# On the Validity of Perceived Social Structure*

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**Abstract**

The validity of survey-based reports of social relationships is a critical assumption for much social network research. Research on informant accuracy has shown that observational data and recalled behavior by informants are imperfectly correlated, which calls into question whether complex relations like friendship and advice-seeking can be accurately measured from individual reports. A class of network inference models, the Bayesian Network Accuracy Models, growing out of the pioneering work of Batchelder and Romney on inference from informant reports, provides a principled basis for inferring network structure given such error-prone data. Using these models, we can gain insight into the accuracy of informants' self and proxy reports of social ties, and more broadly, the reliability and validity of respondents' reports of informal social relations. While existing data does not provide a criterion validity check for inferring most relationships, other notions of validity and/or reliability can still be applied. For instance, if friendship reports are generated from a common underlying network that is perceivable (albeit imperfectly) by all actors, then random subsets of actors should produce estimates that should agree (i.e., split-half reliability). Using informant reports on friendship and advice-seeking networks from four different organizations, we show substantially higher levels of split-half reliability than can be explained by chance, suggesting that models are indeed estimating a common underlying relation. We also show that informants' errors appear to be structured in ways that are consistent with cognitive models of social perception, with greater accuracy on average for large-scale network features rather than fine details, for own versus others' ties, and for core-periphery structures versus bipartitions. Evidence from construct validity checks further suggests the that common networks underlying informants' reports have properties that would be expected of true social structures. Taken together, our findings support the view that informants' mental models of social structure, while error-prone, nevertheless reflect an underlying social reality.

## 1 Introduction

*Network inference* is the problem of inferring the unknown edges of a network from a set of measurements. The validity of network inference models has been tested in biological and digital contexts where there is an observable ground truth against which to compare an inferred network (Dougherty, 2007; Qian and Dougherty, 2013; Howison et al., 2011). However, this is notably more difficult in social contexts, particularly for latent relations that represent a combination of multiple behaviors. Taking friendship as an example, note that while a researcher cannot observe friendship

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relations as directly as he or she could possibly observe online communication or trade (both relations allowing standardized behavioral metrics that an outside omniscient observer could use as a rule to code observed behavior), network members can still be asked to report on their perceptions of friendship ties. This introduces complications, however. Informant-based measurement for social relations requires two assumptions: that the underlying relation exists; and that informants can observe this relation and report it well enough to obtain accurate inference. These assumptions have not gone unchallenged (Bernard et al., 1984; Fischer, 1982). At core, the network measurement problem shares many features with measurement problems in cultural research (Romney et al., 1986), wherein a "cultural consensus" must be inferred from informants of unknown competency with no obvious answer key. The pioneering methodology detailed in Batchelder and Romney (1988) - often referred to as *cultural consensus theory* (CCT) - provides one route to simultaneously inferring the accuracy the informants and the underlying cultural truth on which they are asked to report. Central to the approach is the assumption that informants' reports are imperfect reflections of a common consensus, idiosyncratically modified by individuals' reporting errors; thus, systematic agreement tends to be indicative of accuracy, a broader phenomenon well-known in other areas of psychometric theory (Nunnally, 1994). This methodology has had broad impacts spanning multiple fields, including anthropology (Brewis et al., 2011; Baer et al., 2003; Hadley et al., 2019), psychology (Oravecz et al., 2015; Heshmati et al., 2019; De Puiseau et al., 2017), and sociology (Marcum et al., 2012; Butts, 2003; Koskinen et al., 2013). In a network analytic context, a fully Bayesian adaptation of this measurement approach was introduced by Butts (2003) (the Bayesian Network Accuracy Model (BNAM)), who incorporated general exponential family priors on network structure along with individual level reporting errors under conditionally conjugate (beta) priors; subsequent elaborations have included network priors based on continuous graph mixtures (Butts, 2014), separation of self versus proxy reporting errors (Lee and Butts, 2018), and the use of the BNAMs as the measurement components of hierarchical exponential family random graph models (Koskinen et al., 2019).

While CCT has been validated in various contexts where ground truth data have been available (see e.g. Romney et al., 1987; Agrawal and Batchelder, 2012), use of the BNAM has been motivated largely by a combination of first principles arguments, simulation studies, and appeal to the success of CCT-based approaches in other settings. In that light, it is interesting to consider what other modes of validation can be employed for this approach in the context of relations like friendship, even in the absence of "ground truth" data beyond informants' self and proxy reports. Here, we show evidence for the validity of this model for social measurement through a multi-pronged approach, working in the context of organizational settings wherein both friendship and advice seeking ties were measured. First, we show self-consistency, confirming via simulation that it is possible to reliably recover graph structure given the data generating process assumed under the model. Second, we show that inferred structures from different sets of informants are more similar to each other than can be explained by random chance (split-half reliability (Drost, 2011)). Third, we perform construct validity checks (Cronbach and Meehl, 1955) on inferred networks, showing that they have properties that we would expect for friendship and advice seeking networks from prior theory. Finally, we show that patterns in individual respondent observations (via individual error rates and perceived social group structure) are structured in a manner that is consistent with cognitive theory and prior research on relational perception (Kumbasar et al., 1994; Krackhardt and Kilduff, 1999). Taken together, these results make it difficult to escape the conclusion that, like CCT, the BNAM provides a valid approach to interpersonal network measurement in cases like those studied here.

## 2 Materials and Methods

We utilize the BNAM (Butts, 2003) with graph mixture priors from Butts (2014) and the self vs. proxy error rate decomposition used by Lee and Butts (2018). The foundations of the model are similar to that of cultural consensus theory developed by Batchelder and Romney (1988), with informant reports conditional on an underlying true vector assumed to follow a generalized Condorcet likelihood with individual-specific error rates. The network case requires additional structure, however, particularly with respect to the priors involved. Formally, we represent the underlying true network (the *criterion graph*, corresponding to the unobserved "answer key" of Batchelder and Romney (1988)) by an uncertain dichotomous matrix $\theta$, with $\theta_{jk} = 1$ if node $j$ sends a tie to node $k$. Unlike the "guessing" model of CCT, it is more natural here to parameterize the error process in terms of false positive and false negative rates, with $e^+_{ijk}$ and $e^-_{ijk}$ respectively referring to the probability that informant $i$ will make a false positive or false negative error when reporting on the state of $\theta_{jk}$. As with CCT, errors are assumed to be conditionally independent given $\theta$ and the underlying rates, i.e.

$$\Pr(Y_{ijk} = y_{ijk}|\theta_{jk}, e^+_{ijk}, e^-_{ijk}) = \begin{cases} 1 - e^-_{ijk} & \text{if } y_{ijk} = 1 \wedge \theta_{jk} = 1 \\ e^-_{ijk} & \text{if } y_{ijk} = 0 \wedge \theta_{jk} = 1 \\ e^+_{ijk} & \text{if } y_{ijk} = 1 \wedge \theta_{jk} = 0 \\ 1 - e^+_{ijk} & \text{if } y_{ijk} = 0 \wedge \theta_{jk} = 0 \end{cases} \tag{1}$$

where $y_{ijk} \in \{0, 1\}$ is informant $i$'s report of the state of $\theta_{jk}$.

In general, the set of informants need not be identical to the set of vertices. Typically, however, the BNAM is applied to cognitive social structure data (Krackhardt, 1987a), in which each member of the network is asked to report on the relationships of all network members (including themselves), and here we assume that case. Following Lee and Butts (2018), we allow error rates to vary both across informants, and between informants' reports of their own ties (self-reports) and their reports of third party ties (proxy reports). This amounts to the homogeneity conditions $e^+_{ijk} = e^+_{iS}$ and $e^-_{ijk} = e^-_{iS}$ for all $(j, k)$ such that $i \in (j, k)$, and $e^+_{ijk} = e^+_{iP}$ and $e^-_{ijk} = e^-_{iP}$ for all $(j, k)$ such that $i \notin (j, k)$ with $(e^+_{iS}, e^-_{iS})$ and $(e^+_{iP}, e^-_{iP})$ being informant $i$'s self and proxy error rates (respectively). We place no additional constraints on the error rates, taking them to be a priori independently $\text{Beta}(1, 11)$ distributed (per Lee and Butts, 2018). (Note that this allows for the possibility that some informants may be systematically deceptive, i.e., that the sum of their false positive and false negative rates is greater than 1.) For $\theta$, we employ a Dirichlet categorical graph prior (Butts, 2014) with hyperparameters corresponding to a Jeffreys prior on the dyad census.

Posterior inference is performed by a combination of Gibbs and Metropolis-within-Gibbs Markov chain Monte Carlo sampling, which proceeds very efficiently for this model family. For the analyses performed here, we use a burn-in length of 500 iterations, sampling 1000 subsequent draws from the joint posterior of $\theta$ and the error parameters given the observed array of informant reports. We used 5 replicate chains, assessing convergence using the $\hat{R}$ diagnostic of Gelman and Rubin (1992). Where point estimates of $\theta$ are required we use the marginal posterior mode, with $\hat{\theta}_{ij} = 1$ if $\Pr(\theta_{jk} = 1|Y_{ijk} = y_{ijk}) \geq 0.5$; this is also equivalent to the central graph (Banks and Carley, 1994) of the posterior draws. Note that when only a subset of informants is employed (e.g., for split-half reliability assessment) we still use their full complement of proxy reports, and hence are able to estimate the entire graph structure (including ties among non-informants).

## 2.1 Data

In order to address issues of consistency in reported network data, we use cognitive social structure data Krackhardt (1987a). The cognitive social structure (CSS) for a given individual within a specified group is the complete set of his or her reports on the relationships among all group members (including him or herself). If the relation being elicited has no underlying social reality (and thus cannot be perceived), then we might naively expect that CSS observations would display no consensus when aggregated through the Bayesian Network Accuracy Model. This model family assumes that each informant's CSS report arises as a error-prone reflection of a common underlying network (i.e., $\theta$). If informants' reports are unrelated to each other, then this should manifest in high posterior uncertainty regarding the hypothesized true network and high inferred error rates at the informant level. While the appearance of an inferred consensus thus rules out an extreme form of invalidity, it is nevertheless conceivable that spurious correlations in informant reports might result in an apparent weak consensus. As such, the complete lack of consensus (as estimated by the model) is an imperfect test for validity. The general aim of this paper is to provide a more thorough investigation of the validity of BNAM-based network inference in the context of friendship and advice-seeking relations.

This study focuses on data collected from four environments - one entrepreneurial firm, two technological firms, and one university - respectively referred to as "Silicon Systems" (Krackhardt, 1987a), "High Tech Managers" (Krackhardt, 1992), "Italian University" (Casciaro, 1998), and "Pacific Distributors" (Kilduff and Krackhardt, 2008). Each of these data sets contains CSS data on advice-seeking and friendship relations. High Tech Managers (HM) contains 21 informants. Silicon Systems (SS) involved a network of 36 potential informants, but informants 13, 24, and 35 declined to participate and their data is missing for both friendship and advice networks, for a total of 33 informants. (The three non-responding informants and others' perceptions of their ties were removed upon the release of this dataset to the public by Krackhardt and thus inference cannot be done to approximate the data we would have seen. We treat them as absent by design.) Italian University (IU) contains 25 informants. Pacific Distributors (PD) contains 47 informants. As organizations are common contexts for network studies, we regard these environments to be broadly comparable to others commonly used by network researchers. We model friendship and advice-seeking as directed relations, since both were elicited from respondents in directed form. (Specifically, each informant was asked "Who would [PERSON] go to for help or advice at work?" and "Who would [PERSON] consider to be a personal friend?" for each person in the network (including the informant him or herself), with alters for each question being chosen from a roster of all network members.).

## 2.2 Assessing Validity

Given the lack of behavioral data on friendship or advice seeking in these data[1], we assess the validity of networks inferred from the respective CSS reports using the BNAM via several indirect criteria. Specifically, we consider: (1) inferential self-consistency (i.e., if the true networks and error rates are as we infer them to be, would the BNAM have been likely to give us the answer we observed?); (2) split-half reliability (i.e., are the networks inferred from randomly chosen subsamples of informants similar to each other?); (3) construct validity of the network structures (i.e., do the inferred networks have properties that one would expect on theoretical grounds?); (4) and construct validity of the informants' error rates (i.e., do the inferred informant errors behave as we would

[1] Indeed, we are unaware of any data sets containing both informant CSS reports and high-quality behavioral information on friendship and/or advice-seeking. Such data would greatly advance the field of network measurement.

expect on psychological grounds?). Below, we summarize each in turn.

### 2.2.1 Self Consistency Check

A first requirement for valid inference is that the BNAM should be able to correctly recover network structure where the true structure is known (e.g., under simulation) when the true structure and error rates match those inferred for the observed data. To test this, we begin by estimating the underlying graph and error rates for each observed relation using the BNAM. Treating the posterior mode estimate of $\theta$ as the criterion graph, we then simulate observations of informant reports under the behavioral process assumed by the BNAM, with error rates based on their corresponding posterior mean estimates from the fitted model. Given this synthetic dataset, we again use BNAM to infer the consensus structure. This structure is then compared to the originally inferred graph. To the extent that the model correctly estimates the underlying structure, we can conclude that the model would obtain the correct answer if the generating process were as assumed. If by contrast, the model cannot infer the original criterion graph, then the validity of the original inference cannot be trusted. While this test does not eliminate other possible threats to validity, it does verify that the inferential process is self-consistent. To assess consistency, we examine the Hamming distances[2] between the inferred graphs from the original datasets and the inferred graphs from their respective synthetic datasets.

### 2.2.2 Split-Half Reliability

We next assess the consistency of network inference across sets of informants. We perform a split sample reliability check by randomly sampling half of the informants from the full dataset and pooling their information via the BNAM. The remaining informants also have their reports pooled via a second BNAM. With the use of a correspondence metric, we examine how well the two graph estimates agree. To the extent that the same network is inferred from both report sets, it suggests that, independent of who is selected, the same relation is being measured. If, on the other hand, the network that is inferred depends strongly on the specific choice of informant set, this calls into question the validity of the measurement. We use the Hamming error as a correspondence metric, repeating this procedure for 100 split samples and in each case comparing the observed Hamming error against the Hamming error produced by a random row/column permutation of the original matrix (thus controlling for spurious similarity due e.g. to block structure or degree heterogeneity; see e.g. Krackhardt, 1987b; Hubert, 1987). The row/column permutation (in which the rows and columns of the adjacency matrix are simultaneously and jointly permuted) can functionally be understood as fixing the structure of edges in the graph and permuting the node labels, preserving all of the unlabeled graph properties while randomizing the correspondence of adjacency across informants.

### 2.2.3 Construct Validity of the Inferred Network

We evaluate the construct validity of the inferred networks by examining their graph level properties. If the inferred networks do not accord with theoretical expectations (e.g., friendship should tend towards reciprocity), this calls into question the validity of the inference. We examine the properties of such networks and perform conditional uniform graph (CUG) tests (Anderson et al., 1999) on said properties to compare them against chance. We are conditioning on graph density

[2]The Hamming distance between two graphs is the number of edge changes required to convert one into the other, and is a standard metric for graph comparison (see e.g. Banks and Carley, 1994; Butts and Carley, 2005).

in this study. We also perform quadratic assignment procedure tests (Krackhardt, 1987b) to assess relatedness of the inferred networks to external covariates. Both procedures are simulated for 1000 iterations.

## 2.3 Construct Validity of the Inferred Errors

Finally, we consider the pattern of respondent errors and how well they accord with expectations given previous research on perception of social structure. We reproduce the analysis in Lee and Butts (2018), that separately evaluated proxy and self-report error rates, here examining them in the context of differing theoretical expectations for errors made when assessing one's own ties versus errors made when assessing others' ties. We also consider respondents' ability to perceive higher order structure in the inferred networks. Specifically, we expect that group structures should be more reliably perceived than specific ties, as argued e.g. by Romney and Faust (1982). We assess this by means of spectral decomposition of the CSS and inferred graph adjacency matrices, as discussed below.

### 2.3.1 Defining and Detecting Group Structure

In order to assess whether or not respondents can accurately recover group structure, we must specify what we mean by "group." While many notions of group structure exist, perhaps the most basic is an allocation of individuals to relatively "core" or "peripheral" positions, such that those in the core tend to be adjacent to each other (and often to the periphery) while more peripheral individuals are not adjacent to each other. Many such core/periphery structures can be present within a single network, each of which can be thought of as an underlying group. In the undirected case, group structure in this sense has a direct correspondence with the eigendecomposition of the graph adjacency matrix,

$$A = S\Lambda S^T$$

where $A$ is the graph adjacency matrix, $S$ is a column matrix of eigenvectors, and $\Lambda$ is a diagonal matrix of eigenvalues sorted in descending order. (We here exploit the fact that, for real symmetric $A$, $S^{-1} = S^T$.) Where $\Lambda_{ii} > 0$, the corresponding eigenvector $S_{\cdot i}$ encodes group membership, with $|S_{ji}|$ giving the "strength" of $j$'s membership and the sign of $S_{ji}$ indicating which of the two groups encoded by $S_{\cdot i}$ (for mixed sign eigenvectors) to which $j$ belongs. This principle has been used by e.g. Richards and Seary (2000) and Newman (2006) for group structure detection, in some cases after modifying $A$ by subtracting the effect of (approximately) the principal eigenvector to obtain a reduced matrix $B$ (sometimes called the modularity matrix). In our case, the primary core/periphery structure associated with the principal eigenvector of $A$ is of substantive interest, and hence we do not remove it.

To see how subsequent eigenvectors (hence groups) contribute to structure formation, it is useful to note the edgewise decomposition

$$A_{ij} = \sum_{k=1}^{N} \Lambda_{kk} S_{ik} S_{jk}. \tag{2}$$

For $\lambda_k = \Lambda_{kk} > 0$, the contribution of the $k$th group division to the value of the $i, j$ edge is the product of $i$ and $j$'s respective "coreness" (when they belong to the same group), weighted by the associated eigenvalue. This corresponds to our intuition that core members within a particular group should have a higher propensity to be adjacent. (Note that, when $i$ and $j$ belong to different groups, they are likewise less likely to be adjacent, as indicated by the negative sign of the above product.) Importantly, the overall weight associated with the groups contributed by each eigenvector is controlled by the magnitude of the associated eigenvalue, which we may hence regard as indicating the prominence of the substructures in question.

Equation 2 also suggests an interpretation for the case in which $\lambda_i < 0$. In this case, node pairs with matching $S_{\cdot i}$ signs are *less* likely to interact, while those with opposing signs are *more* likely to interact. This situation corresponds to *bipartite* structure, in which one has sets of nodes that tend to be connected across sets but not within sets. As with core/periphery structure, the strength of a node's involvement in a bipartition is given by the magnitude of its corresponding eigenvector loading, and the overall weight of the bipartition in the graph is governed by the magnitude of its associated eigenvalue.

The above allow us to assess informants' perceptions of group structure as follows. First, we symmetrize both the inferred true network and the informants' CSS reports, treating an $\{i, j\}$ tie as present if $(i, j)$ or $(j, i)$ are present in the original network. Next, we perform an eigendecomposition of both the symmetrized inferred network and the symmetrized CSS networks. We then consider the set of matrices formed by the outer product of the $k$th eigenvector with itself in each symmetrized matrix, and correlate the outer product matrix derived from each CSS network to that derived from the corresponding inferred true network. If informants are better at perceiving general group structure than fine details, then we hypothesize that these matrix correlations will be higher, on average for (1) high-magnitude-$\lambda$ eigenvectors (encoding broad features) than low-magnitude-$\lambda$ eigenvectors (encoding fine details), and (2) positive-$\lambda$ eigenvectors (indicative of groups) versus negative-$\lambda$ eigenvectors (indicative of bipartitions). We assess these hypotheses via vector permutation tests as follows. Let $\lambda^{(\theta)}$ be the vector of eigenvalues of the symmetrized inferred network, with $\lambda^{(i)}$ the vector of eigenvalues for the corresponding symmetrized CSS network produced by informant $i$. Likewise, let $S^{(\theta)}_{\cdot j}$ be the $j$th eigenvector of the symmetrized inferred network, with $S^{(i)}_{\cdot j}$ the $j$th eigenvector of the symmetrized CSS network for informant $i$; we similarly define $M^{(\theta)}_j = S^{(\theta)}_{\cdot j}(S^{(\theta)}_{\cdot j})^T$ and $M^{(i)}_j = S^{(i)}_{\cdot j}(S^{(i)}_{\cdot j})^T$ to be the outer product matrices associated with the $j$th eigenvector in each case. To examine the correspondence between the eigendecomposed elements of the inferred true structure and the informants' views of the network, we compute the matrix correlations $r_{ij} = \rho(M^{(\theta)}_j, M^{(i)}_j)$ (where $\rho$ is the matrix correlation function, per e.g. Butts and Carley, 2005), defining $\overline{r_j} = \frac{1}{N}\sum_i r_{ij}$ to be the mean correlation between the $j$th dimension of the eigendecomposed informants' reports and the $j$th dimension of the eigendecomposed inferred structure. To assess our first hypothesis, we then perform a permutation test of the correlation between the vector $\overline{r}$ and $|\lambda^{(\theta)}|$: if informants are better on average at perceiving broad structural features than fine details, then this correlation should be large relative to what would be expected under random ordering of $\overline{r}$. To assess our second hypothesis, we perform a permutation test of the correlation between $\overline{r}$ and $s = \text{sign}(\lambda^{(\theta)})$: if informants are better on average at perceiving group structure than bipartite structure, then this correlation should likewise be large compared to what would be seen under random permutation of $\overline{r}$. To test the hypothesis that the structures seen by the informants correspond to actual structure in the data (as opposed to being e.g. artifacts of the density of the network), we also repeat the same procedure with synthetic CSS data generated by drawing conditional uniform random graphs with the same densities as the observed CSS structures, allowing us to determine whether the observed correspondences in group and bipartition structure between informant reports and inferred networks are larger than would be expected by chance.

# 3 Results

## 3.1 Model Validity

### 3.1.1 Self Consistency Checks

We find the Hamming distances of the parameterized criterion graph from the corresponding BNAM graph estimates from the simulated datasets to be 0 across all datasets. This shows that the BNAM is self-consistent for the data sets examined here, in the sense that if the true networks and error rates were as inferred, the BNAM would consistently draw the correct inference.

### 3.1.2 Split-Half Reliability Checks

The results of the split-half reliability checks are presented in terms of Hamming distance in Table 1. We present the fraction of simulations where the correspondences between the independent subsplit-estimated consensuses are higher than the correspondences between one of the independent subsplit-estimated consensuses and a random row/column permutation of that consensus. (Intuitively, this compares observed degree of correspondence between split-half estimates with what would be expected from the unlabeled properties of the respective estimates alone (e.g., density, block structure, etc.).) Higher correspondence is characterized through having a lower Hamming distance. As shown, networks inferred from split samples of informants correspond more closely to each other than to row/column permuted networks, on average, in all eight data sets. (The distributions of differences in Hamming distances are shown in Figure 1.) As we can observe, on average, there is a positive difference between the Hamming distance of a consensus and its random graph permutation subtracting the Hamming distance of the two split-half consensuses, indicating a higher correspondence in split-half estimated consensuses, as opposed to a random graph permutation of itself. This demonstrates that there is indeed a common structure underlying the informants' reports, as one would expect if they were imperfectly reporting a real social relation.

Table 1: Proportion of split-half samples with Hamming distances lower than distances obtained under random row/column permutation.

| Organization | Advice-Seeking | Friendship |
|---:|---:|---:|
| SS | 1.00 | 1.00 |
| HM | 1.00 | 0.98 |
| IU | 1.00 | 1.00 |
| PD | 1.00 | 1.00 |

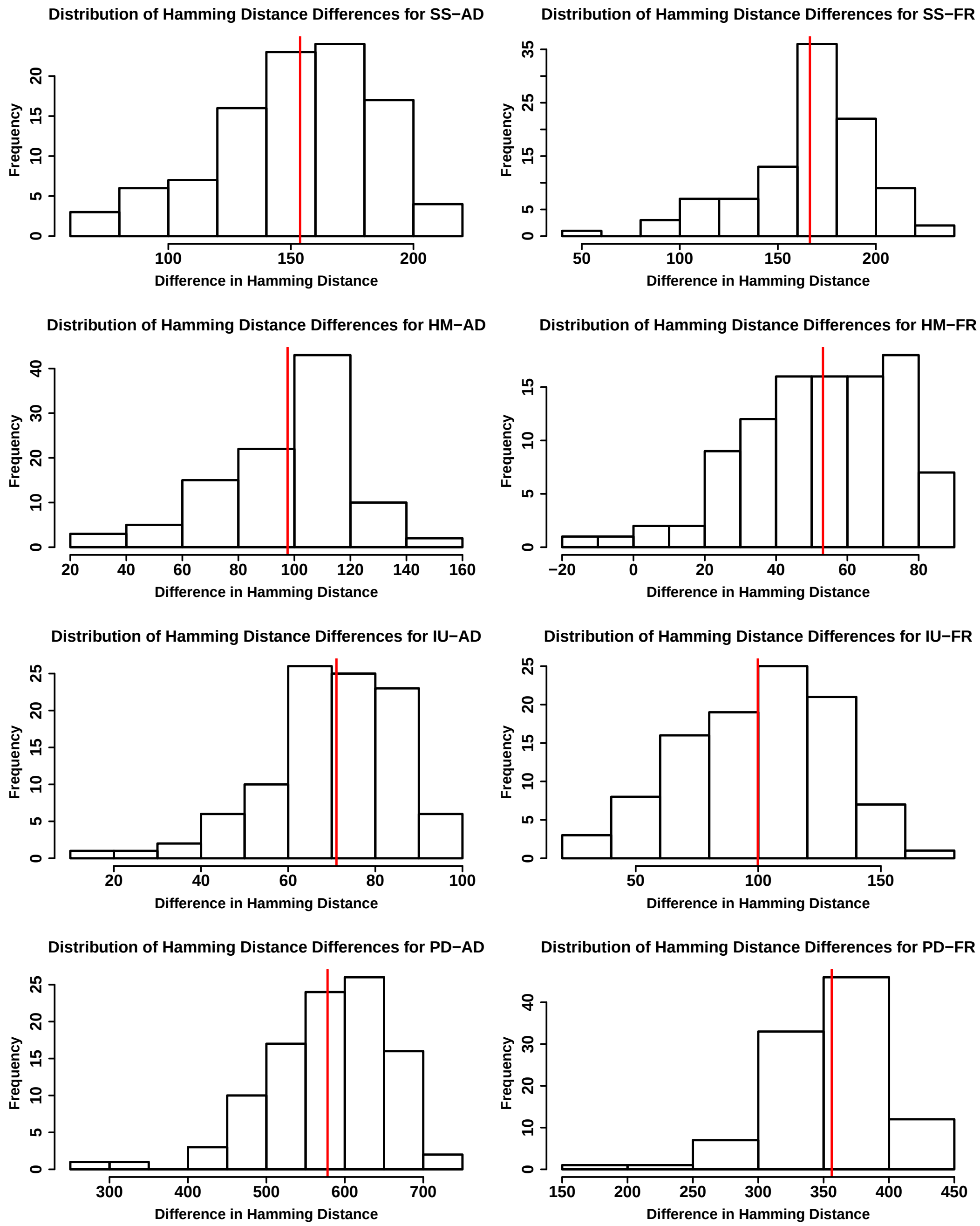


Figure 1: Hamming distance differences for split-half consensus estimates versus the estimate and a random permutation of that estimate. Positive values indicate greater similarity between split-half network estimates than between permuted split-half estimates; red line indicates the sample mean. For all eight networks, split-half estimates of network structure are on average considerably more similar to each other than would be expected from their unlabeled network properties. Study labels refer respectively to High-tech Managers (HM), Silicon Systems (SS), Pacific Distributors (PD), and Italian University (IU). The two relations measured were advice-seeking (AD) and friendship (FR).

### 3.1.3 Network Construct Validity Checks

Having demonstrated self-consistency and reliability, we now turn to construct validity checks on the network. Broadly, is the structure we observe in the inferred networks the sort of structure we would expect to see in friendship and advice networks? As would be expected for informal, non-dominance relations in organizational settings (Newcomb, 1961; Davis and Leinhardt, 1972; Hallinan, 1978), we observe a much higher level of reciprocity than would be expected in an equivalent random structure, with the results presented in Table 2. Reciprocity is calculated as dyadic reciprocity (the fraction of dyads that are symmetric).

Table 2: CUG test for inferred relations on reciprocity; all values significant at $p < 0.001$.

| Organization | Advice-Seeking | Friendship |
|---:|---|---|
| SS | 0.84*** | 0.97*** |
| HM | 0.77*** | 0.96*** |
| IU | 0.98*** | 0.86*** |
| PD | 0.99*** | 0.84*** |

Transitive closure is a widely observed feature of interpersonal networks, and can be generated by multiple mechanisms (Granovetter, 1973; Davis, 1963; Cartwright and Harary, 1956). This phenomenon has been observed not only for reported ties, but also for behaviorally assessed networks (e.g., "friendship" ties on Facebook and other social media sites Alhazmi et al., 2015; Doran et al., 2013). In the case of both advice-seeking and friendship networks, we would likewise expect to see transitivity levels that are higher than typical for random graphs of similar density. As shown in Table 3, transitivity levels are indeed significantly higher (CUG test) than would be expected for all inferred networks. Transitivity is defined as the fraction of all $(i, j, k)$ two-paths for which there exists a $(i, k)$ edge (Wasserman and Faust, 1994).

Table 3: CUG test for inferred relations on transitivity; all values significant at $p < 0.001$.

| Organization | Advice-Seeking | Friendship |
|---:|---|---|
| SS | 0.47*** | 0.38*** |
| HM | 0.58*** | 0.38*** |
| IU | 0.51*** | 0.57*** |
| PD | 0.42*** | 0.57*** |

Previous literature finds that strong indegree centralization in advice-seeking networks is commonly present (e.g. Creswick and Westbrook, 2010; Dearing et al., 2017), being driven by both structural factors such as inequality in seniority, degree of specialization, and expertise and by the incentive to seek information from high-value mentors. As shown in Table 4, all four of our advice-seeking networks show levels of indegree centralization that is higher than would be expected by chance. In the case of friendship, indegree centralization is less clear: while popularity differences exist in some settings, adult friendships in workplace contexts may not in all cases show strong concentration of friendship nominations on a small number of individuals. We would not, however, expect to see significantly *low* levels of centralization in typical friendship networks. As Table 4 shows, three of the four friendship networks are significantly centralized with respect to indegree (albeit less so than the advice-seeking networks), and no network shows a significantly low level of indegree centralization. This behavior is again compatible with what would be expected on a priori grounds for networks of this type.

| Organization | Advice-Seeking | Friendship |
|---|---|---|
| SS | 0.673*** | 0.280** |
| HM | 0.637*** | 0.200 |
| IU | 0.849*** | 0.289* |
| PD | 0.534*** | 0.323*** |

Table 4: CUG test for inferred relations on indegree centralization; * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Study labels refer respectively to High-tech Managers (HM), Silicon Systems (SS), Pacific Distributors (PD), and Italian University (IU).

Finally, two of our cases also contain covariate information that can be used to assess the plausibility of the advice networks. As previous literature has shown advice-seeking to be related to the formal hierarchy (Lomi et al., 2013), we would expect to see some relationship between advice-seeking and the organizational rank. In particular, we consider the graph correlations between the posterior estimate of the advice network and matrices of indicators for similarity in organizational rank in the High-Tech Managers and Silicon Systems organizations. We then perform matrix permutation tests for the correlation of the advice network with the corporate level covariate. We see a clear negative correlation between advice-seeking and being at the same level in the organization (corr= -0.28, $p = 0.003$), indicating that advice ties tend to bridge individuals at different levels (corr= 0.30, $p < 0.0001$). These observations are consistent with what would be expected from an organizational advice network. In the Silicon System advice-seeking network, we find that people seek advice between ranks, as one might expect in a multi-level organization, shown by our matrix permutation test (corr=-0.16, $p = 0.012$). Here again, we thus see that the networks inferred from the informant reports behave as we would expect from prior literature.

### 3.2 Perception of Group Structure

We now consider evidence regarding perceptions of group structure. Let $A$ be the symmetrized adjacency matrix for an estimated network, with $A^{(1)}, \ldots, A^{(N)}$ the corresponding symmetrized CSS observations for that network ($A^{(i)}$ being the CSS matrix for informant $i$). Following the discussion of Section 2.3.1, we define $S^{(i)}_{\cdot j}$ to be the $j$th eigenvector for $A^{(i)}$ and $S^{(\theta)}_{\cdot j}$ to be the $j$th eigenvector for $A$, with $\lambda^{(i)}_j$ and $\lambda^{(\theta)}_j$ the corresponding eigenvalues. From the above, we define the outer product matrices $M^{(\theta)}_j = S^{(\theta)}_{\cdot j}(S^{(\theta)}_{\cdot j})^T$ and $M^{(i)}_j = S^{(i)}_{\cdot j}(S^{(i)}_{\cdot j})^T$ encoding the contribution of each eigenvector to the global network structure. The average correlation between the informant structures generated by eigenvector $j$ and the corresponding structure in the inferred graph $A$ is then assessed by $\overline{r_j} = \frac{1}{N}\sum_i \rho(M^{(\theta)}_j, M^{(i)}, j)$, where $\rho(A, B)$ is the matrix correlation between matrices $A$ and $B$.

$\bar{r} = (\bar{r}_1, \ldots, \bar{r}_N)$ is employed to test for biases in structural perception as follows. First, we may consider $\mathrm{cor}(\bar{r}, \mathrm{sign}(\lambda^{(\theta)}_j))$; when positive, this correlation indicates a tendency to perceive structure associated with groups (local core/periphery structures) more readily than that associated with bipartitions. Similarly, a positive value of $\mathrm{cor}(\bar{r}, |\lambda^{(\theta)}|)$ indicates a tendency to perceive broader structural features of the underlying relation rather than fine details. Below, we assess these correlations via vector permutation tests (1000 replications). To compare the values of $\bar{r}$ themselves against a random baseline, we also compute CUG replications of $\bar{r}$ by generating 100 replicate CSS data sets for each network (with each simulated adjacency matrix drawn from a CUG distribution with identical density to the density of the observed CSS matrix) and computing $\bar{r}$ for each simulated data set. Elements of $\bar{r}$ in the extreme tails of the simulated distribution can be interpreted as reflecting levels of correspondence in excess of what would be expected from chance alone.

The correlations of $\bar{r}$ with the sign and modulus of $\lambda^{(\theta)}$ are shown in Table 5. In general, we observe a higher level of sensitivity to group structure versus bipartite structure across the eight networks, with significant positive correlations in six of eight cases and positive non-significant correlations in the remaining two cases. The tendency to be sensitive to broad versus fine details of the network is even stronger: we see significant correlations of $\bar{r}$ with $|\lambda^{(\theta)}|$ for all eight of our networks, with the correlations themselves being quite large (0.77 to 0.90). Taken together, these results are compatible with a cognitive model similar to those proposed by e.g. Kumbasar et al. (1994), Krackhardt and Kilduff (1999) or Romney and Faust (1982), where large-scale structural features are accurately perceived but details are altered by perceptual or mnemonic errors that tend to impact bipartite structure more than group structure. (It is noteworthy that the tendency to see triadic closure where it is not in fact present, as proposed e.g. by Krackhardt and Killduff (1999) and (Brashears and Quintane, 2015), would be expected to have precisely this sort of disparate impact.)

| Network | Sign Correlation | Sign $p$-value | Modulus Correlation | Modulus $p$-value |
|---|---|---|---|---|
| SS-AD | 0.43 | 0.01 | 0.88 | <0.001 |
| SS-FR | 0.31 | 0.03 | 0.85 | <0.001 |
| HM-AD | 0.49 | 0.02 | 0.85 | <0.001 |
| HM-FR | 0.19 | 0.21 | 0.77 | <0.001 |
| IU-AD | 0.19 | 0.23 | 0.91 | <0.001 |
| IU-FR | 0.43 | 0.01 | 0.88 | <0.001 |
| PD-AD | 0.39 | <0.001 | 0.90 | <0.001 |
| PD-FR | 0.31 | 0.01 | 0.88 | <0.001 |

Table 5: Correlations between $\bar{r}$ and sign($\lambda^{(\theta)}$) and $|\lambda^{(\theta)}|$ (respectively) with permutation $p$-values. Study labels refer respectively to High-tech Managers (HM), Silicon Systems (SS), Pacific Distributors (PD), and Italian University (IU). The two relations measured were advice-seeking (AD) and friendship (FR).

A more detailed view of $\bar{r}$ is shown in Figure 2, which shows the elements of each mean correlation by eigenvector (sorted by signed eigenvalue from most positive to most negative). The range of simulated CUG replicate correlations (vertical intervals) provides a point of comparison for the observed correlation values; mean correlations outside the simulation intervals can be interpreted as showing significant average association between perceived and inferred structure on the associated eigenvectors. Overall, we see a striking pattern across the eight networks in which the first several (typically 4-5) positive eigenvectors and the last 1-2 eigenvectors show relatively strong and significant mean correlations between inferred and informant networks, with structure on the remaining eigenvectors correlating at or near chance levels. Since each eigenvector after the first splits the nodes of the network into two sets, this amounts very roughly to a typical discriminating capacity of 16 to 64 subsets per informant. While such a view is relatively rich on the scale of the networks studied here, it is still substantially compressive in quantitative terms (with the number of eigenvectors with significant correlations being much smaller than the number of vertices). While some individuals may have detailed information on particular parts of the network, informants' average views can be summarized by a relatively low-dimensional structure. This comports well with expectations from prior work regarding cognitive representations of network structure, and is consistent with the notion that informants' reports are providing approximations to a more complex underlying structure (Picek et al., 1975; Fiske et al., 1991; Brashears and Brashears, 2016).

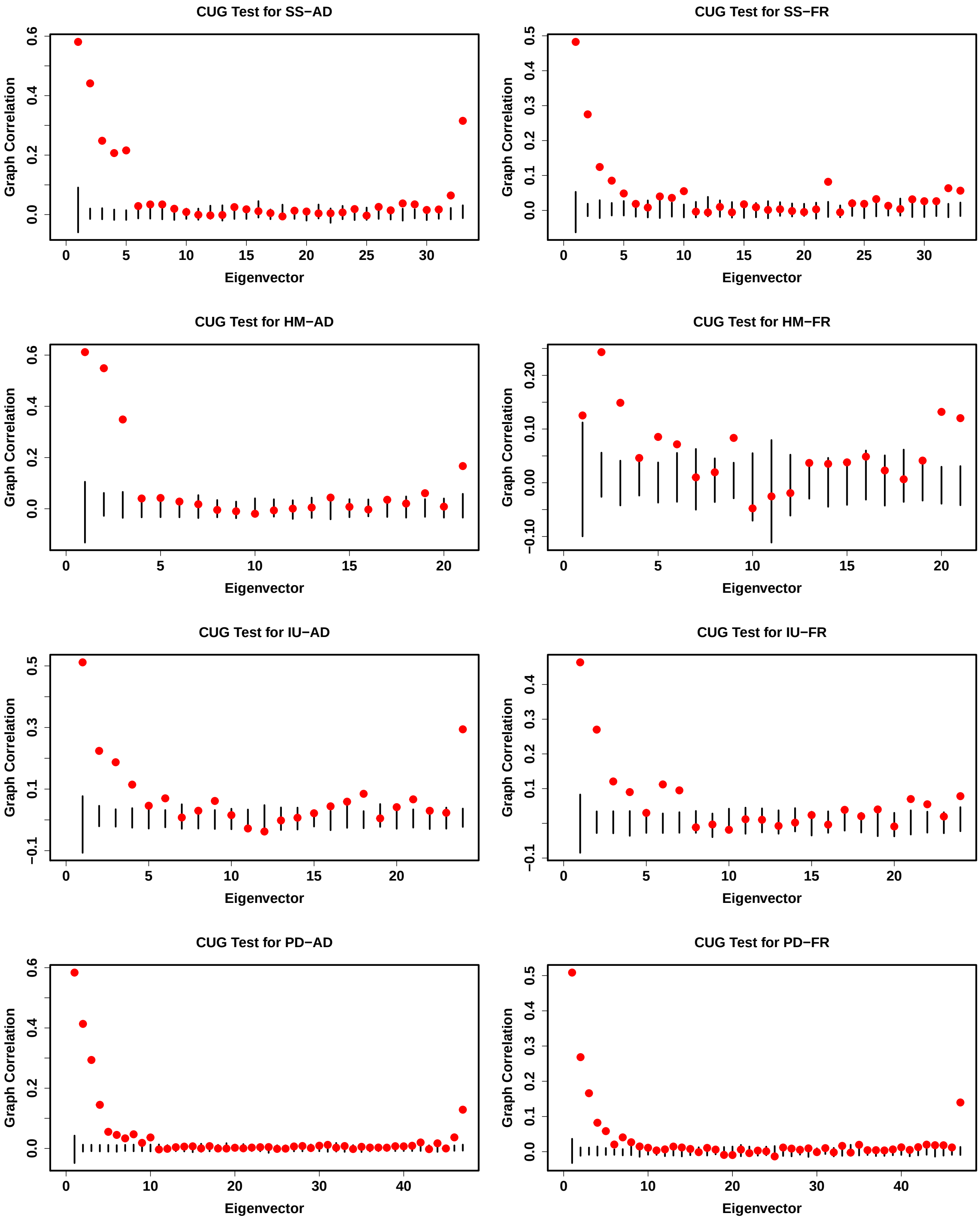


Figure 2: $\bar{r}$ of respective eigenvalue for each observed network (sorted from most positive to most negative eigenvalue); vertical intervals indicate the range of correlations under 100 CUG replicate data sets.

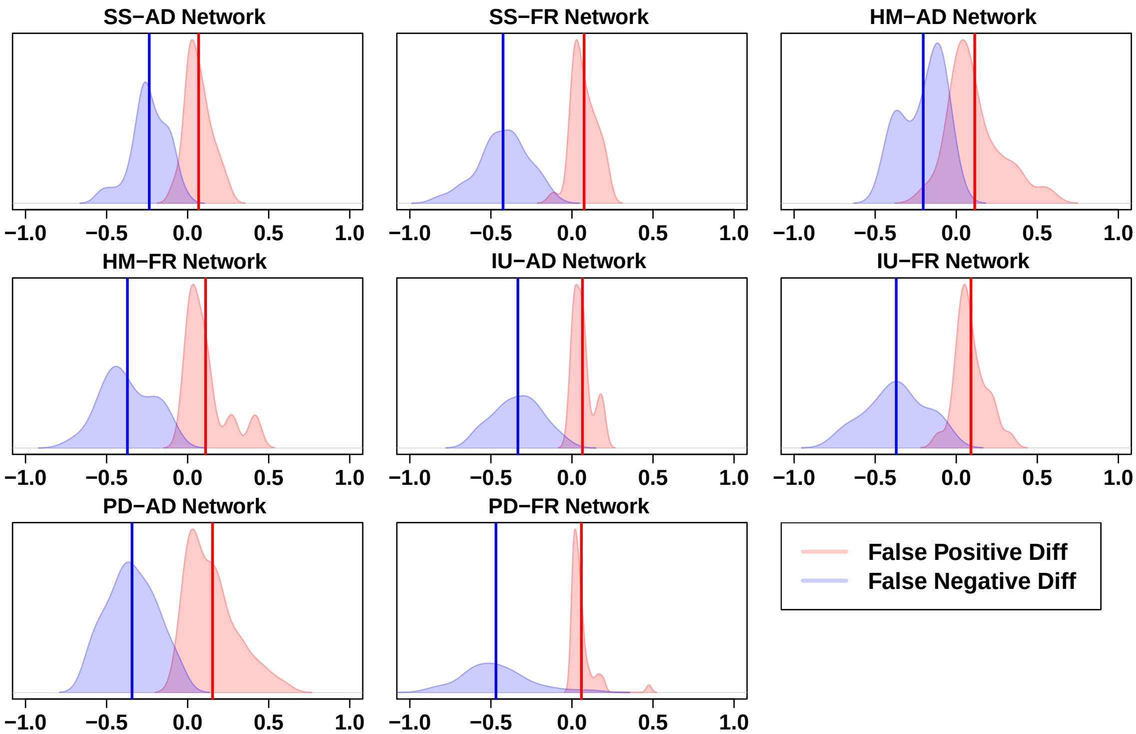


Figure 3: Distribution of differences between informants' posterior mean error rates when reporting on their own ties versus reporting on ties between third parties. Positive error rates indicate higher error rates for ego's own ties versus proxy reports; 0 indicates equal error rates. Means are shown by vertical lines. Study labels refer respectively to High-tech Managers (HM), Silicon Systems (SS), Pacific Distributors (PD), and Italian University (IU). The two relations measured were advice-seeking (AD) and friendship (FR).

### 3.3 Informant Error Rates

Lee and Butts (2018) estimated informant error rates for the networks examined here using BNAMs; here, we briefly revisit these results in light of the question of construct validity. As shown in Figure 3, we see that while informants' false positive rates for proxy reports are on average the same as their false positive rates for self-reports, false negative rates for proxy reports are on average much higher than for self-reports. This is compatible with the notion that informants are reluctant to "invent" ties, but have much less information on ties among third parties (and hence miss them, manifesting as a higher false negative rate). This pattern is typical of proxy reports in other settings (Mingay et al., 1994; Sudman et al., 1996), with informants tending to under-report activities and involvements of others. We also note that this pattern is consistent with work by Kumbasar et al. (1994) showing that informants tend to overestimate their own centrality; the dominant mechanism for this effect would seem to be that informants fail to perceive others' ties, although the right-tail of the false positive differences for some networks in Figure 3 suggests that some individuals may also be prone to exaggerating their own degree.

Examining absolute error levels in Figures 4 (self-reports) and 5 (proxy reports), we see less differentiation between the distribution of false negative and false positive error rates when discussing one's own ties, with most error rate distributions being fairly similar across networks. For proxy error rates, the false negative error rates are substantially higher than the false positive error rates

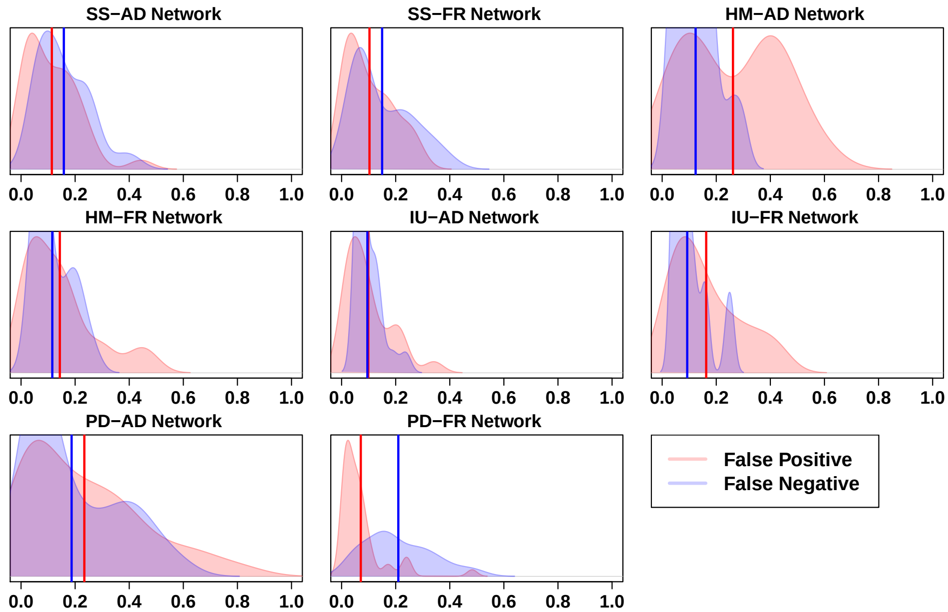


Figure 4: Marginal posterior distributions of informant false positive (FP) and false negative (FN) self-report error rates for each of the 4 organizational settings: Silicon Systems (SS), High-tech Managers (HM), Italian University (IU), and Pacific Distributors (PD). Means indicated by vertical lines.

across all networks, consistent with prior literature (Butts, 2003). It should be noted that, because the number of proxy reports greatly exceeds the number of self-reports per informant, global mean error rates are much closer to the proxy rates than the self-report rates.

# 4 Discussion and Conclusion

To summarize our findings, we see an overall pattern of results that strongly supports the validity of network inference from the BNAM in the eight cases examined here. Our self-consistency check verifies that the model can, in principle, recover an unknown graph structure from noisy observations under the conditions found in our eight data cases. This eliminates the possibility that even if there was a true graph to recover, the method would not be able to reliably recover it at the inferred error rates. Next, our split sample reliability results show that our informants are reporting a common relational structure that is consistent across subsets. This eliminates the possibility that informants are simply reporting idiosyncratic mental models with no underlying common structure (as suggested somewhat theatrically by Bernard et al., 1979). Furthermore, examination of the BNAM inferred graphs shows that they display properties that accord with what we would expect from friendship or advice-seeking networks, including associations with external covariates. This eliminates the possibility that the networks being inferred reflect some shared mental model that is unrelated to social process. Although we cannot definitively rule out the possibility that our informants in each setting are reporting (for each of two different relations) a fictitious consensus that happens to appear like a realistic social network, this would seem at this point to be an unlikely hypothesis. In the absence of positive evidence for such joint confabulation, we thus conclude that the BNAM is able to provide valid estimates of social structure from informant reports.

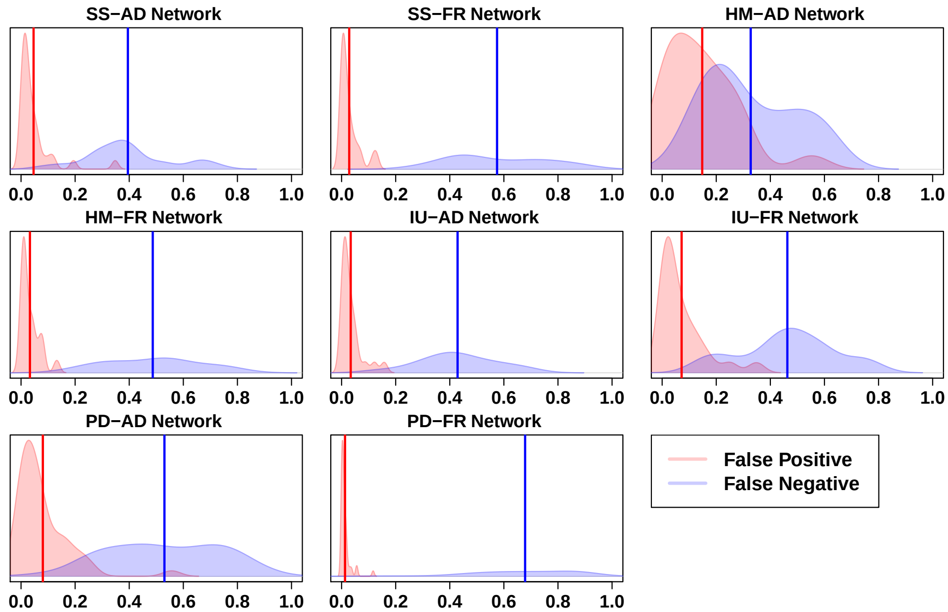


Figure 5: Marginal posterior distributions of informant false positive (FP) and false negative (FN) proxy error rates for each of the 4 organizational settings: Silicon Systems (SS), High-tech Managers (HM), Italian University (IU), and Pacific Distributors (PD). Means indicated by vertical lines.

Further strengthening the above are our findings on the detailed pattern of correspondence between informants' views of network structure and our global estimates. As suggested by work such as that of Kumbasar et al. (1994), Krackhardt and Kilduff (1999), and Romney and Faust (1982), we find that informants' CSS slices agree on average with the consensus estimate with respect to broad structural features (as represented by eigenvectors with high-modulus eigenvalues), while showing more distortion on fine details. Moreover, we see on average a higher tendency towards agreement with group structure than bipartite structure, compatible with cognitive models that predict excess triadic closure. Examination of inferred individual-level error rates supports Kumbasar et al.'s (1994) findings regarding ego's tendency to overestimate his or her centrality relative to others, with the clarification that this arises primarily from ego's much higher false negative rates for proxy reports than for self-reports. As false positive rates tend to be similar for self versus proxy reports, the dominant difference between the two types of reporting accuracy is plausibly due to informants' more limited knowledge of ties among third parties (leading to an over-perception of sparsity elsewhere in the network).

These findings suggest a number of practical implications for research going forward. First, our findings imply that asking proxies to provide information on relationships may prove useful when researchers need to gather information about the underlying network structure, but with very few resources to gather informants: aggregated reports from a subset of network members can provide useful information regarding the entire group. Furthermore, since error rates are non-trivial for informant self-reports, aggregating reports from many network members (including proxy reports) can plausibly provide much more accurate estimates of network structure than can be obtained from simply using each individual's reports of his or her own ties (as is common in current network research). As such, we recommend as a best practice that researchers collect CSS data when circumstances allow (employing methods such as arc sampling designs (Butts, 2003) when complete CSS data is infeasible), followed by aggregation using BNAM or a similar consensus modeling strategy. A more subtle implication of our findings is that broad group structures are likely to be accurately inferred even from fairly noisy network data, but that bipartite structure may easily be obscured. Since phenomena such as bridging and brokerage depend on local bipartitions, this suggests that studies aimed at studying these phenomena may need to pay greater attention to data quality (with the use of greater numbers of informant reports, if feasible). It is unclear whether informants' difficulty in reporting bipartitions extends to two-mode relations that are inherently bipartite (e.g., connections between persons and organizations, or tasks and required resources). Further work on informant accuracy in such settings is needed.

In conclusion, examination of the measurement of social relations from informant reports suggests that valid network inference is possible, using techniques stemming from the tradition of Cultural Consensus Theory pioneered by Batchelder and colleagues. We hope that these insights will improve measurement practices within the social network community, and help to spur further research on the perception and reporting of social relationships.

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