Risk-Sensitive Security-Constrained Economic Dispatch via Critical Region Exploration

Avinash N. Madavan   Subhonmesh Bose   Ye Guo   Lang Tong

Abstract—We propose a risk-sensitive security constrained economic dispatch (R-SCED) problem that allows a system operator to systematically tradeoff between the cost of power procurement and the reliability of power delivery in the event of a contingency. Our formulation includes a parameterized conditional value at risk (CVaR) of the cost across contingencies and allows for re-dispatch of generators and load shedding. Finally, we propose the critical region exploration (CRE) algorithm to solve R-SCED, and discuss its performance on the IEEE 30-bus test system.

I. INTRODUCTION

System operators (SOs) routinely solve a security-constrained economic dispatch (SCED) problem to compute dispatch decisions to meet demand requirements over a transmission network. SOs often seek a dispatch that is robust to all single potential outages of transmission lines, transformers, or generators, to maintain the so-called $N - 1$ *security criterion* for an N-component power system.

SCED tries to balance between the SO's two conflicting goals – minimizing power procurement costs and maintaining reliability of power delivery under a collection of counterfactual scenarios called contingencies. Most formulations in the literature sacrifice cost considerations to prioritize reliability. In this work, we propose a risk-sensitive SCED (R-SCED) problem that provides the SO a tunable parameter to tradeoff between cost and reliability. We also provide a computational procedure to solve R-SCED under linearized power flow models.

SCED formulations abound in the literature; the first of which is *preventive*-SCED (P-SCED). This formulation enforces that the nominal dispatch remains feasible within existing limits for all operational components in every contingency [1]. P-SCED does not consider potential recourse actions following a contingency and the resulting dispatch is overly conservative. *Corrective*-SCED (C-SCED) expands upon P-SCED by allowing active network components to respond to a contingency, e.g., see [2]. It allows re-dispatch of generators with fast-ramping capabilities and some even allow partial load-shedding, e.g., see [3]–[5]. Most C-SCED formulations ignore costs associated with recourse actions. Such costs can be high, especially for potential load shedding modeled via *value of lost load*. To remedy that, authors in [6] associate probabilities to contingencies and advocate to minimize the expected dispatch costs across contingencies. In contrast, our

R-SCED formulation in Section II proposes to minimize the *conditional value at risk* (CVaR) of said costs. CVaR_α of a random variable measures the expected loss in the $1 - \alpha$ fraction of the worst outcomes. In Section III, we explore the properties of R-SCED and illustrate through a two-bus network example, how the SO can express its preference in trading off cost versus reliability through its choice of α in R-SCED.

The R-SCED problem has a much larger problem description compared to a nominal economic dispatch problem owing to the number of contingencies, which leads to computational difficulties that are shared by other C-SCED formulations. To deal with this challenge, many have suggested to pre-filter contingencies; see [7] for a survey. In this work, we consider a decomposition approach to divide the R-SCED problem into smaller subproblems that can potentially be solved in parallel. We propose a critical region exploration (CRE) algorithm in Section IV to solve the R-SCED problem. CRE leverages properties of multiparametric linear programming and has proven effective in the tie-line scheduling problem for multi-area power systems in [8]. We demonstrate the efficacy of our algorithm on the IEEE 30-bus test system in Section V.

II. RISK-SENSITIVE SCED PROBLEM

We formulate the risk-sensitive SCED (R-SCED) problem with the linear DC power flow model and discuss how it generalizes prior formulations. R-SCED can easily be extended to more detailed nonlinear AC power flow equations. In practice, however, SOs often solve a sequence of SCED problems with successive linearizations of power flow equations to handle nonlinearity [9].

A. Network model

We begin by describing our model for the power network. Consider a grid on n buses, labeled $1, \ldots, n$, with m transmission lines. Let each bus be equipped with a dispatchable generator and a nominal load, whose vector values are denoted $\boldsymbol{g} \in \mathbb{R}^n$ and $\overline{\boldsymbol{d}} \in \mathbb{R}^n$, respectively. We adopt a linear power flow model via DC approximations, where the (directed) power flows over the transmission lines are linear maps of the vector of nodal power injections $\boldsymbol{x}$, given by $\boldsymbol{Hx}$. Here, $\boldsymbol{H} \in \mathbb{R}^{2m \times n}$ denotes the injection shift-factor matrix that depends on the topology of the power network and the admittances of the transmission lines. Let the limits on the (directed) power flows be denoted by $\boldsymbol{f} \in \mathbb{R}^{2m}$. The set of allowable nodal power injections then becomes

$$\mathbb{P} := \{\boldsymbol{x} \in \mathbb{R}^n \mid \boldsymbol{Hx} \le \boldsymbol{f}, \ \mathbb{1}^\mathsf{T}\boldsymbol{x} = 0\}, \qquad (1)$$

where $\mathbb{1} \in \mathbb{R}^n$ is a vector of all ones. The equality $\mathbb{1}^\mathsf{T}\boldsymbol{x} = 0$ captures the balance of demand and supply of power across the network. The DC approximations deem the voltage magnitudes to be at their nominal values, ignore transmission line

losses, and assume that voltage phase angle differences across neighboring buses are small.[1] Assume that a linear dispatch cost $c^\mathsf{T} g$ to produce g from the dispatchable generators can vary their outputs within $\mathbb{G} = [\underline{G}, \overline{G}]$. The lack of a generator at bus i can be modeled by letting $\underline{G}_i = \overline{G}_i = 0$.

B. Modeling contingencies

Consider a collection of scenarios, denoted by $1, \ldots, K$, each of which corresponds to a single transmission line failure. In the event of a contingency, we allow the operator to take recourse actions; they may alter generator output within ramping capabilities and shed load. Let δg^k denote the deviation of supply from the generators in contingency k from the nominal case, constrained by ramping limitations modeled as $|\delta g^k| \leq \Delta_g$. Denote the amount of load shed by $\delta d^k \in [0, \overline{d} - \underline{d}] := \Delta_d$ in contingency k.

A line outage alters the network topology, and hence, results in a different injection shift factor matrix H^k that in turn defines a different feasible injection region $\mathbb{P}^k$. Transfer capabilities of transmission lines are primarily determined by thermal considerations, and can exceed their rated power capacities for short durations. Following [11] and prior formulations [3], [5], [12], we adopt dynamic line ratings under contingencies. The *drastic action* limits are adopted immediately following a contingency, but before recourse actions are taken, and the *short-term emergency* limits are adopted 5 minutes after the SO takes the recourse actions. Let the corresponding sets of feasible injections be denoted $\mathbb{P}_{\mathsf{DA}}^k$ and $\mathbb{P}_{\mathsf{SE}}^k$ respectively, where

$$\mathbb{P}^k \subset \mathbb{P}_{\mathsf{SE}}^k \subset \mathbb{P}_{\mathsf{DA}}^k.$$

C. Formulating the risk-sensitive SCED (R-SCED) problem

Our formulation relies on the use of *conditional value at risk* of a random variable. We begin by describing this risk measure and then present R-SCED in (2).
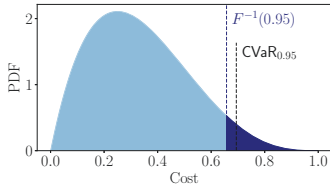

Fig. 1. The probability distribution of random cost with the shaded region denoting the tail of the distribution with probability 0.05.

If χ describes a random cost with a continuous distribution, $\mathsf{CVaR}_\alpha[\chi]$ computes the expected cost of χ in the $(1 - \alpha)$ fraction of worst-case outcomes, or

$$\mathsf{CVaR}_\alpha[\chi] := \mathbb{E}\left[\chi | \chi \geq F^{-1}(\alpha)\right],$$

where F is the cumulative distribution function of χ and $\mathbb{E}$ denotes the expectation computed over that distribution. Figure 1 visualizes the definition for some probability distribution of random cost χ. $\mathsf{CVaR}_{0.95}[\chi]$ is the average value of χ over the distribution of shaded tail where the tail has probability 0.05.

As $\alpha \downarrow 0$, $\mathsf{CVaR}_\alpha[\chi]$ reduces to the expected value of χ. For α close to 1, the tail shrinks to only include the maximum value of χ and $\mathsf{CVaR}_\alpha[\chi]$ yields that maximum.[2]

To present R-SCED formally, associate probabilities $p \in \mathbb{R}^K$ to the contingencies and let $p_0 := 1 - \mathbb{1}^\mathsf{T} p$ as the probability of the nominal state. We arrive at the following optimization problem of the *risk-sensitive* SCED problem.

$$\text{minimize} \quad \mathsf{CVaR}_\alpha\left[c^\mathsf{T} g + C(\delta g, \delta d)\right], \tag{2a}$$

$$\text{subject to} \quad g \in \mathbb{G}, \ g - \overline{d} \in \mathbb{P}, g - \overline{d} \in \mathbb{P}_{\mathsf{DA}}^k, \tag{2b}$$

$$g + \delta g^k \in \mathbb{G}, g + \delta g^k - \overline{d} + \delta d^k \in \mathbb{P}_{\mathsf{SE}}^k, \tag{2c}$$

$$|\delta g^k| \leq \Delta_g, \ \delta d^k \in \Delta_d, \tag{2d}$$

$$\text{for each } k = 1, \ldots, K$$

over g, δg, δd. Here, δg, δd denote the collection of the respective variables across all contingencies. Additionally, $C(\delta g, \delta d)$ is the random recourse cost, assuming a contingency occurs, that takes the value

$$C^k(\delta g^k, \delta d^k) := c^\mathsf{T} \delta g^k + v^\mathsf{T} \delta d^k$$

in contingency k.[3] Here, v measures the vector of nodal values of lost load (VoLL).

In R-SCED , the dispatch cost depends on two factors — the dispatch decisions and the realized contingency. Fixing the decisions, the cost is a random variable over the set of contingencies. Minimizing the expected value of this random variable yields the formulation in [6]. Taking the CVaR of this variable generalizes this to encode an SO's tolerance to higher costs through the parameter α. Choosing α equal to zero, R-SCED treats all contingencies equally and minimizes expected cost as in [6]. As α increases, R-SCED weighs contingencies where the cost is higher more heavily.

For convenience, we denote the dispatch associated with nominal operation, g, as *nominal dispatch* and the associated cost, $c^\mathsf{T} g$, as *nominal dispatch cost*.

D. Comparison to existing SCED formulations

Before delineating the properties of the R-SCED problem in the next section, we briefly discuss its relationship to prior formulations of the SCED problem in the literature. We refer the reader to [5] for a comprehensive survey.

- Preventive SCED (P-SCED) stipulates that the nominal dispatch be feasible after any single line failure, and does not model recourse actions or dynamic line ratings. R-SCED with $\Delta_g = \Delta_d = 0$ and $\mathbb{P}_{\mathsf{DA}}^k = \mathbb{P}_{\mathsf{SE}}^k = \mathbb{P}^k$ reduces to P-SCED.
- Corrective SCED (C-SCED) often does not model recourse costs or load shedding. When they are, e.g., in [6], expected costs are minimized—the case of R-SCED with $\alpha = 0$.

III. PROPERTIES OF THE R-SCED PROBLEM

In this section, we first characterize a property of R-SCED in our first result. The proof of this property proves useful in devising an algorithm to solve it in Section IV. Second,

---

[1] We can alternatively utilize linearization of the power flow equations around the current operating point, possibly using real-time measurements to estimate H, e.g., in [10].

[2] For the definition of $\mathsf{CVaR}_\alpha[\chi]$ for χ with general distributions, see [13].

[3] The cost structure can be altered to distinguish between different costs for regulation up and down, i.e., by replacing $c^\mathsf{T} \delta g^k$ in the recourse cost by $c_+^\mathsf{T} [\delta g^k]^+ + c_-^\mathsf{T} [-\delta g^k]^+$ without adding conceptual difficulties.

we discuss the outcome of R-SCED on a two-bus network example and compare it to that of C-SCED and P-SCED.

Proposition 1. *R-SCED can be formulated as a linear program, linearly parameterized in α. Additionally, the optimal cost of R-SCED in (2) is piecewise affine in $\alpha' := (1 - \alpha)^{-1}$ over any closed interval in $\mathbb{R}_+$, and the optimal nominal dispatch $\boldsymbol{g}^*$ remains constant over sub-intervals where the optimal cost is affine.*

Proof. Following Rockafellar and Uryasev in [13], CVaR of a random variable χ is given by

$$\mathsf{CVaR}_\alpha(\chi) := \min_z \left\{ z + \frac{1}{1-\alpha} \mathbb{E}[\chi - z]^+ \right\}, \quad (3)$$

where $[\cdot]^+$ yields the positive part of its argument. Observe that $\boldsymbol{c}^T \boldsymbol{g} + C(\delta \boldsymbol{g}, \delta \boldsymbol{d})$ takes values in a discrete set with probabilities $\boldsymbol{p}$. Letting $C^0 = 0$, the objective function of (2) using (3) becomes

$$\min_z \left\{ z + \alpha' \sum_{k=0}^{K} p^k \left[\boldsymbol{c}^T \boldsymbol{g} + C^k(\delta \boldsymbol{g}^k, \delta \boldsymbol{d}^k) - z \right]^+ \right\}. \quad (4)$$

Using the epigraph form, (2) then reduces to solving

$$\begin{aligned} \underset{z, \boldsymbol{y}, \boldsymbol{g}, \delta \boldsymbol{g}, \delta \boldsymbol{d}}{\text{minimize}} \quad & z + \alpha' \sum_{k=0}^{K} p^k y^k, \\ \text{subject to} \quad & y^k \geq 0, \\ & y^k \geq \boldsymbol{c}^\mathsf{T} \boldsymbol{g} + C^k(\delta \boldsymbol{g}^k, \delta \boldsymbol{d}^k) - z, \\ & \text{(2b)} - \text{(2d)}, \\ & \text{for each } k = 1, \ldots, K, \end{aligned} \quad (5)$$

where $\boldsymbol{y} := (y_0, \ldots, y_K)^\mathsf{T}$. More compactly, define

$$\boldsymbol{x}^0 := \left(z, y^0, \boldsymbol{g}^\mathsf{T} \right)^\mathsf{T}, \quad \boldsymbol{x}^k := \left(y^k, [\delta \boldsymbol{g}^k]^\mathsf{T}, [\delta \boldsymbol{d}^k]^\mathsf{T} \right)^\mathsf{T},$$

and rewrite (5) as

$$\begin{aligned} \underset{\boldsymbol{x}^0, \boldsymbol{x}^1, \ldots, \boldsymbol{x}^K}{\text{minimize}} \quad & [\boldsymbol{c}^0]^\mathsf{T} \boldsymbol{x}^0 + \alpha' \sum_{k=1}^{K} \left[\boldsymbol{c}^k \right]^\mathsf{T} \boldsymbol{x}^k, \\ \text{subject to} \quad & \boldsymbol{A} \boldsymbol{x}^0 \leq \boldsymbol{b}, \\ & \boldsymbol{A}^k \boldsymbol{x}^0 + \boldsymbol{E}^k \boldsymbol{x}^k \leq \boldsymbol{b}^k, \\ & k = 1, \ldots, K, \end{aligned} \quad (6)$$

for suitably defined $\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{A}^k, \boldsymbol{E}^k, \boldsymbol{b}^k, \boldsymbol{c}^0, \boldsymbol{c}^k$. This is a parametric linear program linearly parameterized by α'. The proof then follows from [14, Theorem 7.2]. □

The above proof demonstrates that R-SCED can be cast as a linear program (6) with a decomposable structure, a property we leverage to design our algorithm in Section IV.

A. R-SCED on a two-bus network example

To gain insights into the properties of R-SCED, we present a simple yet illustrative two-bus network example and contrast the results of R-SCED with that of P-SCED and C-SCED.

Consider the network in Figure 2a with $\Delta_{g_1} = 0.25$ MW/min, $\Delta_{g_2} = 0.2$ MW/min, and $v_1 = v_2 = \$30$/MW. Assume line failures occur with probabilities $p_1 = p_2 = 0.01$

and dynamic line ratings of $\boldsymbol{f}_{DA} = 1.75\boldsymbol{f}$ and $\boldsymbol{f}_{SE} = 1.25\boldsymbol{f}$. The following table captures the nominal dispatch cost under various formulations of economic dispatch, where the nominal case is denoted ED.

Method	g_1^* (MW/hr)	g_2^* (MW/hr)	Nominal Cost (\$/hr)
ED	20.0	0.0	20.0
P-SCED	15.0	5.0	25.0
C-SCED	17.25	2.75	22.75
C-SCED$_{\text{aug}}$	18.75	1.25	21.25
R-SCED (0.1)	18.75	1.25	21.25
R-SCED (0.9)	17.25	2.75	22.75

TABLE I. Comparison of various ED formulations.

C-SCED$_{\text{aug}}$ in Table I is C-SCED augmented with load shedding, where an SO aims to minimize expected cost with recourse. When α is small ($\alpha \approx 0$), the R-SCED solution equals that in the augmented C-SCED solution. Additionally for large α, i.e., $\alpha \approx 1$, R-SCED reduces to expected cost minimization without load shed. For general power networks, the R-SCED solution with $\alpha \approx 1$ is not equal to the C-SCED solution; it minimizes the maximum recourse cost across contingencies balancing the cost associated with load shedding and generator re-dispatch.

Figure I demonstrates that nominal cost and total load shed are piecewise constant in α. Additionally, as α increases, the cost of nominal dispatch increases while load shedding decreases. This illustrates how SO can utilize α to trade-off between between cost and reliability.

We draw attention to the case when the dispatch cost of the expensive generator at bus 2 is reduced from \$2/MW to \$1.5/MW. For a range of α (approximately 0.6-0.7), the nominal dispatch cost with the reduced c_2 is higher than that with the larger c_2. Reduction in c_2 makes the VoLL relatively larger compared to ramping costs. As a result, R-SCED favors lesser load shedding at lower α's, leading to the behavior depicted in Figure 2b and 2c.

IV. SOLVING R-SCED VIA CRITICAL REGION EXPLORATION

The R-SCED problem in (2) can be cast as a linear program (LP). For practical power networks, that LP can be prohibitively large, a property shared by prior C-SCED formulations. We exploit the structure in its reformulation (6) and propose the *critical region exploration* algorithm to solve R-SCED. Our algorithm decomposes the problem into a master problem and a collection of subproblems that can be solved in parallel, and leverages properties of multi-parametric linear programming [8]. To describe the algorithm, begin by noticing that (6) can be written as

$$\begin{aligned} \underset{\boldsymbol{x}^0}{\text{minimize}} \quad & \left[\boldsymbol{c}^0 \right]^\mathsf{T} \boldsymbol{x}^0 + \alpha' \sum_{k=1}^{K} J_*^k \left(\boldsymbol{x}^0 \right), \\ \text{subject to} \quad & \boldsymbol{A} \boldsymbol{x}^0 \leq \boldsymbol{b}, \end{aligned} \quad (7)$$

where

$$\begin{aligned} J_*^k \left(\boldsymbol{x}^0 \right) := \underset{\boldsymbol{x}^k}{\text{minimize}} \quad & \left[\boldsymbol{c}^k \right]^\mathsf{T} \boldsymbol{x}^k, \\ \text{subject to} \quad & \boldsymbol{A}^k \boldsymbol{x}^0 + \boldsymbol{E}^k \boldsymbol{x}^k \leq \boldsymbol{b}^k. \end{aligned} \quad (8)$$
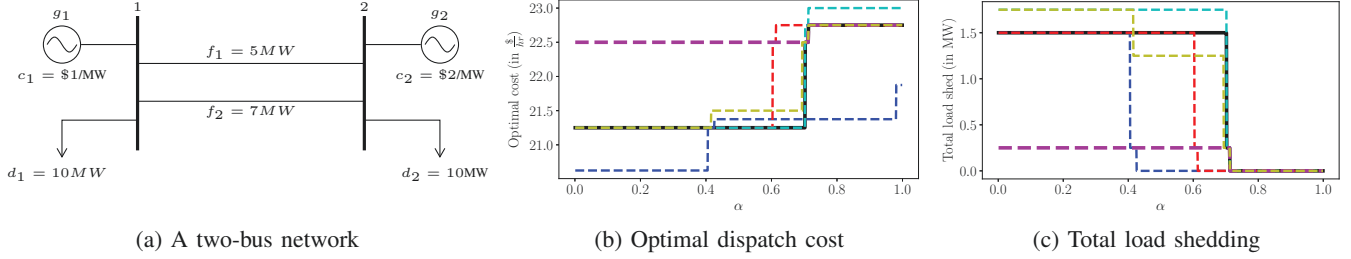
Fig. 2. Nominal cost and total load shedding from either contingency for 2-bus example in (a) is depicted for various parameter choices. Here, (——) denotes the nominal case, and the remaining lines denote the nominal case augmented with (- - -) dispatch cost at bus 2 reduced to $c_2 = 1.5$, (- - -) with VoLL increased to $v = 40\$/MW$, (- - -) for the case with generation ramping limits reduced to $\Delta_{g_1} = 0.2MW, \Delta_{g_2} = 0.15MW$, (- - -) reduces the drastic action limits to $\boldsymbol{f}_{DA} = 1.5\boldsymbol{f}$, and (- - -) corresponds to the line flow limit of the higher capacity line reduced to $f_2 = 6MW$.

Properties of J_*^k are crucial to describe our algorithm. We need additional notation to describe them. Define

$$\mathbb{X}^0 := \{\boldsymbol{x} \mid \boldsymbol{A}\boldsymbol{x} \leq \boldsymbol{b}\}.$$

Assume throughout that (8) is feasible for any $\boldsymbol{x}^0 \in \mathbb{X}^0$. We say a collection of polyhedral sets $\mathbb{S}_1, \ldots, \mathbb{S}_L$ define a *polyhedral partition* of $\mathbb{S}$, if these L sets are polyhedral, their union spans $\mathbb{S}$, and any intersections are only at their boundaries. Given this definition, we record a vital property of J_*^k in the following lemma.

Lemma 1. $J_*^k(\boldsymbol{x}^0)$ *is piecewise affine over* $\mathbb{X}^0$ *and the sets over which it is affine describe a polyhedral partition of* $\mathbb{X}^0$.

Problem (8) is a multi-parametric linear program, linearly parameterized by $\boldsymbol{x}^0$. As a consequence, its proof follows directly from [14, Theorem 7.2]. Hereafter, call the sets in the polyhedral partition as *critical regions*. For a given $\boldsymbol{x}^0 \in \mathbb{X}^0$, one can compute the critical region $\mathbb{C}^k$ that contains $\boldsymbol{x}^0 \in \mathbb{X}^0$ and the affine description of the optimal cost J_*^k over $\mathbb{C}^k$ for each $k = 1, \ldots, K.$[4] More precisely, let the affine description of J_*^k be given by $[\boldsymbol{\rho}^k]^{\mathsf{T}}\boldsymbol{x}^0 + \eta^k$ over $\mathbb{C}^k$. With the affine descriptions of $J_*^1, \ldots, J_*^K$, we can then solve

$$\begin{aligned} \underset{\boldsymbol{x}^0}{\text{minimize}} \quad & [\boldsymbol{c}^0]^{\mathsf{T}}\boldsymbol{x}^0 + \alpha' \sum_{k=1}^{K} [\boldsymbol{\rho}^k]^{\mathsf{T}}\boldsymbol{x}^0 + \alpha'\eta^k, \\ \text{subject to} \quad & \boldsymbol{A}\boldsymbol{x}^0 \leq \boldsymbol{b}, \; \boldsymbol{x}^0 \in \cap_{k=1}^{K}\mathbb{C}_k, \end{aligned} \quad (9)$$

i.e., (7) with the additional constraint $\boldsymbol{x}^0 \in \cap_{k=1}^{K}\mathbb{C}_k$, over which the affine description of J_*^k holds. The above problem can be solved as an LP. We assume that one can determine the *lexicographically smallest* minimizer of (9). This provides a tie-breaking rule in the case the minimizer is not unique. The final consideration is a necessary and sufficient condition for $\boldsymbol{x}^{0,*}$ to be a minimizer of (7). To that end, $\boldsymbol{x}^{0,*}$ is a minimizer for (7) if and only if

$$0 \in \delta J^*(\boldsymbol{x}^{0,*}) + N_{\mathbb{X}^0}(\boldsymbol{x}^{0,*}), \quad (10)$$

where $\delta J^*(\cdot)$ denotes the sub-differential set of the objective function of (7) and $N_{\mathbb{X}^0}$ is the normal cone of $\mathbb{X}^0$. Algorithm 1 presents the CRE algorithm to solve (6). The crucial property of our algorithm is summarized next.

[4]We omit the details of the procedure to determine the critical regions and the associated affine cost description due to space limitations.

Proposition 2. *Algorithm 1 converges to an optimizer of* (6) *in finitely many iterations.*

The proof is largely similar to that of [8, Theorem 1], and is omitted for brevity. The proof requires boundedness of all variables in (7) and (8). Variable z in (5) can be unbounded in general. Taking advantage of the definition of CVaR, we bound it by the minimum and the maximum value of recourse costs across contingencies.

Algorithm 1 CRE algorithm to solve R-SCED.

1: **Initialize:**
$\boldsymbol{x}^0 \in \mathbb{X}^0, J^* \leftarrow \infty, \mathbb{D} \leftarrow \text{empty set}, \epsilon \leftarrow$ small positive number
2: **do**
3:     Given $\boldsymbol{x}^0$, compute $\boldsymbol{\rho}^k, \eta^k, \mathbb{C}^k$ for $k = 1, \ldots, K$.
4:     Solve (9)
5:     $[\boldsymbol{x}^0]^{\text{opt}} \leftarrow$ lexicographically smallest minimizer of step 4.
6:     $J^{\text{opt}} \leftarrow$ optimal cost of step 4
7:     **if** $J^{\text{opt}} < J^*$ **then**
8:       $[\boldsymbol{x}^0]^* \leftarrow [\boldsymbol{x}^0]^{\text{opt}}, J^* \leftarrow J^{\text{opt}}, \mathbb{D} \leftarrow \{\boldsymbol{c}\}$
9:     **else**
10:       $\mathbb{D} \leftarrow \mathbb{D} \cup \{\boldsymbol{c}^0 + \alpha' \sum_{k=1}^{K} \boldsymbol{\rho}^k\}$
11:     **end if**
12:     $\boldsymbol{v}^* \leftarrow \text{argmin}_{\boldsymbol{v} \in \text{conv}(\mathbb{D}) + N_{\mathbb{X}^0}([\boldsymbol{x}^0]^*)} \|\boldsymbol{v}\|^2$
13:     $\boldsymbol{x}^0 \leftarrow [\boldsymbol{x}^0]^{\text{opt}} - \epsilon\boldsymbol{v}^*$
14: **while** $v^* \neq 0$

V. EXPERIMENTS ON THE IEEE 30-BUS TEST SYSTEM

We have implemented CRE for R-SCED on various IEEE test networks. We only report the results on the highly-loaded IEEE 30-bus system from PGLIB v17.08 [15] for space constraints. In our experiments, we assumed drastic action and short-term emergency limits to be 70% and 10% higher than the nominal limits, respectively. Ramping costs were set equal to nominal dispatch costs and ramp limits were uniformly set to 0.2 MW per minute. The system was augmented with generation capacities of 3MW at buses 13, 22, 23, and 27, and costs were set to $1.4, 1.8, 1.6, 1.7$ per MWh, respectively. Line limits were modified according to Table II.
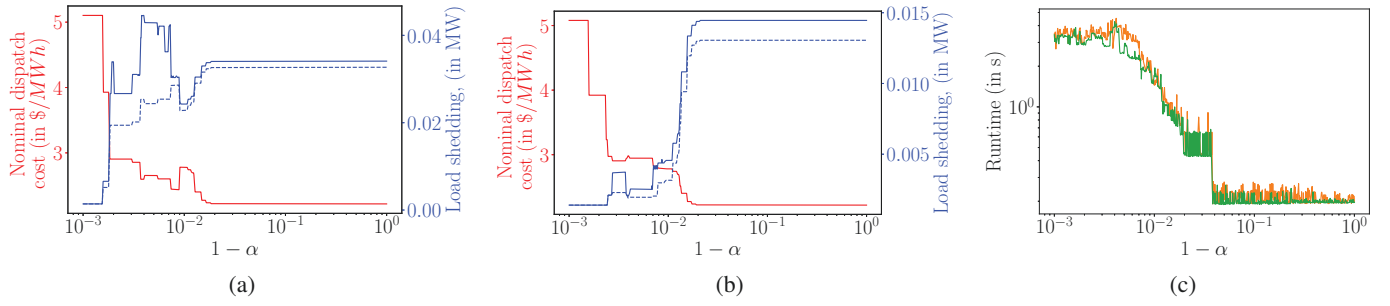
Fig. 3. R-SCED solutions as a function of α for the IEEE 30-bus test network with VoLL uniformly (a) \$90/MWh and (b) \$126/MWh, where plots (a) and (b) show the (——) nominal dispatch cost, (——) total load shed, and (- -) maximum load shed. Plot (c) shows runtime for VoLL of (——) \$90/MWh and (——) \$126/MWh.

Bus 1	Bus 2	Line Limit
12	16	0.33
14	15	0.396
16	17	0.36
15	18	0.319
10	20	0.312

TABLE II. Augmented line limits for IEEE 30-bus example.

Slack variables were added to each subproblem constraint to ensure feasibility, emulating [12]. The algorithm was initialized with the solution of the ED problem with drastic action limits. We formulated the problems in Python, but the CRE algorithm runs on C++. All LPs were solved using Gurobi 8.0. The reported running times are from solutions on a 2015 MacBook Pro with 2.7 GHz Core i5 processor and 8 GB RAM.

As one expects from Proposition 1, Figure 3a illustrates that both nominal dispatch cost and load shedding are piecewise constant. While nominal dispatch cost generally increases with risk-aversion, and maximum load shed generally decreases, this does not occur monotonically. This results from the balance of cost associated with load shedding and regulation. When the relative weight of load shedding is increased in Figure 3b, R-SCED is less willing to shed load and shows fewer increases in load shed as risk aversion increases.

Additionally, notice the large increase in total load shed despite the maximum load shed not increasing significantly in Figure 3a. CVaR considers the tail cost of most expensive contingencies and ignores contingencies whose cost is below the cutoff, allowing for a low level of load-shedding throughout each of the contingencies.

VI. CONCLUSIONS AND FUTURE WORK

In this paper, we defined an alternative formulation of SCED that allows a system operator (SO) to tradeoff between minimization of dispatch cost and reliability of power delivery, and explored its salient properties. Finally, we proposed the critical region exploration (CRE) algorithm to solve it. In future studies, we aim to compare CRE with the popular Bender's decomposition technique on larger power networks for R-SCED, as proposed in [12], [16]–[19]. We also aim to extend our formulation and algorithm to model uncertainty in renewable power production.

REFERENCES

[1] O Alsac and B Stott. Optimal load flow with steady-state security. *IEEE transactions on power apparatus and systems*, (3):745–751, 1974.

[2] A Monticelli, MVF Pereira, and S Granville. Security-constrained optimal power flow with post-contingency corrective rescheduling. *IEEE Transactions on Power Systems*, 2(1):175–180, 1987.

[3] Florin Capitanescu and Louis Wehenkel. Improving the statement of the corrective security-constrained optimal power-flow problem. *IEEE Transactions on Power Systems*, 22(2):887–889, 2007.

[4] Zhigang Li, Wenchuan Wu, Mohammad Shahidehpour, and Boming Zhang. Adaptive robust tie-line scheduling considering wind power uncertainty for interconnected power systems. *IEEE Transactions on Power Systems*, 31(4):2701–2713, 2016.

[5] Florin Capitanescu, JL Martinez Ramos, Patrick Panciatici, Daniel Kirschen, A Marano Marcolini, Ludovic Platbrood, and Louis Wehenkel. State-of-the-art, challenges, and future trends in security constrained optimal power flow. *Electric Power Systems Research*, 81(8):1731–1741, 2011.

[6] Francois Bouffard and Francisco D Galiana. Stochastic security for operations planning with significant wind power generation. In *Power and Energy Society General Meeting-Conversion and Delivery of Electrical Energy in the 21st Century, 2008 IEEE*, pages 1–11. IEEE, 2008.

[7] Florin Capitanescu, Mevludin Glavic, Damien Ernst, and Louis Wehenkel. Contingency filtering techniques for preventive security-constrained optimal power flow. *IEEE Transactions on Power Systems*, 22(4):1690–1697, 2007.

[8] Ye Guo, Subhonmesh Bose, and Lang Tong. On robust tie-line scheduling in multi-area power systems. *IEEE Transactions on Power Systems*, 2017.

[9] O Alsac, J Bright, M Prais, and B Stott. Further developments in lp-based optimal power flow. *IEEE Transactions on Power Systems*, 5(3):697–711, 1990.

[10] Yu Christine Chen, Alejandro D Domínguez-García, and Peter W Sauer. Measurement-based estimation of linear sensitivity distribution factors and applications. *IEEE Transactions on Power Systems*, 29(3):1372–1382, 2014.

[11] Paul Renaud. *Transmission group procedure*. National Grid USA Service Company, Inc., 2010.

[12] Yanchao Liu, Michael C Ferris, and Feng Zhao. Computational study of security constrained economic dispatch with multi-stage rescheduling. *IEEE Transactions on Power Systems*, 30(2):920–929, 2015.

[13] R Tyrrell Rockafellar, Stanislav Uryasev, et al. Optimization of conditional value-at-risk. *Journal of risk*, 2:21–42, 2000.

[14] Francesco Borrelli, Alberto Bemporad, and Manfred Morari. *Predictive control for linear and hybrid systems*. Cambridge University Press, 2017.

[15] The IEEE PES Task Force on Benchmarks for Validation of Emerging Power System Algorithms. Pglib optimal power flow benchmarks. https://github.com/power-grid-lib/pglib-opf, 2018.

[16] T Gomez, IJ Perez-Arriaga, J Lumbreras, and VM Parra. A security-constrained decomposition approach to optimal reactive power planning. *IEEE Transactions on Power Systems*, 6(3):1069–1076, 1991.

[17] Jorge Martínez-Crespo, Julio Usaola, and José L Fernández. Security-constrained optimal generation scheduling in large-scale power systems. *IEEE Transactions on Power Systems*, 21(1):321–332, 2006.

[18] Yuan Li and James D McCalley. Decomposed scopf for improving efficiency. *IEEE Transactions on Power Systems*, 24(1):494–495, 2009.

[19] Dzung Phan and Jayant Kalagnanam. Some efficient optimization methods for solving the security-constrained optimal power flow problem. *IEEE Transactions on Power Systems*, 29(2):863–872, 2014.