II are then simply related by the Clebsch-Gordan coefficients in the first column. In addition to the sum rules Eqs. (14)–(16), we have then

$$\sqrt{2} \mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 S) = \mathcal{A}(\Xi_b^- \rightarrow \Sigma^- S), \quad (\text{isospin sum rule}) \quad (18)$$

$$\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 S) = -\sqrt{6} \mathcal{A}(\Xi_b^0 \rightarrow \Lambda S) \frac{\lambda_{cs}}{\lambda_{cd}}, \quad (SU(3)_F \text{ sum rule}) \quad (19)$$

$$-\sqrt{6} \mathcal{A}(\Xi_b^0 \rightarrow \Lambda S) = \sqrt{2} \mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 S), \quad (SU(3)_F \text{ sum rule}) \quad (20)$$

$$\sqrt{2} \mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 S) = \mathcal{A}(\Lambda_b \rightarrow n S), \quad (SU(3)_F \text{ sum rule}) \quad (21)$$

$$\mathcal{A}(\Lambda_b \rightarrow n S) = \mathcal{A}(\Xi_b^- \rightarrow \Sigma^- S). \quad (SU(3)_F \text{ sum rule}) \quad (22)$$

C. Isospin and U -Spin Decompositions

For comprehensiveness, we give also the isospin and U -spin decompositions of the Hamiltonians, which read

$$\mathcal{H}_{b \rightarrow s} = \lambda_{cs} (0, 0)_I^c + \lambda_{us} ((0, 0)_I^u + (1, 0)_I^u) \quad (23)$$

$$= \lambda_{cs} \left(\frac{1}{2}, -\frac{1}{2} \right)_U^c + \lambda_{us} \left(\frac{1}{2}, -\frac{1}{2} \right)_U^u, \quad (24)$$

and

$$\mathcal{H}_{b \rightarrow d} = \lambda_{cd} \left(\frac{1}{2}, -\frac{1}{2} \right)_I^c + \lambda_{ud} \left(\left(\frac{3}{2}, -\frac{1}{2} \right)_I^u + \left(\frac{1}{2}, -\frac{1}{2} \right)_I^u \right) \quad (25)$$

$$= \lambda_{cd} \left(\frac{1}{2}, \frac{1}{2} \right)_U^c + \lambda_{ud} \left(\frac{1}{2}, \frac{1}{2} \right)_U^u, \quad (26)$$

where we use the notation

$$(i, j)_I^q \equiv \mathcal{O}_{\Delta I_3=j}^{\Delta I=i}, \quad (i, j)_U^q \equiv \mathcal{O}_{\Delta U_3=j}^{\Delta U=i}, \quad (27)$$

where q denotes the quark content of the operator the representation stems from and we absorbed Clebsch-Gordan coefficients into operators.

Using the isospin and U -spin states in Table I, we obtain the isospin decompositions in Tables III and IV and the U -spin decomposition in Table V. We note that the $SU(3)_F$ decomposition includes more information than the isospin and U -spin tables each on their own. An example is the ratio

$$\left| \frac{\mathcal{A}(\Xi_b^0 \rightarrow \Lambda S)}{\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 S)} \right| = \frac{1}{\sqrt{2}} \left| \frac{\langle 0 | \frac{1}{2} | \frac{1}{2} \rangle}{\langle \frac{1}{2} | 0 | \frac{1}{2} \rangle} \right| \left| \frac{\lambda_{cd}}{\lambda_{cs}} \right|. \quad (28)$$

where the appearing reduced matrix elements are not related, e.g. the final states belong to different isospin representations. That means we really need $SU(3)_F$ to find the relation Eq. (19).

We can make this completely transparent by writing out the implications of Eq. (14) for the corresponding U -spin decomposition. From Table V and Eq. (14) it follows for the U -spin matrix elements

$$-\frac{\sqrt{3}}{2\sqrt{2}} \left\langle 0 \left| \frac{1}{2} \right| \frac{1}{2} \right\rangle^c + \frac{1}{2\sqrt{2}} \left\langle 1 \left| \frac{1}{2} \right| \frac{1}{2} \right\rangle^c = 0. \quad (29)$$

Inserting this relation into the U -spin decomposition of the decay $\Xi_b^0 \rightarrow \Lambda S$ in Table V, we obtain

$$\mathcal{A}(\Xi_b^0 \rightarrow \Lambda S) = \frac{1}{\sqrt{6}} \lambda_{cd} \left\langle 1 \left| \frac{1}{2} \right| \frac{1}{2} \right\rangle^c. \quad (30)$$

Comparing this expression with the U -spin decomposition of the decay $\Xi_b^0 \rightarrow \Xi^0 S$ in Table V, we arrive again at the sum rule Eq. (19).

In order that Eq. (19) holds we need not only the suppression of other $SU(3)_F$ limit contributions as discussed above, but also the suppression of both isospin and U -spin violating contributions. A non-vanishing dynamic isospin breaking contribution to $\Lambda_b \rightarrow \Sigma^0 S$ would also be reflected in isospin and $SU(3)_F$ -breaking violations of Eq. (19). We make this correlation explicit in Sec. II E.

D. CP Asymmetry Sum Rules

Due to a general sum rule theorem given in Ref. [64] that relates direct CP asymmetries of decays connected by a complete interchange of d and s quarks [64–67], we can directly write down two U -spin limit sum rules:

$$\frac{a_{CP}^{\text{dir}}(\Xi_b^0 \rightarrow \Xi^0 S)}{a_{CP}^{\text{dir}}(\Lambda_b \rightarrow n S)} = -\frac{\tau(\Xi_b^0)}{\tau(\Lambda_b)} \frac{\mathcal{B}(\Lambda_b \rightarrow n S)}{\mathcal{B}(\Xi_b^0 \rightarrow \Xi^0 S)}, \quad (31)$$

$$\frac{a_{CP}^{\text{dir}}(\Xi_b^- \rightarrow \Xi^- S)}{a_{CP}^{\text{dir}}(\Xi_b^- \rightarrow \Sigma^- S)} = -\frac{\mathcal{B}(\Xi_b^- \rightarrow \Sigma^- S)}{\mathcal{B}(\Xi_b^- \rightarrow \Xi^- S)}, \quad (32)$$

where the branching ratios imply CP averaging. Note that the general U -spin rule leading to Eqs. (31) and (32) also applies to multi-body final states, as pointed out in Refs. [26, 64, 68]. It follows that Eqs. (31) and (32) apply also when S is a multi-body state like $S = l^+ l^-$.

Note that although the quark content of the Λ and Σ is uds , this does not mean that a complete interchange of d and s quarks gives the identity. The reason is given by the underlying quark wave functions [69]

$$|\Lambda\rangle \sim \frac{1}{\sqrt{2}} (ud - du) s, \quad |\Sigma^0\rangle \sim \frac{1}{\sqrt{2}} (ud + du) s, \quad (33)$$

where we do not write the spin wave function. Eq. (33) shows explicitly that a complete interchange of d and s quarks in Λ or Σ^0 does not result again in a Λ or Σ^0 wave function, respectively. This is similar to the situation for η and η' , where no respective particles

correspond to a complete interchange of d and s quarks [70], see e.g. the quark wave functions given in Ref. [71].

We can put this into a different language, namely that in the U -spin basis the large mixing of $|1, 0\rangle_U$ and $|0, 0\rangle_U$ to the U -spin states of Λ and Σ^0 , see Table I, destroys two sum rules which exist for the U -spin eigenstates. To be explicit, we define U -spin eigenstates which are not close to mass eigenstates

$$|X\rangle = |0, 0\rangle_U, \quad |Y\rangle = |1, 0\rangle_U. \quad (34)$$

For these, we obtain the U -spin decomposition given in Table VI. From that it is straightforward to obtain another two CP asymmetry sum rules. These are however impractical, because there is no method available to prepare Λ and Σ^0 as U -spin eigenstates, instead of approximate isospin eigenstates. Consequently, we are left only with the two CP asymmetry sum rules Eqs. (31) and (32).

Note that CKM-leading $SU(3)_F$ breaking by itself cannot contribute to CP violation, because it comes only with relative strong phases but not with the necessary relative weak phase. Therefore, the individual CP asymmetries can be written as

$$a_{CP}^{\text{dir}} = \text{Im} \frac{\lambda_{uq}}{\lambda_{cq}} \text{Im} \frac{A_u}{A_c}, \quad (35)$$

where $A_{u,c}$ have only a strong phase and to leading order in Wolfenstein- λ we have [58]

$$\text{Im} \left(\frac{\lambda_{us}}{\lambda_{cs}} \right) \approx \lambda^2 \bar{\eta} \approx 0.02, \quad \text{Im} \left(\frac{\lambda_{ud}}{\lambda_{cd}} \right) \approx \bar{\eta} \approx 0.36. \quad (36)$$

Additional suppression from rescattering implies that on top of Eq. (36) we have $|A_u| \ll |A_c|$, i.e. the respective imaginary part is also expected to be small. This implies that we do not expect to see a nonvanishing CP asymmetry in these decays any time soon. The other way around, this prediction is also a test of our assumption that the λ_{uq} -amplitude is suppressed.

E. $SU(3)_F$ Breaking

We consider now isospin and $SU(3)_F$ breaking effects in the CKM-leading part of the $b \rightarrow s$ and $b \rightarrow d$ Hamiltonians. This will become useful once we have measurements of several b -baryon decays that are precise enough to see deviations from the $SU(3)_F$ limit

sum rules. $SU(3)_F$ breaking effects for charm and beauty decays have been discussed in the literature for a long time [26, 52, 72–86]. They are generated through the spurion

$$\begin{pmatrix} \frac{m_u}{\Lambda} - \frac{2}{3}\alpha & 0 & 0 \\ 0 & \frac{m_d}{\Lambda} + \frac{1}{3}\alpha & 0 \\ 0 & 0 & \frac{m_s}{\Lambda} + \frac{1}{3}\alpha \end{pmatrix} = \frac{1}{3} \frac{m_u + m_d + m_s}{\Lambda} \mathbb{1} - \frac{1}{2} \left(\frac{m_d - m_u}{\Lambda} + \alpha \right) \lambda_3 + \frac{1}{2\sqrt{3}} \left(\frac{m_u + m_d - 2m_s}{\Lambda} - \alpha \right) \lambda_8, \quad (37)$$

with the unity $\mathbb{1}$ and the Gell-Mann matrices λ_3 and λ_8 . The part of Eq. (37) that is proportional to $\mathbb{1}$ can be absorbed into the $SU(3)_F$ limit part. It follows that the isospin and $SU(3)_F$ -breaking tensor operator is given as

$$\delta (\mathbf{8})_{1,0,0} + \varepsilon (\mathbf{8})_{0,0,0}, \quad (38)$$

with

$$\delta = \frac{1}{2} \left(\frac{m_d - m_u}{\Lambda} + \alpha \right), \quad \varepsilon = \frac{1}{2\sqrt{3}} \left(\frac{m_u + m_d - 2m_s}{\Lambda} - \alpha \right), \quad (39)$$

where α is the electromagnetic coupling and we generically expect the size of isospin and $SU(3)_F$ breaking to be $\delta \sim 1\%$ and $\varepsilon \sim 20\%$, respectively. Note that the scale-dependence of the quark masses, as well as the fact that we do not know how to define the scale Λ make it impossible to quote decisive values for δ and ε . Eventually, they will have to be determined experimentally for each process of interest separately as they are not universal.

For the tensor products of the perturbation with the CKM-leading $SU(3)_F$ limit operator it follows:

$$(\mathbf{8})_{1,0,0} \otimes (\mathbf{3})_{0,0,-\frac{2}{3}}^c = \sqrt{\frac{1}{2}} (\bar{\mathbf{6}})_{1,0,-\frac{2}{3}} + \sqrt{\frac{1}{2}} (\mathbf{15})_{1,0,-\frac{2}{3}}, \quad (40)$$

$$(\mathbf{8})_{0,0,0} \otimes (\mathbf{3})_{0,0,-\frac{2}{3}}^c = \frac{1}{2} (\mathbf{3})_{0,0,-\frac{2}{3}} + \frac{\sqrt{3}}{2} (\mathbf{15})_{0,0,-\frac{2}{3}}, \quad (41)$$

$$\begin{aligned} (\mathbf{8})_{1,0,0} \otimes (\mathbf{3})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}}^c &= \frac{\sqrt{3}}{4} (\mathbf{3})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}} - \sqrt{\frac{1}{8}} (\bar{\mathbf{6}})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}} \\ &\quad - \sqrt{\frac{1}{48}} (\mathbf{15})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}} + \sqrt{\frac{2}{3}} (\mathbf{15})_{\frac{3}{2},-\frac{1}{2},\frac{1}{3}}, \end{aligned} \quad (42)$$

$$(\mathbf{8})_{0,0,0} \otimes (\mathbf{3})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}}^c = -\frac{1}{4} (\mathbf{3})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}} - \sqrt{\frac{3}{8}} (\bar{\mathbf{6}})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}} + \frac{3}{4} (\mathbf{15})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}}, \quad (43)$$

so that we arrive at the $SU(3)_F$ breaking Hamiltonians

$$\mathcal{H}_X^{b \rightarrow s} \equiv \lambda_{cs} \delta \left(\sqrt{\frac{1}{2}} (\bar{\mathbf{6}})_{1,0,-\frac{2}{3}} + \sqrt{\frac{1}{2}} (\mathbf{15})_{1,0,-\frac{2}{3}} \right) + \lambda_{cs} \varepsilon \left(\frac{1}{2} (\mathbf{3})_{0,0,-\frac{2}{3}} + \frac{\sqrt{3}}{2} (\mathbf{15})_{0,0,-\frac{2}{3}} \right), \quad (44)$$

$$\mathcal{H}_X^{b \rightarrow d} \equiv \lambda_{cd} \delta \left(\frac{\sqrt{3}}{4} (\mathbf{3})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}} - \sqrt{\frac{1}{8}} (\bar{\mathbf{6}})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}} - \sqrt{\frac{1}{48}} (\mathbf{15})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}} + \sqrt{\frac{2}{3}} (\mathbf{15})_{\frac{3}{2},-\frac{1}{2},\frac{1}{3}} \right) + \lambda_{cd} \varepsilon \left(-\frac{1}{4} (\mathbf{3})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}} - \sqrt{\frac{3}{8}} (\bar{\mathbf{6}})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}} + \frac{3}{4} (\mathbf{15})_{\frac{1}{2},-\frac{1}{2},\frac{1}{3}} \right). \quad (45)$$

This gives rise to three additional matrix elements

$$B^3, \quad B^{\bar{6}}, \quad B^{15}. \quad (46)$$

The CKM-leading decomposition for $b \rightarrow s$ and $b \rightarrow d$ decays including isospin and $SU(3)_F$ breaking is given in Table VIII. The complete 4×4 matrix of the $b \rightarrow s$ matrix has rank four, i.e. there is no $b \rightarrow s$ sum rule to this order. As discussed in Sec. II after Eq. (30) we see from Table VIII explicitly that isospin breaking contributions to $\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 S)$ lead at the same time to a deviation of the ratio $|\mathcal{A}(\Xi_b^0 \rightarrow \Lambda S)|/|\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 S)|$ from the result Eq. (19).

Comparing to results present in the literature, in Ref. [14] two separate coefficient matrices of $b \rightarrow s$ and $b \rightarrow d$ decays are given in terms of the isoscalar coefficients, i.e. where the isospin quantum number is still kept in the corresponding reduced matrix element. We improve on that by giving instead the $SU(3)_F$ Clebsch-Gordan coefficient table that makes transparent the corresponding sum rules in a direct way and furthermore reveals directly the correlations between $b \rightarrow s$ and $b \rightarrow d$ decays. We also find the complete set of sum rules, and discuss how further assumptions lead to additional sum rules. We note that the first two sum rules in Eq. (43) in Ref. [14] are sum rules for coefficient matrix vectors but do not apply to the corresponding amplitudes because of the different CKM factors involved.

III. Σ^0 - Λ MIXING IN Λ_b DECAYS

A. General Considerations

In this section we study the ratio

$$R \equiv \frac{\mathcal{A}(\Lambda_b \rightarrow \Sigma_{\text{phys}}^0 J/\psi)}{\mathcal{A}(\Lambda_b \rightarrow \Lambda_{\text{phys}} J/\psi)} = \frac{\langle J/\psi \Sigma_{\text{phys}}^0 | \mathcal{H} | \Lambda_b \rangle}{\langle J/\psi \Lambda_{\text{phys}} | \mathcal{H} | \Lambda_b \rangle}. \quad (47)$$

In order to do this we need the matrix elements appearing in Eq. (47). In the limit where isospin is a good symmetry and Σ_{phys}^0 is an isospin eigenstate, R vanishes, and therefore we are interested in the deviations from that limit. We study leading order effects in isospin breaking.

We first note that we can neglect the deviation of Λ_b from its isospin limit. The reason is that regarding the mixing of heavy baryons, for example Σ_b - Λ_b , Ξ_c^0 - $\Xi_c'^0$ or Ξ_c^+ - $\Xi_c'^+$, in the quark model one obtains a suppression of the mixing angle with the heavy quark mass [87–92]. It follows that for our purposes we can safely neglect the mixing between Λ_b and Σ_b as it is not only isospin suppressed but on top suppressed by the b quark mass.

We now move to discuss the mixing of the light baryons. It has already been pointed out in Ref. [91], that a description with a single mixing angle captures only part of the effect. The reason is because isospin breaking contributions will affect not only the mixing between the states but also the decay amplitude. The non-universality is also reflected in the fact that the $\Lambda_b \rightarrow \Sigma^0$ transition amplitude vanishes in the heavy quark limit at large recoil, i.e. in the phase space when Σ^0 carries away a large fraction of the energy [47], see also Ref. [25] for the heavy quark limit of similar classes of decays.

To leading order in isospin breaking we consider two effects, the mixing between Λ and Σ^0 as well as the correction to the Hamiltonian. We discuss these two effects below.

Starting with the wave function mixing angle θ_m , this is defined as the mixing angle between the isospin limit states $|\Sigma^0\rangle = |1, 0\rangle_I$ and $|\Lambda\rangle = |0, 0\rangle_I$, see Eq. (33), into the physical states (see Refs. [93–98])

$$|\Lambda_{\text{phys}}\rangle = \cos \theta_m |\Lambda\rangle - \sin \theta_m |\Sigma^0\rangle, \quad (48)$$

$$|\Sigma_{\text{phys}}^0\rangle = \sin \theta_m |\Lambda\rangle + \cos \theta_m |\Sigma^0\rangle. \quad (49)$$

The effect stems from the non-vanishing mass difference $m_d - m_u$ as well as different electromagnetic charges [69] which lead to a hyperfine mixing between the isospin limit states.

A similar mixing effect takes place for the light mesons in form of singlet octet mixing of π^0 and $\eta^{(\prime)}$ [99–104].

As for the Hamiltonian, we write $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$ where $\mathcal{H}_0$ is the isospin limit one and $\mathcal{H}_1$ is the leading order breaking. In general for decays into final states Λf and $\Sigma^0 f$ we can write

$$\begin{aligned} \langle f \Sigma_{\text{phys}}^0 | \mathcal{H} | \Lambda_b \rangle &= \sin \theta_m \langle f \Lambda | \mathcal{H} | \Lambda_b \rangle + \cos \theta_m \langle f \Sigma^0 | \mathcal{H} | \Lambda_b \rangle \approx \\ &\quad \theta_m \langle f \Lambda | \mathcal{H}_0 | \Lambda_b \rangle + \langle f \Sigma^0 | \mathcal{H}_1 | \Lambda_b \rangle , \\ \langle f \Lambda_{\text{phys}} | \mathcal{H} | \Lambda_b \rangle &= \cos \theta_m \langle f \Lambda | \mathcal{H} | \Lambda_b \rangle - \sin \theta_m \langle f \Sigma^0 | \mathcal{H} | \Lambda_b \rangle \approx \langle f \Lambda | \mathcal{H}_0 | \Lambda_b \rangle , \end{aligned} \quad (50)$$

where we use the isospin eigenstates $|\Lambda\rangle$ and $|\Sigma^0\rangle$. It follows that we can write

$$R \approx \theta_f \equiv \theta_m + \theta_f^{\text{dyn}} , \quad \theta_f^{\text{dyn}} \equiv \frac{\langle f \Sigma^0 | \mathcal{H}_1 | \Lambda_b \rangle}{\langle f \Lambda | \mathcal{H}_0 | \Lambda_b \rangle} . \quad (51)$$

We learn that the angle θ_f has contributions from two sources: A universal part θ_m from wave function overlap, which we call “static” mixing, and a non-universal contribution θ_f^{dyn} that we call “dynamic” mixing. We can think of θ_f as a decay dependent “effective” mixing angle relevant for the decay $\Lambda_b \rightarrow \Sigma^0 f$. It follows

$$\frac{\mathcal{B}(\Lambda_b \rightarrow \Sigma^0 J/\psi)}{\mathcal{B}(\Lambda_b \rightarrow \Lambda J/\psi)} = \frac{\mathcal{P}(\Lambda_b, \Sigma^0, J/\psi)}{\mathcal{P}(\Lambda_b, \Lambda, J/\psi)} \times |\theta_f|^2 . \quad (52)$$

Our aim in the next section is to find θ_f .

B. Anatomy of Σ^0 – Λ Mixing

We start with θ_m . Because of isospin and $\text{SU}(3)_F$ breaking effects, the physical states $|\Lambda_{\text{phys}}\rangle$ and $|\Sigma_{\text{phys}}^0\rangle$ deviate from their decomposition into their $\text{SU}(3)_F$ eigenstates both in the U -spin and in the isospin basis. As isospin is the better symmetry, we expect generically the scaling

$$\theta_m \sim \frac{\delta}{\varepsilon} . \quad (53)$$

This scaling can be seen explicitly in some of the estimates of the effect. In the quark model, the QCD part of the isospin breaking corrections comes from the strong hyperfine interaction generated by the chromomagnetic spin-spin interaction as [89]

$$\theta_m = \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - (m_u + m_d)/2} , \quad (54)$$

see also Refs. [69, 105–109], and where constituent quark masses are used. Eq. (54) agrees with our generic estimate from group-theory considerations, Eq. (53). The same analytic result, Eq. (54), is also obtained in chiral perturbation theory [106, 110].

Within the quark model, the mixing angle can also be related to baryon masses via [89, 94, 96]

$$\tan \theta_m = \frac{(m_{\Sigma^0} - m_{\Sigma^+}) - (m_n - m_p)}{\sqrt{3}(m_{\Sigma} - m_{\Lambda})}, \quad (55)$$

or equally [89, 96, 111]

$$\tan \theta_m = \frac{(m_{\Xi^-} - m_{\Xi^0}) - (m_{\Xi^{*-}} - m_{\Xi^{*0}})}{2\sqrt{3}(m_{\Sigma} - m_{\Lambda})}. \quad (56)$$

In Ref. [89] Eqs. (54)-(56) have been derived within the generic “independent quark model” [112, 113]. Furthermore, Ref. [89] provides SU(3)-breaking corrections to Eq. (55) within this model.

Note that Eqs. (55) and (56) automatically include also QED corrections through the measured baryon masses. Recently, lattice calculations of θ_m have become available that include QCD and QED effects [98], and which we consider as the most reliable and robust of the quoted results.

The various results for the mixing angle from the literature are summarized in Table VII. It turns out that the quark-model predictions agree quite well with modern lattice QCD calculations. Note however, that the lattice result of Ref. [98] (see Table VII) demonstrates that the QED correction is large, contrary to the quark model expectation in Ref. [69], and amounts to about 50% of the total result [98].

In the literature the mixing angle has often been assumed to be universal and employed straight forward for the prediction of decays, see Refs. [94, 109, 114, 115]. It was already pointed out in Refs. [109, 116] that the Σ^0 – Λ mixing angle can also be extracted from semileptonic $\Sigma^- \rightarrow \Lambda l^- \nu$ decays. The angle has also been directly related to the π – η mixing angle [115, 117]. The ratio on the right hand side of Eq. (54) can be extracted from $\eta \rightarrow 3\pi$ decays [117] or from the comparison of $K^+ \rightarrow \pi^0 e^+ \nu_e$ and $K_L^0 \rightarrow \pi^- e^+ \nu_e$ [117]. For pseudoscalar mesons it has been shown [118, 119] that the reduction of isospin violation from $(m_d - m_u)/(m_d + m_u)$ to the ratio in Eq. (54) is related to the Adler-Bell-Jackiw anomaly [120, 121] of QCD.

Note that in principle also θ_m is scale dependent [109], as was observed for the similar case of π^0 – η mixing in Ref. [122]. Furthermore, θ_m has an electromagnetic component.

Depending on the relevant scale of the process in principle the QED correction can be large. We see from the lattice results in Table VII that this is the case for θ_m . Very generally, at high energy scales electromagnetic interactions will dominate over QCD ones [123].

C. The Dynamic Contribution

The dynamical contributions to isospin breaking can be parametrized as part of the isospin- and $SU(3)_F$ -breaking expansion, see Sec. II E. Explicitly we found

$$\theta_{J/\psi}^{\text{dyn}} \equiv \frac{\langle J/\psi \Sigma^0 | \mathcal{H}_1 | \Lambda_b \rangle}{\langle J/\psi \Lambda | \mathcal{H}_0 | \Lambda_b \rangle} = \delta \times \left[\sqrt{\frac{1}{5}} \frac{B^{15}}{A_c^3} - \sqrt{\frac{1}{2}} \frac{B^{\bar{6}}}{A_c^3} \right]. \quad (57)$$

We expect that $B^{15} \sim B^{\bar{6}} \sim A_c^3$. The important result is that these effects are order δ . Taking everything into account, very schematically we expect therefore the power counting

$$\theta_f \sim \left(\frac{\delta}{\varepsilon} \right)_m + \delta_f \sim \theta_m [1 + \mathcal{O}(\varepsilon_f)] . \quad (58)$$

where δ_f and ε_f refer to isospin and $SU(3)$ breaking parameters that depend on f .

D. Prediction for $\mathcal{B}(\Lambda_b \rightarrow \Sigma^0 J/\psi)$

We see from the power counting in Eq. (58) that the static component θ_m dominates, as it is relatively enhanced by the inverse of the size of $SU(3)_F$ breaking. Employing this assumption we obtain for

$$\theta_f \sim \theta_m \sim 1^\circ \quad (59)$$

the prediction

$$\left| \frac{\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 J/\psi)}{\mathcal{A}(\Lambda_b \rightarrow \Lambda J/\psi)} \right| = |\theta_f| \sim 0.02 . \quad (60)$$

A confirmation of our prediction would imply the approximate universality of the Σ^0 - Λ mixing angle in b -baryon decays. In that case we would expect that likewise

$$\left| \frac{\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 J/\psi)}{\mathcal{A}(\Lambda_b \rightarrow \Lambda J/\psi)} \right| = \left| \frac{\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 \gamma)}{\mathcal{A}(\Lambda_b \rightarrow \Lambda \gamma)} \right| = \left| \frac{\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 l^+ l^-)}{\mathcal{A}(\Lambda_b \rightarrow \Lambda l^+ l^-)} \right| = \left| \frac{\mathcal{A}(\Sigma_b^0 \rightarrow \Lambda J/\psi)}{\mathcal{A}(\Sigma_b^0 \rightarrow \Sigma^0 J/\psi)} \right| \sim 0.02 , \quad (61)$$

up to $SU(3)_F$ breaking. Note that Λ_b and Σ_b are not in the same $SU(3)_F$ multiplet, so that there is no relation between their reduced matrix elements.

The above predictions are based on the assumption that the dynamic contribution is smaller by a factor of the order of the $SU(3)$ breaking. In practice, these effects may be large enough to be probed experimentally. Thus, we can hope that precise measurements of these ratios will be able to test these assumptions.

IV. COMPARISON WITH RECENT DATA

We move to compare the general results of Sections II and III to the recent LHCb data for the case $S = J/\psi$ [6]. Particularly relevant to the experimental findings is the sum rule Eq. (19) which we rephrase as

$$\left| \frac{\mathcal{A}(\Xi_b^0 \rightarrow \Lambda J/\psi)}{\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 J/\psi)} \right| = \frac{1}{\sqrt{6}} (1 + \mathcal{O}(\varepsilon)) \left| \frac{\lambda_{cd}}{\lambda_{cs}} \right| \approx 0.41 \left| \frac{\lambda_{cd}}{\lambda_{cs}} \right|, \quad (62)$$

where in the last step we only wrote the central value. The error is expected to be roughly of order $\varepsilon \sim 20\%$. The estimate in Eq. (62) agrees very well with the recent measurement [6]

$$\left| \frac{\mathcal{A}(\Xi_b^0 \rightarrow \Lambda J/\psi)}{\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 J/\psi)} \right| = (0.44 \pm 0.06 \pm 0.02) \left| \frac{\lambda_{cd}}{\lambda_{cs}} \right|. \quad (63)$$

This suggests that the assumptions made in Sec. II are justified. However, from the $SU(3)_F$ -breaking contributions which we calculated in Sec. II E, we expect generically an order 20% correction to Eq. (62). The measurement Eq. (63) is not yet precise enough to probe and learn about the size of these corrections. However, $SU(3)_F$ breaking seems also not to be enhanced beyond the generic 20%.

The only other theory result for the ratio Eq. (63) that we are aware of in the literature can be obtained from the branching ratios provided in Ref. [9], where a covariant confined quark model has been employed. From the branching ratios given therein we extract the central value

$$\left| \frac{\mathcal{A}(\Xi_b^0 \rightarrow \Lambda J/\psi)}{\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 J/\psi)} \right| \sim 0.34 \frac{\lambda_{cd}}{\lambda_{cs}}, \quad (64)$$

where an error of $\sim 20\%$ is quoted in Ref. [9] for the branching ratios. This estimate is also in agreement with the data, Eq. (63) (see for details in Ref. [9]).

Finally, our prediction Eq. (60)

$$\left| \frac{\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 J/\psi)}{\mathcal{A}(\Lambda_b \rightarrow \Lambda J/\psi)} \right| = |\theta_f| \sim 0.02, \quad (65)$$

is only about a factor two below the bound provided in Ref. [6],

$$\left| \frac{\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 J/\psi)}{\mathcal{A}(\Lambda_b \rightarrow \Lambda J/\psi)} \right| < 1/20.9 = 0.048 \quad \text{at 95\% CL.} \quad (66)$$

A deviation from Eq. (60) would indicate the observation of a non-universal contribution to the effective mixing angle, i.e. an enhancement of isospin violation in the dynamical contribution $\theta_{J/\psi}^{\text{dyn}}$. It seems that a first observation of isospin violation in Λ_b decays is feasible for LHCb in the near future.

V. CONCLUSIONS

We perform a comprehensive $\text{SU}(3)_F$ analysis of two-body $b \rightarrow c\bar{c}s(d)$ decays of the b -baryon antitriplet to baryons of the light octet and an $\text{SU}(3)_F$ singlet, including a discussion of isospin and $\text{SU}(3)_F$ breaking effects as well as Σ^0 - Λ mixing. Our formalism allows us to interpret recent results for the case $S = J/\psi$ by LHCb, which do not yet show signs of isospin violation or $\text{SU}(3)_F$ breaking. We point out several sum rules which can be tested in the future and give a prediction for the ratios $|\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 J/\psi)|/|\mathcal{A}(\Lambda_b \rightarrow \Lambda J/\psi)| \sim 0.02$ and $|\mathcal{A}(\Xi_b^0 \rightarrow \Lambda J/\psi)/\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 J/\psi)| \approx 1/\sqrt{6} |V_{cb}^* V_{cd}/(V_{cb}^* V_{cs})|$. More measurements are needed in order to probe isospin and $\text{SU}(3)_F$ breaking corrections to these and many more relations that we laid out in this work.

ACKNOWLEDGMENTS

We thank Sheldon Stone for discussions. The work of YG is supported in part by the NSF grant PHY1316222. SS is supported by a DFG Forschungsstipendium under contract no. SCHA 2125/1-1.

Particle	Quark Content	$SU(3)_F$ State	Isospin	U -spin	Hadron Mass [MeV]
u	u	$ \mathbf{3}\rangle_{\frac{1}{2}, \frac{1}{2}, \frac{1}{3}}$	$ \frac{1}{2}, \frac{1}{2}\rangle_I$	$ 0, 0\rangle_U$	n/a
d	d	$ \mathbf{3}\rangle_{\frac{1}{2}, -\frac{1}{2}, \frac{1}{3}}$	$ \frac{1}{2}, -\frac{1}{2}\rangle_I$	$ \frac{1}{2}, \frac{1}{2}\rangle_U$	n/a
s	s	$ \mathbf{3}\rangle_{0, 0, -\frac{2}{3}}$	$ 0, 0\rangle_I$	$ \frac{1}{2}, -\frac{1}{2}\rangle_U$	n/a
$ \Lambda_b\rangle$	udb	$ \bar{\mathbf{3}}\rangle_{0, 0, \frac{2}{3}}$	$ 0, 0\rangle_I$	$ \frac{1}{2}, \frac{1}{2}\rangle_U$	5619.60 ± 0.17
$ \Xi_b^-\rangle$	dsb	$ \bar{\mathbf{3}}\rangle_{\frac{1}{2}, -\frac{1}{2}, -\frac{1}{3}}$	$ \frac{1}{2}, -\frac{1}{2}\rangle_I$	$ 0, 0\rangle_U$	5797.0 ± 0.9
$ \Xi_b^0\rangle$	usb	$ \bar{\mathbf{3}}\rangle_{\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}}$	$ \frac{1}{2}, \frac{1}{2}\rangle_I$	$ \frac{1}{2}, -\frac{1}{2}\rangle_U$	5791.9 ± 0.5
$ \Lambda\rangle$	uds	$ \mathbf{8}\rangle_{0, 0, 0}$	$ 0, 0\rangle_I$	$\frac{\sqrt{3}}{2} 1, 0\rangle_U - \frac{1}{2} 0, 0\rangle_U$	1115.683 ± 0.006
$ \Sigma^0\rangle$	uds	$ \mathbf{8}\rangle_{1, 0, 0}$	$ 1, 0\rangle_I$	$\frac{1}{2} 1, 0\rangle_U + \frac{\sqrt{3}}{2} 0, 0\rangle_U$	1192.642 ± 0.024
$ \Sigma^-\rangle$	dds	$ \mathbf{8}\rangle_{1, -1, 0}$	$ 1, -1\rangle_I$	$ \frac{1}{2}, \frac{1}{2}\rangle_U$	1197.449 ± 0.0030
$ \Sigma^+\rangle$	uus	$ \mathbf{8}\rangle_{1, 1, 0}$	$ 1, 1\rangle_I$	$ \frac{1}{2}, -\frac{1}{2}\rangle_U$	1189.37 ± 0.07
$ \Xi^0\rangle$	uss	$ \mathbf{8}\rangle_{\frac{1}{2}, \frac{1}{2}, -1}$	$ \frac{1}{2}, \frac{1}{2}\rangle_I$	$ 1, -1\rangle_U$	1314.86 ± 0.20
$ \Xi^-\rangle$	dss	$ \mathbf{8}\rangle_{\frac{1}{2}, -\frac{1}{2}, -1}$	$ \frac{1}{2}, -\frac{1}{2}\rangle_I$	$ \frac{1}{2}, -\frac{1}{2}\rangle_U$	1321.71 ± 0.07
$ n\rangle$	udd	$ \mathbf{8}\rangle_{\frac{1}{2}, -\frac{1}{2}, 1}$	$ \frac{1}{2}, -\frac{1}{2}\rangle_I$	$ 1, 1\rangle_U$	939.565413 ± 0.000006
$ p\rangle$	uud	$ \mathbf{8}\rangle_{\frac{1}{2}, \frac{1}{2}, 1}$	$ \frac{1}{2}, \frac{1}{2}\rangle_I$	$ \frac{1}{2}, \frac{1}{2}\rangle_U$	$938.2720813 \pm 0.0000058$
$ J/\psi\rangle$	$c\bar{c}$	$ \mathbf{1}\rangle_{0, 0, 0}$	$ 0, 0\rangle_I$	$ 0, 0\rangle_U$	3096.900 ± 0.006

TABLE I. $SU(3)_F$, isospin and U -spin wave functions [54, 58, 124–126] and masses [58]. For the indices of the $SU(3)_F$ states we use the convention $|\mu\rangle_{I, I_3, Y}$. The lifetimes of the members of the heavy baryon triplet are $\tau_{\Lambda_b}/\tau_{B^0} = 0.964 \pm 0.007$, where $\tau_{B^0} = (1519 \pm 4) \times 10^{-15} s$, $\tau_{\Xi_b^0} = (1.480 \pm 0.030) \times 10^{-12} s$, $\tau_{\Xi_b^-} = (1.572 \pm 0.040) \cdot 10^{-12} s$. [58]. Note that the exact form of the Σ^0 – Λ mixing is phase convention dependent [126]. Our convention agrees with Refs. [126, 127]. Another convention can be found e.g. in Ref. [128] in the form of $|\Lambda\rangle = \frac{\sqrt{3}}{2} |1, 0\rangle_U - \frac{1}{2} |0, 0\rangle_U$ and $|\Sigma^0\rangle = -\frac{1}{2} |1, 0\rangle_U - \frac{\sqrt{3}}{2} |0, 0\rangle_U$.

Decay ampl. $\mathcal{A}$	A_c^3	A_u^3	$A_u^{\bar{6}}$	A_u^{15}
$b \rightarrow s$				
$\mathcal{A}(\Lambda_b \rightarrow \Lambda S)$	$\sqrt{\frac{2}{3}}\lambda_{cs}$	$\sqrt{\frac{2}{3}}\lambda_{us}$	0	$\sqrt{\frac{6}{5}}\lambda_{us}$
$\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 S)$	0	0	$-\sqrt{\frac{2}{3}}\lambda_{us}$	$2\sqrt{\frac{2}{5}}\lambda_{us}$
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 S)$	λ_{cs}	λ_{us}	$\sqrt{\frac{1}{3}}\lambda_{us}$	$\sqrt{\frac{1}{5}}\lambda_{us}$
$\mathcal{A}(\Xi_b^- \rightarrow \Xi^- S)$	λ_{cs}	λ_{us}	$-\sqrt{\frac{1}{3}}\lambda_{us}$	$-\frac{3}{\sqrt{5}}\lambda_{us}$
$b \rightarrow d$				
$\mathcal{A}(\Xi_b^0 \rightarrow \Lambda S)$	$-\sqrt{\frac{1}{6}}\lambda_{cd}$	$-\sqrt{\frac{1}{6}}\lambda_{ud}$	$-\frac{1}{\sqrt{2}}\lambda_{ud}$	$\sqrt{\frac{3}{10}}\lambda_{ud}$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 S)$	$\frac{1}{\sqrt{2}}\lambda_{cd}$	$\frac{1}{\sqrt{2}}\lambda_{ud}$	$-\frac{1}{\sqrt{6}}\lambda_{ud}$	$\sqrt{\frac{5}{2}}\lambda_{ud}$
$\mathcal{A}(\Lambda_b \rightarrow n S)$	λ_{cd}	λ_{ud}	$\frac{1}{\sqrt{3}}\lambda_{ud}$	$\frac{1}{\sqrt{5}}\lambda_{ud}$
$\mathcal{A}(\Xi_b^- \rightarrow \Sigma^- S)$	λ_{cd}	λ_{ud}	$-\frac{1}{\sqrt{3}}\lambda_{ud}$	$-\frac{3}{\sqrt{5}}\lambda_{ud}$

TABLE II. $SU(3)_F$ -limit decomposition.

$b \rightarrow s$						
Decay Ampl. $\mathcal{A}$	$\langle 0 0 0\rangle^c$	$\langle \frac{1}{2} 0 \frac{1}{2}\rangle^c$	$\langle 0 0 0\rangle^u$	$\langle 1 1 0\rangle^u$	$\langle \frac{1}{2} 1 \frac{1}{2}\rangle^u$	$\langle \frac{1}{2} 0 \frac{1}{2}\rangle^u$
$\mathcal{A}(\Lambda_b \rightarrow \Lambda S)$	λ_{cs}	0	λ_{us}	0	0	0
$\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 S)$	0	0	0	λ_{us}	0	0
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 S)$	0	λ_{cs}	0	0	$-\sqrt{\frac{1}{3}}\lambda_{us}$	λ_{us}
$\mathcal{A}(\Xi_b^- \rightarrow \Xi^- S)$	0	λ_{cs}	0	0	$\sqrt{\frac{1}{3}}\lambda_{us}$	λ_{us}

TABLE III. Isospin decomposition for $b \rightarrow s$ transitions.

$b \rightarrow d$							
Decay ampl. $\mathcal{A}$	$\langle 0 \frac{1}{2} \frac{1}{2}\rangle^c$	$\langle 0 \frac{1}{2} \frac{1}{2}\rangle^u$	$\langle 1 \frac{1}{2} \frac{1}{2}\rangle^c$	$\langle 1 \frac{1}{2} \frac{1}{2}\rangle^u$	$\langle \frac{1}{2} \frac{1}{2} 0\rangle^c$	$\langle \frac{1}{2} \frac{1}{2} 0\rangle^u$	$\langle 1 \frac{3}{2} \frac{1}{2}\rangle^u$
$\mathcal{A}(\Xi_b^0 \rightarrow \Lambda S)$	$-\frac{1}{\sqrt{2}}\lambda_{cd}$	$-\frac{1}{\sqrt{2}}\lambda_{ud}$	0	0	0	0	0
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 S)$	0	0	$\frac{1}{\sqrt{2}}\lambda_{cd}$	$\frac{1}{\sqrt{2}}\lambda_{ud}$	0	0	$-\frac{1}{\sqrt{2}}\lambda_{ud}$
$\mathcal{A}(\Lambda_b \rightarrow n S)$	0	0	0	0	λ_{cd}	λ_{ud}	0
$\mathcal{A}(\Xi_b^- \rightarrow \Sigma^- S)$	0	0	λ_{cd}	λ_{ud}	0	0	$\frac{1}{2}\lambda_{ud}$

TABLE IV. Isospin decomposition for $b \rightarrow d$ transitions.

Decay ampl. $\mathcal{A}$	$\langle 0 \frac{1}{2} \frac{1}{2} \rangle^c$	$\langle 0 \frac{1}{2} \frac{1}{2} \rangle^u$	$\langle 1 \frac{1}{2} \frac{1}{2} \rangle^c$	$\langle 1 \frac{1}{2} \frac{1}{2} \rangle^u$	$\langle \frac{1}{2} \frac{1}{2} 0 \rangle^c$	$\langle \frac{1}{2} \frac{1}{2} 0 \rangle^u$
$b \rightarrow s$						
$\mathcal{A}(\Lambda_b \rightarrow \Lambda S)$	$\frac{1}{2\sqrt{2}}\lambda_{cs}$	$\frac{1}{2\sqrt{2}}\lambda_{us}$	$\frac{\sqrt{3}}{2\sqrt{2}}\lambda_{cs}$	$\frac{\sqrt{3}}{2\sqrt{2}}\lambda_{us}$	0	0
$\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 S)$	$-\frac{\sqrt{3}}{2\sqrt{2}}\lambda_{cs}$	$-\frac{\sqrt{3}}{2\sqrt{2}}\lambda_{us}$	$\frac{1}{2\sqrt{2}}\lambda_{cs}$	$\frac{1}{2\sqrt{2}}\lambda_{us}$	0	0
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 S)$	0	0	λ_{cs}	λ_{us}	0	0
$\mathcal{A}(\Xi_b^- \rightarrow \Xi^- S)$	0	0	0	0	λ_{cs}	λ_{us}
$b \rightarrow d$						
$\mathcal{A}(\Xi_b^0 \rightarrow \Lambda S)$	$-\frac{1}{2\sqrt{2}}\lambda_{cd}$	$-\frac{1}{2\sqrt{2}}\lambda_{ud}$	$\frac{\sqrt{3}}{2\sqrt{2}}\lambda_{cd}$	$\frac{\sqrt{3}}{2\sqrt{2}}\lambda_{ud}$	0	0
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 S)$	$\frac{\sqrt{3}}{2\sqrt{2}}\lambda_{cd}$	$\frac{\sqrt{3}}{2\sqrt{2}}\lambda_{ud}$	$\frac{1}{2\sqrt{2}}\lambda_{cd}$	$\frac{1}{2\sqrt{2}}\lambda_{ud}$	0	0
$\mathcal{A}(\Lambda_b \rightarrow n S)$	0	0	λ_{cd}	λ_{ud}	0	0
$\mathcal{A}(\Xi_b^- \rightarrow \Sigma^- S)$	0	0	0	0	λ_{cd}	λ_{ud}

TABLE V. U -spin decomposition.

Decay ampl. $\mathcal{A}$	$\langle 0 \frac{1}{2} \frac{1}{2} \rangle^c$	$\langle 0 \frac{1}{2} \frac{1}{2} \rangle^u$	$\langle 1 \frac{1}{2} \frac{1}{2} \rangle^c$	$\langle 1 \frac{1}{2} \frac{1}{2} \rangle^u$	$\langle \frac{1}{2} \frac{1}{2} 0 \rangle^c$	$\langle \frac{1}{2} \frac{1}{2} 0 \rangle^u$
$b \rightarrow s$						
$\mathcal{A}(\Lambda_b \rightarrow X S)$	$-\frac{1}{\sqrt{2}}\lambda_{cs}$	$-\frac{1}{\sqrt{2}}\lambda_{us}$	0	0	0	0
$\mathcal{A}(\Lambda_b \rightarrow Y S)$	0	0	$\frac{1}{\sqrt{2}}\lambda_{cs}$	$\frac{1}{\sqrt{2}}\lambda_{us}$	0	0
$b \rightarrow d$						
$\mathcal{A}(\Xi_b^0 \rightarrow X S)$	$\frac{1}{\sqrt{2}}\lambda_{cd}$	$\frac{1}{\sqrt{2}}\lambda_{ud}$	0	0	0	0
$\mathcal{A}(\Xi_b^0 \rightarrow Y S)$	0	0	$\frac{1}{\sqrt{2}}\lambda_{cd}$	$\frac{1}{\sqrt{2}}\lambda_{ud}$	0	0

TABLE VI. (Unpractical) U -spin decomposition for the U -spin eigenstates $|X\rangle$ and $|Y\rangle$, see Eq. (34) and discussion in the text.

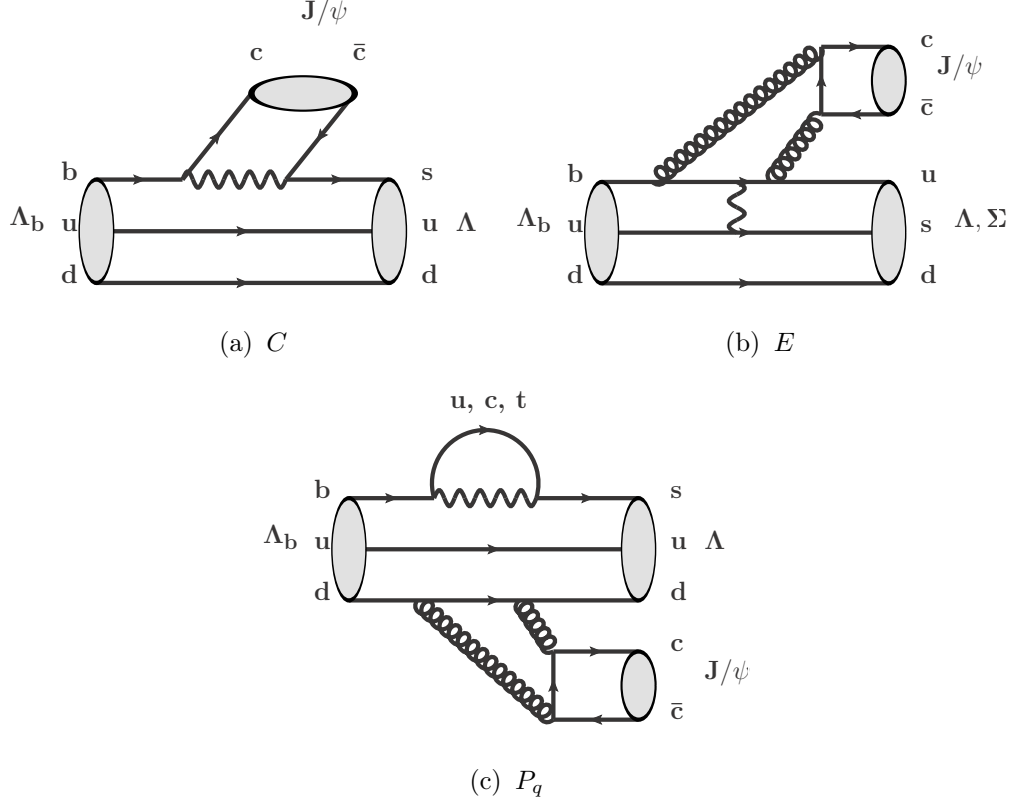


FIG. 1. Topological diagrams for the decays $\Lambda_b \rightarrow \Lambda J/\psi$ and $\Lambda_b \rightarrow \Sigma J/\psi$. Note that in the exchange diagram one gluon alone can not create the J/ψ because it is a color singlet.

Method	Mixing Angle θ_m [°]	Ref.
Quark model: Relation to Baryon masses	0.86 ± 0.06	[94, 96]
Quark model: Hyperfine splitting + EM interactions	$\simeq 0.57$	[69]
Lattice QCD+QED A	1.00 ± 0.32	[98]
Lattice QCD+QED B	0.96 ± 0.31	[98]
Lattice QCD without QED	0.55 ± 0.03	[98]

TABLE VII. Results for the Σ^0 – Λ mixing angle. Note that we adjusted the sign conventions for the results to match always the one of Ref. [94], see also the corresponding comment in Ref. [98]. For older lattice results for the “QCD only” scenario see Refs. [95, 97]. Note that with alternate quark mass input taken from Ref. [129] the result for the “Lattice QCD without QED” scenario is changed to $\theta_m = 0.65 \pm 0.03$ [98].

Decay ampl. $\mathcal{A}$	A_c^3	B^3	B^{15}	$B^{\bar{6}}$
$b \rightarrow s$				
$\mathcal{A}(\Lambda_b \rightarrow \Lambda S)/\lambda_{cs}$	$\sqrt{\frac{2}{3}}$	$\frac{1}{2}\sqrt{\frac{1}{3}}\varepsilon$	$\sqrt{\frac{3}{10}}\varepsilon$	0
$\mathcal{A}(\Lambda_b \rightarrow \Sigma^0 S)/\lambda_{cs}$	0	0	$\sqrt{\frac{2}{15}}\delta$	$-\sqrt{\frac{1}{3}}\delta$
$\mathcal{A}(\Xi_b^0 \rightarrow \Xi^0 S)/\lambda_{cs}$	1	$\frac{1}{2}\varepsilon$	$\sqrt{\frac{1}{15}}\delta - \frac{1}{2\sqrt{5}}\varepsilon$	$\sqrt{\frac{1}{6}}\delta$
$\mathcal{A}(\Xi_b^- \rightarrow \Xi^- S)/\lambda_{cs}$	1	$\frac{1}{2}\varepsilon$	$-\sqrt{\frac{1}{15}}\delta - \frac{1}{2\sqrt{5}}\varepsilon$	$-\sqrt{\frac{1}{6}}\delta$
$b \rightarrow d$				
$\mathcal{A}(\Xi_b^0 \rightarrow \Lambda S)/\lambda_{cd}$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{4\sqrt{2}}\delta + \frac{1}{4\sqrt{6}}\varepsilon$	$-\frac{1}{4\sqrt{10}}\delta + \frac{3}{4}\sqrt{\frac{3}{10}}\varepsilon$	$-\frac{1}{4}\delta - \frac{\sqrt{3}}{4}\varepsilon$
$\mathcal{A}(\Xi_b^0 \rightarrow \Sigma^0 S)/\lambda_{cd}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{4}\sqrt{\frac{3}{2}}\delta - \frac{1}{4\sqrt{2}}\varepsilon$	$\frac{11}{4\sqrt{30}}\delta - \frac{1}{4\sqrt{10}}\varepsilon$	$-\frac{1}{4\sqrt{3}}\delta - \frac{1}{4}\varepsilon$
$\mathcal{A}(\Lambda_b \rightarrow n S)/\lambda_{cd}$	1	$\frac{\sqrt{3}}{4}\delta - \frac{1}{4}\varepsilon$	$-\frac{1}{4\sqrt{15}}\delta + \frac{3}{4\sqrt{5}}\varepsilon$	$\frac{1}{2\sqrt{6}}\delta + \frac{1}{2\sqrt{2}}\varepsilon$
$\mathcal{A}(\Xi_b^- \rightarrow \Sigma^- S)/\lambda_{cd}$	1	$\frac{\sqrt{3}}{4}\delta - \frac{1}{4}\varepsilon$	$-\frac{1}{4}\sqrt{\frac{5}{3}}\delta - \frac{1}{4\sqrt{5}}\varepsilon$	$-\frac{1}{2\sqrt{6}}\delta - \frac{1}{2\sqrt{2}}\varepsilon$

TABLE VIII. CKM-leading $SU(3)_F$ decomposition including isospin- and $SU(3)_F$ -breaking.

-
- [1] A. Cerri *et al.*, (2018), arXiv:1812.07638 [hep-ph].
 - [2] R. Aaij *et al.* (LHCb), Phys. Lett. **B724**, 27 (2013), arXiv:1302.5578 [hep-ex].
 - [3] G. Aad *et al.* (ATLAS), Phys. Rev. **D89**, 092009 (2014), arXiv:1404.1071 [hep-ex].
 - [4] R. Aaij *et al.* (LHCb), Nature Phys. **13**, 391 (2017), arXiv:1609.05216 [hep-ex].
 - [5] R. Aaij *et al.* (LHCb), Phys. Rev. Lett. **123**, 031801 (2019), arXiv:1904.06697 [hep-ex].
 - [6] R. Aaij *et al.* (LHCb), (2019), arXiv:1912.02110 [hep-ex].
 - [7] M. B. Voloshin, (2015), arXiv:1510.05568 [hep-ph].
 - [8] Fayyazuddin and M. J. Aslam, Phys. Rev. **D95**, 113002 (2017), arXiv:1705.05106 [hep-ph].
 - [9] T. Gutsche, M. A. Ivanov, J. G. Körner, and V. E. Lyubovitskij, Phys. Rev. **D98**, 074011 (2018), arXiv:1806.11549 [hep-ph].
 - [10] Y. K. Hsiao, P. Y. Lin, C. C. Lih, and C. Q. Geng, Phys. Rev. **D92**, 114013 (2015), arXiv:1509.05603 [hep-ph].
 - [11] Y. K. Hsiao, P. Y. Lin, L. W. Luo, and C. Q. Geng, Phys. Lett. **B751**, 127 (2015), arXiv:1510.01808 [hep-ph].
 - [12] J. Zhu, Z.-T. Wei, and H.-W. Ke, Phys. Rev. **D99**, 054020 (2019), arXiv:1803.01297 [hep-ph].
 - [13] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, V. V. Lyubushkin, and P. Santorelli, Phys. Rev. **D96**, 013003 (2017), arXiv:1705.07299 [hep-ph].
 - [14] S. Roy, R. Sinha, and N. G. Deshpande, (2019), arXiv:1911.01121 [hep-ph].
 - [15] M. Gronau and J. L. Rosner, Phys. Rev. **D89**, 037501 (2014), [Erratum: Phys. Rev. **D91**, no.11, 119902 (2015)], arXiv:1312.5730 [hep-ph].
 - [16] M. He, X.-G. He, and G.-N. Li, Phys. Rev. **D92**, 036010 (2015), arXiv:1507.07990 [hep-ph].
 - [17] X.-G. He and G.-N. Li, Phys. Lett. **B750**, 82 (2015), arXiv:1501.00646 [hep-ph].
 - [18] R. Arora, G. K. Sidana, and M. P. Khanna, Phys. Rev. **D45**, 4121 (1992).
 - [19] D.-S. Du and D.-X. Zhang, Phys. Rev. **D50**, 2058 (1994).
 - [20] J. G. Korner, M. Kramer, and D. Pirjol, Prog. Part. Nucl. Phys. **33**, 787 (1994), arXiv:hep-ph/9406359 [hep-ph].
 - [21] Y. K. Hsiao and C. Q. Geng, Phys. Rev. **D91**, 116007 (2015), arXiv:1412.1899 [hep-ph].
 - [22] M. Gronau and J. L. Rosner, Phys. Rev. **D93**, 034020 (2016), arXiv:1512.06700 [hep-ph].
 - [23] M. Gronau and J. L. Rosner, Phys. Lett. **B757**, 330 (2016), arXiv:1603.07309 [hep-ph].

- [24] D. A. Egolf, R. P. Springer, and J. Urban, Phys. Rev. **D68**, 013003 (2003), arXiv:hep-ph/0211360 [hep-ph].
- [25] A. K. Leibovich, Z. Ligeti, I. W. Stewart, and M. B. Wise, Phys. Lett. **B586**, 337 (2004), arXiv:hep-ph/0312319 [hep-ph].
- [26] Y. Grossman and S. Schacht, Phys. Rev. **D99**, 033005 (2019), arXiv:1811.11188 [hep-ph].
- [27] C.-D. Lü, W. Wang, and F.-S. Yu, Phys. Rev. **D93**, 056008 (2016), arXiv:1601.04241 [hep-ph].
- [28] C. Q. Geng, Y. K. Hsiao, C.-W. Liu, and T.-H. Tsai, Phys. Rev. **D99**, 073003 (2019), arXiv:1810.01079 [hep-ph].
- [29] C.-P. Jia, D. Wang, and F.-S. Yu, (2019), arXiv:1910.00876 [hep-ph].
- [30] M. J. Savage and R. P. Springer, Phys. Rev. **D42**, 1527 (1990).
- [31] M. J. Savage and R. P. Springer, Int. J. Mod. Phys. **A6**, 1701 (1991).
- [32] R.-M. Wang, M.-Z. Yang, H.-B. Li, and X.-D. Cheng, Phys. Rev. **D100**, 076008 (2019), arXiv:1906.08413 [hep-ph].
- [33] C. Q. Geng, Y. K. Hsiao, C.-W. Liu, and T.-H. Tsai, Eur. Phys. J. **C78**, 593 (2018), arXiv:1804.01666 [hep-ph].
- [34] C. Q. Geng, Y. K. Hsiao, C.-W. Liu, and T.-H. Tsai, Phys. Rev. **D97**, 073006 (2018), arXiv:1801.03276 [hep-ph].
- [35] C. Q. Geng, Y. K. Hsiao, C.-W. Liu, and T.-H. Tsai, JHEP **11**, 147 (2017), arXiv:1709.00808 [hep-ph].
- [36] H. J. Zhao, Y.-L. Wang, Y. K. Hsiao, and Y. Yu, (2018), arXiv:1811.07265 [hep-ph].
- [37] M. Gronau, J. L. Rosner, and C. G. Wohl, Phys. Rev. **D97**, 116015 (2018), [Addendum: Phys. Rev.D98,no.7,073003(2018)], arXiv:1808.03720 [hep-ph].
- [38] D. Ebert and W. Kallies, Phys. Lett. **131B**, 183 (1983), [Erratum: Phys. Lett.148B,502(1984)].
- [39] I. I. Y. Bigi, Z. Phys. **C9**, 197 (1981).
- [40] I. I. Bigi, (2012), arXiv:1206.4554 [hep-ph].
- [41] N. Isgur and M. B. Wise, Nucl. Phys. **B348**, 276 (1991).
- [42] A. Khodjamirian, C. Klein, T. Mannel, and Y. M. Wang, JHEP **09**, 106 (2011), arXiv:1108.2971 [hep-ph].
- [43] Y.-M. Wang and Y.-L. Shen, JHEP **02**, 179 (2016), arXiv:1511.09036 [hep-ph].

- [44] T. Husek and S. Leupold, (2019), arXiv:1911.02571 [hep-ph].
- [45] C. Granados, S. Leupold, and E. Perotti, Eur. Phys. J. **A53**, 117 (2017), arXiv:1701.09130 [hep-ph].
- [46] B. Kubis and U. G. Meissner, Eur. Phys. J. **C18**, 747 (2001), arXiv:hep-ph/0010283 [hep-ph].
- [47] T. Mannel and Y.-M. Wang, JHEP **12**, 067 (2011), arXiv:1111.1849 [hep-ph].
- [48] W. Detmold, C. J. D. Lin, S. Meinel, and M. Wingate, Phys. Rev. **D87**, 074502 (2013), arXiv:1212.4827 [hep-lat].
- [49] W. Detmold and S. Meinel, Phys. Rev. **D93**, 074501 (2016), arXiv:1602.01399 [hep-lat].
- [50] F. U. Bernlochner, Z. Ligeti, D. J. Robinson, and W. L. Sutcliffe, Phys. Rev. **D99**, 055008 (2019), arXiv:1812.07593 [hep-ph].
- [51] D. Zeppenfeld, Z. Phys. **C8**, 77 (1981).
- [52] M. Jung, Phys. Rev. **D86**, 053008 (2012), arXiv:1206.2050 [hep-ph].
- [53] M. Jung and S. Schacht, Phys. Rev. **D91**, 034027 (2015), arXiv:1410.8396 [hep-ph].
- [54] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963), [Erratum: Rev. Mod. Phys.37,326(1965)].
- [55] T. A. Kaeding, Atom. Data Nucl. Data Tabl. **61**, 233 (1995), arXiv:nucl-th/9502037 [nucl-th].
- [56] T. A. Kaeding and H. T. Williams, Comput. Phys. Commun. **98**, 398 (1996), arXiv:nucl-th/9511025 [nucl-th].
- [57] T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, and P. Santorelli, Phys. Rev. **D88**, 114018 (2013), arXiv:1309.7879 [hep-ph].
- [58] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. **D98**, 030001 (2018).
- [59] O. F. Hernandez, D. London, M. Gronau, and J. L. Rosner, Phys. Lett. **B333**, 500 (1994), arXiv:hep-ph/9404281 [hep-ph].
- [60] M. Gronau, J. L. Rosner, and D. London, Phys. Rev. Lett. **73**, 21 (1994), arXiv:hep-ph/9404282 [hep-ph].
- [61] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Phys. Rev. **D50**, 4529 (1994), arXiv:hep-ph/9404283 [hep-ph].
- [62] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Phys. Rev. **D52**, 6356 (1995), arXiv:hep-ph/9504326 [hep-ph].
- [63] M. Neubert and J. L. Rosner, Phys. Rev. Lett. **81**, 5076 (1998), arXiv:hep-ph/9809311 [hep-ph].
- [64] M. Gronau, Phys. Lett. **B492**, 297 (2000), arXiv:hep-ph/0008292 [hep-ph].

- [65] R. Fleischer, Phys. Lett. **B459**, 306 (1999), arXiv:hep-ph/9903456 [hep-ph].
- [66] M. Gronau and J. L. Rosner, Phys. Lett. **B482**, 71 (2000), arXiv:hep-ph/0003119 [hep-ph].
- [67] X.-G. He, Eur. Phys. J. **C9**, 443 (1999), arXiv:hep-ph/9810397 [hep-ph].
- [68] B. Bhattacharya, M. Gronau, and J. L. Rosner, Phys. Lett. **B726**, 337 (2013), arXiv:1306.2625 [hep-ph].
- [69] N. Isgur, Phys. Rev. **D21**, 779 (1980), [Erratum: Phys. Rev.D23,817(1981)].
- [70] D. Wang, Eur. Phys. J. **C79**, 429 (2019), arXiv:1901.01776 [hep-ph].
- [71] B. Bhattacharya and J. L. Rosner, Phys. Rev. **D81**, 014026 (2010), arXiv:0911.2812 [hep-ph].
- [72] G. Hiller, M. Jung, and S. Schacht, Phys. Rev. **D87**, 014024 (2013), arXiv:1211.3734 [hep-ph].
- [73] Y. Grossman and S. Schacht, JHEP **07**, 020 (2019), arXiv:1903.10952 [hep-ph].
- [74] S. Müller, U. Nierste, and S. Schacht, Phys. Rev. Lett. **115**, 251802 (2015), arXiv:1506.04121 [hep-ph].
- [75] S. Müller, U. Nierste, and S. Schacht, Phys. Rev. **D92**, 014004 (2015), arXiv:1503.06759 [hep-ph].
- [76] J. Brod, Y. Grossman, A. L. Kagan, and J. Zupan, JHEP **10**, 161 (2012), arXiv:1203.6659 [hep-ph].
- [77] Y. Grossman, A. L. Kagan, and J. Zupan, Phys. Rev. **D85**, 114036 (2012), arXiv:1204.3557 [hep-ph].
- [78] Y. Grossman, A. L. Kagan, and Y. Nir, Phys. Rev. **D75**, 036008 (2007), arXiv:hep-ph/0609178 [hep-ph].
- [79] A. F. Falk, Y. Grossman, Z. Ligeti, and A. A. Petrov, Phys. Rev. **D65**, 054034 (2002), arXiv:hep-ph/0110317 [hep-ph].
- [80] M. J. Savage, Phys. Lett. **B257**, 414 (1991).
- [81] I. Hinchliffe and T. A. Kaeding, Phys. Rev. **D54**, 914 (1996), arXiv:hep-ph/9502275 [hep-ph].
- [82] M. Jung and T. Mannel, Phys. Rev. **D80**, 116002 (2009), arXiv:0907.0117 [hep-ph].
- [83] M. Gronau, Y. Grossman, G. Raz, and J. L. Rosner, Phys. Lett. **B635**, 207 (2006), arXiv:hep-ph/0601129 [hep-ph].
- [84] D. Pirtskhalava and P. Uttayarat, Phys. Lett. **B712**, 81 (2012), arXiv:1112.5451 [hep-ph].
- [85] F. Buccella, A. Paul, and P. Santorelli, Phys. Rev. **D99**, 113001 (2019), arXiv:1902.05564 [hep-ph].
- [86] Y. Grossman and D. J. Robinson, JHEP **04**, 067 (2013), arXiv:1211.3361 [hep-ph].

- [87] L. A. Copley, N. Isgur, and G. Karl, Phys. Rev. **D20**, 768 (1979), [Erratum: Phys. Rev.D23,817(1981)].
- [88] K. Maltman and N. Isgur, Phys. Rev. **D22**, 1701 (1980).
- [89] J. Franklin, D. B. Lichtenberg, W. Namgung, and D. Carydas, Phys. Rev. **D24**, 2910 (1981).
- [90] M. J. Savage and M. B. Wise, Nucl. Phys. **B326**, 15 (1989).
- [91] C. G. Boyd, M. Lu, and M. J. Savage, Phys. Rev. **D55**, 5474 (1997), arXiv:hep-ph/9612441 [hep-ph].
- [92] M. Karliner, B. Keren-Zur, H. J. Lipkin, and J. L. Rosner, Annals Phys. **324**, 2 (2009), arXiv:0804.1575 [hep-ph].
- [93] S. R. Coleman and S. L. Glashow, Phys. Rev. Lett. **6**, 423 (1961).
- [94] R. H. Dalitz and F. Von Hippel, Phys. Lett. **10**, 153 (1964).
- [95] R. Horsley, J. Najjar, Y. Nakamura, H. Perlt, D. Pleiter, P. E. L. Rakow, G. Schierholz, A. Schiller, H. Stüben, and J. M. Zanotti, Phys. Rev. **D91**, 074512 (2015), arXiv:1411.7665 [hep-lat].
- [96] A. Gal, Phys. Rev. **D92**, 018501 (2015), arXiv:1506.01143 [nucl-th].
- [97] R. Horsley, J. Najjar, Y. Nakamura, H. Perlt, D. Pleiter, P. E. L. Rakow, G. Schierholz, A. Schiller, H. Stüben, and J. M. Zanotti (QCDSF-UKQCD), Phys. Rev. **D92**, 018502 (2015), arXiv:1507.07825 [hep-lat].
- [98] Z. R. Kordov, R. Horsley, Y. Nakamura, H. Perlt, P. E. L. Rakow, G. Schierholz, H. Stüben, R. D. Young, and J. M. Zanotti, (2019), arXiv:1911.02186 [hep-lat].
- [99] T. Feldmann and P. Kroll, *Production, interaction and decay of the eta meson. Proceedings, Workshop, Uppsala, Sweden, October 25-27, 2001*, Phys. Scripta **T99**, 13 (2002), arXiv:hep-ph/0201044 [hep-ph].
- [100] T. Feldmann, P. Kroll, and B. Stech, Phys. Lett. **B449**, 339 (1999), arXiv:hep-ph/9812269 [hep-ph].
- [101] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. **D58**, 114006 (1998), arXiv:hep-ph/9802409 [hep-ph].
- [102] T. Feldmann and P. Kroll, Eur. Phys. J. **C5**, 327 (1998), arXiv:hep-ph/9711231 [hep-ph].
- [103] J. J. Dudek, R. G. Edwards, B. Joo, M. J. Peardon, D. G. Richards, and C. E. Thomas, Phys. Rev. **D83**, 111502 (2011), arXiv:1102.4299 [hep-lat].

- [104] K. Ottnad, in *9th International Workshop on Chiral Dynamics (CD18) Durham, NC, USA, September 17-21, 2018* (2019) arXiv:1905.08385 [hep-lat].
- [105] A. De Rujula, H. Georgi, and S. L. Glashow, Phys. Rev. **D12**, 147 (1975).
- [106] J. Gasser and H. Leutwyler, Phys. Rept. **87**, 77 (1982).
- [107] J. F. Donoghue, E. Golowich, and B. R. Holstein, Phys. Rept. **131**, 319 (1986).
- [108] J. F. Donoghue, Ann. Rev. Nucl. Part. Sci. **39**, 1 (1989).
- [109] G. Karl, Phys. Lett. **B328**, 149 (1994), [Erratum: Phys. Lett.B341,449(1995)].
- [110] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
- [111] A. Gal and F. Scheck, Nucl. Phys. **B2**, 110 (1967).
- [112] P. Federman, H. R. Rubinstein, and I. Talmi, Phys. Lett. **22**, 208 (1966).
- [113] H. R. Rubinstein, F. Scheck, and R. H. Socolow, Phys. Rev. **154**, 1608 (1967).
- [114] E. M. Henley and G. A. Miller, Phys. Rev. **D50**, 7077 (1994), arXiv:hep-ph/9408227 [hep-ph].
- [115] K. Maltman, Phys. Lett. **B345**, 541 (1995), arXiv:hep-ph/9504253 [hep-ph].
- [116] G. Karl, in *Hyperon physics symposium : Hyperon 99, September 27-29, 1999, Fermi National Accelerator Laboratory, Batavia, Illinois* (1999) pp. 41–42, arXiv:hep-ph/9910418 [hep-ph].
- [117] E. S. Na and B. R. Holstein, Phys. Rev. **D56**, 4404 (1997), arXiv:hep-ph/9704407 [hep-ph].
- [118] D. J. Gross, S. B. Treiman, and F. Wilczek, Phys. Rev. **D19**, 2188 (1979).
- [119] P. Kroll, Mod. Phys. Lett. **A20**, 2667 (2005), arXiv:hep-ph/0509031 [hep-ph].
- [120] S. L. Adler, Phys. Rev. **177**, 2426 (1969), [,241(1969)].
- [121] J. S. Bell and R. Jackiw, Nuovo Cim. **A60**, 47 (1969).
- [122] K. Maltman, Phys. Lett. **B313**, 203 (1993), arXiv:nucl-th/9211006 [nucl-th].
- [123] G. Berlاد, A. Dar, G. Eilam, and J. Franklin, Annals Phys. **75**, 461 (1973).
- [124] A. Ali, C. Hambrock, A. Ya. Parkhomenko, and W. Wang, Eur. Phys. J. **C73**, 2302 (2013), arXiv:1212.3280 [hep-ph].
- [125] W. Roberts and M. Pervin, Int. J. Mod. Phys. **A23**, 2817 (2008), arXiv:0711.2492 [nucl-th].
- [126] W. Greiner and B. Muller, *Theoretical physics. Vol. 2: Quantum mechanics. Symmetries* (1989).
- [127] J. L. Rosner, Prog. Theor. Phys. **66**, 1422 (1981).
- [128] Yu. V. Novozhilov, *Introduction to Elementary Particle Theory* (1975).
- [129] S. Aoki *et al.* (Flavour Lattice Averaging Group), (2019), arXiv:1902.08191 [hep-lat].