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Condition-based maintenance optimization for multi-component systems subject to a system reliability requirement

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ABSTRACT

Prognostic methods for remaining useful life and reliability prediction have been extensively studied in the past decade. However, the use of prognostic information and methods in maintenance decision-making for complex systems is still an underexplored area. In this paper, using a rolling-horizon approach, we develop a condition-based maintenance decision-framework for a multi-component system subject to a system reliability requirement. The system is inspected periodically and new degradation information on components is obtained upon inspection. The new degradation observations are used to update the posterior distributions of the failure model parameters via Bayesian updating, providing more accurate and customized predictive reliabilities. If the predictive system reliability is below the reliability requirement, a novel dynamic-priority-based heuristic algorithm is used to identify a group of components for preventive maintenance. Numerical results show that significant cost savings and improved system reliabilities can be obtained by using more accurate predictive information in maintenance decision-making. We further illustrate the modeling flexibility of the proposed framework by considering dynamic environmental information in decision-making and investigate the potential benefits of incorporating dynamic contexts.

1. Introduction

Continuous monitoring of system's health conditions has been playing an increasingly important role in preventing or delaying system failures. The advances in sensor technologies have greatly accelerated the use of real-time monitoring and condition assessment for complex systems. Prognostic methods based on sensor information for remaining useful life (RUL) prediction have been extensively studied in the past decade [1–3]. It has been shown that incorporating prognostic information in maintenance planning can help make more informed maintenance decisions for single-component systems [4, 5]. On the other hand, the complexity of modern systems keeps increasing and their operational environments are often dynamic. Many complex systems consist of a large number of interconnected technological elements such as subsystems, components, etc. There often exist one or more types of interactions between components, such as economic, structural, and stochastic dependence. Due to the presence of these dependences, maintenance optimization models for single-component systems are no longer applicable. However, the use of prognostic/predictive information in maintenance decisions for complex multi-component system is an underexplored area [6, 7].

Maintenance optimization for multi-component systems is a challenging problem, since it combines the stochastic failure processes of components with the combinatorial optimization problem regarding the grouping of maintenance activities [8]. Due to the mathematical difficulties in modeling and analysis of multi-component maintenance problems, many existing multi-component models are developed based on time-based maintenance (TBM) policies [8–12]. A major drawback of TBM is that unnecessary maintenance actions may be performed. Aiming to perform preventive maintenance (PM) just in time, condition-based maintenance (CBM) has gained much popularity in the past decades. However, the majority of the CBM literature focuses on single-component systems [13–17], and CBM for multi-component systems has received much less attention. Tian and Liao [18] present a CBM policy for multi-component systems based on proportional hazards model. Xu et al. [19] develop a CBM optimization model for a two-component repairable system and consider a policy that replaces the two components if the system-level hazard rate reaches a replacement threshold. Zhu and Xiang [20] develop an analytical CBM model for multi-component systems with economic dependence using a stochastic

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programming approach. Keizer et al. [21] consider a joint optimization of CBM and spare part inventory for multi-component systems and formulate the problem as a Markov decision process. More papers on CBM for multi-component systems can be found in a recent survey by Keizer et al. [22].

Most aforementioned CBM-based multi-component maintenance models make simple use of current state/degradation information. Huynh et al. [7] propose three predictive maintenance policies for single-component systems, and their models make inspection and replacement decisions using the predictive reliability and the mean RUL. Nguyen et al. [23] develop a joint optimization model of predictive maintenance and spare part inventory for systems consisting of multiple non-identical components. Predictive maintenance and spare part ordering for each component are triggered if its predictive reliability is below their respective thresholds. Incorporating component's predictive reliability/mean residual life in maintenance decision-making for multi-component systems can also be found in Cheng et al. [24] and Vu et al. [25]. However, these studies assume that failure distributions are known (e.g., estimated from historical observations, obtained from experts' knowledge) and remain fixed during the maintenance decision process; they do not leverage online condition monitoring data (e.g., degradation level and health state) to update components' prior failure information, which may lead to inferior predictive results and result in significantly degraded system performances.

To obtain more accurate and customized RUL/reliability prediction for more effective maintenance decisions, several studies [26–30] use Bayesian updating methods to update posterior distributions of unknown failure parameters based on online data. Si et al. [27] consider a condition-based replacement model for a single-component system. Although observed degradation information is used to update posterior distributions of unknown degradation parameters and estimate the RUL, the maintenance policy considered in [27] is still time-based (i.e., age-based maintenance). Omshi et al. [28] propose a dynamic auto-adaptive inspection and predictive maintenance policy for single-component systems. Their model re-schedules the next inspection interval based on the updated system's RUL so that the probability of failure before the next inspection remains lower than a pre-specified requirement. Walter and Flapper [30] propose a dynamic maintenance policy for multi-component systems, which determines the possible age-based PM time using the sequentially updated RUL. Our review shows that limited works make use of online condition monitoring data and dynamic environmental information for more accurate predictive results and more informed CBM decisions. Moreover, the few studies that incorporate dynamic information in maintenance decision-making mainly focus on single-unit systems. There is a lack of CBM models for multi-component systems that leverage multi-source dynamic information for effective inspection and maintenance planning.

To fill this gap, we develop a condition-based maintenance decision-framework for a general multi-component system which has serially connected k-out-of-n subsystems subject to a reliability requirement. The objective is to minimize the total maintenance cost over a finite planning horizon and ensure that the system reliability meets the pre-specified requirement. The system is inspected periodically. Since true degradation model parameters are usually unknown due to limited historical data, at each inspection, when desired, a Bayesian updating procedure is used to leverage short-term information, e.g., component deterioration level, environmental condition, to obtain more accurate predictive reliabilities for better informed maintenance decision-making. If PM is needed, a dynamic-priority-based heuristic algorithm is used to select a group of components for PM. Sensitivity analysis is conducted to assess the potential benefits of using predictive information and methods in maintenance decision-making for multi-component systems. The main contributions of this paper are twofold.

(1) The proposed CBM decision framework for multi-component systems leverages multi-source dynamic information (e.g., online deterioration data, environmental conditions) to update posterior distributions of unknown degradation parameters using a Bayesian updating approach. Thus, more accurate and customized reliability prediction pertaining to in-service components can be obtained and used for better informed maintenance decisions.
(2) An effective dynamic-priority-based heuristic algorithm is designed to seek the optimal maintenance grouping. The heuristic employs a composite index that jointly considers the importance measure and the maintenance cost of each component. In particular, the importance measure used in the priority index is dynamic and identifies the importance of a component based on its evolving conditions, instead of static importance measures commonly used in the literature.

This study has a wide range of real-world applications, such as high-speed trains [31], oil and gas pipeline systems [32], and power systems [33]. For example, offshore pipeline systems laid on the seabed are usually exposed to seabed mobility and unsteady ocean waves, and various techniques such as ultrasonic tools have been used to inspect conditions of pipeline systems routinely [34]. Power systems are often operating in season-dependent weather conditions, and a large number of sensors and instruments are installed in power systems for condition monitoring. The proposed method can help accurately predict the future performance of these complex systems and recommend effective inspection schedules and maintenance decisions. With the advances in sensor technologies for condition monitoring and the decreases in the cost of sensors, the proposed model is of greater relevance to improve the reliability of complex engineering systems at reduced costs.

The remainder of the paper is organized as follows. Section 2 describes system degradation with uncertainties and the maintenance problem. The details of the proposed maintenance decision framework are provided in Section 3. Sensitivity analysis is presented in Section 4. Conclusions and future research work are summarized in Section 5.

2. Problem formulation

Notation

N	Number of subsystems
n_i	Number of components in subsystem i
T	Finite-time horizon
θ	Vector of unknown degradation parameters
$\phi(\theta)$	Prior distribution of unknown degradation parameters θ
$X_i(t)$	Cumulative degradation level of a component in subsystem i in time $[0, t]$
$f_i(x; x_0, t, \theta)$	Probability density function (pdf) of $X_i(t)$ given the initial degradation level x_0 and the unknown degradation parameters θ
$F_i(x; x_0, t, \theta)$	Cumulative distribution function (cdf) of $X_i(t)$ given the initial degradation level x_0 and the unknown degradation parameters θ
c_s	Setup cost
$c_{i,\text{insp}}$	Inspection cost per component of subsystem i
$c_{i,\text{pm}}$	PM cost per component of subsystem i
$c_{i,\text{cm}}$	Corrective maintenance (CM) cost per component of subsystem i
N_{insp}	Number of inspections performed in time $[0, T]$

N_{mx}	Number of inspections with maintenance performed in time [0, T]
$N_{i,j,\text{pm}}$	Number of PM actions performed on component j of subsystem i in time [0, T]
$N_{i,j,\text{cm}}$	Number of CM actions performed on component j of subsystem i in time [0, T]
R_0	System reliability requirement
R_s	System reliability
γ_s	Total expected cost over a finite planning horizon T
z_s	Maintenance decisions over the entire planning horizon
δ_s	System inspection interval
δ_i	Inspection interval of subsystem i
ξ_i	PM threshold for subsystem i when n_i =1 (1-out-of-1 subsystem)
ω_i	Minimal number of failed components needed to trigger PM for subsystem i when $n_i \geq 2$ (k-out-of-n subsystem)
$D_{\text{f},i}$	Failure threshold of components in subsystem i
t_ν	Time at the ν^{th} inspection
γ_i	Expected cost of subsystem i
η_i	Expected cycle length of subsystem i
m	Number of the Bayesian updating procedures performed
$q_{s,s'}$	Transition rate from environmental state s to state s'
κ	Environment stress
ζ	Coefficient of Arrhenius reaction rate model
ψ	Exponential coefficient of Arrhenius reaction rate model

2.1. System description

Consider a system consisting of N subsystems, which are serially connected. Each subsystem is a k-out-of-n system ($1 \leq k < n$) with n_i identical components and at least k_i functioning components to ensure the functionality of each subsystem, $i \in \{1,\dots, N\}$. Note that a single-component subsystem, i.e., 1-out-of-1 subsystem, is a special case when $n_i = 1$. Fig. 1 illustrates the system structure of such a system. We assume that component failure is hidden and can only be detected through inspection. It is also assumed that maintenance of any component requires the shutdown of the system. For example, in many process industries, the entire production line needs to be shut down for maintenance. Another example is drilling systems that operate in oil and gas industries. Drilling equipment failures require taking the equipment out of the hole for replacement, resulting in nonproductive rig time. Preventive and corrective maintenance have a component-dependent cost and a system-dependent cost. Component-dependent cost is much lower for PM than for CM, i.e., $c_{i,\text{pm}} < c_{i,\text{cm}}$. The system-dependent cost is often referred to as the setup cost and is the same for all activities regardless of the number of components maintained and the type of maintenance actions. The shared setup cost is common due to crew travelling, scaffolding, shutdown, etc., and it may also consist of the downtime cost due to production loss if the system cannot be used

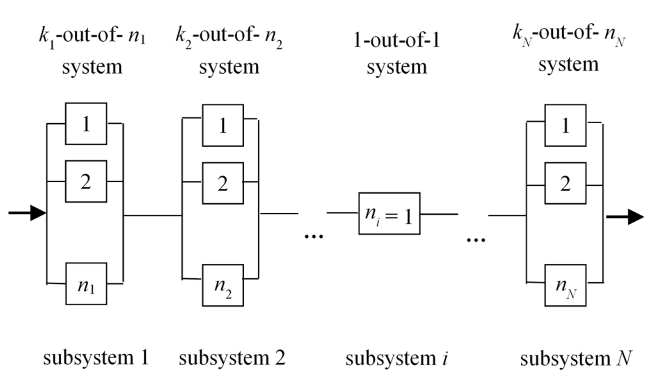

Fig. 1. System configuration.

during maintenance. If $\lambda_1(\lambda_2)$ components in subsystem i are preventively (correctively) maintained, this setup cost incurs once and the maintenance cost is $c_s + \lambda_1 c_{i,\text{pm}} + \lambda_2 c_{i,\text{cm}}$. This implies that executing $\lambda_1 + \lambda_2$ maintenance activities jointly can save a cost of $(\lambda_1 + \lambda_2 - 1)c_s$.

For notational convenience, the subscript i for the index of the subsystem is omitted in the analysis of the degradation model. We assume that components are subject to stochastic degradation, and the cumulative deterioration level is characterized by $X(t)$. The cumulative damage model using a damage accumulation function is given by [35]

$$C(X_{\nu+1}) = C(X_\nu) + D_\nu h(X_\nu), \tag{1}$$

where X_ν denotes the cumulative damage after ν increments, D_ν denotes the cumulative damage occurred at the $(\nu + 1)^{\text{th}}$ increment, and $h(\cdot)$ is the damage model function, and $C(\cdot)$ is the damage accumulation function. The cumulative damage model in a continuous version is given by [35]

$$\int_0^t \frac{1}{h(X(u))} dC(X(u)) = \int_0^t dD(u) = D(t) - D(0), \tag{2}$$

where $D(t)$ denotes the cumulative damage at time t. Several models for degradation can be obtained by selecting various forms of the functions $C(\cdot)$ and $h(\cdot)$ with an appropriate stochastic process $D(\cdot)$, e.g., Gamma process and geometric Brownian process. Note that we consider unknown parameters θ in the stochastic degradation model with prior knowledge of the distribution $\phi(\theta)$ due to limited historical data.

2.2. Maintenance cost model

The objective of the maintenance planning is to minimize the total expected cost subject to a system reliability requirement over a finite planning horizon. The reliability requirement at the system level considered in this paper is required for the entire time horizon. A pre-specified reliability requirement is often determined by customers in agreement with system manufacturers. For example, the minimal reliability requirement of an aircraft is set by the airline company in agreement with the aircraft manufacturer [36]. Let γ_s denote the total expected cost, T denote the finite planning horizon, and R_0 denote the pre-specified reliability requirement. We denote the total number of inspections performed in the finite planning horizon by N_{insp} and the number of inspections with maintenance performed by N_{mx}. The total setup cost is the product of the N_{mx} and c_s. Let $N_{i,j,\text{pm}}$ and $N_{i,j,\text{cm}}$ represent the numbers of PM and CM actions performed on component j in subsystem i, respectively, $i \in \{1,\dots, N\}$ and $j \in \{1,\dots, n_i\}$. Our decisions concern the system inspection interval (δ_s) and the maintenance decision (z_s) made in the entire planning horizon. The maintenance optimization model is formulated as follows.

Model 1

$$\min_{\delta_s, z_s} \quad \gamma_s = \sum_i \left(c_{i,\text{insp}} \times n_i \times E(N_{\text{insp}}) + \sum_j (c_{i,\text{pm}} \times E(N_{i,j,\text{pm}}) + c_{i,\text{cm}} \times E(N_{i,j,\text{cm}})) \right) + c_s \times E(N_{\text{mx}})$$

$$\text{s. t.} \quad R_s \geq R_0$$

Due to the combined stochastic failure processes and combinatorial optimization, it is difficult to derive explicitly analytic expressions for the optimal total cost and the maintenance grouping decision. The use of the predictive method in the decision process further complicates the model. We therefore propose a four-phase maintenance decision framework to find high-quality solutions. Details of this decision framework are presented in the next section.

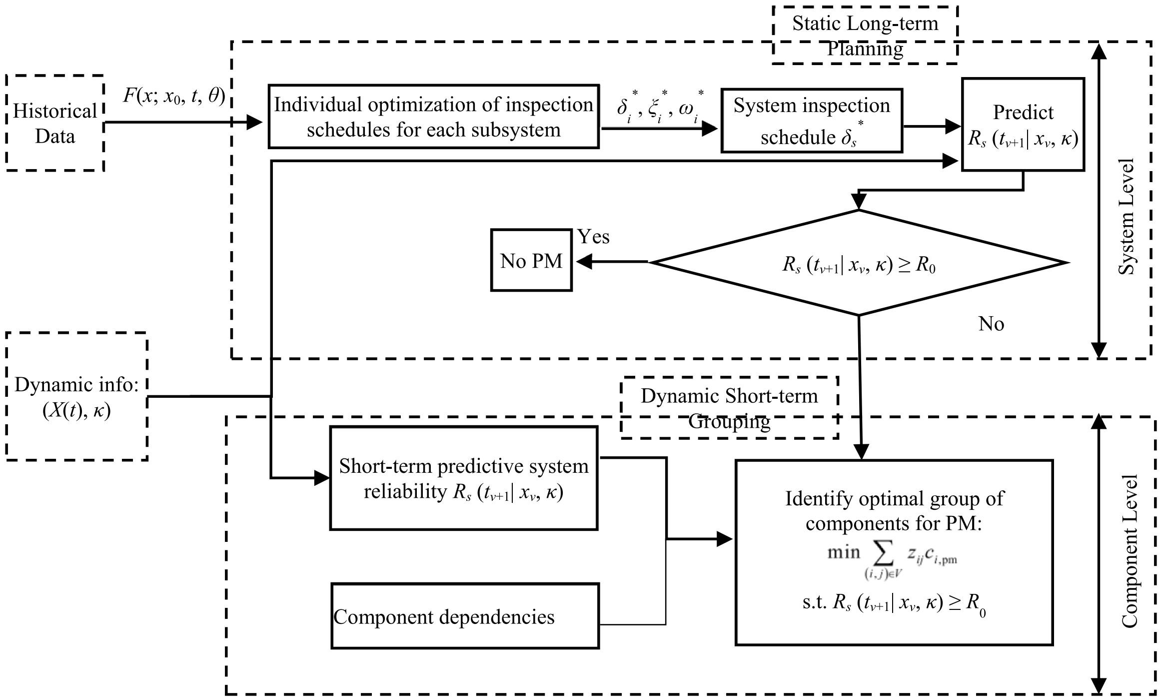

Fig. 2. A Unified Maintenance Decision-making Framework.

3. Condition-based maintenance decision framework

In this section, we develop a condition-based maintenance decision framework to combine static long-term inspection planning and dynamic short-term maintenance grouping. Specifically, the proposed approach starts with maintenance decisions at the system level regarding when to inspect and whether maintenance is needed. At each inspection, if needed, a Bayesian updating procedure is performed to update the posterior distributions of the failure model parameters with new degradation observations. The predictive system reliability is then computed and triggers PM when it is below the requirement. If the need for maintenance is identified at the system level, the next step is to dynamically select a group of components for PM actions based on information that becomes available on the short term. The joint long-term inspection planning and the short-term maintenance grouping are illustrated in Fig. 2.

This maintenance decision process consists of four phases. In Phase 1, we seek the optimal system inspection interval using a decomposition method. Phase 2 uses a Bayesian updating procedure leveraging short-term information. The optimal group of components for PM is identified in Phase 3. Finally, Phase 4 addresses the rolling horizon.

3.1. Phase 1: long-term inspection planning

In Phase 1, we use a decomposition method to seek the optimal system inspection interval. Specifically, we first find the optimal periodic inspection/replacement policy for each subsystem without considering their structural/economic dependences. The minimum of the optimal inspection intervals of all subsystems is used as the optimal system inspection interval [37]. Note that we only use the subsystems' inspection intervals in Phase 1 to determine the system inspection schedule, and maintenance decisions are determined using the dynamic-priority-based heuristic algorithm in Phase 3. For notational convenience, we omit the subscript i that represents the index of a subsystem in the inspection optimization model for each subsystem.

3.1.1. Optimal inspection schedule of a 1-out-of-1 subsystem

We first optimize the inspection policy for a 1-out-of-1 subsystem (i.e., a single-component subsystem). At each inspection, if the cumulative deterioration is between the PM threshold (ξ) and failure threshold (D_f), PM is performed; CM is performed if the cumulative deterioration is above the failure threshold; otherwise, do nothing. Let $f(x; x_0, t, \theta)$ and $F(x; x_0, t, \theta)$ denote the pdf and cdf of $X(t)$, respectively. Let γ denote the expected maintenance cost, η denote the expected length of a cycle. The optimal inspection interval δ^* and PM threshold ξ^* are obtained by minimizing the cost rate:

$$(\delta^*, \xi^*) = \arg\min_{\delta,\xi} \frac{\gamma(\delta, \xi)}{\eta(\delta, \xi)}. \tag{3}$$

We show the detailed derivations of the expected cost $\gamma(\delta, \xi)$ and the expected operational time $\eta(\delta, \xi)$ in Eq. (3) in Appendix A1.

3.1.2. Optimal inspection schedule for a k-out-of-n subsystem

We now optimize the inspection schedule for a k-out-of-n subsystem ($n \geq 2$). There are different maintenance policies for a k-out-of-n system. The one considered in this paper is similar to the policies in [38–40]. At each inspection, we perform CM on all components (failed or not failed) if the subsystem is found in a failure state (the number of failed components is above $n-k$), or perform PM on all components if the number of failed components reaches a pre-specified threshold (ω, $1 \leq \omega \leq n-k$). The pre-specified threshold is the minimal number of failed components needed to trigger PM. Thus, the decision variables in the inspection optimization for a k-out-of-n system are (δ, ω). The optimal inspection schedules (δ^*, ω^*) are determined by:

$$(\delta^*, \omega^*) = \arg\min_{\delta,\omega} \frac{\gamma(\delta, \omega)}{\eta(\delta, \omega)}, \tag{4}$$

The derivations of the expected cost $\gamma(\delta, \omega)$ and the expected length of a cycle $\eta(\delta, \omega)$ in Eq. (4) are shown in Appendix A2.

The derivatives of the two objective functions in Eqs. (3) and (4) are usually difficult to obtain, and therefore a numerical search algorithm without using derivative is employed to seek the optimal decision. Pattern search is used in this paper since it provides a useful exploratory tool and has good global behaviors, which is often useful when the objective function is not smooth and the derivatives of the objective function are either not available or not reliable [41]. To improve the solution quality, we adopt a multi-start-points search approach.

After obtaining the optimal inspection interval for each subsystem, we determine the optimal system inspection interval δ_s* as the minimal inspection interval of the N subsystems, that is, $\delta_s^* = \min\{\delta_i^*;\ i = 1, \ldots, N\}$.

3.2. Phase 2: Bayesian updating

At each inspection, we obtain the latest deterioration information of each component, and use the Bayesian approach to improve the estimation of the unknown parameters (θ) when needed. The subscript for subsystems is also omitted in the following discussion. Let $\Delta x_1,\ldots, \Delta x_n$ represent the degradation increments of all components during an inspection interval. We update the posterior distribution of the parameter vector θ as follows:

$$\phi'(\theta|\Delta x_1, \ldots,\Delta x_n) = \frac{L(\theta|\Delta x_1, \ldots,\Delta x_n)\phi(\theta)}{\varphi(\Delta x_1, \ldots,\Delta x_n)}, \tag{5}$$

where $\varphi(\Delta x_1, \ldots,\Delta x_n) = \int L(\theta|\Delta x_1, \ldots,\Delta x_n)\phi(\theta)d\theta$ and $L(\theta|\Delta x_1, \ldots, \Delta x_n)$is the likelihood function. The integral, $\varphi(\Delta x_1,\ldots, \Delta x_n)$, is tractable for conjugate families. A widely used distribution family that provides conjugate priors is the exponential family. For a conjugate prior, the family of the prior distribution is chosen such that prior-to-posterior updating yields a posterior that is also in the family. However, to appropriately describe a failure process, one may need to use non-conjugate priors which leads to intractable Bayesian computing of the posterior distribution. When the Bayesian computing of the posterior distribution is intractable, we can resort to numerical methods such as Markov Chain Monte-Carlo (MCMC) sampling methods.

We assume different components work independently, and their updating procedures can be done independently for each subsystem. Based on the posterior distribution of θ for each subsystem, the predictive reliability of component j, $R_j(t \mid \Delta x_1,\ldots, \Delta x_n)$, can be computed as follows:

$$R_j(t|\Delta x_1,\ldots,\Delta x_n) = \int F(D_f;\, x_j,\, t,\, \theta)\phi'(\theta|\Delta x_1,\ldots,\Delta x_n)d\theta, \tag{6}$$

where x_j is the degradation level of component j at the current inspection time.

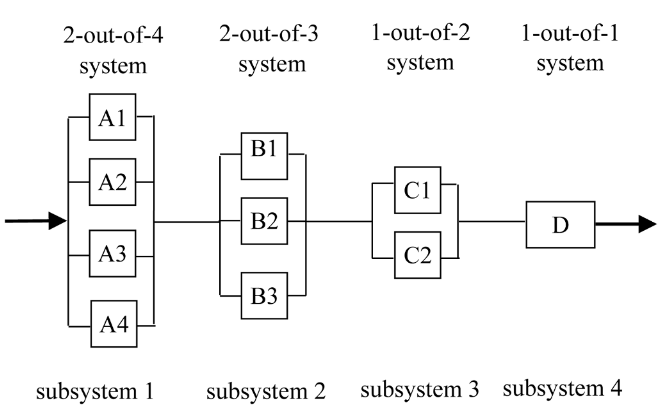

Fig. 3. System configuration in sensitivity analysis.

Table 1
The degradation and cost parameters for all subsystems.

Subsystem	α_i	β_i	a_i	$D_{f,i}$	$c_{i,\mathrm{pm}}$
1	2.74	4.20	5.30	48.72	61.70
2	1.10	3.30	5.27	30.52	92.03
3	3.20	2.05	5.62	70.99	20.01
4	2.74	6.19	5.53	18.71	50.23

3.3. Phase 3: grouping maintenance activities

3.3.1. Preventive maintenance grouping problem

At the system level, components in any failed subsystem are all replaced upon inspection and the system reliability is computed. If the system reliability is above the reliability requirement, no PM is needed. Otherwise, PM actions are desired and the next step is to find the optimal group of components to be preventively maintained. Note that the reliability requirement for the entire time horizon is aimed to be met by imposing such a requirement for each inspection interval.

Based on the updated predictive reliability of each component as described in Phase 2, we can compute the predictive reliability of each subsystem and obtain the system predictive reliability. We now compute the predictive reliability for a k-out-of-n subsystem. Since components may have different predictive reliabilities in each subsystem, we use the symmetric switching function approach provided by Rushdi [42] to compute the subsystem's reliability recursively as follows:

$$R_s(i, j) = R_jR_s(i - 1, j - 1) + (1 - R_j)R_s(i, j - 1),\ 1 \le i \le k,\ 1 \le j \le n, \tag{7}$$

with boundary conditions $R_s(0, j) = 1$ and $R_s(j + 1, j) = 0$, where R_j denotes the reliability of component j, and $R_s(i, j)$ represents the system reliability for an i-out-of-j system.

If a PM decision is triggered at the system level ($R_s < R_0$), a group of components that are not performed with CM needs to be selected for PM. The objective here is to select a group of components for PM to best ensure the reliability requirement of the system at minimal costs. Let z_{ij} be a binary decision variable, indicating whether component j in subsystem i is selected for PM. The maintenance grouping problem at an inspection is formulated as follows:

Model 2

$$\min \gamma_{\mathrm{pm}} = \sum_{(i,j)\in V} z_{ij}c_{i,\mathrm{pm}} \quad \text{s.t.} \quad R_s \ge R_0, \quad z_{ij} \in \{0,\, 1\},$$

where γ_{pm} is the total PM cost at the current decision time and V

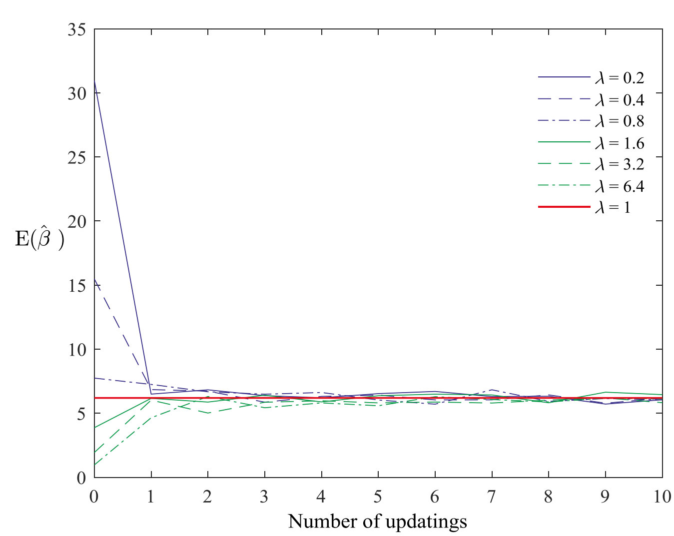

Fig. 4. Impacts of the number of Bayesian updatings on the posterior mean of β.

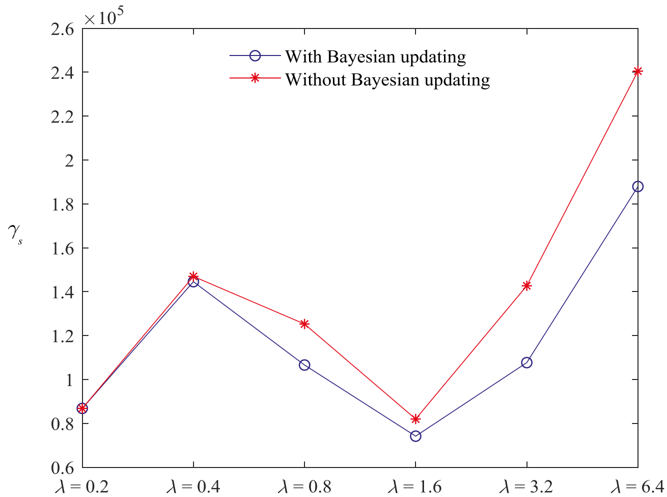

Fig. 5. Comparison of the total costs with and without Bayesian updating under different λ values.

represents the set of components that are not performed with CM. Model 2 is a combinatorial optimization. Suppose the cardinality of the set V is m ($|V| = m$), the number of possible combinations for maintenance grouping is $2^m - 1$. It is not feasible to enumerate all possible groups to obtain the optimal group decision. Therefore, we design an efficient heuristic algorithm based on the priority measure for component selection.

3.3.2. Dynamic-priority-based heuristic algorithm for component selection

In the dynamic-priority-based heuristic algorithm, we first develop an importance measure that ranks components' priorities dynamically. Importance measures are generally used to quantify the contribution of individual elements of a system to the overall system performance [43, 44]. Traditional reliability importance measures include Birnbaum reliability importance measure [45], Criticality Importance [43], Reliability Reduction Worth [46], and Reliability Achievement Worth [46]. However, the majority of the existing criticality measures have been developed for components with specified finite mission times, and are static without considering components' changing conditions. In order to account for the component's current deterioration information, a modified Reliability Achievement Worth (RAW) measure, denoted by σ, is proposed in this paper. The standard RAW of component i in subsystem j (σ_{ij}^{o}) is the ratio of the actual system reliability obtained when the component is always in perfect functioning ($R_{ij}(t) = 1$) to the actual value of the system reliability ($R_s(t)$). This measure quantifies the maximum possible percentage increase in system reliability generated by a component, and is defined as follows:

$$\sigma_{ij}^{o} = \frac{R_s(t;\, R_{ij}(t) = 1)}{R_s(t)}. \tag{8}$$

To include a component's current reliability (deterioration), the modified RAW of component j in subsystem i at the νth inspection, denoted by σ_{ij}, is defined as:

$$\sigma_{ij} = \frac{R_s(t_{\nu+1};\, X_{ij}(t_\nu) = 0) - R_s(t_{\nu+1};\, X_{ij}(t_\nu) = x_{ij}^{\nu})}{R_s(t_{\nu+1};\, X_{ij}(t_\nu) = x_{ij}^{\nu})}. \tag{9}$$

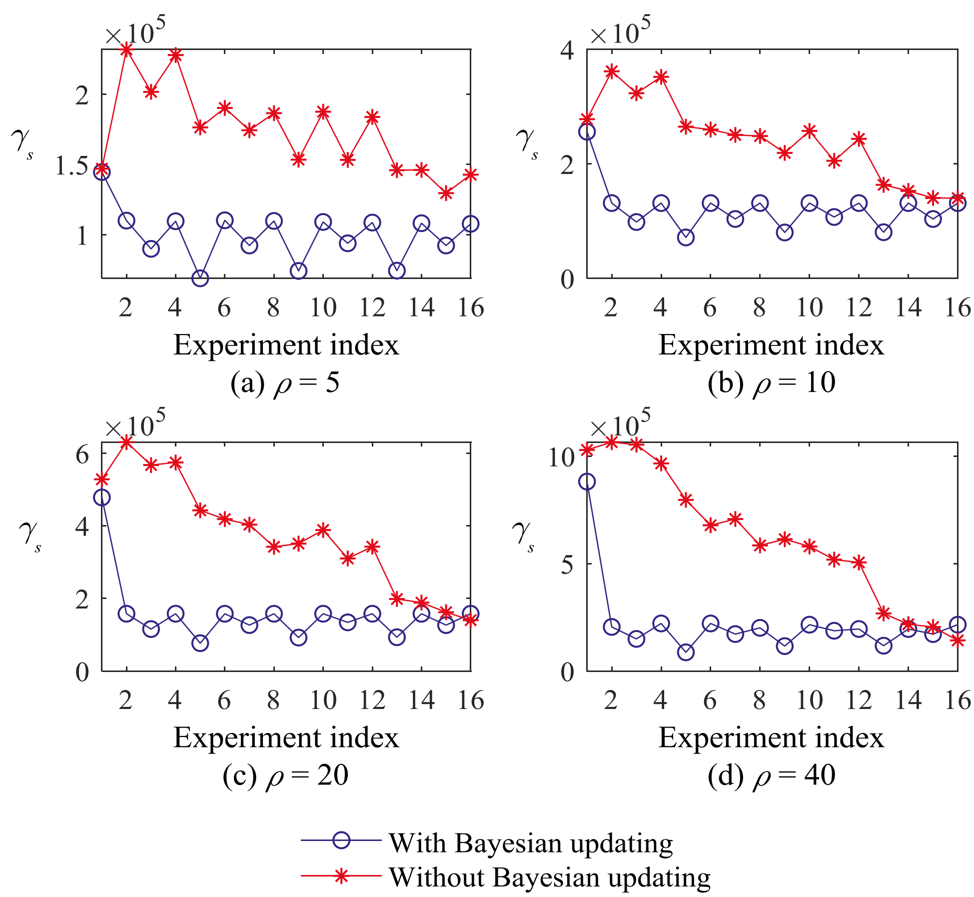

Fig. 6. Comparison of the total costs with and without Bayesian updating.

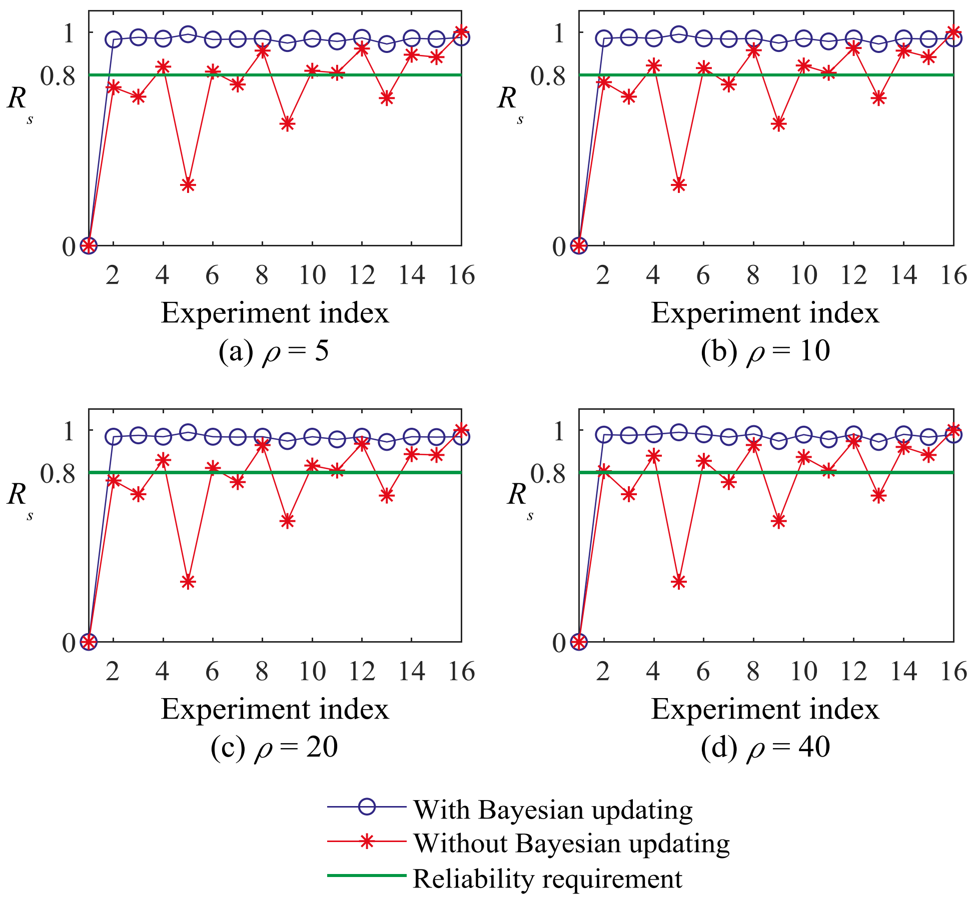

Fig. 7. Comparison of the system reliabilities with and without Bayesian updating.

In the modified RAW, the component's current deterioration is taken into consideration, where a less reliable component is given more attention. Meeting the system requirement at reduced costs requires accounting for a component's σ_{ij} and cost, and therefore, we use the ratio of σ_{ij} to its PM cost, denoted by π_{ij}, to set the priority of a component, i.e., $\pi_{ij} = \sigma_{ij} / c_{i,\mathrm{pm}}$. Note that the priority measure used in the proposed dynamic-priority-based heuristic algorithm is a composite index that considers components' dynamic importance in system reliability and maintenance costs. Common reliability importance measures mentioned previously only consider components' importance in terms of reliability. Selecting components for maintenance entirely based on a reliability importance measure may lead to a reliability improvement at much higher costs. On the other hand, selecting components for maintenance based only on costs may not effectively improve system's reliability.

In the heuristic algorithm, we rank the components in descending order with respect to the ratio and select the component for PM sequentially until the system reliability requirement is met. Note that every time a component is selected for PM, we re-calculate the modified RAW's for the remaining components. This is because the contribution of each component to the improvement of system reliability may change after a component being selected for PM. For example, for a subsystem consisting of three components in parallel, if one of the three components is selected for PM, we can expect the importance of the other two components to decrease. Algorithm 1 presents the component selection procedure in detail.

3.4. Phase 4: rolling-horizon step

A rolling-horizon approach is considered in this paper that maintenance decisions are repeatedly optimized at each inspection when new information on component degradation becomes available [47]. This means that at each inspection, if the Bayesian updating is desired, the procedure starts from Phase 2 again. The posterior distributions are updated, and the reliability of the system at the next inspection is predicted with the updated posterior distributions and the (expected) future environmental condition and the planned usage. If the degradation model parameters are accurate enough and no more updating is needed, the procedure starts from Phase 3. The maintenance grouping is sequentially determined based on the updated predictive reliability.

4. Sensitivity analysis

In this section, we assess the benefits of the Bayesian updating by comparing the total costs and the system reliabilities with and without performing the updating procedure in the maintenance decision framework. We also investigate whether the Bayesian updating affects the maintenance decision at the system level (i.e., system inspection interval). Furthermore, we demonstrate the flexibility of the proposed framework by incorporating dynamic environmental information in decision making and examine the effects of the Bayesian updating in a dynamic environment.

Consider a system that consists of four subsystems in series. The

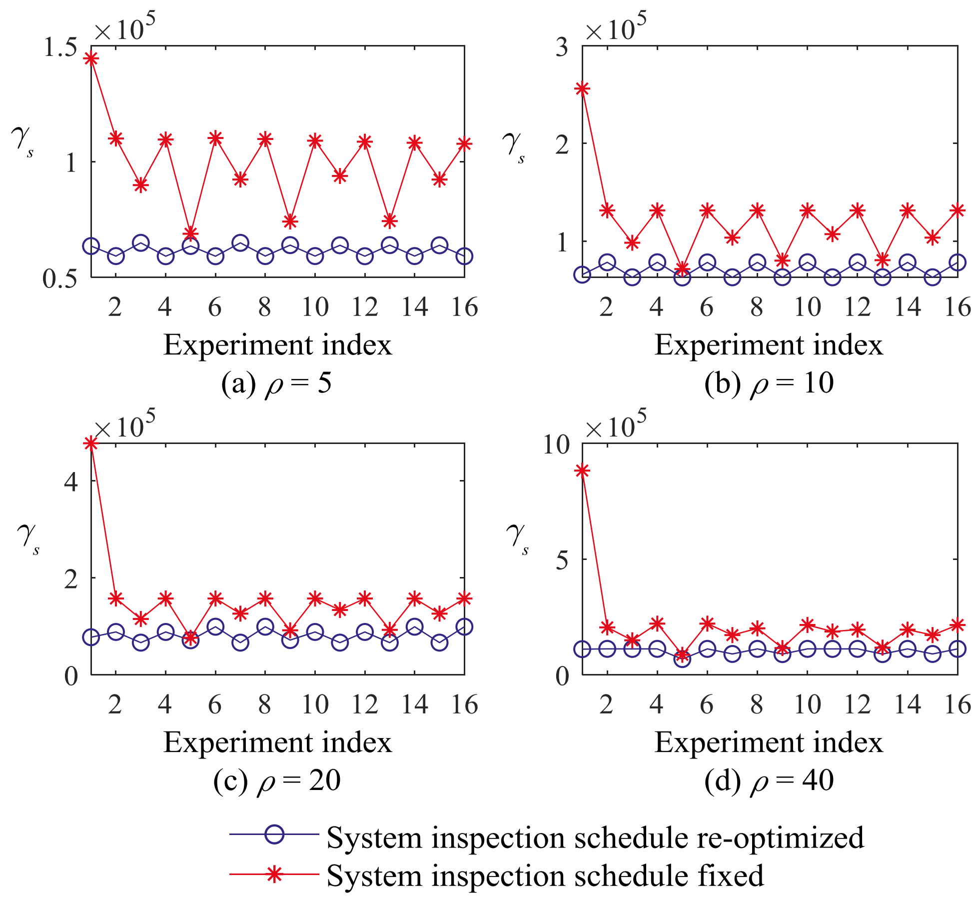

Fig. 8. Comparison of the total costs with and without system inspection schedule re-optimized.

subsystems are a 2-out-of-4 system, a 2-out-of-3 system, a 1-out-of-2 system, and a 1-out-of-1 system (illustrated in Fig. 3). We assume that the degradation process of the component in subsystem i is described by a Gamma process, Gamma ($\alpha_i t$, β_i), i = 1, 2, 3, and 4, and omit the script i in the following discussion. The pdf of degradation increment during the system inspection interval $X(t + \delta_s{}^*) - X(t)$ is

$$Ga(x|\alpha\delta_s^*, \beta) = \frac{\beta^{\alpha\delta_s^*}}{\Gamma(\alpha\delta_s^*)} x^{\alpha\delta_s^*-1} e^{-\beta x}. \tag{10}$$

Assume that the shape parameter α is known, the rate parameter β is unknown. We use the conjugate prior since it provides an analytically tractable posterior of the rate parameter. The conjugate prior ϕ(.), is a Gamma distribution of β with a shape parameter a and a rate parameter b [48], i.e., $\beta \sim$ Gamma (a, b). By Eq. (5), we obtain the posterior distribution of β given degradation increments $\Delta x_1,\ldots, \Delta x_n$, which is a Gamma distribution with the shape parameter $a' = a + \alpha\delta_s{}^* n$ and the rate parameter $b' = b + \Sigma\, \Delta x_i$. The analytical form of the posterior predictive reliability of component j in a k-out-of-n subsystem before the next inspection time is given as follows:

$$R_j(\delta_s^*|\Delta x_1, \ldots, \Delta x_n) = \int_0^\infty \int_{x_j}^{D_f} \frac{\beta^{\alpha\delta_s^*}}{\Gamma(\alpha\delta_s^*)} x^{\alpha\delta_s^*-1} e^{-\beta x} \frac{(b + \sum \Delta x_i)^{a+\alpha\delta_s^* n}}{\Gamma(a + \alpha\delta_s^* n)} \beta^{a+\alpha\delta_s^* n-1} e^{-(b+\sum \Delta x_i)\beta} dx d\beta. \tag{11}$$

Since the conjugate prior is used in the Bayesian updating, resulting in the analytical form of the posterior distribution, it does not require much computational effort. We therefore perform the updating procedure continuously at each inspection to obtain more accurate parameter estimation when new information becomes available.

In the sensitivity analysis, suppose that the inspection cost per component $c_{i,\mathrm{insp}}$ is 10 for all subsystems, the set-up cost c_s is 50, and the system reliability threshold R_0 is 0.8. The value of the shape parameter α_i is drawn from U(1, 5), and β_i from U(0, 10). The initial estimation of the shape parameter a_i, drawn from U(5, 6), is fixed for all components. The failure threshold $D_{f,i}$ of components in subsystem i is drawn from U (10, 80). The PM cost $c_{i,\mathrm{pm}}$ is drawn from U(20, 120), and the CM cost $c_{i,\mathrm{cm}}$ is determined as $c_{i,\mathrm{cm}} = \rho c_{i,\mathrm{pm}}$, ρ = 5, 10, 20, and 40. The details of the parameter values are provided in Table 1.

4.1. Effects of Bayesian updating in stationary environment

We first conduct the sensitivity analysis for the system operating in a stationary environment. The parameters of interest include the initial estimation of the unknown rate parameter and the cost ratio of CM to PM.

4.1.1. Effects of Bayesian updating on system performance

To examine the effects of the Bayesian updating, we parameterize the distribution of the rate parameter β with different initial estimations of b and keep the value of a constant. We model the initial value of b as λb_0 such that the prior mean is the true β value when $\lambda = 1$. Six different λ values are considered, λ = 0.2, 0.4, 0.8, 1.6, 3.2, and 6.4. When $\lambda < 1$, the prior mean is larger than the true β value, and vice versa.

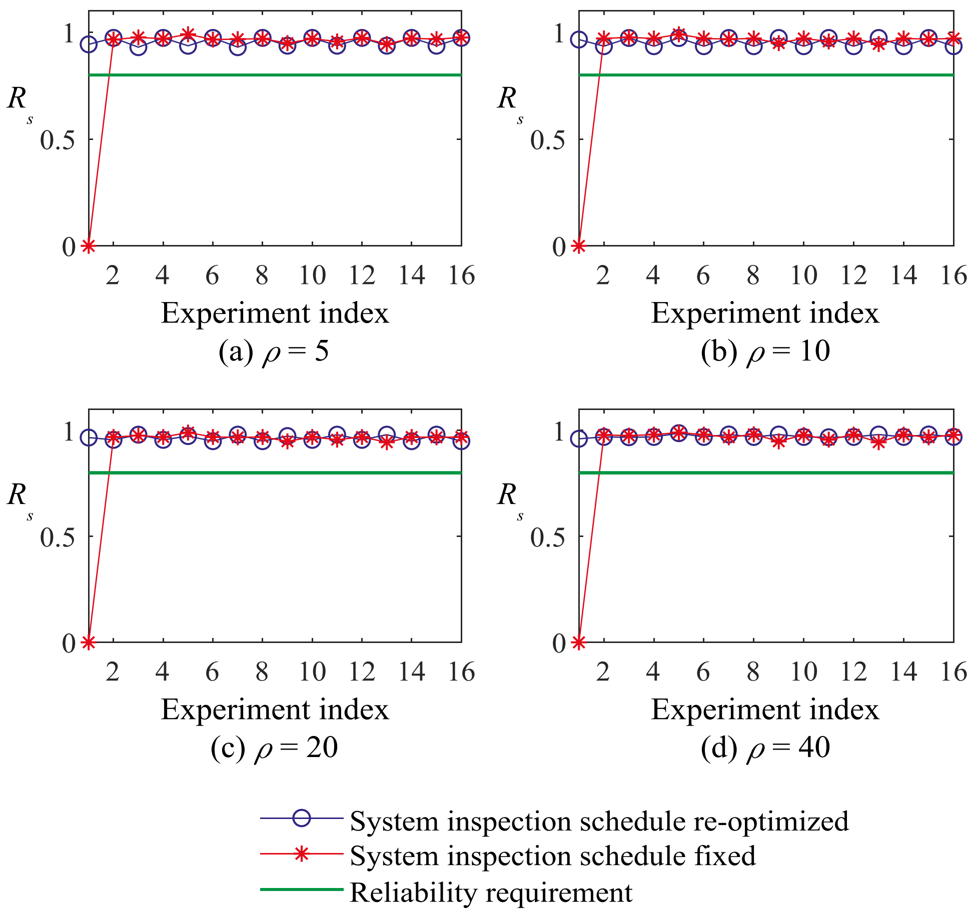

Fig. 9. Comparison of the system reliabilities with and without system inspection schedule re-optimized.

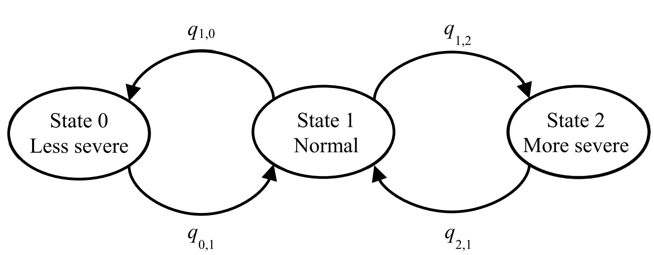

Fig. 10. State transition diagram of the dynamic environment.

We first examine how the posterior mean changes as the number of the Bayesian updating procedure performed increases. We arbitrarily use component D (shown in Fig. 3) to illustrate this impact. From Fig. 4, we can see that the posterior means have significant improvements when the Bayesian updating is performed, and converge to the true value after performing several updating procedures.

We further investigate the impacts of the Bayesian updating on the total cost. Suppose the ratio of CM cost to PM cost is 5 for all components ($\rho = 5$), and the time horizon is 5000 ($T = 5000$). We compare the total costs with and without the Bayesian updating. The same set of λ values is considered and the value of λ is kept the same for all components. From Fig. 5, we can see that the total costs with updating performed are lower for all the experiments considered except for $\lambda = 0.2$ and $\lambda = 0.4$ when the prior mean of the rate parameter (β) is largely overestimated. This is because a significant underestimation of component degradation due to the overestimation of the rate parameter leads to an inappropriately long inspection, resulting in a large number of unexpected system failures. We also observe that the more the prior mean is underestimated, the larger the improvement becomes.

In the previous analysis, we only consider the experiments where λ is the same for all components, meaning the prior means are either all larger or smaller than the true β values. In practice, it is likely that the prior mean is larger than the true model parameter for components in some subsystems and smaller for components in other subsystems. To better assess the benefits from the Bayesian updating on system performances in terms of the total cost and the system reliability, we consider two levels of λ, low ($\lambda = 0.4$) and high ($\lambda = 3.2$) for each subsystem. We also consider four different cost ratios of CM to PM, $\rho = 5$, 10, 20, and 40. There are four subsystems in the system, and we thus have 16 experiments for each ρ value. Before analyzing the experimental results, we define the system reliability as the ratio of the number of inspections with at least one CM action performed (N_{cm}) to the total number of inspections (N_{insp}) in the finite time horizon considered.

Fig. 6 compares the total costs with and without the Bayesian updating. We can see that the experiments with the Bayesian updating outperform those without for the majority of the 64 experiments investigated. Fig. 7 illustrates the impacts of the Bayesian updating on the system reliabilities. It is clear that all the experiments with updating performed have higher or equal system reliabilities than those without updating.

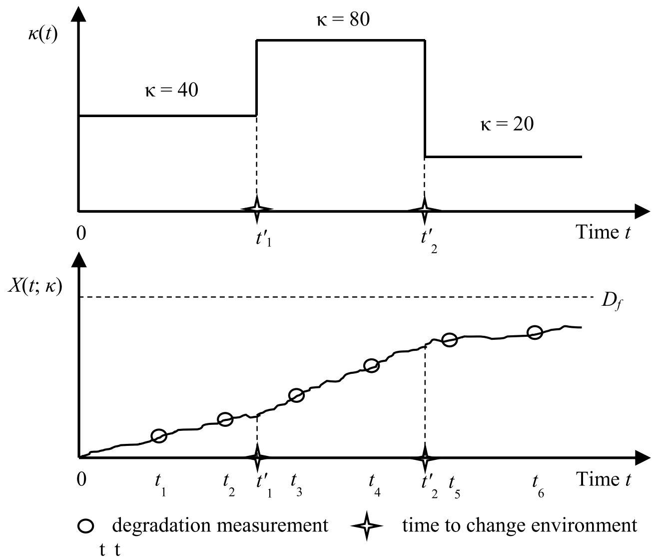

Fig. 11. Illustration of the degradation path in a dynamic environment.

Table 2
Shape parameters under different environmental states.

State	Subsystems 1	2	3	4
0	1.23	0.88	1.33	1.23
1	2.31	1.65	2.49	2.30
2	3.15	2.25	3.40	3.15

Algorithm 1
Dynamic-priority-based heuristic.

Input: Current system reliability R_s, reliability requirement R_0, component set for PM V, and PM cost for each subsystem $c_{i,\text{pm}}$, $i \in \{1, 2, \ldots, N\}$;
Output: Optimal PM decision V_{pm};
Initialize $V_{\text{pm}} \leftarrow \varnothing$, $\gamma_{\text{pm}} \leftarrow 0$;
while $R_s \le R_0$ or $V \ne \varnothing$ **do**
Compute π_{ij} based on Eq. (9), $\forall\ (i, j) \in V$;
Select component j^* in subsystem i^* for PM from $(i^*, j^*) = \arg\max_{(i,j)\in V} \{\pi_{ij}\}$;
$V \leftarrow V \setminus \{(i^*, j^*)\}$;
$V_{pm} \leftarrow V_{pm}\{(i^*, j^*)\}$;
$\gamma_{pm} \leftarrow \gamma_{pm} + c_{i*,pm}$;
Update R_s;
end while

4.1.2. Effects of Bayesian updating on system inspection interval

As we keep updating the distribution of the unknown parameters, a better estimation of the degradation distribution is obtained. This naturally leads to an important question: Do we need to re-optimize the system inspection interval which is obtained based on the prior means of the unknown parameters? We investigate this necessity by comparing the total costs and the system reliabilities under a fixed system inspection schedule with those under a dynamic inspection schedule that is sequentially optimized. We re-optimize the system inspection schedule with the latest posterior mean of the distribution of β at the first ten inspections. The comparison results are demonstrated in Figs. 8 and 9. From Figs. 8 and 9, we can see that all experiments with re-optimizing the system inspection interval have lower total costs than those without, and the difference between the system reliability is small except for the case when initial parameters of all subsystems are underestimated. This is because extremely long inspection interval due to initially inaccurate parameter estimation leads to a large number of system failures and a low system reliability when the system inspection interval is not re-optimized with the updated degradation information. Our analysis shows that it is worthy to re-optimize the system inspection interval with the short-term degradation information.

4.2. Effects of Bayesian updating in dynamic environment

In this section, we consider systems operating in a dynamic environment. Consider an operating environment whose transition is governed by a continuous-time Markov chain (CTMC). We assume that the environment has three states (illustrated in Fig. 10): 0, 1, and 2 representing less severe, normal, and more severe environments, respectively. Let $q_{s,s'}$ denote the transition rate from state s to state s', $s, s' \in \{0, 1, 2\}$. Suppose $q_{0,2} = q_{2,0} = 0$, $q_{1,0} = q_{1,2} = 1/500$, $q_{0,1} = 1/200$, and $q_{2,1} = 1/300$. We choose these transition rates so that the expected sojourn time in one state is longer than the inspection interval (illustrated in Fig. 11).

The degradation rates are different at different environmental states. We use the Arrhenius reaction rate model to describe the relationship between the environmental condition and the degradation rate [49]. Let κ represent the environment stress. We assume that the shape parameter α of the degradation process is a function of the environment stress κ, denoted as $\alpha(\kappa) = \zeta e^{\psi/\kappa}$. Suppose that the environment stress has three levels, which are 20, 40, and 80 corresponding to states 0, 1, and 2, respectively. The value of coefficient ψ is assumed to be the same for all components, i.e., $\psi = -25$, and the value of ζ varies among components, which follows $U(3, 6)$. The values of α under different environments for all components are presented in Table 2.

In order to explore the benefits of incorporating dynamic environmental information, we compare the total costs and the system reliabilities with and without considering the environmental impacts. Assume that the environmental state can only be revealed through inspection. At the end of an inspection, say the ν^{th} inspection, we update the prior distribution of β with the observed degradation increments during the ν^{th} inspection interval and the environmental state at the beginning of the ν^{th} inspection. We re-optimize the system inspection interval for better system performances at each inspection when new information (i.e., degradation levels and environmental condition) becomes available. We use the environmental state observed at the beginning of the $(\nu+1)^{\text{th}}$ inspection to re-optimize the system inspection interval and compute the predictive reliability. Notice that the change of the environment during an inspection interval is ignored. We expect this neglect to have little impact, since the expected duration time in any state is longer than the inspection interval length.

Fig. 12 compares the total costs with and without considering environmental information. We can see that the total costs with consideration of short-term environmental information are better than those without in most cases, and about 93.75% of the experiments have lower costs when the environmental information is incorporated. Fig. 13 shows that all experiments with environmental information incorporated satisfy the reliability requirement while those ignoring environmental information fail to meet this requirement. Our analysis shows that taking account of this information can lead to better system performances.

5. Conclusion

In this paper, we develop a condition-based maintenance framework for a multi-component system. The proposed framework joins the long-term system inspection schedule based on the historical data and the short-term dynamic grouping based on the newly observed information available. More importantly, this framework allows the Bayesian

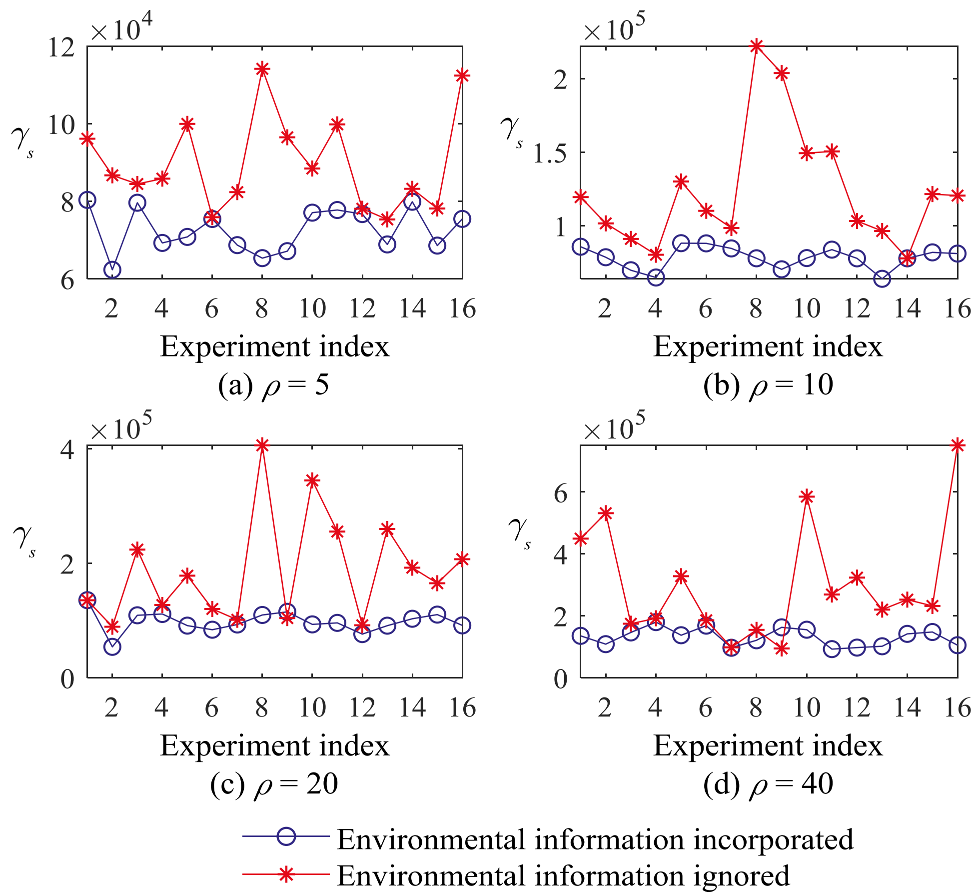

Fig. 12. Comparison of the total costs with and without dynamic environmental information incorporated.

updating on the underlying failure process, which makes the reliability prediction more accurate and customized for the in-service units. An efficient dynamic-priority-based heuristic algorithm is developed to quickly identify the components for group maintenance to meet the reliability requirement at the minimum cost at each inspection. Our research findings show that it is necessary to perform the Bayesian updating especially when the prior mean largely excesses the true model parameter value. We also find that there is a need to re-optimize the system inspection interval sequentially because the inaccurately initial estimation leads to unexpected inspection schedules. Lastly, our analysis shows that the proposed framework is flexible and can incorporate dynamic information (e.g., environmental condition), and such an incorporation is essential to lower the total cost and satisfy the system reliability requirement.

Future extensions of this work will focus on extending the current model to a more complex system such as systems with a more complex configuration (e.g., network systems) or systems with a combination of different dependences such as structural and stochastic dependences. In addition, it is worth considering a dynamic or adaptive system inspection schedule rather than the static one considered in this paper.

CRediT authorship contribution statement

Yue Shi: Writing - original draft, Software, Investigation, Data curation, Visualization. **Weihang Zhu:** Software, Investigation, Writing - review & editing. **Yisha Xiang:** Conceptualization, Methodology, Supervision, Funding acquisition, Writing - review & editing. **Qianmei Feng:** Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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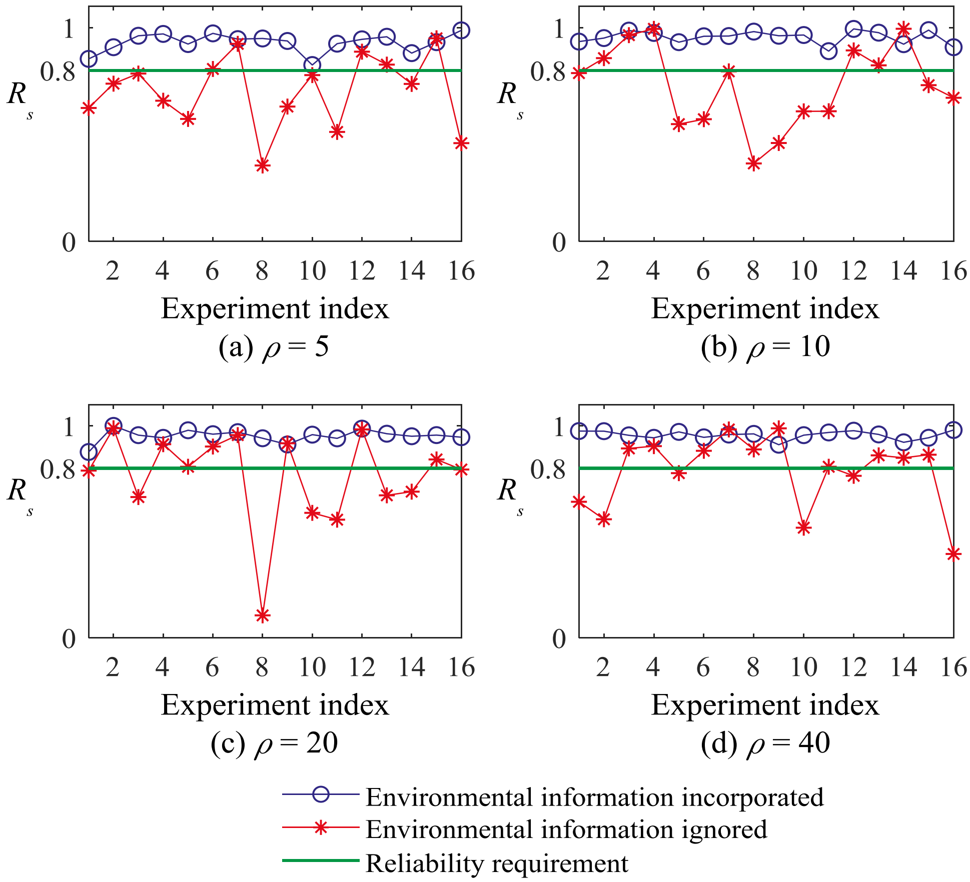

Fig. 13. Comparison of the system reliabilities with and without dynamic environmental information incorporated.

Appendix

A1. Derivations of γ (δ, ξ) and η(δ, ξ) in Eq. (3)

We first derive the expected cost incurred during a cycle $\gamma\,(\delta, \xi)$ in Eq. (3):

$$\begin{aligned}\gamma(\delta, \xi) &= \sum_{\nu=1}^{\infty}(c_{\text{pm}} + \nu c_{\text{insp}})\Pr(\text{PM at the } \nu^{\text{th}} \text{ inspection}) \\ &\quad + \sum_{\nu=1}^{\infty}(c_{\text{cm}} + \nu c_{\text{insp}})\Pr(\text{CM at the } \nu^{\text{th}} \text{ inspection}) \\ &= \sum_{\nu=1}^{\infty}(c_{\text{pm}} + \nu c_{\text{insp}})\int_{x_0}^{\xi} f(u; x_0, (\nu-1)\delta, \theta)(F(D_f - u; 0, \delta, \theta) - F(\xi - u; 0, \delta, \theta))du \\ &\quad + \sum_{\nu=1}^{\infty}(c_{\text{cm}} + \nu c_{\text{insp}})\int_{x_0}^{\xi} f(u; x_0, (\nu-1)\delta, \theta)(1 - F(D_f - u; 0, \delta, \theta))du.\end{aligned} \tag{12}$$

Similarly, we derive the expected operation duration time $\eta(\delta, \xi)$ in Eq. (3):

$$\begin{aligned}\eta(\delta, \xi) &= \sum_{\nu=1}^{\infty}\nu\delta(\Pr(\text{PM at the} \nu^{\text{th}} \text{ inspection}) + \Pr(\text{CM at the} \nu^{\text{th}} \text{ inspection})) \\ &= \sum_{\nu=1}^{\infty}\nu\delta\left(\begin{aligned}&\int_{x_0}^{\xi} f(u; x_0, (\nu-1)\delta, \theta)(F(D_f - u; 0, \delta, \theta) - F(\xi - u; 0, \delta, \theta))du \\ &+ \int_{x_0}^{\xi} f(u; x_0, (\nu-1)\delta, \theta)(1 - F(D_f - u; 0, \delta, \theta))du\end{aligned}\right) \\ &= \sum_{\nu=1}^{\infty}\nu\delta\left(\int_{x_0}^{\xi} f(u; x_0, (\nu-1)\delta, \theta)(1 - F(\xi - u; 0, \delta, \theta))du\right).\end{aligned} \tag{13}$$

A2. Derivations of γ (δ, ω) and η(δ, ω) in Eq. (4)

The expected cost and the expected operation time in Eq. (4) are given as follows

$$\gamma(\delta,\omega) = \sum_{\nu=1}^{\infty} n(c_{\rm pm}+\nu c_{\rm insp}){\rm Pr}(\text{PM at the} \nu^{\rm th} \text{ inspection}) + \sum_{\nu=1}^{\infty} n(c_{\rm cm}+\nu c_{\rm insp}){\rm Pr}(\text{CM at the } \nu^{\rm th} \text{ inspection}), \tag{14}$$

$$\eta(\delta,\omega) = \sum_{\nu=1}^{\infty} \nu\delta({\rm Pr}(\text{PM at the } \nu^{\rm th} \text{ inspection}) + {\rm Pr}(\text{CM at the } \nu^{\rm th} \text{ inspection})). \tag{15}$$

In order to obtain the probabilities that CM and PM occur at the $\nu^{\rm th}$ inspection, we first derive the probability that a component fails before the $\nu^{\rm th}$ inspection, denoted by H_ν, as follows:

$$H_\nu = 1 - F(D_{\rm f}; x_0, \nu\delta, \theta),\ \nu \geq 1. \tag{16}$$

We then compute the probability that a component fails between the $(\nu-1)^{\rm th}$ and $\nu^{\rm th}$ inspections, represented by G_ν, as:

$$G_\nu = \begin{cases} H_1, & \nu = 1 \\ 1 - H_{\nu-1} - \int_{x_0}^{D_{\rm f}} F(D_{\rm f}-u; 0, \delta, \theta) f(u; x_0, (\nu-1)\delta, \theta)du, & \nu \geq 2. \end{cases} \tag{17}$$

Based on H_ν and G_ν, we have the probability that PM occurs at the $\nu^{\rm th}$ inspection:

$$\Pr\{\text{PM at the } \nu^{\rm th} \text{ inspection}\} = \begin{cases} \sum_{i=\omega}^{n-k} \binom{n}{i}(G_1)^i(1-G_1)^{n-i}, & \nu = 1 \\ \sum_{i=\omega}^{n-k}\sum_{j=0}^{\omega-1} \binom{n}{j}(H_{\nu-1})^j(1-H_{\nu-1})^{n-j}\binom{n-j}{i-j}(G_\nu)^{i-j}(1-H_{\nu-1}-G_\nu)^{n-i}, & \nu \geq 2. \end{cases} \tag{18}$$

We also obtain the probability that CM occurs at the $\nu^{\rm th}$ inspection as follows:

$$\Pr\{\text{CM at the } \nu^{\rm th} \text{ inspection}\} = \begin{cases} \sum_{i=n-k+1}^{n} \binom{n}{i}(G_1)^i(1-G_1)^{n-i}, & \nu = 1 \\ \sum_{i=n-k+1}^{n}\sum_{j=0}^{\omega-1} \binom{n}{j}(H_{\nu-1})^j(1-H_{\nu-1})^{n-j}\binom{n-j}{i-j}(G_\nu)^{i-j}(1-H_{\nu-1}-G_\nu)^{n-i}, & \nu \geq 2. \end{cases} \tag{19}$$

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