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Effective theories of dark mesons with custodial symmetry

Graham D. Kribs,[a] Adam Martin[b] and Tom Tong[a]

[a] *Department of Physics, University of Oregon, 1371 E. 13th Ave., Eugene, OR, 97403 U.S.A.*
[b] *Department of Physics, University of Notre Dame, 225 Nieuwland Science Hall, South Bend, IN, 46556 U.S.A.*

E-mail: kribs@uoregon.edu, amarti41@nd.edu, ttong2@uoregon.edu

Abstract: Dark mesons are bosonic composites of a new, strongly-coupled sector beyond the Standard Model. We consider several dark sectors with fermions that transform under the electroweak group, as arise from a variety of models including strongly-coupled theories of dark matter (e.g., stealth dark matter), bosonic technicolor (strongly-coupled indcued electroweak symmetry breaking), vector-like confinement, etc. We consider theories with two and four flavors under an SU(N) strong group that acquire variously chiral, vector-like, and mixed contributions to their masses. We construct the non-linear sigma model describing the dark pions and match the ultraviolet theory onto a low energy effective theory that provides the leading interactions of the lightest dark pions with the Standard Model. We uncover two distinct classes of effective theories that are distinguishable by how the lightest dark pions decay: "Gaugephilic": where $\pi^0 \to Zh$, $\pi^\pm \to Wh$ dominate once kinematically open, and "Gaugephobic": where $\pi^0 \to \bar{f}f$, $\pi^\pm \to \bar{f}'f$ dominate. Custodial SU(2) plays a critical role in determining the "philic" or "phobic" nature of a model. In dark sectors that preserve custodial SU(2), there is no axial anomaly, and so the decay $\pi^0 \to \gamma\gamma$ is highly suppressed. In a companion paper, we study dark pion production and decay at colliders, obtaining the constraints and sensitivity at the LHC.

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1 Introduction

We consider extensions of the Standard Model that incorporate a new, strongly-coupled, confining gauge theory with fermion representations that transform under the electroweak group. The notion of a new sector of fields transforming under a new, strongly-coupled, confining group is a fascinating possibility for physics the Standard Model. All of the new sector's scales are either natural (the new confinement scale) or technically natural (new fermion masses), and so such a scenario is, at a minimum, no worse off than the Standard Model from a naturalness point of view.

There are a wide variety of uses of a new, strongly-coupled, confining group. One use is to at least partially break electroweak symmetry dynamically, such as bosonic technicolor [1–9] and the closely related ideas on strongly-coupled induced electroweak symmetry breaking [10–20]. Composite Higgs theories also posit a new strongly-coupled sector in which at least an entire Higgs doublet emerges in the low energy effective theory (the literature is far too vast to survey, for a review see e.g., [21]). There is also a interesting connection to the relaxation of the electroweak scale [22] using a new strongly-coupled sector, e.g., [19, 20, 22–24].

Dark matter can emerge as a composite meson or baryon of a strongly-coupled theory, often with an automatic accidental symmetry that protects against its decay. Since the early days of technicolor there was a possibility of dark matter emerging as technibaryons [25–31]. There is now a growing literature that has studied strongly-coupled dark matter as dark pions [32–53], dark quarkonia-like states [54–58], as well as dark baryons and related candidates [32, 36, 40, 55, 59–86] (for a review, see [87]).

Another use is to simply characterize generic strongly-coupled-like signals as targets for LHC and future colliders. Vector-like confinement [88] pioneered this study in the context of vector-like fermions that transform under part of the SM group as well as under a new, strongly-coupled group with scales near or above the electroweak scale. Further explorations into the phenomenology and especially the meson sector included [20, 38, 55, 73, 89–97]. In theories with somewhat lower confinement scales, the dark sector may lead to dark showers and related phenomena [98–104], displaced signals [105, 106] and potentially intriguing spectroscopy [47, 107, 108]. Spectacular "quirky" signals can arise in theories with a very low confinement scale [109, 110]. The latter theories may also lead to a high multiplicity of soft particles that are tricky to observe [111–113].

The dark sector theory that is of particular interest to us is Stealth Dark Matter [75]. In this theory, there is a new, strongly-coupled "dark sector" that consists of vector-like fermions that transform under both the new "dark group" group as well as the electroweak part of the SM, and crucially, also permit Higgs interactions. Others have also pursued dark sectors with vector-like fermions that permit Higgs interactions for a variety of purposes [20, 23, 43, 85, 114]. The meson sector of the Stealth Dark Matter theory, however, has several intriguing properties due to the accidental symmetries of the model.

One might think a dark meson sector whose low energy effective theory is a set of scalars with electroweak quantum numbers has already been fully (or mostly) covered by the wide range of existing search strategies. As we show in our companion paper [115], this is *not* the case. There we find that a dark vector meson could be as light as about 300 GeV, something that, at first glance, seems hard to believe given the multi-TeV bounds on new Z' bosons from LHC data. The dark vector meson can mediate dark pion pair production (just like $\rho \to \pi\pi$ in QCD), and in some models, the bounds on the dark pion mass could be as small as 130 GeV. Clearly, the LHC easily has the energy to produce these states, and so it really comes down to finding search strategies that maximize sensitivity. We believe substantial improvements are possible, providing impetus and breathing new life into LHC searches in the hundreds of GeV regime.

The difficulty with strongly-coupled physics is that it is strongly-coupled. However, many years ago Kilic, Okui, and Sundrum pioneered the study of a new strongly-coupled sector's phenomenology for collider physics [88]. Their insight was to determine the leading interactions of an effective theory of pseudo-Nambu Goldstone (pNGB) mesons with vector mesons (both composite and fundamental). They were motivated by imagining QCD scaled

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up to weak scale energies, except, and here is the key point, their BSM fermion masses were taken to be purely vector-like.

In this paper, we generalize vector-like confinement by permitting specific interactions between the strong sector fermions and the Standard Model. In some models, these interactions are renormalizable Yukawa couplings of the dark sector fermions with the Higgs of the SM. In others that do not permit Yukawa couplings, we also consider higher dimensional operators (that also involve the Higgs sector in some way). These interactions lead to dark pion decay. And, what is distinct in the vector-like theories we consider is that there is no axial anomaly contribution to neutral dark pion decay. We use a non-linear sigma model (NLSM) to describe the pNGB mesons, which we carry out in detail in the paper. Equally important, the fact that we break the flavor symmetries of the strong sector with Higgs interactions necessarily locks the strong sector flavor symmetries to the $O(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ global symmetry of the Higgs potential. As a result, the strong sector fields can be grouped into multiplets of this symmetry, with different assignments possessing qualitatively different phenomenology.

The structure of this paper is as follows. First (section 2), we briefly remind the reader of the ingredients in the type of strong sector we want to consider. Next, in section 3 we discuss custodial SU(2) of the Higgs sector, emphasizing the role of hypercharge and the difference between up-type and down-type fermion Yukawa couplings that act as the spurions for custodial SU(2) violation. This will greatly assist us in understanding and classifying the dynamics of dark mesons in the set of theories we consider. In section 5, we discuss two-flavor theories, one chiral and two vector-like scenarios. Understanding the dynamics of these relatively simple theories provides a warmup to theories with more flavors. Next we consider vector-like four-flavor theories, that are the smallest field content that permit vector-like masses and Higgs interactions at the renormalizable level. The model was first proposed in [75, 76] where baryonic sector of these theories was extensively studied since the lightest baryon is a viable dark matter candidate. Our main goal is to determine the dark pion interactions with the SM, and to understand the results in terms of limits when two of the flavors are decoupled and the theory reduces to just a two-flavor theory with higher dimensional interactions. Finally, to emphasize the role of custodial SU(2), we discuss a vector-like two-flavor theories where custodial symmetry is violated, and the consequences for the dark pion decays. In appendix A, we review the case of a general two-Higgs doublet model and the "gaugephobic" decays of its $A^0, H^{\pm}$ states.

2 Defining the dark sector

Throughout this paper, we will refer to the new strong sector as the "dark" sector. It consists of a strongly-coupled "dark gauge group" $\mathrm{SU}(N_D)$ with its own "dark confinement scale", and "dark fermions" or "dark flavors" that transform under the dark group as well as the electroweak part of the Standard Model. Below the dark confinement scale, the effective theory description of the composites includes "dark mesons"; the latter breaking up into "dark vector mesons" and "dark pions". Despite the naming convention, we emphasize that the new states are certainly not "dark" to collider experiments [115].

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When describing the fermionic content of the dark sector (in the UV), we will work entirely with left-handed fields, meaning (1/2, 0) under the Lorentz group. We will distinguish between theories by the number of dark fermion flavors, where each flavor corresponds to one (two-component) fermion in the fundamental of the dark color group and one anti-fundamental. We will generically refer to dark color fundamentals as F, and anti-fundamental as $\hat{F}$. Throughout this paper, we will assume that all dark fermions are inert under SM $\mathrm{SU}(3)_c$ while at least some of them interact electroweakly. Other references that have pursued dark sectors transforming under $\mathrm{SU}(3)_c$ can be found in [35, 88, 116].

In the absence of other interactions, the symmetry of the dark sector is $\mathrm{SU}(N_{\mathrm{fund}}) \times \mathrm{SU}(N_{\mathrm{anti}})$. Turning on electroweak interactions, some of these flavor symmetries are explicitly broken. The majority of the dark sectors we'll study are vector-like, which — in terms of two-component fermions — implies that if F_i is a fundamental of dark color and transforms under EW representation G, then the theory also includes a dark-color anti-fundamental $\hat{F}_j$ also residing in EW representation $\hat{G}$. This charge assignment permits mass terms of the form $M_{ij} F_i \hat{F}_j$. In addition, we can form dimension > 3 operators connecting dark fermions with the Higgs boson. Interactions with the Higgs force us to connect flavor symmetries of the fermionic sector with the $O(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ global symmetry Higgs potential. If F are EW doublets and $\hat{F}$ are EW singlets, then the interactions take a form familiar from SM Yukawas, $y F \hat{F} \mathcal{H}$. For other representations of $F, \hat{F}$, the interactions only come about at the non-renormalizable level, e.g., $F \hat{F} H^\dagger H/\Lambda$.

Once we cross below the dark confining scale, the low energy effective theory is described in terms of the composite mesons of this sector. Provided that the vector-like dark fermion masses are $< 4\pi f$, the leading interactions of the dark pions can be determined using non-linear sigma model language analogous to the real pions of QCD. Confinement spontaneously breaks the chiral symmetry of the dark fermions down to the diagonal subgroup: $\mathrm{SU}(N_{\mathrm{fund}}) \times \mathrm{SU}(N_{\mathrm{anti}}) \to \mathrm{SU}(N)_V$, with the dark pion multiplets falling into representations of $\mathrm{SU}(N)_V$. Whether or not $\mathrm{SU}(N)_V$ is gauged and how it connects with the Higgs potential symmetries depends on the setup. In the IR, interactions between dark fermions and Higgses become interactions between the dark pions and the Higgs. For example, the two examples used above become $\mathrm{tr}(\Sigma \mathcal{H}^\dagger) + \mathrm{h.c.}$ and $\mathrm{Tr}(\Sigma \mathcal{H}^\dagger \mathcal{H} + \mathrm{h.c.})$ respectively, where Σ is the NLSM field.

At this point it is useful to distinguish between the dark sectors that we consider in this paper and early proposals for dynamical electroweak symmetry breaking (technicolor). Simply put, in the extension we consider, we *assume* there is a Higgs doublet in the low energy effective theory that acquires an electroweak breaking vev that is responsible for (most) of electroweak symmetry breaking in the Standard Model.

3 Custodial SU(2)

A critical part of the classification of effective theories of dark mesons is whether custodial SU(2) is preserved or violated by the dark sector dynamics. Custodial SU(2) is the residual accidental global symmetry of the Higgs multiplet after it acquires an expectation value, $O(4) \cong \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to O(3) \cong \mathrm{SU}(2)_C$.

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Custodial SU(2) arises automatically once the matter content and interactions are (at least formally) promoted to become $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ invariant. We will use the terminology $\mathrm{SU}(2)_L$, $\mathrm{SU}(2)_R$ frequently in this paper and emphasize that this will always refer to internal symmetries of the theory and never to Lorentz symmetry. It will become very convenient to utilize a manifestly $\mathrm{SU}(2)_C$ symmetric formalism for writing interactions of the dark sector with the Higgs multiplet. The basic notions are well-known, though not necessarily exploited in the ways that we will be doing. A manifestly custodially $\mathrm{SU}(2)_C$ symmetric formalism promotes $\mathrm{U}(1)_Y$ to $\mathrm{SU}(2)_R$, where only the t_3 generator of $\mathrm{SU}(2)_R$ is gauged.

To establish notation, the Higgs doublet of the Standard Model

$$H = \begin{pmatrix} G^+ \\ (v + h + iG^0)/\sqrt{2} \end{pmatrix} , \tag{3.1}$$

can be re-expressed in terms of a $(\mathbf{2}, \mathbf{2})$ bifundamental scalar field under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ as

$$\mathcal{H}_{i_L i_R} = \frac{1}{\sqrt{2}} \begin{pmatrix} (v + h - iG^0)/\sqrt{2} & G^+ \\ -G^- & (v + h + iG^0)/\sqrt{2} \end{pmatrix} . \tag{3.2}$$

In principle, all custodially-symmetric interactions can be written in terms of powers of $\mathcal{H}$, and suitable $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ contractions. The notation becomes much more compact when we utilize the definition

$$\mathcal{H}^\dagger_{i_R i_L} \equiv \epsilon_{i_R j_R} \epsilon_{i_L j_L} \mathcal{H}_{i_L i_R} \tag{3.3}$$

which matches the naive complex conjugation and transpose of the 2×2 matrix definition in eq. (3.2). In this form, the Standard Model Higgs potential becomes simply

$$V = m_H^2 \mathrm{Tr}\, \mathcal{H}^\dagger \mathcal{H} + \frac{\lambda}{4} \left(\mathrm{Tr}\, \mathcal{H}^\dagger \mathcal{H} \right)^2 . \tag{3.4}$$

The absence of any explicit t_R^3 signals the absence of any explicit custodial symmetry violation. When the Higgs gets a vev and $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ breaks to the diagonal $\mathrm{SU}(2)_C$, the original $(\mathbf{2},\mathbf{2})$ of Higgs states decomposes into a singlet (radial mode) plus a triplet (Goldstones) of the diagonal $\mathrm{SU}(2)_C$.

The full covariant derivative for the Higgs multiplet, eq. (3.2), does not respect $\mathrm{SU}(2)_R$ due to gauging hypercharge, i.e., just the t_R^3 generator is gauged. This is straightforwardly handled by writing the covariant derivative as

$$D_\mu \mathcal{H}_{i_L i_R} = \partial_\mu \mathcal{H}_{i_L i_R} - i g W_\mu^a (t_L^a \mathcal{H})_{i_L i_R} - i g' B_\mu (\mathcal{H} t_R^3)_{i_L i_R} \tag{3.5}$$

making kinetic term of the bi-doublet $\mathcal{H}$:

$$\mathrm{Tr}\, D_\mu \mathcal{H}^\dagger D^\mu \mathcal{H} . \tag{3.6}$$

The explicit t_R^3 can be thought of as $2Y t_R^3$, where the Higgs doublet $H_{Y=1/2}$ and its complex conjugate $H^*_{Y=-1/2}$ are embedded as the two components of an $\mathrm{SU}(2)_R$ doublet. In the limit $g'Y \to 0$, the last term of eq. (3.5) vanishes, restoring the full $\mathrm{SU}(2)_R$ global symmetry.

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In this way, we see that $g'Yt_R^3$ acts as a spurion for custodial SU(2) violation. One could instead promote $B_\mu t_R^3 \to W_R^a t_R^a$, formally gauging the full $\mathrm{SU}(2)_R$ symmetry. In this case, we would need an explicit $\mathrm{SU}(2)_R$-breaking mass term in order to remove the $W_R^{1,2}$ gauge bosons and recover the Standard Model. Moreover, as is well-known from left-right models, an additional U(1) is required to obtain the correct hypercharge of the left-handed and right-handed quarks and leptons (e.g., for a review, see [117]).

Yukawa couplings are another source of custodial breaking. In terms of the usual Higgs doublet H, the up and down Yukawa couplings are

$$y_{ij}^u Q_{Li}\epsilon H u_{Rj} + y_{ij}^d Q_{Li} H^\dagger d_{Rj} + \text{h.c.} \tag{3.7}$$

Grouping $Q_{Ri} = \{u_i^c, d_i^c\}$ together, we can rewrite the up and down quark Yukawas in terms of $\mathcal{H}$ as

$$y_{ij}^u Q_{Li}\mathcal{H}\, P_u Q_{Rj} + y_{ij}^d Q_{Li}\mathcal{H}\, P_d Q_{Rj} + \text{h.c.} \tag{3.8}$$

where $P_{u,d} = (\mathbb{1}_R \mp 2t_R^3)/2$ are matrices in $\mathrm{SU}(2)_R$ space that project out the up-type or down-type right-handed fermion. In fact, it is useful to rewrite eq. (3.8) as the sum of a custodial symmetric Yukawa plus a custodial violating term:

$$\mathscr{L}_{\text{Yuk}} = \mathcal{Y}_{ij}^{C} Q_{Li}\mathcal{H}\frac{\mathbb{1}_R}{2} Q_{Rj} + \mathcal{Y}_{ij}^{\not{C}} Q_{Li}\mathcal{H} t_R^3 Q_{Rj} + \text{h.c.} \tag{3.9}$$

where

$$\begin{aligned} \mathcal{Y}_{ij}^{C} &= y_{ij}^u + y_{ij}^d \\ \mathcal{Y}_{ij}^{\not{C}} &= y_{ij}^u - y_{ij}^d\,. \end{aligned} \tag{3.10}$$

There is no loss of generality from the SM, i.e., $\mathcal{Y}_{ij}^{C}$ and $\mathcal{Y}_{ij}^{\not{C}}$ are independent matrices. In the special case where $y_{ij}^u = y_{ij}^d$ and thus $\mathcal{Y}_{ij}^{\not{C}} = 0$, the Yukawa couplings are custodially symmetric. Later in the paper when we write higher dimensional operators involving the SM fermions, we will always assume a form of minimal flavor-violation (MFV) where operators involving $Q_{Li}\mathcal{H}(\mathbb{1}_R/2)Q_{Rj}$ are accompanied by $\mathcal{Y}_{ij}^{C}$ and operators involving $Q_{Li}\mathcal{H}t_R^3 Q_{Rj}$ are accompanied by $\mathcal{Y}_{ij}^{\not{C}}$.

Looking beyond the SM, we will use the same logic we applied to SM Yukawas when writing down interactions between the dark fermions and the Higgs. Specifically, in addition to grouping dark fermions into multiplets of (gauged) $\mathrm{SU}(2)_W \equiv \mathrm{SU}(2)_L$, we also assign them to multiplets of $\mathrm{SU}(2)_R$ then classify interactions in $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ language. Put another way, interactions among the SM Higgs multiplet and dark fermions break the combination of the $\mathrm{SU}(2)_R$ Higgs potential symmetry and the $\mathrm{SU}(N_{\text{fund}})$ (or $\mathrm{SU}(N_{\text{anti}})$) flavor symmetries of the dark fermions down to a common SU(2), which we relabel as $\mathrm{SU}(2)_R$. New custodial violating breaking interactions/spurions must be proportional to t_R^3, as that is the only choice consistent with gauging $\mathrm{SU}(2)_L$ and the t^3 generator $[\mathrm{U}(1)_Y]$ of $\mathrm{SU}(2)_R$ [118]. Thus, in $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ language, strong sectors that respect custodial

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symmetry contain no terms with explicit t^3_R, while a generic custodial violating dark sector can have one or more such terms.[1]

4 Effective interactions of dark pions

The dark sectors of greatest interest to us in this paper preserve custodial SU(2), so all deviations from exact custodial symmetry can be traced to $g'Y$ or the differences among SM Yukawas. Consequently, dark pions transform in representations of $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$. Once the Higgs gets a vacuum expectation, these pions will break up into multiplets of custodial SU(2). The smallest, and therefore lightest, non-trivial $\mathrm{SU}(2)_C$ representation the pions can fill is the triplet. Heavier dark pions in larger representations are possible, as are higher spin composites such spin-1 dark rho mesons. In general, these states rapidly decay into the lightest dark pions. While this is certainly highly relevant for phenomenology [88, 115], it the lightest dark pion decays that are the main concern for this paper.

4.1 Dark pion triplet interactions in custodial preserving strong sectors

Suppose we have an $\mathrm{SU}(2)_C$ triplet of dark pions π^a, that we have already motivated as arising in a wide class of interesting class of dark sector theories, and we wish to understand its interactions. The most phenomenologically relevant interactions to determine are those with a single dark pion since they will govern decays. As we will show below, single pion interactions can be understood from symmetry considerations alone.

First, let's consider a "toy" Standard Model that is fully $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ symmetric — meaning we set $g'=0$ and $\mathcal{Y}^{\not C}_{ij}=0$, in the presence of a dark sector that produces a (custodially symmetric) triplet of dark pions. In this limit, the $\{u^c, d^c\}$ quarks of the SM can be written in terms of a $\mathrm{SU}(2)_R$ doublet as in eq. (3.8) and the $\mathrm{SU}(2)_W$ gauge bosons lie in a $\mathrm{SU}(2)_L$ triplet. When EWSB occurs, $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\to\mathrm{SU}(2)_C$, so we can reclassify all fields into $\mathrm{SU}(2)_C$ multiplets and form invariants from them. Contracting π^a with $\mathrm{SU}(2)_C$ triplets formed from SM fields, the lowest dimension operators involving a single dark pion are:

$$\mathcal{Y}^C_{ij}\left(\frac{v}{v_\pi}\right)\pi^a\left(Q_{Li}t^aQ_{Rj}\right)+\xi\,g\left(\frac{v}{v_\pi}\right)W^a_\mu\left(h\overleftrightarrow{\partial}^\mu\pi^a\right), \tag{4.1}$$

where t^a are the generators of $\mathrm{SU}(2)_C$. (A similar expression for the first term is also present for the leptons of the SM.) As both terms require electroweak symmetry breaking, they must be proportional to the mass of the SM fields. Therefore, we need another dimensionful parameter v_π to balance dimensions. For the fermion terms, we have assumed the flavor structure obeys minimal flavor violation with a (lowest order) coefficient of $\mathcal{Y}^C_{ij}$. The factor ξ parameterizes the relative strength between the interactions of pions with fermions versus the gauge/Higgs sector. We will explore the size and origin of v_π and ξ in specific theories shortly. The presence of the Higgs boson in the second term is also easy to motivate. While

[1] Additionally, dark sector theories with $\mathrm{SU}(2)_L$ multiplets with hypercharge, as well as $\mathrm{SU}(2)_R$ multiplets with hypercharge not proportional to t^3_R, require an additional $\mathrm{U}(1)_X$. We do not consider such theories in this paper.

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$W^a_\mu \partial_\mu \pi^a$ is custodially symmetric, by itself this is a mixing involving longitudinal W^a and π^a would indicate that we have not properly gauge fixed. Hence, we need to add a $\mathrm{SU}(2)_C$ singlet, and h is the option with the lowest dimension in the broken phase of the SM.

One might wonder how ξ could be different from unity, given what we have described thus far. When dark pions transform in representations that are larger than a triplet, there is a possibility of dark pion-Higgs boson mixing. For example, dark pions in the complex representation $(\mathbf{2}, \mathbf{2})$ under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ contain a "Higgs-like" dark pion state ($\mathrm{SU}(2)_C$ singlet) that can — and generically does — mix with the SM ($\mathrm{SU}(2)_C$ singlet) Higgs boson. This implies additional contributions to the gauge/Higgs boson/dark pion interactions arise from the covariant derivative of the dark pions. These interactions turn out to be *critical* to understanding the phenomenology of models with more than two flavors of dark fermions.

Let's now re-introduce the custodial SU(2) violation in the SM. This involves the difference between up and down Yukawas,

$$\mathcal{Y}^{\not{C}}_{ij} \left(\frac{v}{v_\pi} \right) \pi^a \left(Q_{L i} t^a t^3_R Q_{R j} \right) \tag{4.2}$$

as well as $g' \neq 0$,

$$\xi \, g' \left(\frac{v}{v_\pi} \right) B_\mu \left(h \overleftrightarrow{\partial}^\mu \pi^0 \right) . \tag{4.3}$$

With these terms, the simple lagrangian eq. (4.1) becomes somewhat more complicated. If we focus our attention on just one generation of quarks, and convert from two-component fermions to four-component notation, the effective lagrangian for dark pion decay becomes:

$$\begin{aligned} \mathscr{L}_{\text{decay}} = \frac{\sqrt{2}}{v_\pi} \Bigg[& {\pi_D}^+ \bar{\psi}_u (m_d P_R - m_u P_L) \psi_d + {\pi_D}^- \bar{\psi}_d (m_d P_L - m_u P_R) \psi_u \\ & + \frac{i}{\sqrt{2}} {\pi_D}^0 (m_u \, \bar{\psi}_u \gamma_5 \psi_u - m_d \, \bar{\psi}_d \gamma_5 \psi_d) \Bigg] \\ - \xi \frac{m_W}{v_\pi} \Bigg[& (W^-_\mu \, h \overleftrightarrow{\partial}^\mu {\pi_D}^+) + (W^+_\mu \, h \overleftrightarrow{\partial}^\mu {\pi_D}^-) + \frac{1}{\cos\theta_W} (Z_\mu \, h \overleftrightarrow{\partial}^\mu {\pi_D}^0) \Bigg] . \end{aligned} \tag{4.4}$$

The effective theories of dark mesons that we consider below will give specific predictions for these couplings. We find two qualitatively distinct possibilities:

$$\begin{aligned} \xi &\sim 1 \quad \text{"gaugephilic"} \\ \xi &\ll 1 \quad \text{"gaugephobic"} . \end{aligned} \tag{4.5}$$

Equation (4.4), which has been argued purely from custodial symmetry and assumptions about the most relevant connections between the dark sector and the SM, is our first main result. The main purpose of the rest of this paper is to determine how dark pion interactions in different dark sector theories with (or without) custodial $\mathrm{SU}(2)_C$ map into eq. (4.4) and, especially, whether they fall into the gaugephilic or gaugephobic category.

Before jumping head first into strongly-coupled dark sectors, the interactions of a custodial SU(2) triplet given in eq. (4.4) are perhaps most familiar from two-Higgs doublet models. We take a brief look at this in the next section, leaving a detailed discussion to appendix A.

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4.2 Two-Higgs doublet models

As a point of reference, it is helpful to consider the couplings of $(H^{\pm}, A^0)$ in two-Higgs doublet models (2HDMs). The couplings to the fermions are model-dependent; for illustration here let's consider the so-called Type I 2HDM where the fermions couple to just one Higgs doublet as that is the 2HDM setup that most closely resembles the our dark pion theories. In Type I 2HDM theory, one obtains [119]

$$\frac{1}{v_\pi} = \frac{1}{v}\cot\beta$$

where we have neglected the CKM mixing for the charged Higgs couplings. For the gauge/Higgs sector,

$$\frac{\xi}{v_\pi} = \frac{1}{v}\cos(\beta-\alpha).$$

Here, $\cot\beta$ is the usual ratio of the expectation values in two Higgs doublets. In the decoupling limit, the coupling to gauge/Higgs boson is well-known to scale as [120]

$$|\cos(\beta-\alpha)| \sim \frac{v^2}{m_A^2}\,. \tag{4.6}$$

Since the coupling of fermions does not have a similar scaling, we see that 2HDMs are gaugephobic regardless of the Type of 2HDM.

There is an interesting story about utilizing the custodially-symmetric basis for 2HDMs. In the decoupling limit a 2HDM becomes custodially symmetric, and the decays of its heavy states $(H^{\pm}, A^0)$ to SM particles in this limit are gaugephobic. Details are presented in appendix A.

4.3 Neutral dark pion decay to diphotons

Finally, it is interesting to discuss the coupling of π^0 to $\gamma\gamma$. The usual axial anomaly contribution to this decay mode, $\pi^0 F_{\mu\nu}\widetilde{F}^{\mu\nu}/f$, whose leading contribution is proportional to $\mathrm{Tr}\,Q^2 t_a^3$ [where t_a^3 is the generator of the axial U(1)] is conspicuously absent from eq. (4.4). The reason for this is that in a dark sector where the $\mathrm{SU}(N)_V$, preserved by strong interactions, is an exact symmetry, this contribution must vanish. For example, in a two-flavor dark sector, invariance under an exact $\mathrm{SU}(2)_V$ would enforce the two flavors of dark fermion masses are equal. Gauging the full $\mathrm{SU}(2)_V$ [as in $\mathrm{SU}(2)_L$] or just the t^3 subgroup [as in $\mathrm{U}(1)_Y$] implies the dark fermion electric charges are equal and opposite. In this case, $\mathrm{Tr}\,Q^2 t_a^3$ vanishes, as do higher order $\pi_D^0\gamma\gamma$ operators proportional to the differences of dark fermion masses.

Nevertheless, there is a very small, residual contribution to $\pi_D^0 \to \gamma\gamma$, due to the interactions with the SM in eq. (4.4). That is, even though custodial SU(2) is preserved by the dark sector interactions with the SM, the SM itself violates custodial SU(2). The dark pion interactions with the SM fermion axial current generate a one-loop suppressed π_D^0-γ-γ coupling proportional to $m_f/(16\pi^2 v_\pi)$. We can calculate the amplitude for the

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rate by borrowing the standard results for A^0 decay in two-Higgs doublet models [121] and suitably substituting couplings:

$$\mathcal{A}(\pi_D^0 \to \gamma\gamma) = \sum_f \frac{\alpha}{4\pi} N_c Q_f^2 \left(\frac{m_f}{v_\pi}\right) \sqrt{\tau_f} f(\tau_f) \tag{4.7}$$

where N_c is the number of colors, Q_f is the electric charge, $\tau_f = 4m_f^2/m_\pi^2$, and

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4}\left[\log \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & \tau < 1\,. \end{cases} \tag{4.8}$$

In the limit $\tau \ll 1$,

$$f(\tau) \to -(1/4)[\log(4/\tau) - i\pi]^2\,, \tag{4.9}$$

and thus we see an additional suppression of the π_D^0 decay amplitude of roughly $\sqrt{\tau_f} = 2m_f/m_{\pi_D}$ when $m_{\pi_D} \gg m_f$ (neglecting the τ dependence of the log). Hence, while there is π_D^0 decay to $\gamma\gamma$ due to the custodial SU(2) breaking in the SM, the decay rate is suppressed by roughly $\alpha^2/(16\pi^2) \times (4m_f^2/m_{\pi_D}^2)$ that is $\simeq 10^{-6} \times m_f^2/m_{\pi_D}^2$ smaller than the direct decay to fermions. This is so small as to be phenomenologically irrelevant.

5 Two-flavor theories

The simplest anomaly-free dark sector theories that we consider have two flavors of dark fermions. We refer to the dark color fundamentals as F_i and anti-fundamentals as $\hat{F}_i$ transforming under SU(N_D) with flavor index $i = 1, 2$. The global symmetry of the flavors is $\mathrm{SU}(2)_{\mathrm{fund}} \times \mathrm{SU}(2)_{\mathrm{anti}}$. Once we include interactions between the dark fermions and the Higgs multiplet, we will be forced to connect the fermion flavor symmetries to the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry of the Higgs potential. This connection can be made in a few different ways, two vector-like and one chiral. In the vector-like assignments, both F_i and $\hat{F}_i$ must be doublets of the same SU(2) — either $\mathrm{SU}(2)_L$ or $\mathrm{SU}(2)_R$, while in the chiral assignment, F and $\hat{F}$ transform under different SU(2)s.[2] However, in all of these cases, $\mathrm{SU}(2)_{\mathrm{fund}} \times \mathrm{SU}(2)_{\mathrm{anti}}$ is broken to the diagonal $\mathrm{SU}(2)_V$ by strong dynamics, just as in two-flavor QCD. Also just like QCD, the dark pions form a triplet of the diagonal $\mathrm{SU}(2)_V$, which ultimately becomes (π^+, π^0, π^-) after electroweak breaking down to just $\mathrm{U}(1)_{\mathrm{em}}$. This is the custodial SU(2) symmetric triplet that we discussed in the previous section.

Prior to electroweak breaking scale, all three pions $\pi^\pm, \pi^0$ are stable. Once electroweak symmetry is broken, electromagnetic corrections split the multiplets by [122]

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{(3\ln 2)}{2\pi}\,\alpha\, m_\rho^2 \tag{5.1}$$

where α is the electroweak coupling constant, and m_ρ is the mass of the vector resonances of the dark sector. This mass splitting allows the weak decay of $\pi^\pm \to \pi^0 \bar{f}' f$. Whether this decay is competitive (or not) with direct decays $\pi \to$ SM will depend on the π-SM-SM coupling strength proportional to $1/v_\pi$ in the effective theory.

We now consider each of these theories in turn.

[2] Anomaly cancellation requires N_D to be even for the chiral case.

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Field	$(\mathrm{SU}(N_D), \mathrm{SU}(2)_L, \mathrm{SU}(2)_R)$
F	$(\mathbf{N}, \mathbf{2}, \mathbf{1})$
$\hat{F}$	$(\overline{\mathbf{N}}, \mathbf{1}, \mathbf{2})$

Table 1. Two-flavor fermion content of the chiral theory.

5.1 Two-flavor chiral theory

The two-flavor chiral theory contains the matter content in table 1. $\mathrm{SU}(2)_L$ is embedded as $\mathrm{SU}(2)_{\rm fund}$ while $\mathrm{U}(1)_Y$ is the t^3 generator of $\mathrm{SU}(2)_{\rm anti}$. Confinement breaks the global symmetry to $\mathrm{SU}(2)_V$, of which only the gauged $\mathrm{U}(1)_{\rm em}$ survives.

Identifying the flavor symmetries $\mathrm{SU}(2)_{\rm fund}, \mathrm{SU}(2)_{\rm anti}$ with $\mathrm{SU}(2)_L, \mathrm{SU}(2)_R$ respectively, we can write a Yukawa interaction between the Higgs bi-doublet and the dark fermions

$$\mathscr{Y}_{\rm Yuk} = yF\mathcal{H}\hat{F} + \text{h.c.}\,. \tag{5.2}$$

Once the Higgs acquires a vev, this will give gives equal contributions to the masses of the "up-type" and "down-type" dark fermions. In the absence of a fundamental Higgs, this theory is minimal technicolor. Including the Higgs (and Yukawa coupling), the two-flavor chiral theory dynamics "induces" electroweak symmetry breaking even when the Higgs multiplet $(\text{mass})^2$ is positive. This theory is better known as bosonic technicolor [1, 2] or strongly-coupled induced electroweak symmetry breaking [10, 11].

Now that we have established how dark fermions transform under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ we can consider a more general set of interactions that arise with higher dimensional operators. These terms can involve more Higgs fields, derivatives, SM quarks and/or leptons. Examples at dimension-6 include:

$$c_{6A}\,\frac{(F\,\hat{F})(Q_L\hat{Q}_R)}{\Lambda^2},\; c_{6B}\,\frac{(F^\dagger\bar{\sigma}^\mu\,F)(\mathcal{H}\,D_\mu\mathcal{H})}{\Lambda^2},\; c_{6C}\,\frac{(F^\dagger\bar{\sigma}^\mu\,t^a_L\,F)(\mathcal{H}\,t^a_L\,D_\mu\mathcal{H})}{\Lambda^2},\cdots \tag{5.3}$$

where t^a_L are the generators of $\mathrm{SU}(2)_L$ that pick out the triplet combination of the two doublets. We will use t^a_L and the $\mathrm{SU}(2)_R$ counterpart t^a_R throughout this paper.

The translation to the NLSM involves

$$F\hat{F} \to 4\pi f^3\Sigma, \quad \Sigma = \exp\left[i\frac{2\pi^a t^a}{f}\right]. \tag{5.4}$$

The covariant derivative acts on Σ identically to the Higgs bi-doublet, eq. (3.5), leading to interactions of the dark pions with the electroweak gauge bosons. While there is a systematic way to transmute interactions between a strong, chiral symmetry breaking sector and external fields into interactions involving pNGBs [123, 124], we do not need the full machinery since we are interested in the additional (higher dimensional) terms in the dark sector chiral lagrangian that, after expanding Σ, involve a single power of π^a. This criteria selects out operators whose dark sector components are i.) Lorentz invariant, as we want operators with π_D, not $\partial_\mu\pi_D$, and ii.) that transform non-trivially under $\mathrm{SU}(2)_C$ — as discussed in section 4.1, the dark pion decay terms involve connecting $\mathrm{SU}(2)_C$ triplets

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in the strong sector with SM $\mathrm{SU}(2)_C$ triplets. In the chiral case, these criteria tell us to ignore operators containing $F^\dagger \bar{\sigma}^\mu F$ (inert under $\mathrm{SU}(2)_C$ and not a Lorentz invariant) in favor of operators containing $F\hat{F}$.

Performing the translation to pNGB form and focusing on the most relevant interactions between the dark fermions and the Higgs/SM, the theory becomes

$$\begin{aligned}\mathscr{L} = &\frac{f^2}{4}\mathrm{Tr}\,(D_\mu\Sigma)^\dagger D^\mu\Sigma \\ &+ 4\pi f^3 y\,\mathrm{Tr}\,(\mathcal{H}\Sigma^\dagger + \mathrm{h.c.}) + \text{higher dimensional terms}\,,\end{aligned} \tag{5.5}$$

where Σ contains the triplet of dark pions and we have written only the leading terms in the chiral lagrangian relevant to our discussion below. Here "higher dimensional" refers to operators such as eq. (5.3) that are non-renormalizable when written in the UV, in terms of the underlying dark fermions. We assume $4\pi f \ll \Lambda$ throughout our discussion of this model, so the higher dimensional operators are subdominant, and so we can ignore them for now. But as we will see in later sections, in other models they are vital to connect the dark sector to the SM.

With the pNGB description of the theory in hand, we can now work out how these Σ interactions map into interactions among dark pions to SM fields in eq. (4.4). The term linear in Σ expresses the explicit chiral symmetry breaking that arises from the Yukawa interactions. Expanding the linear term out to quadratic order in the dark pion fields,

$$4\pi f^3 y\,\mathrm{Tr}\,(\mathcal{H}\Sigma^\dagger + \mathrm{h.c.}) \supset 8\pi f^2 y\, G^a\pi^a + 4\pi f y v\,\pi^a\pi^a\,, \tag{5.6}$$

we see the dark pions acquire masses $m_\pi^2 = 8\pi f y v$ and mixing between the would-be Goldstones of the Higgs doublet and the triplet of dark pion fields. The Goldstone-dark pion mixing is independent of a, as it should be given the custodially-symmetric origin of the Yukawa couplings.

Defining the physical pions and Goldstones as

$$\begin{pmatrix} G^a_{\mathrm{phys}} \\ \pi^a_{\mathrm{phys}} \end{pmatrix} = V \begin{pmatrix} G^a \\ \pi^a \end{pmatrix} \tag{5.7}$$

where the mixing angle is determined by diagonalizing the mass matrix

$$M^2_{\mathrm{diag}} = V M^2 V^T \tag{5.8}$$

with

$$M^2 = \begin{pmatrix} 8\pi f^3 y/v & 8\pi f^2 y \\ 8\pi f^2 y & 8\pi f y v \end{pmatrix}, \quad V = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \tag{5.9}$$

and $\theta = \arctan(f/v)$ is the mixing angle. The nonzero entry for Goldstone part of the mass matrix (G^aG^a) arises after minimizing the Higgs potential to include the contributions from the dark sector (see [14] for details). Inserting the diagonalized eigenstates back into the Lagrangian leads to a shift of the electroweak vev

$$v^2 + f^2 = v_{246}^2 \simeq (246\,\mathrm{GeV})^2\,. \tag{5.10}$$

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This leads to well-known corrections to Higgs couplings [14]. For our purposes, the couplings of the physical pions to $\bar{f}f$, Zh and $\bar{f}'f$, Wh become

$$\begin{aligned}
(\pi^{\pm}_{\rm phys}\partial_\mu h - h\partial_\mu \pi^{\pm}_{\rm phys})W^{\mu,\mp} &: \; \frac{M_W}{v} s_\theta \\
(\pi^{0}_{\rm phys}\partial_\mu h - h\partial_\mu \pi^{0}_{\rm phys})Z^{\mu} &: \; \frac{M_Z}{v} s_\theta \\
\pi^{\pm}_{\rm phys}\bar{f}'f &: \; \sqrt{2}\left(\frac{m_{f'}}{v}P_L - \frac{m_f}{v}P_R\right)(2T_3^f)\, s_\theta \\
\pi^{0}_{\rm phys}\bar{f}f &: \; i\left(\frac{m_f}{v}\gamma_5\right)(2T_3^f)\, s_\theta
\end{aligned} \tag{5.11}$$

where $2T_3^f = \pm 1$ is the isospin of the fermion. The mixing angle is

$$s_\theta = \frac{f}{v_{246}}\,. \tag{5.12}$$

We see that custodially-symmetric two-flavor chiral theories have couplings to fermions and gauge bosons that are parametrically comparable — $M_{W,Z}$ versus m_f. From the couplings we can identify

$$\frac{1}{v_\pi} \simeq \frac{1}{v} \times \left(\frac{f}{v_{246}}\right), \qquad \xi = 1 \tag{5.13}$$

so the couplings are "gaugephilic" according to eq. (4.4). While this provides an excellent example of "gaugephilic" dark pion interactions, there is no way to formally separate the Goldstone/pion mixing from the dark pion mass itself — both are proportional to the Yukawa coupling y. Consequently, there is no limit where the mixing between the Goldstone and the dark pion can be taken small while simultaneously holding the dark pion mass fixed.

We should emphasize that in the two-flavor chiral model we arrive at eq. (4.4) through the mixing of the dark pions with the triplet of Goldstone bosons. This mixing was possible only because of the Yukawa term, which is the only allowed renormalizable coupling. Had we included the higher dimensional terms in the chiral lagrangian, we would find that they can still be parameterized by the effective lagrangian eq. (4.4). In two-flavor vector-like models, which we explore next, the dark pion-Goldstone mixing is not present, however we will still recover eq. (4.4).

Finally, the absence of π^0-γ-γ coupling critically relied on the renormalizable coupling between the dark sector and the SM, eq. (5.2), being custodially symmetric. If there had been an explicit custodial violation of the dark sector with Higgs multiplet, e.g., $y^{\not{C}} F\mathcal{H}t_R^3\hat{F}$, the pions would acquire different masses as well as different mixings with the Goldstones. This would re-introduce $\pi^0 \to \gamma\gamma$ and a more detailed calculation would be needed to determine the branching fractions of π^0.

5.2 Two-flavor vector-like theories

Vector-like confinement [88] popularized the possibility that a new strong sector contains fermions in vector-like representations so that contributions to electroweak precision corrections are negligible, and (bare) vector-like masses for the dark fermions are allowed.

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$\mathrm{SU}(2)_L$ model	
Field	$(\mathrm{SU}(N_D), \mathrm{SU}(2)_L, \mathrm{SU}(2)_R)$
F	$(\mathbf{N}, \mathbf{2}, \mathbf{1})$
$\hat{F}$	$(\overline{\mathbf{N}}, \mathbf{2}, \mathbf{1})$

$\mathrm{SU}(2)_R$ model	
Field	$(\mathrm{SU}(N_D), \mathrm{SU}(2)_L, \mathrm{SU}(2)_R)$
F	$(\mathbf{N}, \mathbf{1}, \mathbf{2})$
$\hat{F}$	$(\overline{\mathbf{N}}, \mathbf{1}, \mathbf{2})$

Table 2. Two-flavor fermion content of $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ vector-like theories.

There are two versions of two-flavor vector-like theories, shown in table 2, depending on whether the dark fermions transform under just $\mathrm{SU}(2)_L$ or just $\mathrm{SU}(2)_R$. We will refer to these as the "$\mathrm{SU}(2)_L$ model" and "$\mathrm{SU}(2)_R$ model", respectively.

Vector-like theories permit dark fermion masses,

$$\mathscr{L}_{\text{mass}} = M F \hat{F} + \text{h.c.}\,. \tag{5.14}$$

The global $\mathrm{SU}(2)_{\text{fund}} \times \mathrm{SU}(2)_{\text{anti}}$ symmetries are broken to $\mathrm{SU}(2)_V$ that is identified either with the fully gauged $\mathrm{SU}(2)_L$ or $\mathrm{SU}(2)_R$ (with, as usual, just $\mathrm{U}(1)_Y$ gauged).

Now we begin to add interactions between the dark fermions and the SM fields, working in a $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ invariant manner. Unlike the two-flavor chiral model, in the vector-like models we cannot write a renormalizable interaction between $F, \hat{F}$ and $\mathcal{H}$. To write down interactions between the Higgs and the dark fermions, we need to consider higher dimensional operators. Interactions involving two Higgs bifundamentals lead to the group contractions

$$(\mathbf{2}_L, \mathbf{2}_R) \otimes (\mathbf{2}_L, \mathbf{2}_R) = (\mathbf{1}_L, \mathbf{1}_R) \oplus (\mathbf{3}_L, \mathbf{3}_R) \tag{5.15}$$

where the surviving combinations are $\mathrm{Tr}\,\mathcal{H}^\dagger \mathcal{H}$ and $\mathrm{Tr}\,\mathcal{H}^\dagger t_L^a \mathcal{H} t_R^{a'}$. The would-be $(\mathbf{3}_L, \mathbf{1}_R)$ or $(\mathbf{1}_L, \mathbf{3}_R)$ involve $\mathrm{Tr}\,\mathcal{H}^\dagger t_L^a \mathcal{H}$ or $\mathrm{Tr}\,\mathcal{H}^\dagger \mathcal{H} t_R^a$ that both exactly vanish for any a. The permitted fermion contractions are either pure singlet $(F\hat{F})$, $\mathrm{SU}(2)_L$-triplet $(F t_L^a \hat{F})$ in the $\mathrm{SU}(2)_L$ model, or $\mathrm{SU}(2)_R$-triplet $(F t_R^a \hat{F})$ in the $\mathrm{SU}(2)_R$ model. From these Higgs and fermion field bilinears, the only operator at dimension-5, in both models, is the "singlet" contribution

$$c_{5M} \frac{(F\hat{F})\,\mathrm{Tr}\,\mathcal{H}^\dagger \mathcal{H}}{\Lambda} + \text{h.c.} \tag{5.16}$$

After electroweak symmetry breaking, this operator leads to a $\sim v^2/\Lambda$ contribution to the dark fermion masses but does not influence their decays. This is because the first non-zero interactions arising from expanding out eq. (5.16) must contain a singlet, i.e., at least two dark pions.

Hence, to find an operator contributing to dark pion decay we need to go beyond dimension-5 in order to involve a non-singlet contraction of F and $\hat{F}$. In the case of the $\mathrm{SU}(2)_L$ model, this is $F t_L^a \hat{F}$. In the case of the $\mathrm{SU}(2)_R$ model, invariance under the full $\mathrm{SU}(2)_R$ allows just $F t_R^a \hat{F}$. Of course given that just $\mathrm{U}(1)_Y$ is gauged, the term $F t_R^3 \hat{F}$ is gauge-invariant but not $\mathrm{SU}(2)_R$ invariant. If we insist that the dark sector preserves custodial $\mathrm{SU}(2)$, this combination is forbidden.

In the Standard Model, there are no dimension-3 operators of the form $Q t_L^a Q'$ since, of course, the SM fermions transform under a chiral representation of the electroweak group.

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By dimension-4 we can write, e.g., $Q_L t^a_L \mathcal{H} \hat{Q}_R$, which can be combined with the $F t^a_{L,R} \hat{F}$ from the dark sector to obtain dimension-7 operators including:

$$
\begin{aligned}
\mathrm{SU}(2)_L \text{ model}: \qquad & \mathscr{L} = \mathcal{Y}^C_{ij} \frac{(F t^a_L \hat{F}) \left(Q_{Li} t^a_L \mathcal{H} \frac{\mathbb{1}_R}{2} \hat{Q}_{Rj} \right)}{\Lambda^3} + \mathcal{Y}^{\not{C}}_{ij} \frac{(F t^a_L \hat{F})(Q_{Li} t^a_L \mathcal{H} t^3_R \hat{Q}_{Rj})}{\Lambda^3} \,, \\
\mathrm{SU}(2)_R \text{ model}: \qquad & \mathscr{L} = \mathcal{Y}^C_{ij} \frac{(F t^a_R \hat{F})(Q_{Li} \mathcal{H} t^a_R \hat{Q}_{Rj})}{\Lambda^3} + \mathcal{Y}^{\not{C}}_{ij} \frac{(F t^a_R \hat{F})(Q_{Li} \mathcal{H} t^a_R t^3_R \hat{Q}_{Rj})}{\Lambda^3} \,.
\end{aligned}
\tag{5.17}
$$

As we discussed in section 3, we have included the SM Yukawa couplings as coefficients to these operators in order to maintain minimal flavor violation.

Focusing on just one generation of SM fermions, these dimension-7 operators become

$$
\begin{aligned}
\mathrm{SU}(2)_L \text{ model}: \qquad \mathscr{L} &= c_{7f} \frac{(F t^a_L \hat{F})(Q_L t^a_L \mathcal{H} Y_{ud} \hat{Q}_R)}{\Lambda^3} \,, \\
\mathrm{SU}(2)_R \text{ model}: \qquad \mathscr{L} &= c_{7f} \frac{(F t^a_R \hat{F})(Q_L \mathcal{H} t^a_R Y_{ud} \hat{Q}_R)}{\Lambda^3} \,.
\end{aligned}
\tag{5.18}
$$

where Y_{ud} is a 2×2 matrix in $\mathrm{SU}(2)_R$ space with the form $Y_{ud} = (y_u + y_d)\mathbb{1}_R/2 + (y_u - y_d)t^3_R$. After electroweak symmetry breaking, this operator mixes $(F t^a_{L,R} \hat{F})$ with a triplet combination of SM fermions $(y_d u_L d^c_R,\ y_u u_L u^c_L - y_d d_L d^c_L,\ y_u d_L u^c_L)$. Passing to the non-linear sigma model formalism, the dimension-7 operator becomes

$$
\begin{aligned}
\mathrm{SU}(2)_L \text{ model}: \qquad \mathscr{L} &= c_{7f} \frac{4\pi f^3}{\Lambda^3} \left(\mathrm{Tr} \Sigma_L t^a_L \right) Q_L t^a_L \mathcal{H} Y_{ud} \hat{Q}_R \\
\mathrm{SU}(2)_R \text{ model}: \qquad \mathscr{L} &= c_{7f} \frac{4\pi f^3}{\Lambda^3} \left(\mathrm{Tr} \Sigma_R t^a_R \right) Q_L \mathcal{H} t^a_R Y_{ud} \hat{Q}_R \,,
\end{aligned}
\tag{5.19}
$$

where $\Sigma_{L,R}$ is in terms of the $\mathrm{SU}(2)_{L,R}$ generators, $\Sigma_{L,R} = \exp[i 2\pi^a t^a_{L,R}/f]$. Notice that Σ_L transforms as an adjoint under the $\mathrm{SU}(2)_V$ [that is fully gauged as $\mathrm{SU}(2)_L$], hence the combination $\mathrm{Tr}\Sigma_L t^a_L$ expands to π^a/f to leading order in π^a. Using this expansion, we obtain the interactions:

$$
\begin{aligned}
\pi^{\pm}_{\mathrm{phys}} \bar{f}' f: &\quad \sqrt{2}\,(m_{f'} P_L - m_f P_R)(2T^f_3) \times \left(c_{7f} \frac{\sqrt{2}\pi f^2}{\Lambda^3} \right) \\
\pi^0_{\mathrm{phys}} \bar{f} f: &\quad i\,(m_f \gamma_5)(2T^f_3) \times \left(c_{7f} \frac{\sqrt{2}\pi f^2}{\Lambda^3} \right).
\end{aligned}
\tag{5.20}
$$

From this we can identify

$$
\frac{1}{v_\pi} = c_{7f} \frac{\sqrt{2}\pi f^2}{\Lambda^3} \,. \tag{5.21}
$$

Notice that the interactions are otherwise identical regardless of whether the underlying theory is $\mathrm{SU}(2)_L$ or $\mathrm{SU}(2)_R$.

If we extend the effective theory to even higher dimension operators, we encounter operators involving the triplet combination $F t^a_{L,R} \hat{F}$ with the Higgs multiplet. The lowest

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dimension operator involving $Ft^a_{L,R}\hat{F}$ and custodially symmetric contractions of powers of $\mathcal{H}$ occurs at dimension-9:

$$\begin{aligned}
\mathrm{SU}(2)_L \text{ model}: \qquad \mathscr{L} &= c_{9C}\epsilon_{abc}\delta_{de}\frac{(Ft^a_L\hat{F})\mathrm{Tr}\left[(D_\mu\mathcal{H})^\dagger t^b_L(D^\mu\mathcal{H})t^d_R\mathcal{H}^\dagger t^c_L\mathcal{H}t^e_R\right]}{\Lambda^5} \\
\mathrm{SU}(2)_R \text{ model}: \qquad \mathscr{L} &= c_{9C}\epsilon_{abc}\delta_{de}\frac{(Ft^a_R\hat{F})\mathrm{Tr}\left[(D_\mu\mathcal{H})^\dagger t^d_L(D^\mu\mathcal{H})t^b_R\mathcal{H}^\dagger t^e_L\mathcal{H}t^c_R\right]}{\Lambda^5}\,. \qquad (5.22)
\end{aligned}$$

Passing to the low energy effective theory, the non-linear sigma model acquires the same kinetic and mass terms as in eq. (7.2) with an interaction term

$$\begin{aligned}
\mathrm{SU}(2)_L \text{ model}: \quad \mathscr{L} &= c_{9C}\frac{4\pi f^3}{\Lambda^5}\epsilon_{abc}\delta_{de}\mathrm{Tr}\left[\Sigma_L t^a_L\right]\mathrm{Tr}\left[(D_\mu\mathcal{H})^\dagger t^b_L(D^\mu\mathcal{H})t^d_R\mathcal{H}^\dagger t^c_L\mathcal{H}t^e_R\right] \\
\mathrm{SU}(2)_R \text{ model}: \quad \mathscr{L} &= c_{9C}\frac{4\pi f^3}{\Lambda^5}\epsilon_{abc}\delta_{de}\mathrm{Tr}\left[\Sigma_R t^a_R\right]\mathrm{Tr}\left[(D_\mu\mathcal{H})^\dagger t^d_L(D^\mu\mathcal{H})t^b_R\mathcal{H}^\dagger t^e_L\mathcal{H}t^c_R\right] \quad (5.23)
\end{aligned}$$

where $\Sigma_{L,R}$ is as before. Expanding the interaction in unitary gauge to leading order in π^a we obtain:

$$\mathscr{L} \;=\; c_{9C}\frac{\pi f^2}{16\Lambda^5}(v+h)^3\left[gW^\mp_\mu\left(\pi^\pm\partial^\mu h - h\partial^\mu\pi^\pm\right) + \sqrt{g^2+g'^2}Z_\mu\left(\pi^0\partial^\mu h - h\partial^\mu\pi^0\right)\right] \qquad (5.24)$$

and thus the couplings are

$$\begin{aligned}
(\pi^\pm_{\rm phys}\partial_\mu h - h\partial_\mu\pi^\pm_{\rm phys})W^{\mu,\mp}:\; M_W \times \left(c_{9C}\frac{\pi f^2}{\Lambda^3}\right)\times\left(\frac{v^2}{8\Lambda^2}\right) \\
(\pi^0_{\rm phys}\partial_\mu h - h\partial_\mu\pi^0_{\rm phys})Z^\mu:\; M_Z \times \left(c_{9C}\frac{\pi f^2}{\Lambda^3}\right)\times\left(\frac{v^2}{8\Lambda^2}\right). \qquad (5.25)
\end{aligned}$$

Compare to the fermion couplings, we then obtain

$$\frac{1}{v_\pi} = c_{7f}\frac{\sqrt{2}\pi f^2}{\Lambda^3}\,, \qquad \xi = \left(\frac{c_{9C}}{c_{7f}}\right)\times\left(\frac{v^2}{8\sqrt{2}\Lambda^2}\right). \qquad (5.26)$$

The single dark pion interactions with the Standard Model can be precisely characterized by the effective lagrangian eq. (4.4). Unlike the two-flavor chiral model, in the vector-like models there is no Goldstone/dark pion mixing connecting the dark sector with the Standard Model. Instead, this is fully characterized by the higher dimensional interactions that, by assumption, preserve custodial SU(2).

Notice also that the coefficient of the π-V-h interaction is *suppressed* relative to the π-f-f interaction by an amount $\xi \propto v^2/\Lambda^2$. In this particular model, the suppression arises because custodial symmetry demanded that operators involving the Higgs multiplets appear at a dimension that is *two powers* higher than that for SM fermions. Thus, dark pions preferentially interact with (and ultimately decay primarily to) SM fermions — these theories are gaugephobic — in two-flavor, vector-like, custodially-preserving dark sector theories.

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Field	$(\mathrm{SU}(N_D), \mathrm{SU}(2)_L, \mathrm{SU}(2)_R)$
F_L	$(\mathbf{N}, \mathbf{2}, \mathbf{1})$
$\hat{F}_L$	$(\overline{\mathbf{N}}, \mathbf{2}, \mathbf{1})$
F_R	$(\mathbf{N}, \mathbf{1}, \mathbf{2})$
$\hat{F}_R$	$(\overline{\mathbf{N}}, \mathbf{1}, \mathbf{2})$

Table 3. Four-flavor, custodially-symmetric dark sector fermion content.

6 Four-flavor theories

The main disadvantage to limiting ourselves to two flavors of fermions is that we are forced to choose between either having chiral masses or vector-like masses for fermions at the renormalizable level. With four flavors, we can engineer the electroweak quantum numbers to permit both vector-like and chiral masses, governed by Lagrangian parameters that are fully adjustable.

Large chiral masses with small vector-like masses will tend to cause the dark sector to substantially break electroweak symmetry (and violate bounds from the S parameter as well as Higgs coupling measurements). Therefore, we focus on the opposite case — the parameter space where the dark sector fermion masses are *mostly* vector-like with small chiral masses where $yv/M \ll 1$. In this way, these theories are automatically safe from electroweak precision constraints and Higgs coupling measurements. Yet, the presence of both vector-like and small chiral masses in general means that the dark sector flavor symmetries are broken to $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{\text{dark baryon}}$. The existence of baryons stabilized by the accidental $\mathrm{U}(1)_{\text{dark baryon}}$ was exploited by the Stealth Dark Matter model [75]. In that theory, with $N(\geq 4, \text{even})$, the lightest baryon was shown to be a viable dark matter candidate. In this paper, we focus solely on the mesons of the dark sector that was of only peripheral interest in the dark matter papers.

The field content of our prototype four-flavor, custodially-symmetric theory is given in table 3. At dimension-3, the vector-like masses for the dark fermions are

$$\mathscr{L} = M_{12} F_L \hat{F}_L + M_{34} F_R \hat{F}_R + \text{h.c.}\,. \tag{6.1}$$

At dimension-4, the chiral masses for the dark fermions are

$$\mathscr{L} = y_{14} F_L \mathcal{H} \hat{F}_R + y_{23} \hat{F}_L \mathcal{H} F_R + \text{h.c.}\,. \tag{6.2}$$

With fully general M_{12}, M_{34}, y_{14}, y_{23}, and the gauging of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, we see that vector-like and chiral masses arise at the renormalizable level, unlike the case of the two-flavor theories.[3] We could also include higher dimensional operators that we considered earlier in section 5. But like the two-flavor chiral theory, we anticipate the renormalizable interactions with the SM Higgs sector above will dominate over the higher dimensional ones, and so we won't consider them further in this section.

[3] We have switched notation $(F_1, F_2, F_3, F_4) \to (F_L, \hat{F}_L, F_R, \hat{F}_R)$ but retained the same mass and Yukawa coupling parameter names as ref. [75].

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After electroweak symmetry breaking, the mass matrix for the dark fermions can be written in a fully Higgs field-dependent way as

$$\mathscr{L}_{\text{mass}} = -\begin{pmatrix} F_L^u & -iF_L^d & F_R^u & -iF_R^d \end{pmatrix} \mathcal{M} \begin{pmatrix} \hat{F}_L^d \\ -i\hat{F}_L^u \\ \hat{F}_R^d \\ -i\hat{F}_R^u \end{pmatrix} + \text{h.c.}\,, \tag{6.3}$$

where

$$\mathcal{M} = \begin{pmatrix} M_{12} & 0 & \frac{y_{23}(-iG^0+h+v)}{\sqrt{2}} & -iy_{23}G^+ \\ 0 & M_{11} & -iy_{23}G^- & \frac{y_{23}(iG^0+h+v)}{\sqrt{2}} \\ \frac{y_{14}(iG^0+h+v)}{\sqrt{2}} & iy_{14}G^+ & M_{34} & 0 \\ iy_{14}G^- & \frac{y_{14}(-iG^0+h+v)}{\sqrt{2}} & 0 & M_{34} \end{pmatrix}. \tag{6.4}$$

The field-independent mass terms break up into two 2×2 mass matrices — one for the $Q = +1/2$ fermions and one for the $Q = -1/2$ fermions that are identical due to custodial symmetry. It is very convenient to rewrite $y_{14} = y(1+\epsilon)$ and $y_{23} = y(1-\epsilon)$ since, as we will see, contributions to electroweak precision observables is proportional to $(\epsilon y)^2$. Using this parameterization, the 2×2 mass matrices are

$$M_u \;=\; M_d = \begin{pmatrix} M_{12} & y(1-\epsilon)v/\sqrt{2} \\ y(1+\epsilon)v/\sqrt{2} & M_{34} \end{pmatrix}. \tag{6.5}$$

The mass matrix can be diagonalized by a biunitary transformation involving

$$\begin{aligned} \tan 2\theta_1 &= -\frac{\sqrt{2}yv\,(\Delta\epsilon - M)}{2\Delta M + \epsilon y^2 v^2} \\ \tan 2\theta_2 &= \frac{\sqrt{2}yv\,(\Delta\epsilon + M)}{2\Delta M - \epsilon y^2 v^2} \end{aligned} \tag{6.6}$$

where $M \equiv (M_{12}+M_{34})/2$, $\Delta \equiv (M_{34}-M_{12})/2$, and θ_1 (θ_2) diagonalizes $M_u M_u^T$ ($M_u^T M_u$). The diagonalized fermion masses are

$$m_{1,2} = M \mp \sqrt{\Delta^2 + \frac{y^2(1-\epsilon^2)v^2}{2}}\,. \tag{6.7}$$

We can use these results to rotate eq. (6.3) into the mass basis. The field-independent parts of the mass matrix are, of course, fully diagonalized. But the field-dependent ones are not. We need the field-dependence to determine the dark pion/Goldstone mixing.

Passing to the non-linear sigma model, we use

$$\Sigma = \exp\left[\frac{2\,i\pi^a\,t_{15}^a}{f}\right], \tag{6.8}$$

where the π^a are in the adjoint representation of $\text{SU}(4)_V$. Decomposing π^a into multiplets of $\text{SU}(2)_L \times \text{SU}(2)_R$, we have

$$\mathbf{15} \to (\mathbf{3},\mathbf{1}) \oplus (\mathbf{2},\mathbf{2})_a \oplus (\mathbf{2},\mathbf{2})_b \oplus (\mathbf{1},\mathbf{3}) \oplus (\mathbf{1},\mathbf{1})\,, \tag{6.9}$$

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where a and b are two separate bi-doublets. After rotating into the mass eigenstates of the dark mesons, we have

$$\Sigma = \exp\left[\frac{i}{f}\begin{pmatrix} \pi_1^0 + \frac{\eta}{\sqrt{2}} & \sqrt{2}\pi_1^+ & K_A^0 & -\sqrt{2}K_B^+ \\ \sqrt{2}\pi_1^- & -\pi_1^0 + \frac{\eta}{\sqrt{2}} & -\sqrt{2}K_A^- & K_B^0 \\ \bar{K}_A^0 & -\sqrt{2}K_A^+ & \pi_2^0 - \frac{\eta}{\sqrt{2}} & \sqrt{2}\pi_2^+ \\ -\sqrt{2}K_B^- & \bar{K}_B^0 & \sqrt{2}\pi_2^- & -\pi_2^0 - \frac{\eta}{\sqrt{2}} \end{pmatrix}\right] \tag{6.10}$$

where we use $\pi_{1,2}$ to denote the dark pions transforming as $(\mathbf{3},\mathbf{1})$ and $(\mathbf{1},\mathbf{3})$, $K_{A,B}$ to denote the "dark kaons" that are in $(\mathbf{2},\mathbf{2})$ representations, and η to denote the "dark eta" singlet.

The lowest dimension terms in the NLSM lagrangian are:

$$\mathscr{L}_\chi = \frac{f^2}{4}\operatorname{Tr}(D_\mu\Sigma(D^\mu\Sigma)^\dagger) + 4\pi c_D f^3 \operatorname{Tr}\left(\mathcal{L}\mathcal{M}\mathcal{R}^\dagger\Sigma^\dagger + \text{h.c.}\right), \tag{6.11}$$

where c_D is an $\mathcal{O}(1)$ coefficient from the strong dynamics. As we discussed in section 5.1, these terms are sufficient to capture the leading interactions of the dark pions, and in particular, will allow us to characterize the single dark pion interactions with the SM that lead to dark pion decay. The mixing matrices are formed from the angles eq. (6.6)

$$\mathcal{L} = \begin{pmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ 0 & \cos\theta_1 & 0 & -\sin\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & \sin\theta_1 & 0 & \cos\theta_1 \end{pmatrix} \tag{6.12}$$

$$\mathcal{R} = \begin{pmatrix} \cos\theta_2 & 0 & -\sin\theta_2 & 0 \\ 0 & \cos\theta_2 & 0 & -\sin\theta_2 \\ \sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & \sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix}. \tag{6.13}$$

In the field-independent limit,

$$\mathcal{L}\mathcal{M}\mathcal{R}^\dagger = \begin{pmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & m_2 \end{pmatrix}, \tag{6.14}$$

and so the dark pion masses are

$$m_{\pi_1} = 4\pi c_D f(2m_1) \tag{6.15}$$

$$m_K = 4\pi c_D f(m_1 + m_2) \tag{6.16}$$

$$m_{\pi_2} = 4\pi c_D f(2m_2)\,. \tag{6.17}$$

Finally, the covariant derivative for Σ involves the weak currents

$$D_\mu\Sigma = \partial_\mu\Sigma - i\,g W_\mu^\alpha\left(j_\alpha^V + j_\alpha^A\right)\Sigma - i\,g' B_\mu\left(j_Y^V + j_Y^A\right)\Sigma \tag{6.18}$$

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where it is convenient to express the vector and axial currents explicitly

$$j_\alpha^{V,A} = \mathcal{L}^\dagger t^\alpha \mathcal{L} \pm \mathcal{R}^\dagger t^\alpha \mathcal{R} \quad (\alpha = 1 \ldots 3) \tag{6.19}$$
$$j_Y^{V,A} = \mathcal{L}^\dagger t^{15} \mathcal{L} \pm \mathcal{R}^\dagger t^{15} \mathcal{R}\,. \tag{6.20}$$

Expanding the covariant derivatives to extract only the non-derivative contributions — the mass terms for W^μ and Z^μ — we find the contributions of the dark sector to electroweak symmetry breaking for two flavors:

$$v_{246}^2 = v^2 \left(1 + \frac{\epsilon^2 y^2 f^2}{M^2} + \ldots\right)\,. \tag{6.21}$$

Here we have written the leading result in a small ϵ expansion. Obviously the correction from the dark sector, $\epsilon^2 y^2 f^2/M^2$, should be small to avoid constraints from the electroweak precision observables as well as Higgs coupling measurements. In particular, the dark sector's contribution to the S parameter can be estimated [75] utilizing QCD and large N_D,

$$S \sim \frac{1}{6\pi} N_F N_D \left(\frac{\epsilon y f}{M}\right)^2 \simeq 0.1 \frac{N_F}{4}\frac{N_D}{4}\left(\frac{\epsilon y}{0.3}\right)^2 \left(\frac{f}{M}\right)^2\,. \tag{6.22}$$

Since $M < 4\pi f$ for the NLSM effective theory to be valid, in general we need $|\epsilon y|$ small to ensure the dark sector condensate is aligned nearly (but not completely) in an electroweak preserving direction.

While eq. (6.21) is reminiscent of eq. (5.10) in the two-flavor chiral case, there are some crucial differences. In eq. (5.10), we could not take f — the EWSB contribution from the strong sector — to be arbitrarily small without making the dark pions dangerously light. As a result, there is a minimum f that we can take, and therefore a minimum deviation in Higgs coupling and precision electroweak observables, see ref. [14]. In the four flavor case, we have more freedom. The fact that the fermions are vector-like means we can take f (more correctly yf) as small as we like without worrying about m_{π_D}. This allows us to explore a parameter space where the renormalizable coupling between the Higgs and the dark sector has negligible role on EWSB yet still acts as a portal for the dark pions to decay through.

6.1 Mixing with the Higgs and goldstones

We have chosen a basis for our dark pions such that they do not acquire an expectation value. This is evident by expanding the linear term, eq. (6.11), where one finds no terms linear in the dark pion fields, i.e., contributions of the form $\mathscr{L} \subset (\text{constant})\pi$ are absent.

There are, however, dark pion mixing terms with both the Higgs field h and (prior to gauge-fixing) the Higgs Goldstone fields $G^\pm, G^0$. Disentangling the mixing among the Higgs and dark pion fields is somewhat involved, and in full generality would need to be done numerically. In the following, we have calculated the mixing to leading order in ϵy, where we can obtain analytic expressions. Since we know ϵy must in general be small to ensure electroweak symmetry breaking occurs mostly from the fundamental Higgs field, this is a good choice of an expansion parameter.

One unique combination of the dark pion fields mixes with the Higgs boson h,

$$4\pi c_D f^3 \,\mathrm{Tr}\left(\mathcal{L}\mathcal{M}\mathcal{R}^\dagger\Sigma^\dagger + \mathrm{h.c.}\right) \supset 4\sqrt{2}\pi c_D \epsilon y f^2 h\, \mathrm{Im}(K_A^0 + K_B^0)\,. \tag{6.23}$$

This will turn out to be *critical* to understand the effective couplings of the lightest dark pions to the SM gauge sector.

The dark pions also mix with the Higgs Goldstones,

$$\begin{aligned}
&4\pi c_D f^3 \,\mathrm{Tr}\left(\mathcal{L}\mathcal{M}\mathcal{R}^\dagger\Sigma^\dagger + \mathrm{h.c.}\right) \supset \\
&\quad\frac{8\pi c_D f^2 \epsilon y}{M}\Bigg[\Big(G^-\left(s_m(2m_1\pi_1^+ - 2m_2\pi_2^+) + c_m(m_1+m_2)(K_A^+ - K_B^+)\right) + \mathrm{h.c.}\Big) \\
&\qquad\qquad + G^0\left(s_m(2m_1\pi_1^0 - 2m_2\pi_2^0) + c_m(m_1+m_2)\mathrm{Re}(K_A^0 - K_B^0)\right)\Bigg]\,,
\end{aligned} \tag{6.24}$$

where

$$s_m \equiv \sin\theta_m \equiv \frac{\sqrt{2}yv}{\sqrt{2(yv)^2 + 4\Delta^2}} \tag{6.25}$$

$$c_m \equiv \cos\theta_m \equiv \frac{2\Delta}{\sqrt{2(yv)^2 + 4\Delta^2}}\,, \tag{6.26}$$

are mixing angles among combinations of the dark pions. The dark pion/Goldstone mass mixing can be perturbatively diagonalized to leading order in ϵy,

$$G^{\pm,0}_{\mathrm{phys}} = G^{\pm,0} + \frac{\epsilon y f}{M}\left(s_m(\pi_1^{\pm,0} - \pi_2^{\pm,0}) + c_m\,\mathrm{Re}(K_A - K_B)\right) \tag{6.27}$$

$$\pi^{\pm,0}_{1,\mathrm{phys}} = \pi_1^{\pm,0} + \frac{\epsilon y f}{M} s_m G^{\pm,0} \tag{6.28}$$

$$\pi^{\pm,0}_{2,\mathrm{phys}} = \pi_2^{\pm,0} - \frac{\epsilon y f}{M} s_m G^{\pm,0} \tag{6.29}$$

$$\frac{\mathrm{Re}\left(K^{\pm,0}_{A,\mathrm{phys}} - K^{\pm,0}_{B,\mathrm{phys}}\right)}{\sqrt{2}} = \frac{\mathrm{Re}\left(K_A^{\pm,0} - K_B^{\pm,0}\right)}{\sqrt{2}} + \frac{\epsilon y f}{M} c_m G^{\pm,0} \tag{6.30}$$

$$\frac{\mathrm{Re}\left(K^{\pm,0}_{A,\mathrm{phys}} + K^{\pm,0}_{B,\mathrm{phys}}\right)}{\sqrt{2}} = \frac{\mathrm{Re}\left(K_A^{\pm,0} + K_B^{\pm,0}\right)}{\sqrt{2}}\,. \tag{6.31}$$

In addition, diagonalizing the Higgs boson/dark pion mixing one obtains

$$h_{\mathrm{phys}} = h - \frac{\epsilon y f m_K}{M}\frac{\mathrm{Im}(K_A^0 + K_B^0)}{m_h^2 - m_K^2} \tag{6.32}$$

$$\mathrm{Im}\left(K^0_{A,\mathrm{phys}} + K^0_{B,\mathrm{phys}}\right) = \mathrm{Im}\left(K_A^0 + K_B^0\right) + \frac{\epsilon y f m_K}{M}\frac{h}{m_h^2 - m_K^2} \tag{6.33}$$

where m_K is given by eq. (6.16).

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6.2 Dark pion couplings to the SM

We now calculate the couplings of dark pions to the gauge sector of the Standard Model. These couplings arise when the interaction eigenstates (G, π, K) are rotated into the physical states ($G_{\rm phys}$, $\pi_{\rm phys}$, $K_{\rm phys}$). Gauge-fixing in unitary gauge removes all terms involving $G_{\rm phys}$, leaving just the interactions with the "physical" (mass eigenstate) dark pions.

It is clear from eqs. (6.7) that a non-zero Yukawa coupling necessarily splits the fermion masses, and thus there is always some (possibly small) mass hierarchy between π_1, K, and π_2 (and η), see eqs. (6.15)–(6.17). While it is straightforward to calculate the couplings of all of the dark pions to the Standard Model, here we focus only on the lightest pions. For instance, strong decays of $\pi_{\rm heavy}, K \to \pi_{\rm light} + X$ are expected to be rapid so long as the dark pion mass differences are large enough that phase space does not severely limit their rates.

The two-pion interactions with the SM gauge sector take the form

$$W^{\mu,\mp}\left(\pi^{\pm}_{\rm phys}\partial_\mu\pi^0_{\rm phys} - \pi^0_{\rm phys}\partial_\mu\pi^{\pm}_{\rm phys}\right) :\, = g\,\frac{1+c_m}{2}\,. \tag{6.34}$$

Several limits are interesting. First, for $\Delta > 0$ and $\Delta \gg yv$, then $c_m \simeq 1$, and so the coupling of the dark pions to the gauge bosons becomes $\simeq g$ — exactly the coupling expected for three SU(2)$_L$-triplets to interact via the SU(2) anti-symmetric tensor contraction. This is not surprising — in this limit the lightest pions are a nearly exactly an SU(2)$_L$ triplet with only $(yv)/\Delta$-suppressed mixings into the other dark pions.

Next consider $\Delta < 0$, while still $|\Delta| \gg yv$. Now $c_m \simeq -1$, and the coupling of the dark pions to the gauge bosons becomes $\simeq 0$. This is again unsurprising — in this limit the lightest pions are a nearly exact SU(2)$_R$ triplet that does not couple with SU(2)$_L$ gauge bosons.

Finally, when $\Delta \ll yv$ (and thus $c_m \simeq 0$) the splittings among the dark pions are dominated by electroweak symmetry breaking contributions. In this case, the would-be SU(2)$_L$ triplet and SU(2)$_R$ triplets are fully mixed, and each share an approximately $g/2$ coupling to SU(2)$_L$ gauge bosons.

Single pion interactions with one gauge boson and one Higgs boson are the most interesting (and most relevant for pion decay). We obtain:

$$\begin{aligned}
(\pi^{\pm}_{\rm phys}\partial_\mu h - h\partial_\mu\pi^{\pm}_{\rm phys})W^{\mu,\mp} :\;& \frac{M_W}{v}\times\left(\sqrt{2}c_D\epsilon y s_m\frac{4\pi f^2}{m_K^2}\right)\times\left(\frac{m_h^2}{m_K^2-m_h^2}\right)\\
(\pi^{0}_{\rm phys}\partial_\mu h - h\partial_\mu\pi^{0}_{\rm phys})Z^{\mu} :\;& \frac{M_Z}{v}\times\left(\sqrt{2}c_D\epsilon y s_m\frac{4\pi f^2}{m_K^2}\right)\times\left(\frac{m_h^2}{m_K^2-m_h^2}\right)\\
\pi^{\pm}_{\rm phys}\bar{f}'f :\;& \sqrt{2}\left(\frac{m_{f'}}{v}P_L - \frac{m_f}{v}P_R\right)(2T_3^f)\times\left(\sqrt{2}c_D\epsilon y s_m\frac{4\pi f^2}{m_K^2}\right)\\
\pi^{0}_{\rm phys}\bar{f}f :\;& i\left(\frac{m_f}{v}\gamma_5\right)(2T_3^f)\times\left(\sqrt{2}c_D\epsilon y s_m\frac{4\pi f^2}{m_K^2}\right).
\end{aligned} \tag{6.35}$$

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From these expressions, we can identify

$$\frac{1}{v_\pi} = \frac{1}{v} \times \left(\sqrt{2} c_D \epsilon y s_m \frac{4\pi f^2}{m_K^2}\right), \qquad \xi = \frac{m_h^2}{m_K^2 - m_h^2}. \tag{6.36}$$

This is the main result for the four-flavor theory. We find that the pion interactions with the gauge bosons and Higgs boson are suppressed relative to the fermion couplings by a factor $m_h^2/(m_K^2 - m_h^2)$ that becomes roughly m_h^2/m_K^2 for larger dark kaon masses. This relative suppression in gauge/Higgs boson couplings to the fermion couplings is exactly what happened in the two-flavor, custodially-symmetric model.

The four-flavor model is, essentially, one ultraviolet completion of the two-flavor theory with higher-dimensional operators that are both custodially symmetric and minimal flavor violating. The dimension-7 operators that lead to interactions with the fermions are matched at $\Lambda^3 = 4\pi f m_K^2$; the dimension-9 operator that leads to the interactions with the gauge bosons and Higgs boson is matched at $\Lambda^5 = 4\pi f^3 m_K^2$; with the coefficient $c_{9C} \propto \lambda_h$ the quartic coupling of the Higgs sector.

7 Dark sector custodial violation

We have focused on dark sectors that preserve custodial SU(2). In practice this means that renormalizable and higher dimensional operators involving dark fermions do not involve explicit t_R^3 — this only appears from the custodially violating SM spurions proportional to $g'Y$ or $\mathcal{Y}_{ij}^{\not{c}}$.

Naturally, it is interesting to consider what happens when explicit t_R^3 is introduced. In the $\mathrm{SU}(2)_R$ model, this is possible already at the renormalizable level. One can include $M' F t_R^3 \hat{F}$ in addition to $MF\hat{F}$. This is equivalent to simply writing different dark fermion masses for the $Y = +1/2$ and $Y = -1/2$ states under $\mathrm{U}(1)_Y$.

In the $\mathrm{SU}(2)_L$ model, gauge invariance forbids a dimension-3 term violating custodial SU(2). At dimension-5 there is an interaction:

$$\mathscr{L} = c_{5V} \frac{(F_1 t_L^a F_2) \mathrm{Tr}\, \mathcal{H}^\dagger t_L^a \mathcal{H} t_R^3}{\Lambda} \tag{7.1}$$

that violates custodial SU(2). With two Higgs bifundamentals, the group contractions were given in eq. (5.15) where the surviving combinations are precisely those in eqs. (5.16) and (7.1). (And as we mentioned earlier, the would-be $(\mathbf{3}_L, \mathbf{1}_R)$ or $(\mathbf{1}_L, \mathbf{3}_R)$ involving $\mathrm{Tr}\, \mathcal{H}^\dagger t_L^a \mathcal{H}$ or $\mathrm{Tr}\, \mathcal{H}^\dagger \mathcal{H} t_R^a$ both simply vanish.) The only way we can write a gauge-invariant term of the form eq. (7.1) is to use t_R^3 of $\mathrm{SU}(2)_R$, and hence is custodially violating.

The low energy effective theory including higher dimensional operators up to $\mathcal{O}(v^2/\Lambda)$ can again be described by a non-linear sigma model,

$$\begin{aligned}\mathscr{L} = {} & \frac{f^2}{4} \mathrm{Tr}\, (D_\mu \Sigma)^\dagger D^\mu \Sigma + 4\pi f^3 \left(M + c_{5M} \frac{v^2}{\Lambda}\right) \mathrm{Tr}\, (\Sigma^\dagger + \mathrm{h.c.}) \\ & + c_{5V} \frac{4\pi f^3}{\Lambda} \mathrm{Tr}(\Sigma_L t_L^a) \mathrm{Tr}\, (\mathcal{H}^\dagger t_L^a \mathcal{H} t_R^3 + \mathrm{h.c.}) .\end{aligned} \tag{7.2}$$

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Expanding the non-linear sigma model up to $\mathcal{O}(\pi^2)$, we obtain

$$\mathscr{L} = \mathrm{Tr}\, D_\mu \pi^a D^\mu \pi^a - \frac{1}{2} m_\pi^2 \pi^a \pi^a - c_{5V} \frac{4\pi f^2}{\Lambda} H^\dagger \pi^a t_L^a H \tag{7.3}$$

where $m_\pi^2 = 4\pi f(M + \mathcal{O}(v^2/\Lambda))$, and we have written the single pion — Higgs interaction in the more familiar form using Higgs doublet notation. This Lagrangian is precisely that of a "crappy triplet model",[4] e.g. [125]. That is, the two-flavor $\mathrm{SU}(2)_L$ dark sector with a dimension-5 custodially-violating interaction with the SM Higgs sector provides an ultraviolet completion of the SM extended to include a real triplet. Higher order terms in the chiral Lagrangian lead to the usual pion self-interactions as well as interactions of multiple pions with Higgs fields.

In this theory, we see that the dark pion interactions with the SM arise at a comparatively low dimension operator, eq. (7.1). The explicit custodial violation causes the dark pions to acquire a "triplet" vev

$$v_T \equiv \langle \pi^a \rangle \sim c_{5V} \frac{f v^2}{\Lambda M} \,. \tag{7.4}$$

Obviously this is highly constrained by electroweak precision data. Nevertheless, following ref. [125] one can proceed as usual, shift to the new vacuum, and extract the effective interactions from eq. (7.3). The result is that there is a neutral singlet that mixes with the Higgs boson and a charged scalar that mixes with the charged Higgs Goldstones. Diagonalizing these interactions leads to

$$\begin{pmatrix} G^\pm_{\mathrm{phys}} \\ \pi^\pm_{\mathrm{phys}} \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} G^\pm \\ \pi^\pm \end{pmatrix} \tag{7.5}$$

where $\sin\delta \sim v_T/\sqrt{v^2 + v_T^2}$. The interactions of the charged dark pions are obtained by replacing G^+ with $\pi^\pm_{\mathrm{phys}}$. Just like in the two-flavor chiral model, this leads to gaugephilic branching ratios. However, unlike the two-flavor chiral model, there is no neutral dark pion/neutral Goldstone mixing.

8 Discussion

In this paper we have studied dark sectors that arise from a new, strongly-coupled confining gauge group $\mathrm{SU}(N_D)$ with dark fermions transforming under the electroweak part of the SM. In dark sectors that preserve custodial SU(2) in their interactions with the SM, a custodial triplet of dark pions appears in the low energy effective theory. The low energy effective interactions with the SM can be classified by the custodial symmetry, leading to two distinct possibilities: "Gaugephilic": where $\pi_D^0 \to Zh$, $\pi_D^\pm \to Wh$ dominate once kinematically open, and "Gaugephobic": where $\pi_D^0 \to \bar{f}f$, $\pi_D^\pm \to \bar{f}'f$ dominate. These classifications assume the only sources of custodial SU(2) breaking are from the SM: the gauging of hypercharge, $g'Y$, and the difference between the up-type and down-type Yukawa couplings, $\mathcal{Y}_{ij}^{\not{C}}$.

[4] "Crappy" in the sense that the linear term for π^a causes it to acquire a custodially-violating vev.

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The simplest theories that exhibited the gaugephobic and gaugephilic classifications contained two-flavors, and we examined one chiral theory and two vector-like theories. The chiral theory is familiar from bosonic technicolor/strongly-coupled induced electroweak symmetry breaking. There, the dominant source of dark pion interactions with the SM is from Goldstone-pion mixing and leads to a gaugephilic decay pattern. In the vector-like theories, dark pion interactions with the SM arise through higher dimensional operators. If we demand custodial SU(2) invariance in these higher dimensional operators, we find that interactions between the π_D and gauge bosons first occur at dimension-9 (in the UV) while $\pi_D \bar{f} f$ operators can be written at dimension-7. The mismatch in operator dimension means the vector-like theories are gaugephobic.

Next, we examined a four-flavor theory. With the proper electroweak charge assignment, this scenario can have both vector-like and chiral masses among its dark fermions, and is therefore a hybrid of the chiral and vector scenarios. The most phenomenologically interesting limit is when the chiral mass is small compared to the vector-like mass. In this case, we find the lightest custodial SU(2) triplet of dark pions have gaugephobic interactions with the SM in which $\pi^0 \to Zh$, $\pi^\pm \to Wh$ are suppressed by $\simeq m_h^2/m_K^2$ relative to fermionic decays. In the chiral lagrangian for the full multiplet of 15 dark pions, this arises through a cancellation between the dark pion mixing with the Goldstones of the SM and dark pion mixing with the Higgs boson of the SM. Decoupling the heavier dark pion multiplets such that only the lightest triplet remains, the four-flavor theory maps into a two-flavor theory with higher dimensional operators that preserve custodial SU(2) and are minimally flavor violating. The custodial SU(2) symmetry of these interactions automatically leads to the operator suppression $\simeq v^2/\Lambda^2$, in agreement with what we found by explicit calculation of the four-flavor theory.

In theories that preserve custodial SU(2), the neutral dark pion decays to the SM through "gaugephobic" or "gaugephilic" interactions with a suppressed rate of $\pi^0 \to \gamma\gamma$. In each of the theories considered, there is no axial anomaly contribution to the decay. However, since the dark pions do have interactions with SM, and the SM fermions have an anomalous axial-vector current, the decay $\pi^0 \to \gamma\gamma$ does occur, but is suppressed by the same $1/v_\pi$ that suppresses the direct decay $\pi^0 \to \mathrm{SM\,SM}$. In the Standard Model, the analogy would be to imagine that the up and down quarks have an exact custodial SU(2) symmetry, i.e., $Q_u = -Q_d = 1/2$ and $y_u = y_d$. In this case, the anomaly contribution to $\pi^0 \to \gamma\gamma$ in the Standard Model would vanish. However, even without the anomaly, the SM π^0 decays through the mode $\pi^0 \to e^+e^-$ proportional to the electron Yukawa coupling. This interaction has the same form as the two-flavor chiral theory we considered in this paper. Now there remains a one-loop suppressed contribution $\pi^0 \to \gamma\gamma$ through the electron Yukawa coupling, but this is highly suppressed compared with the fermionic decay, which is precisely what happens with the π^0 of the custodial SU(2) symmetric dark sector theories that we have considered in this paper.

Finally, the astute reader may have noticed that all of the vector-like dark sector theories with custodially symmetric interactions with the SM were gaugephobic. The only gaugephilic case presented in the paper is the two-flavor chiral theory, which might give the reader the impression that vector-like theories are automatically gaugephobic. This is not

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the case. As an explicit counter-example, the custodial triplets in Georgi-Machacek models have gaugephilic couplings (e.g. [126]). It will come as no surprise that we have already constructed strongly-coupled models based on coset theories that generate the scalar sector of Georgi-Machacek theories as dark pions with gaugephilic couplings with the SM. The details will be presented in ref. [127].

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A Gaugephobic 2HDMs

We review the application of the $(\mathbf{2},\mathbf{2})$ custodial symmetry formalism in the context of general two-Higgs doublet models (2HDM) [128–131]. We'll focus on a general CP-conserving 2HDM.

The most general 2HDM potential can be written as [120, 132]

$$\begin{aligned} V_{\rm 2HDM} = & \; m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) \\ & - m_{12}^2(\phi_1^\dagger\phi_2) - (m_{12}^2)^*(\phi_2^\dagger\phi_1) \\ & + \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\ & + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_5^*(\phi_2^\dagger\phi_1)^2] \\ & + [\lambda_6(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_2^\dagger\phi_1)](\phi_1^\dagger\phi_1) \\ & + [\lambda_7(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_1)](\phi_2^\dagger\phi_2) \end{aligned} \tag{A.1}$$

where m_{11}^2, m_{22}^2, $\lambda_{1,2,3,4}$ are real parameters and m_{12}^2, $\lambda_{5,6,7}$ complex. And ϕ_1 and ϕ_2 are two complex scalar doublets

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \qquad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}, \tag{A.2}$$

In general, m_{11}^2, m_{22}^2, and $\lambda_{1,2,3,4}$ are real parameters while m_{12}^2 and $\lambda_{5,6,7}$ can be complex. Nevertheless, in this study we restrict our discussion to CP-conserving models, by assuming all the parameters of $V_{\rm 2HDM}$ are real [120]. And we also assume the parameters are chosen to make $V_{\rm 2HDM}$ bounded below so that each of the ϕ_i acquires a VEV, denoted as v_1 and v_2 which satisfy

$$v_1^2 + v_2^2 = v^2 = (246\,\text{GeV})^2 \tag{A.3}$$

and we define

$$t_\beta \equiv \tan\beta \equiv \frac{v_2}{v_1}\,. \tag{A.4}$$

The goal of this section is to demonstrate explicitly that it's possible to write a general 2HDM potential in terms of a $(\mathbf{2},\mathbf{2})$ custodial symmetry formalism, by introducing matrices M_{ij} similar to eq. (3.2)

$$M_{ij} \equiv (\tilde{\phi}_i, \phi_j) = \begin{pmatrix} \phi_i^{0\star} & \phi_j^+ \\ -\phi_i^- & \phi_j^0 \end{pmatrix} \tag{A.5}$$

where $i, j = 1, 2$

It is crucial to our approach that we define the following $\mathbf{K}$-terms [128–130]

$$\mathbf{K} = \begin{pmatrix} K_0 \\ K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} \phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 \\ \phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1 \\ i(\phi_2^\dagger\phi_1 - \phi_1^\dagger\phi_2) \\ \phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2 \end{pmatrix}. \tag{A.6}$$

Given eqs. (A.5)–(A.6), we may write $\mathbf{K}$ in two different ways, with either M_{11} and M_{22}, or M_{21} alone

$$\begin{aligned}
K_0 &= \frac{1}{2}\,\mathrm{Tr}(M_{11}^\dagger M_{11} + M_{22}^\dagger M_{22}) = \mathrm{Tr}(M_{21}^\dagger M_{21}) \\
K_1 &= \mathrm{Tr}(M_{11}^\dagger M_{22}) = 2\,\mathrm{Re}(\det M_{21}^\dagger) \\
K_2 &= (-i)\,\mathrm{Tr}(M_{11}\tau_3 M_{22}^\dagger) = -2\,\mathrm{Im}(\det M_{21}) \\
K_3 &= \frac{1}{2}\,\mathrm{Tr}(M_{11}^\dagger M_{11} - M_{22}^\dagger M_{22}) = -\,\mathrm{Tr}(M_{21}\tau_3 M_{21}^\dagger)\,.
\end{aligned} \tag{A.7}$$

Then it is straightforward to verify that V_{2HDM} can be written in terms of $\mathbf{K}$ in a compact form of

$$V_{\mathrm{2HDM}} = \boldsymbol{\xi}^{\mathrm{T}}\mathbf{K} + \mathbf{K}^{\mathrm{T}}\mathbf{E}\mathbf{K} \tag{A.8}$$

where the mass parameter vector $\boldsymbol{\xi}$ and the coupling parameter matrix $\mathbf{E}$ are [130]

$$\boldsymbol{\xi} = \begin{pmatrix} \frac{1}{2}(m_{11}^2 + m_{22}^2) \\ -\,\mathrm{Re}(m_{12}^2) \\ \mathrm{Im}(m_{12}^2) \\ \frac{1}{2}(m_{11}^2 - m_{22}^2) \end{pmatrix} \tag{A.9}$$

$$\mathbf{E} = \frac{1}{4}\begin{pmatrix} \frac{1}{2}(\lambda_1+\lambda_2)+\lambda_3 & \mathrm{Re}(\lambda_6+\lambda_7) & -\,\mathrm{Im}(\lambda_6+\lambda_7) & \frac{1}{2}(\lambda_1-\lambda_2) \\ \mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}(\lambda_5) & -\,\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6-\lambda_7) \\ -\,\mathrm{Im}(\lambda_6+\lambda_7) & -\,\mathrm{Im}(\lambda_5) & \lambda_4-\mathrm{Re}(\lambda_5) & -\,\mathrm{Im}(\lambda_6-\lambda_7) \\ \frac{1}{2}(\lambda_1-\lambda_2) & \mathrm{Re}(\lambda_6-\lambda_7) & -\,\mathrm{Im}(\lambda_6-\lambda_7) & \frac{1}{2}(\lambda_1+\lambda_2)-\lambda_3 \end{pmatrix}. \tag{A.10}$$

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As a consequence of eq. (A.7), there are actually two types of custodial transformations to the potential [133]: Type I: M_{11} and M_{22} transform as

$$M_{ii} \longrightarrow L M_{ii} R^\dagger \text{ for } i = 1,2 \tag{A.11}$$

where L and R are $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ matrices. Type II: In this case, it's M_{21} which transforms as

$$M_{21} \longrightarrow L M_{21} R^\dagger\,. \tag{A.12}$$

The potential V_{2HDM} preserves custodial symmetry if it is invariant under either type of the custodial transformations.

Nevertheless, recall that there is an explicit τ_3 in eq. (A.7). In fact, it's a $(\tau_3)_R$ which appears either in the K_2 term under the Type I custodial transformation, or in the K_3 term for the Type II. Since $(\tau_3)_R$ breaks custodial symmetry explicitly, K_2 term should be absent from V_{2HDM} with Type I custodial symmetry, same as K_3 term for Type II. Apparently, to meet this requirement the corresponding entries in $\boldsymbol{\xi}$ and $\mathbf{E}$ must vanish.

With the argument above, the conditions for a custodial symmetric 2HDM potential can be summarized as:

Type I:

$$\boldsymbol{\xi}_I = \begin{pmatrix} \cdot \\ \cdot \\ 0 \\ \cdot \end{pmatrix}, \qquad \mathbf{E}_I = \begin{pmatrix} \cdot & \cdot & 0 & \cdot \\ \cdot & \cdot & 0 & \cdot \\ 0 & 0 & 0 & 0 \\ \cdot & \cdot & 0 & \cdot \end{pmatrix}. \tag{A.13}$$

Type II:

$$\boldsymbol{\xi}_{II} = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix}, \qquad \mathbf{E}_{II} = \begin{pmatrix} \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A.14}$$

For a CP-conserving 2HDM:

$$\boldsymbol{\xi}_{\mathrm{CP}} = \begin{pmatrix} \frac{1}{2}(m_{11}^2 + m_{22}^2) \\ -m_{12}^2 \\ 0 \\ \frac{1}{2}(m_{11}^2 - m_{22}^2) \end{pmatrix} \tag{A.15}$$

$$\mathbf{E}_{\mathrm{CP}} = \frac{1}{4}\begin{pmatrix} \frac{1}{2}(\lambda_1+\lambda_2)+\lambda_3 & \lambda_6+\lambda_7 & 0 & \frac{1}{2}(\lambda_1-\lambda_2) \\ \lambda_6+\lambda_7 & \lambda_4+\lambda_5 & 0 & \lambda_6-\lambda_7 \\ 0 & 0 & \lambda_4-\lambda_5 & 0 \\ \frac{1}{2}(\lambda_1-\lambda_2) & \lambda_6-\lambda_7 & 0 & \frac{1}{2}(\lambda_1+\lambda_2)-\lambda_3 \end{pmatrix}. \tag{A.16}$$

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Compare eqs. (A.15)–(A.16) to (A.13), we see that to preserve Type-I custodial symmetry, the condition required is

$$\lambda_4 = \lambda_5\,. \tag{A.17}$$

Similarly, the conditions for a Type-II custodial symmetry are

$$\begin{aligned} m_{11}^2 &= m_{22}^2 \\ \lambda_1 &= \lambda_2 \\ \lambda_6 &= \lambda_7 \\ \lambda_3 &= \frac{1}{2}(\lambda_1+\lambda_2) = \lambda_1\,. \end{aligned} \tag{A.18}$$

As is well-known, the observable that measures custodial violation is the ρ-parameter. Assuming the first three conditions of eq. (A.18), the one-loop contributions to $\Delta\rho$ [126] from either Type-I or Type-II models can be calculated to the leading order in v^2 as

$$\Delta\rho = \frac{1}{192\pi^2}\left(\frac{v^2}{m_A^2}\right)(\lambda_4-\lambda_5)(\lambda_1-\lambda_3) \tag{A.19}$$

where m_A is the mass of the heavy pseudoscalar Higgs state A^0 in 2HDM. We explicitly see that $\Delta\rho$ is proportional to $(\lambda_4-\lambda_5)$ *and* $(\lambda_1-\lambda_3)$, which can be identified with the contribution from Type-I and Type-II, correspondingly.

We can also map the general Type II 2HDM model onto the minimal supersymmetric model (MSSM) where the λ_i are [120]

$$\begin{aligned} \lambda_1 &= \lambda_2 = \frac{1}{4}(g^2+g'^2) \\ \lambda_3 &= \frac{1}{4}(g^2-g'^2) \\ \lambda_4 &= -\frac{1}{2}g^2 \\ \lambda_5 &= \lambda_6 = \lambda_7 = 0\,. \end{aligned} \tag{A.20}$$

The contribution to $\Delta\rho$ is then

$$\Delta\rho = \frac{1}{192\pi^2}\left(\frac{v^2}{m_A^2}\right)\left(-\frac{1}{2}g^2\right)\left(\frac{1}{2}g'^2\right)\,. \tag{A.21}$$

The 2HDM potential of the MSSM contains custodial symmetry violation with a small but non-zero correction to the ρ-parameter. The correct is, nevertheless, proportional to g'^2 that is precisely the SM violation of custodial symmetry by gauging hypercharge.

Phenomenologically, the heavy Higgs states in a 2HDM may decay into SM particles if kinematically allowed. Comparing to our study of dark mesons, we are particularly interested in the branching fractions of the charged Higgs $H^\pm$ and the pseudoscalar A^0 decaying into SM fermion pairs or gauge boson and Higgs pairs, especially in the decoupling limit $m_A \gg v$. In this limit, eq. (A.19) indicates that $\Delta\rho$ is always suppressed by two powers of the heavy mass scale m_A, which means the amount of possible custodial symmetry violation is restricted to be relatively small. As a result, one can say that 2HDM becomes custodially symmetric in the decoupling limit.

As for the decay branching fractions, though the couplings of $H^\pm$ and A^0 to SM fermions are usually model dependent, their values are proportional to $\tan\beta$ or $\cot\beta$ [126]

$$C_{ff} \propto g\frac{m_f}{m_W}(\tan\beta \;\text{ or }\; \cot\beta)\,. \tag{A.22}$$

On the other hand, the couplings to SM gauge bosons and SM Higgs are proportional to $\cos(\beta-\alpha)$ [126]

$$C_{Wh} \propto g\cos(\beta-\alpha) \tag{A.23}$$

where α is the CP-even scalar mixing angle, and in the decoupling limit,

$$\cos(\beta-\alpha) = \mathcal{O}\left(\frac{v^2}{m_A^2}\right). \tag{A.24}$$

Compare eq. (A.22) to eq. (A.23), we see that to the leading order in v^2,

$$\frac{C_{Wh}}{C_{ff}} \propto \cos(\beta-\alpha) \propto \mathcal{O}\left(\frac{v^2}{m_A^2}\right)\,. \tag{A.25}$$

Therefore, in the decoupling limit a 2HDM becomes custodially symmetric, and the decays of its heavy states to SM particle in this limit are gaugephobic.

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