

Transportation Research Record

ELECTRIC VEHICLE TRAVELLING SALESMAN PROBLEM WITH DRONE

--Manuscript Draft--

Full Title:	ELECTRIC VEHICLE TRAVELLING SALESMAN PROBLEM WITH DRONE
Abstract:	<p>The idea of deploying electric vehicle and unmanned aerial vehicles, also known as drones, to perform "last-mile" delivery in logistics operations has attracted increasing attention in the past few years. In this paper, an Electric Vehicle Travelling Salesman Problem with Drone (EVTSPD) is formulated as a mixed integer linear program to aid logistic organizations with a new method of delivering parcels which can extend the driving range of both vehicles and decrease the operation cost. An iterative solution heuristic is also developed. Results of numerical experiments show that the heuristic is much more efficient than solving the problem via a standard solver. Incorporating UAVs into EV based routing were found to reduce average delivery times by up to 40% for the instances tested.</p>
Manuscript Classifications:	Operations and Traffic Management; Freight Transportation Planning and Logistics AT015; Logistic; Transit, Freight, and Logistics Modeling ADB30SA; General
Manuscript Number:	20-01548
Article Type:	Presentation
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1 **ELECTRIC VEHICLE TRAVELLING SALESMAN PROBLEM WITH DRONE**

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18 Word Count: 4663 words + 2 table(s) \times 250 = 5163 words

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25 Submission Date: August 1, 2019

Under Review

1 ABSTRACT

2 The idea of deploying electric vehicle and unmanned aerial vehicles, also known as drones, to
3 perform "last-mile" delivery in logistics operations has attracted increasing attention in the past
4 few years. In this paper, an Electric Vehicle Travelling Salesman Problem with Drone (EVTSPD)
5 is formulated as a mixed integer linear program to aid logistic organizations with a new method of
6 delivering parcels which can extend the driving range of both vehicles and decrease the operation
7 cost. An iterative solution heuristic is also developed. Results of numerical experiments show that
8 the heuristic is much more efficient than solving the problem via a standard solver. Incorporating
9 UAVs into EV based routing were found to reduce average delivery times by up to 40% for the
10 instances tested.

11

12 *Keywords:* Travelling Salesman Problem, Electric Vehicle, Unmanned Aerial Vehicles

Under Review

1 INTRODUCTION

2 Many government and corporate policies aim to promote transportation modes with lower emis-
3 sions of pollution and greenhouse gases. Electric vehicles (EVs) are an emerging alternative to
4 internal combustion engines. Besides, unmanned aerial vehicles (UAVs) have also been proposed
5 to assist in delivery of goods.

6 In the last few years, several companies have started to use EVs in their operations. For
7 example, in 2018 FedEx announced a fleet expansion, adding 1,000 electric delivery vehicles to
8 operate commercial and residential pick-up and delivery services in the United States¹. Switching
9 to electric fleets not only has long-term effects on mitigating the impact of climate change. There
10 may also be an immediate financial incentive, as fuel cost accounts for 39% to 60% of operat-
11 ing costs in the trucking sector (1). Compared to conventional internal combustion engines and
12 petroleum-fuel powered vehicles, EVs are much more energy efficient and require less mainte-
13 nance, which indicates potential savings to freight and logistics companies. On the other hand, in
14 recent years, UAVs have also been integrated into the operation of e-commerce and on-demand
15 item delivery. The use of UAVs for “last-mile” parcel delivery promises to change the landscape
16 of the logistics industry. Amazon, Google, DHL all announced their project to use UAVs to deliver
17 small packages and has conducted thousands of test flights in Australia. The past few years has
18 witnessed a dramatic increase in UAV applications. According to Teal Group, commercial use of
19 UAVs will grow eightfold over the decade to reach \$7.3 billion in 2027².

20 However, EVs and UAVs face limitations due to driving range and availability of recharging
21 stations. A commercial UAV has a range of about only 10 miles which indicates a small service
22 area. For the most common EVs used in service operations, the minimum charging time is 0.5 h
23 and the battery capacity is around 22 kWh which indicates a nominal driving range of 142 km,
24 which is approximately one fourth of a petroleum powered vehicle (2). Moreover, the driving
25 range is also affected by road slope, increased speed, loading capacity and the use of peripherals
26 (3). Although this problem could be alleviated in the future by carrying higher-capacity batteries,
27 at present an EV may need to visit charging stations to charge its battery and extend its driving
28 range.

29 In this paper, we investigate a new problem called the Electric Vehicle Traveling Sales-
30 man Problem with Drone (EVTSPD), where an electric truck (or other types of electric vehicle)
31 performs deliveries with an UAV, in a cooperative way. In this problem, the EV and UAV could
32 perform delivery tasks independently, while the EV also serves as a UAVs hub, where it can refresh
33 its battery and load new parcels. Due to driving range limits, EV may need to visit multiple charg-
34 ing stations between customer visits during its daily operation. Note that some charging stations
35 may be visited multiple times, while others may not be visited at all. Another key assumption
36 in this problem — and the primary distinction from recent work on the “flying sidekick traveling
37 salesman problem” — is that the EV and the UAV share their electricity, that is, there is a bat-
38 tery capacity for both EV and UAV and when the UAV is launched from the EV, the remaining
39 electricity of EV also decreases. The main contributions of the paper are:

- 40 • The Electric Vehicle Traveling Salesman Problem with Drone (EVTSPD) is introduced
41 and formulated;

¹<https://about.van.fedex.com/newsroom/fedex-acquires-1000-change-electric-vehicles/> Last accessed 07/09/2018

²<https://www.tealgroup.com/index.php/pages/press-releases/54-teal-group-predicts-worldwide-civil-UAV-production-will-soar-over-the-next-decade> Last accessed 07/09/2018

- 1 • An efficient heuristic algorithm is proposed to solve EVTSPD;
- 2 • The paper demonstrates through computational experiments the improvement of delivery
- 3 time by utilizing UAV;
- 4 • The numerical analysis indicate the proposed heuristic algorithm can obtain good feasible
- 5 solution in a much shorter time than a commercial solver.

6 The rest of the paper is organized as follows. First, a literature review of EVRP and UAV
7 is given, focusing on current approaches of using EV and UAV to perform delivery tasks. This
8 is followed by the problem description of EVTSPD and its mixed integer programming (MIP)
9 formulation. Next, an efficient iterative heuristic algorithm is proposed to solve the EVTSPD,
10 followed by numerical analysis, computational experiments on random instances, and performance
11 comparisons with a commercial solver.

12 Literature Review

13 The vehicle routing problem (VRP) and its special case, travelling salesman problem (TSP) are
14 among the most well-studied optimization problems in operations research. This problem was
15 first proposed by (4). Since then, many variants have been considered, incorporating service time
16 windows, capacities, maximum route lengths, distinguishing pickups and deliveries, fleet inho-
17 mogeneities, and so forth. Various exact and heuristic methods have been proposed to solve the
18 problem, e.g. (5) and (6).

19 As a variant of the classical VRP problem, Green Vehicle Routing Problem (G-VRP) was
20 introduced by (7), in which a battery capacity constraint was added, along with the option of
21 recharging at a station with a constant time. They assumed a full-charge policy and aim to minimize
22 the total travel distance. Since the charging stations are scarce in the network, the vehicle could
23 visit the same station multiple times, modeled through a network transformation. However, this
24 approach also increased the scale of the network and complicates the solution process. They also
25 proposed two heuristics to solve the problem. (8) extended the previous G-VRP models to consider
26 nonlinear charging functions and a hybrid metaheuristic is proposed to solve the problem. (9)
27 formulated the problem of locating charging stations and also designing EV routes as a discrete
28 integer programming optimization problem, based on the classic VRP.

29 Meanwhile, an increasing number of studies investigate the efficiency of delivery systems
30 that deploy UAVs. (10) introduced the "Flying Sidekick Traveling Salesman Problem" (FSTSP),
31 which assumes that a truck can launch its UAV at the depot or customer node and remains on its
32 own route, while the UAV delivers one small parcel to another customer before meeting again at
33 a rendezvous location (another customer node on the truck's route). (11) presented an iterative
34 algorithm that is based on a decomposition approach to minimize delivery completion time of
35 Traveling Salesman Problem with Drone (TSP-D). (12) provide a randomized variable neighbor-
36 hood descent heuristic to solve FSTSP, in which the initial solution is created from the optimal TSP
37 solution obtained by the Concorde solver. Next, an implementation of the Randomized Variable
38 Neighborhood Descent (RVND) heuristic is used as a local search to obtain the problem solution.
39 (13) studied combinations of truck and UAV routes between each possible launch and pickup nodes
40 and refer to each combination as an "operation" and propose an operation-based formulation. Lo-
41 cal search and dynamic programming are presented as heuristic algorithms. (14) investigated the
42 delivery cost of the TSP-D for the first time. The delivery cost includes transportation cost and
43 waiting cost. Two different heuristics are provided, both of which are inspired from the route first-
44 cluster second heuristic, which is based on local search and greedy randomized adaptive search

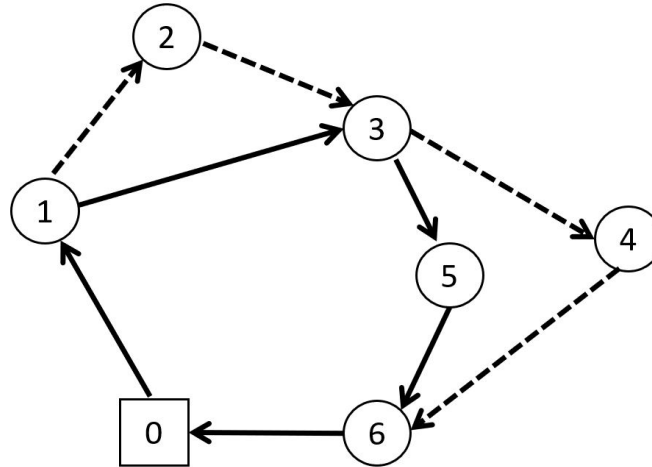


FIGURE 1 A simple representation of the EV-UAV coordinated route

1 procedure (GRASP). TSP with multiple UAVs are also studied. (15) considered a more general
 2 case with multi-trucks and multi-UAVs, in which one or more UAVs could travel with every truck.
 3 The objective function is to minimize the delivery completion time, that is, the time when the last
 4 truck or UAV return to the depot.

5 **PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION**

6 The EVTSPD is defined on an undirected, complete graph $G = (V, E)$, with a vertex set V con-
 7 sisting of the customer set $I = \{v_1, v_2, \dots, v_c\}$, the depot v_0 , and a set of charging stations $S =$
 8 $\{v_{c+1}, v_{c+2}, \dots, v_{c+s}\}$. The vertex set is thus $V = \{v_0\} \cup I \cup F = \{v_0, v_1, \dots, v_{c+s}\}$ and $|V| = c + s + 1$.
 9 It is assumed that all charging stations have unlimited capacities. The edge set $E = \{(v_i, v_j) : v_i, v_j \in$
 10 $V, i < j\}$ contains the edges connecting vertices of V . Each edge (v_i, v_j) is associated with two non-
 11 negative travel time t_{ij} and t'_{ij} , which corresponds to the travel time needed for the EV and UAV
 12 to travel from node i to node j , respectively. In addition, no limit is set on the number of stops
 13 that can be made for recharging. When recharging is undertaken, it is assumed that the battery is
 14 charged to its full capacity (this “full-charge” policy assumption is common in the literature).

15 The EVTSPD is to find a coordinated tour that starts and ends at the depot, visits a subset of
 16 vertices (including charging stations when necessary) such that the total delivery time is minimized.
 17 It is assumed that all customers should be served either by the electric vehicle or the UAV. At the
 18 start of the tour, all parcels are loaded into the electric vehicle. During the tour, the electric vehicle
 19 can launch the UAV at a customer node and retrieve the UAV later at another customer node. The
 20 electric vehicle and the UAV works independently after UAV is launched. Note that both the launch
 21 node and retrieve node should be in the EV’s route. Since it is assumed that there is only one UAV
 22 in the EV, so the UAV cannot be relaunched before it is retrieved. As a result, the final feasible
 23 solution consists of the EV route and several, nonoverlapping, UAV routes which start and end on
 24 the EV route. A simple example is shown in Figure 1, where the EV route is $\{0 - 1 - 3 - 5 - 6 - 0\}$
 25 while UAV route is $\{1 - 2 - 3, 3 - 4 - 6\}$.

26 We assume that the electric vehicle and the UAV share their electricity. There is a battery
 27 on the EV which can be used by both the EV and the UAV. If the UAV is launched from the EV at

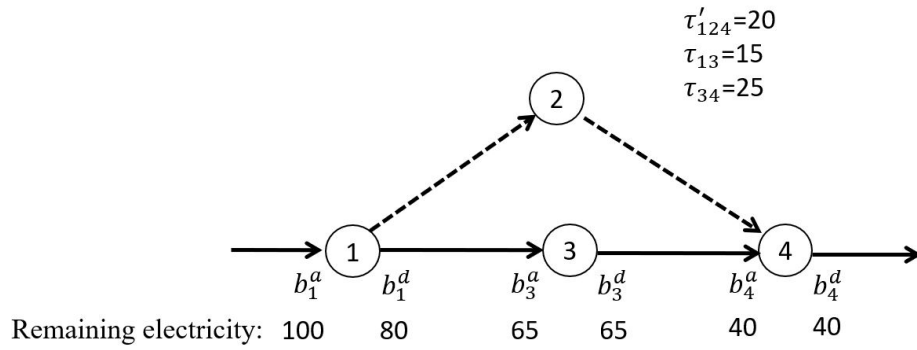


FIGURE 2 A simple representation of the electricity assumption

1 a customer node, the electricity required for the UAV route is deducted from the remaining battery
 2 level of the EV. To explain this, denote b_i^a as the remaining electricity of EV upon arrival of node
 3 i and b_i^d as the remaining electricity of EV upon departure of node i . Using the network in Figure
 4 2, assume the electric vehicle launches the UAV at node 1, travels to node 3 and retrieves the UAV
 5 at node 4 while the UAV serves the customer 2. If $b_1^a = 100$, which indicates that EV's remaining
 6 battery level upon arrival at node 1 is 100, then EV's battery level upon departure of node 1 could
 7 be calculated as $b_1^d = 100 - \tau'_{124} = 100 - 20 = 80$, where τ'_{124} represents the required electricity of
 8 the UAV to be launched at node 1, serves customer 2 and returns to node 4. The battery level of
 9 the other nodes are also shown in Figure 2.

10 Since the battery level of EV should be non-negative along its route, the EV may need to
 11 visit charging stations to charge its battery to full capacity when necessary. It is worth noting that
 12 in EVTSPD, the final solution might contain more than one visit to some specific charging station
 13 nodes, while some other charging station nodes might not in EV's route and are never visited. To
 14 permit multiple (and possibly zero) visits to the charging station nodes, while requiring exactly one
 15 visit to the customer nodes, graph G is augmented to create $G_0 = (V_0, E_0)$ with a set of s' dummy
 16 vertices, $\Phi = \{v_{c+s+1}, v_{c+s+2}, \dots, v_{c+s+s'}\}$, one for each potential visit to an charging station node.
 17 Denote V' as the augmented vertices set and $V' = V \cup \Phi$. The number of dummy vertices associated
 18 with each charging station, n_s , is set to the number of times the associated $v_s \in S$ can be visited. n_s
 19 should be set as small as possible so as to reduce the network size, but large enough to not restrict
 20 multiple beneficial visits.

21 Additional notation used in formulating the EVTSPD is defined next.

22 Sets

23 I : Set of all customers in the problem, $I = \{v_1, v_2, \dots, v_c\}$ and $|I| = c$.

24 I' : Subset of customers that are available to UAV delivery service, $I' \subset I$.

25 S : Set of all charging stations in the original network, $S = \{v_{c+1}, v_{c+2}, \dots, v_{c+s}\}$ and $|S| = s$.

26 S' : Augmented set of all charging stations, including the m copies of set S . $S' = \{v_{c+1}, \dots, v_{c+s}, v_{c+s+1}, \dots, v_{c+ms}\}$
 27 and $|S'| = ms$. Note that in the augmented network each node could be visited at most once.

28 N : Set of all nodes in the augmented network, $N = S' \cup C \cup \{0, c + ms + 1\}$, where 0 and $c + ms + 1$
 29 both represents the depot and $|N| = c + ms + 1$.

30 N_0 : Set of nodes from which a vehicle may depart in the augmented network. $N_0 = \{v_0, v_1, \dots, v_c\} \cup$

- 1 S' .
 2 N_+ : Set of nodes to which a vehicle may visit in the augmented network. $N_+ = \{v_1, v_2, \dots, v_c\} \cup$
 3 $S' \cup \{v_{c+ms+1}\}$.
 4 N' : Set of all customer nodes and charging station nodes in the augmented network. $N' = I \cup S'$.
 5 D : Set of tuples of the UAV's feasible route $\langle i, j, k \rangle$, where the UAV is launched from node i , travels
 6 to node j and returns to node k . $D = \{\langle i, j, k \rangle : i \in N_0, j \in C', k \in N_+, i \neq j, j \neq k, i \neq k, \tau'_{ij} + \tau'_{jk} \leq$
 7 $Q_d\}$.

8 Parameters

- 9 τ_{ij} : Travel time cost for electric vehicle to travel from node i to node j
 10 τ'_{ij} : Travel time cost for UAV to travel from node i to node j
 11 τ'_{ijk} : Travel time cost for UAV to launch from node i , serves node j and return to node k
 12 Q_d : Driving time limit of the UAV, which is measured in time unit.
 13 Q : Driving time limit of the electric vehicle, which is measured in time unit.
 14 M : A positive large number which is greater than the upper bound of total travel time
 15

16 Decision variables:

- 17 $x_{ij} \in \{0, 1\}$: Equals one if the electric vehicle travels from node i to node j and zero otherwise,
 18 where $i \neq j$ and $i \in N_0, j \in N_+$.
 19 $y_{ijk} \in \{0, 1\}$: Equals one if the UAV is launched from node i , travels to node j and returns to the
 20 EV at node k and zero otherwise.
 21 $p_{ij} \in \{0, 1\}$: Equals one if customer node i is visited before customer j in the EV's path and zero
 22 otherwise.
 23 u_i : Position of node i in the EV's path.
 24 b_i^a : Remaining battery charge of EV upon arrival of node i which indicates the remaining maxi-
 25 mum driving time of EV.
 26 b_i^d : Remaining battery charge of EV upon departure of node i which indicates the remaining max-
 27 imum driving time of UAV.
 28 t_j : Time when electric vehicle arrives at node j .
 29 t'_j : Time when UAV arrives at node j .
 30

31 Mathematical Formulation

Objective :

$$\min z = t_{c+ms+1} \quad (1)$$

- 32 The objective is to minimize the time when both vehicles return to the depot after serving all the
 33 customers. There are a large number of constraints in the problem; these are introduced below,
 34 interspersed with descriptions.

Routing Constraints:

$$\sum_{\substack{i \in N_0 \\ i \neq j}} x_{ij} + \sum_{\substack{i \in N_0 \\ i \neq j}} \sum_{\substack{k \in N_+ \\ \langle i, j, k \rangle \in D}} y_{ijk} = 1 \quad \forall j \in I \quad (2)$$

$$\sum_{j \in N_+} x_{0j} = 1 \quad (3)$$

$$\sum_{i \in N_0} x_{i,c+1} = 1 \quad (4)$$

$$u_i - u_j + 1 \leq (c + ms + 2)(1 - x_{ij}) \quad \forall i \in N', j \in N_+, j \neq i \quad (5)$$

$$\sum_{\substack{i \in N_0 \\ i \neq j}} x_{ij} = \sum_{\substack{k \in N_+ \\ k \neq j}} x_{jk} \quad \forall j \in N' \quad (6)$$

$$\sum_{j \in C'} \sum_{\substack{k \in N_+ \\ \langle i, j, k \rangle \in D}} y_{ijk} \leq 1 \quad \forall i \in N_0 \quad (7)$$

$$\sum_{i \in N_0} \sum_{\substack{j \in C' \\ \langle i, j, k \rangle \in D}} y_{ijk} \leq 1 \quad \forall k \in N_+ \quad (8)$$

$$2y_{ijk} \leq \sum_{\substack{h \in N_0 \\ h \neq i}} x_{hi} + \sum_{\substack{l \in N_0 \\ l \neq k}} x_{lk} \quad \forall i \in N_0, j \in C', k \in N_+, \langle i, j, k \rangle \in D \quad (9)$$

$$y_{0jk} \leq \sum_{\substack{h \in N_0 \\ h \neq k}} x_{hk} \quad \forall j \in C', k \in N_+, \langle 0, j, k \rangle \in D \quad (10)$$

$$u_k - u_i \geq 1 - (c + ms + 2)(1 - \sum_{\substack{j \in C' \\ \langle i, j, k \rangle \in D}} y_{ijk}) \quad \forall i \in N_0, k \in N_+, k \neq i \quad (11)$$

1 Constraints (2)-(11) are associated with the routing of the two vehicles. In particular, con-
 2 straint (2) guarantees that each customer node is visited once by either an EV or a UAV. Constraints
 3 (3) and (4) state that the EV must start from and return to the depot. Constraint (5) is a sub-tour
 4 elimination constraint. Constraint (6) indicates that if the EV visits j then it must also depart from
 5 j . Constraints (7) and (8) state that each node can either launch or retrieve the UAV at most once.
 6 Constraint (9) ensures that if there exists a UAV route $\langle i, j, k \rangle$, then EV should travels between
 7 i and k . Constraint (10) states that if the UAV is launched from the depot and returned to node k ,
 8 then node k should be visited by the EV. Constraint (11) is also a sub-tour elimination constraint.

Battery Constraints:

$$b_j^a \leq b_i^d - \tau_{ij} x_{ij} + M(1 - x_{ij}) \quad \forall i \in N_0, j \in N_+, i \neq j \quad (12)$$

$$b_0^a = Q \quad (13)$$

$$b_i^d = Q - \sum_{\substack{j \in C' \\ j \neq i}} \sum_{\substack{k \in N_+ \\ \langle i, j, k \rangle \in D}} y_{ijk} * \tau'_{ijk} \quad \forall i \in S' \cup \{0, c + ms + 1\} \quad (14)$$

$$b_i^d = b_i^a - \sum_{\substack{j \in C' \\ j \neq i}} \sum_{\substack{k \in N_+ \\ \langle i, j, k \rangle \in D}} y_{ijk} \tau'_{ijk} \quad \forall i \in I \quad (15)$$

$$b_i^a \geq 0 \quad \forall i \in N \quad (16)$$

$$b_i^d \geq 0 \quad \forall i \in N \quad (17)$$

1 Constraints (12)-(17) are associated with the battery electricity level. In particular, con-
 2 straint (12) states that if electric vehicle travels from node i to node j , then the electricity level be-
 3 fore arriving node i is less or equal to the electricity level after leaving node j , regardless whether
 4 node i, j are customer node or charging station. Constraint (13) ensures that when EV departs
 5 from the depot it is fully charged. Constraint (14) states that if EV departs from a charging station
 6 node i and there is a UAV route that starts at node i , then when EV departs from i it is no longer
 7 fully charged and the UAV route electricity consumption should be deducted from full-charged bat-
 8 tery. Constraint (15) states the same situation as constraint (14) except when node i is a customer.
 9 Constraint (16) and (17) ensures that the remaining battery should be non-negative.

Coordination Constraints:

$$t'_i \geq t_i - M(1 - \sum_{\substack{j \in C' \\ j \neq i}} \sum_{\substack{k \in N_+ \\ \langle i, j, k \rangle \in D}} y_{ijk}) \quad \forall i \in N_0 \quad (18)$$

$$t'_i \leq t_i + M(1 - \sum_{\substack{j \in C' \\ j \neq i}} \sum_{\substack{k \in N_+ \\ \langle i, j, k \rangle \in D}} y_{ijk}) \quad \forall i \in N_0 \quad (19)$$

$$t'_k \geq t_k + M(1 - \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C' \\ \langle i, j, k \rangle \in D}} y_{ijk}) \quad \forall k \in N_+ \quad (20)$$

$$t'_k \leq t_k - M(1 - \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C' \\ \langle i, j, k \rangle \in D}} y_{ijk}) \quad \forall k \in N_+ \quad (21)$$

$$t_k \geq t_h + \tau_{hk} + S_L * \sum_{\substack{l \in C' \\ l \neq k}} \sum_{\substack{m \in N_+ \\ \langle k, l, m \rangle \in D}} y_{klm} + S_R \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C' \\ \langle i, j, k \rangle \in D}} y_{ijk} - M(1 - x_{hk})$$

$$\forall h \in N_0, k \neq h \quad (22)$$

$$t'_j \geq t'_i + \tau'_{ij} - M(1 - \sum_{\substack{k \in N_+ \\ \langle i, j, k \rangle \in D}} y_{ijk}) \quad \forall j \in C', i \in N_0, i \neq j \quad (23)$$

$$t'_k \geq t'_j + \tau'_{ij} + S_R - M(1 - \sum_{\substack{i \in N_0 \\ \langle i, j, k \rangle \in D}} y_{ijk}) \quad \forall j \in C', k \in N_+, k \neq j \quad (24)$$

10 Constraints (18)-(24) are associated with travel time of the two vehicles. In particular, con-
 11 straints (18)–(21) ensure that the travel time and UAV range limit are correctly handled. Constraint
 12 (22) indicate that if the electric vehicle travels from $h \in N_0$ to $k \in N_+$. The electric vehicle's effec-
 13 tive arrival time at k must incorporate the electric vehicle's arrival time at h , the electric vehicle's
 14 travel time from h to k , the launch time at node h and the retrieve time at node k . This constraint is
 15 not binding if the electric vehicle does not travel from h to k . Constraint (23) and (24) are associ-
 16 ated with the arrival time of UAV. If there is a UAV route of $\langle i, j, k \rangle$, then the arrival time of node
 17 j and k should be related to the UAV's travel time between i and j and j and k .

Ordering Constraints:

$$t'_k - t'_j + \tau'_{ij} \leq e + M(1 - y_{ijk})$$

$$\forall k \in N_+, j \in C', i \in N_0, \langle i, j, k \rangle \in D \quad (25)$$

$$u_i - u_j \leq -1 + (c + ms + 2)(1 - p_{ij}) \quad \forall i, j \in N', i \neq j \quad (26)$$

$$p_{ij} + p_{ji} = 1 \quad \forall i, j \in N', i \neq j \quad (27)$$

$$t'_l \geq t'_k - M(3 - \sum_{\substack{j \in C' \\ \langle i, j, k \rangle \in D \\ j \neq l}} y_{ijk} - \sum_{\substack{m \in C' \\ m \neq i \\ m \neq k \\ m \neq l}} \sum_{\substack{n \in N_+ \\ \langle l, m, n \rangle \in D \\ n \neq i \\ n \neq k}} y_{lmn} - p_{il})$$

$$\forall i, l \in N_0, k \in N_+, i \neq k \neq l, \langle i, j, k \rangle \in D, \langle l, m, n \rangle \in D \quad (28)$$

1 Constraints (25)-(28) are associated with ordering the two vehicles. Constraint (25) ensures
 2 that the UAV route should be within the UAV's flight range. Constraint (26) is a sub-tour elimina-
 3 tion constraint and constraint (27) ensures the correct ordering of two different nodes. Constraint
 4 (28) indicates that if there exists two UAV route deliveries $\langle i, j, k \rangle$ and $\langle l, m, n \rangle$ and node i is
 5 visited before node l by the EV, then node l must be visited after node k .

Domain Constraints:

$$t_0 = 0 \quad (29)$$

$$p_{0j} = 1 \quad \forall j \in N_+ \quad (30)$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in N_0 \quad (31)$$

$$y_{ijk} \in \{0, 1\} \quad \forall i \in N_0 \quad (32)$$

$$1 \leq u_i \leq c + ms + 2 \quad \forall i \in N_+ \quad (33)$$

$$t_i \geq 0 \quad \forall i \in N \quad (34)$$

$$t'_i \geq 0 \quad \forall i \in N \quad (35)$$

$$p_{ij} \in \{0, 1\} \quad \forall i \in N_0, j \in N_+, j \neq i \quad (36)$$

6 Constraints (29)-(36) specify the domain of all the decision variables.

7 SOLUTION METHODS

8 There are several complications in EVTSPD: the vehicle driving range limitations, the existence of
 9 a charging station that can possibly be visited multiple times or not at all, and planning the UAV's
 10 route alongside that of the EV, introduce major complications to the problem. In the previous re-
 11 search which did not include electricity constraints, (11) report a solution time of about 3 hours
 12 to solve a network containing 10 nodes, using a commercial solver. Thus, heuristics designed for
 13 EVTSPD are necessary to solve a problem with practical size. In this section, an iterative three-step
 14 decomposition heuristic algorithm is presented to solve the proposed EVTSPD, based on that pro-
 15 posed by (11). The heuristic algorithm decomposes the problem into three subproblems: electric
 16 vehicle path construction, UAV node insertion and final solution feasibility check. However, since
 17 there are a variety of different combination of electric vehicle node and UAV node, an iterative
 18 approach is used to improve the result and record the best-known solution.

19 The iterative heuristic algorithm

20 With the objective to minimize delivery time, the UAV should be deployed as much as possible
 21 since it generally has higher travel speed. However, the number of customer nodes that could be
 22 served by the UAV is limited. Since it is hard to pre-determine which nodes should be served by
 23 the electric vehicle and which nodes should be served by the UAV, the algorithm partitions V into

1 two subsets: a set of customer nodes that is served by the UAV N_U (the *UAV nodes*), and a set of
 2 remaining customer nodes that should be served by EV plus the depot N_E (the *EV nodes*). Thus
 3 $N_U \cup N_T = I \cup \{0\}$. Since there is a finite (albeit exponential) number of such partitions, a subset
 4 of them can be explored for route generation. The specific steps of the algorithm are as follows:

- 5 • **Step 1** For all the customer nodes $i : i \in C$, list all the combinations of the EV nodes and
 6 UAV nodes and store the combinations in set B . Set the optimal cost to infinity, $Cost^{opt} \leftarrow$
 7 ∞ . (The number of combinations grows exponentially with network size; if necessary
 8 another heuristic rule can be used to select a subset of all possible combinations).
- 9 • **Step 2** While set B is nonempty, pick the first combination $Comb$ in set B and identify
 10 the EV nodes set N_E and UAV nodes set N_U . Delete $Comb$ from B and set $B \leftarrow B \setminus Comb$.
- 11 • **Step 3** For all the nodes in the combined set $N_E \cup S$, solve the Electrical Vehicle Travel-
 12 ling Salesman Problem (EVTSP) using the MCWS heuristic proposed in (7) and obtain
 13 an feasible EV route $Route_{EV}$ and route cost $Cost_{EV}$. (See below for details.) If no
 14 feasible solution exists or $Cost_{EV} \geq Cost^{opt}$, return to Step 2.
- 15 • **Step 4** With set N_U and EV's temporary route $Route_{EV}$, solve the UAV inserting problem
 16 (see below for details) and obtain the optimal UAV route $Route_{UAV}$ and corresponding
 17 waiting time $Wait_{UAV}$. If $Cost_{EV} + Wait_{UAV} \geq Cost^{opt}$, return to Step 2.
- 18 • **Step 5** Check the battery feasibility of the final solution. If $Route_{EV}$ and $Route_{UAV}$
 19 satisfies the battery level constraint, update the optimal cost $Cost^{opt} \leftarrow Cost_{EV} + Wait_{UAV}$.
 20 Otherwise, return to Step 2.

21 The algorithm terminates with an empty set B . The latest $Cost^{opt}$ and its corresponding
 22 $Route_{EV}$ and $Route_{UAV}$ are returned as the best solution found by the algorithm. The detailed
 23 explanation of three key steps are presented below.

24 *Solve EVTSP*

25 In Step 3 of the algorithm, an EVTSP should be solved with with nodes $\{i : i \in N_T \cup S\}$. Note that
 26 N_E is a subset of customer set C . The purpose of solving EVTSP is to construct a feasible EV route
 27 that visits all the customer nodes in set N_E , satisfies the EV driving range constraint and minimizing
 28 the total travel time at the same time. In this paper, a Modified Clarke-Wright Savings algorithm
 29 (MCWS) is implemented to solve the problem. MCWS is proposed in (7) and it is a greedy-based
 30 maximum saving algorithm. To solve EVTSP a augmented network is constructed, where the set of
 31 charging station S should be replicated to ensures that a single charging station could be potentially
 32 visited multiple times. For all the generated instances in this paper the charging station set S are
 33 replicated twice in the network.

34 *UAV node insertion*

35 In Step 4 of the algorithm, a subproblem of UAV node insertion is implemented and solved to
 36 obtain the optimal insertion type such that every customer node in set N_U and the total waiting
 37 time of the EV is minimized. The input data of this problem is the feasible EV route obtained in
 38 step 3 and all the possible UAV routes along the EV route which satisfied the UAV's flight range
 39 constraint. The MILP formulation of this subproblem is proposed in (11) and the reader could
 40 refer to it for more information.

1 *Final solution feasibility check*

2 In step 5 of the heuristic algorithm the feasibility check of the solution obtained from previous
 3 steps is implemented. Note that although the EV's route and UAV's route both satisfy all the con-
 4 straints separately, the final solution that incorporates both of them might not be feasible because
 5 of the shared electricity assumption. More specifically, the electricity level constraint might be
 6 violated when UAV's routes are incorporated into EV's route and the electricity consumption of
 7 UAV's route $\langle i, j, k \rangle$ is deducted from the b_i^d . If the final coordinated route still satisfies the elec-
 8 tricity constraint, it is then compared to the best-known solution. Otherwise it is discarded and the
 9 algorithm moves to step 2 to explore another combination in set B .

10 **NUMERICAL ANALYSIS**

11 In order to validate and evaluate the performance of the proposed heuristic algorithm, numerical
 12 analysis is conducted and described in this section. The experimental setting is described in the
 13 first subsection, while the second one presents the computational result of the instances.

14 **Experimental Setting**

15 The purpose of the experiments is to compare the performance of the proposed heuristic algorithm
 16 against solving the MILP formulation directly via a commercial solver. However, since both the
 17 related problems of EVRP and TSP-D are introduced in recent years, benchmark instances are
 18 limited and most of them are not available publicly. Furthermore, according to (11), commercial
 19 solvers can only obtain optional solutions for instances containing up to 10 customers in two hours.
 20 Because of the complexity of the TSP-D and the extra complexity incurred by adding the driving
 21 range limit constraint, the feasible instance that could be solved by a commercial solver is rela-
 22 tively in small scale. So in this paper, we conduct the numerical analysis on randomly generated
 23 instances. Most of the experimental settings are adopted from (11). For all the generated instances,
 24 the single depot is located at (0, 0) and the coordinates of the customer nodes are uniformly dis-
 25 tributed between -10 km and 10 km. The EV travels at the speed of 40 km/h while the UAV's speed
 26 is 56 km/h. EV has a driving time limit of 4000 seconds while the UAV has a driving time limit
 27 of 1200 seconds, which is adopted from (10). The launch/retrieve time and charging time are set
 28 to zero in these instances. Distances for the electric vehicle are calculated employing Manhattan
 29 metric, while the Euclidean metric is chosen for the UAV, reflecting the greater mobility options
 30 available to a drone.

31 The MILP model is implemented in Pyomo with ILOG's CPLEX Concert Technology
 32 solver (version 12.6.3). The proposed heuristic algorithm is coded in Python. All numerical exper-
 33 iments are run on a 2.8 GHz Intel Core i5 quad-core machine with 16 GB RAM.

34 **Computational Results**

35 The first experimental test aims to evaluate the performance of the proposed heuristic by comparing
 36 it to an exact solver. The computational results for ten randomly generated instances are shown
 37 in Table 1. For the case name of "5C2S2R01", "5C" represents that this instance includes five
 38 different customers, "2S" represents that this instance contains two different charging stations,
 39 "2R" represents that the augmented network contains two copies of all the charging stations and
 40 "01" is the index for a specific type of instance. As a result, in the augmented network of case
 41 "5C2S2R01" the actual number of nodes is 11 including the depot.

42 As can be seen from Table 1, for instances containing 5 customers the average computation

1 time of CPLEX is 2202 seconds while that of the proposed heuristic algorithm is less than 1 second.
2 On average the optimality gap of the heuristic algorithm is 0.162. However for instances including
3 10 customers CPLEX fails to find the optimal solution within two hours while the heuristic find
4 a feasible solution with an average of 26.5 seconds. This result shows that the proposed heuristic
5 is capable of obtain a good feasible solution with a much less computational time compared to
6 solve the MILP formulation via a commercial solver. However, since the computational time
7 grows exponentially with the scale of the network, although it can obtain a good feasible result
8 in a relatively short time for small instances, for larger instances the computational time might be
9 prohibitive. In this situation, a heuristic that could pre-decide the number of EV nodes and UAV
10 nodes could be implemented to decrease the number of combinations and computational time.

11 The second experimental test aims to evaluate the improvement of the final solution with
12 the assistance of UAV delivery. The final solution of EVTSPD is compared with solving the same
13 instance as a electric travelling salesman problem (EVTSP). Two different groups of instances are
14 generated where the instances in the first group have 5 customers and the instances in the second
15 one has 10 customers. The heuristic algorithm for EVTSP is MCWS as proposed in (7). The
16 results are shown in Table 2. As illustrated, the utilization of UAV delivery would greatly decrease
17 the delivery time. On average the final delivery time of EVTSPD is 39.17% lower than EVTSP
18 solution for 5-customer instances and 28.55% lower for 10-customer instances. This improvement
19 indicates that the logistics company might have a great business benefit by utilizing UAV delivery
20 considering not only the delivery time decrease but also UAV could be operated without a human
21 pilot at a lower operation cost.

22 CONCLUSION

23 In this paper, the EVTSPD is formulated and an efficient iterative heuristic algorithm is proposed.
24 This problem is to find a coordinated EV-UAV route minimizing total travel time, to serve a set of
25 customers while incorporating stops at charging stations in route plans to ensure sufficient charge.
26 The case study on randomly generated instances indicates that the proposed algorithm performs
27 well compared to exact solution methods, with significantly less computational time. We therefore
28 plan to apply this solution method to solve larger problem instances. The ability to formulate the
29 EVTSPD, along with the solution techniques, will aid organizations with EV fleets in overcoming
30 difficulties that exist as a result of limited recharging infrastructure. The new delivery concept
31 of using UAV and EV to perform last-mile delivery would result in financial and environmental
32 benefits when considering the reduced operation cost of fueling and switching to UAV which does
33 not require a human pilot. This also allows deliveries companies to understand the potential impact
34 of their decision on daily operations and costs.

35 Additional topics for future research include developing heuristics with relaxed charging
36 assumptions, allowing partial recharges. The EVTSPD could also be extended to a fleet of electri-
37 cal vehicles while seeks optimal tours and considering fuel prices and heterogeneous properties.

38 ACKNOWLEDGEMENTS

39 This research is based on work supported by the National Science Foundation under Grant Nos.
40 1826230, 1562109/1562291, 1562109, 1826337, 1636154 and 1254921. This work is also sup-
41 ported by the Center for Advanced Multimodal Mobility Solutions and Education (CMMSE).

TABLE 1 Computational results for performance evaluation

Case Name	Computational Time (s)		Heur. Solution (s)	Opt. Solution (s)	Opt. Gap
	CPLEX	Heur. Algorithm			
5C2S2R01	240	< 1	4950	4390	0.127
5C2S2R02	486	< 1	4890	4500	0.087
5C2S2R03	2103	< 1	5000	4600	0.087
5C2S2R04	2329	< 1	2730	2720	0.004
5C2S2R05	5912	< 1	4960	4140	0.198
5C2S2R06	259	< 1	6650	6460	0.029
5C2S2R07	2775	< 1	3760	2930	0.2835
5C2S2R08	771	< 1	6350	4780	0.328
5C2S2R09	3038	< 1	4430	3970	0.116
5C2S2R10	4107	< 1	4010	2930	0.369
Average	2202	< 1	4773	4142	0.162
10C4S2R01	>7200	42	4210	None	None
10C4S2R02	>7200	36	5670	None	None
10C4S2R03	>7200	17	6400	None	None
10C4S2R04	>7200	32	6130	None	None
10C4S2R05	>7200	17	5630	None	None
10C4S2R06	>7200	19	5740	None	None
10C4S2R07	>7200	16	6680	None	None
10C4S2R08	>7200	10	5160	None	None
10C4S2R09	>7200	52	5260	None	None
10C4S2R10	>7200	24	6030	None	None
Average	>7200	26.5	5691	None	None

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TABLE 2 Computational results for final delivery time comparison

Case Name	EVTSPD Solution (s)	MCWS Solution (s)	Difference Gap	#UAV Routes
5C4S2R01	5320	5680	6.77%	3
5C4S2R02	4510	5930	31.49%	2
5C4S2R03	5000	8000	60.00%	2
5C4S2R04	2030	3690	81.77%	2
5C4S2R05	3560	None	None	4
5C4S2R06	4520	6010	32.96%	1
5C4S2R07	3590	4790	33.43%	4
5C4S2R08	4360	6900	58.26%	2
5C4S2R09	4130	4800	16.22%	3
5C4S2R10	3100	4080	31.61%	4
Average	4012	5542	39.17%	2.7
10C5S2R01	5320	7920	48.87%	3
10C5S2R02	6400	7240	13.13%	7
10C5S2R03	6490	8150	25.58%	2
10C5S2R04	5730	7060	23.21%	7
10C5S2R05	5690	6990	22.85%	3
10C5S2R06	7250	9100	25.52%	4
10C5S2R07	5270	6350	20.49%	3
10C5S2R08	5440	7330	34.74%	3
10C5S2R09	5410	8020	48.24%	7
10C5S2R10	5860	7200	22.87%	7
Average	5886	7536	28.55%	4.6

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Under Review