Transient Snap-Through of a Bistable Composite Laminate Under Asymmetric Point Load

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ABSTRACT

Bistable fiber composite laminates have promising capabilities for shape morphing and have found applications in advanced airframes, energy harvesting, and robotics. Their bistability originates from an asymmetric ply layout and can create highly complex deformations during the snap-through from one stable state to the other. Therefore, it is essential to understand the transient behaviors of these laminates under different loading conditions. Although simple symmetric loading conditions are well understood, asymmetric loading conditions received far less attention. In this study, we investigate the transient deformation of a [0°/90°] square laminate subjected to an asymmetric point load at different locations. Finite element simulation and experimental testing both show that, depending on the loading position, snap-through can either be a one-step or two-step process, while each step is related to the curvature change of a laminate edge. Also, at some loading positions, snap-through is unattainable regardless of the input magnitude. The results of this study would help us obtain a more comprehensive understanding of the nonlinear mechanics of bistable composite laminates showing transient response to different external forces.

Keywords: Bistable Composite, Asymmetric Loading, Transient Response, Snap-through

1. INTRODUCTION

With the complex and dynamic changes in our working environment comes the need for adaptive structures that could respond to their surroundings and perform multiple functions, and shape morphing is one such critical function.[1,2] There have been many efforts to impart these morphing capabilities into structures, which could provide the platform for performance improvement for various applications like high-performance airframe,[3,4] automobile vehicle,[5] renewable energy infrastructure,[2,3] and energy harvesting.[6] To this end, carbon fiber composite laminates with asymmetric fiber layout have shown promising potentials due to their lightweight, high strength, and multi-stability for shape morphing.[6] These composite laminates can settle into two different configurations (or stable states) because of the internal thermal stress developed during curing, and each state can show a uniquely customized shape. To achieve morphing, one can use internal or external actuators to deform the laminate away from one stable state. When this deformation reaches a critical stage, elastic-instability will occur, and the laminate will rapidly deform to the other stable state—a process commonly referred to as the "snap-through." Moreover, once the snap-through completes, the laminates do not require any continuous energy supply to maintain their shape change between the stable states. Therefore, researchers have been attempting to apply these adaptive composites to many morphing systems over the past two decades.

To fully materialize the potentials of these adaptive composite laminates, it is critical to understand the deformation characteristics and nonlinear mechanics of their snap-through from one stable state to the other. Lots of progress has been make to this end. Dano and Hyer, in their seminal paper, predicted the stable states of simple asymmetric laminates using the classical lamination theory.[7] In the following paper, they went on to study the snap-through behavior of a [0°/90°] laminate with symmetric boundary conditions by extending the classical lamination theory and introducing Von-Karman geometrical non-linearity.[8] Although this study successfully predicted the principal curvature change induced by the applied moment, it did not examine the intermediate shape changes during the snap-through process. In a recent review paper, Emam and Inman discussed the

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assessment of energy harvesting and shape morphing capabilities of CFRP laminates.[6] In particular, they integrated piezoelectric transducers (PZT) on the laminate to evaluate the energy output from snap-through by varying the percentage of the PZT area over the total area of the laminate. Similar efforts were taken by Pedro and Weaver, who studied the actuation of laminates using piezoelectric patches.[9] Dano et al. attempted to recreate an finite element model that could simulate the actuation of distributed PZT integrated composite laminates and estimated the voltage required to snap-through.[10]

However, the aforementioned studies focus more on the external shapes of the stable states and corresponding applications.[4] That is, they focus on understanding the structural mechanics *before and after* the snap-through. Moreover, almost all of the current studies are limited to symmetric loading conditions. There are only a few studies that examined the transient deformations *during* snap-through in detail. Potter et al., in their paper regarding the bifurcations in bistable composite laminates, observed that the snap-through occurs as a combination of two coupled bifurcations instead of one. They further approximated the energy requirement for achieving these bifurcations but did not investigate the transient deformation between the two bifurcations.[11] Also, Cantera et al. showed the experimental load-displacement curves for square laminates of varying side lengths. Their paper showed the intermediate shapes (pictures of actual laminates) of the laminate when it is loaded by a point force at the center and supported at its four corners.[12]

Therefore, there is a still a critical lack of understanding of the transient behavior of bistable laminates. To address these gaps, we aim to use both finite element analysis (FEA) and experimental testing to carefully examine their transient deformations during snap-through under unsymmetric loading conditions. We also study the critical force and displacement needed to complete the snap-through of these laminates. More specifically, we test square-shaped, [0°/90°] composite laminates fixed at its center, and apply transverse point load at different locations on the boundary or within this laminate. The results show that, depending on the loading position, snap-through can either be a one-step or two-step process (aka. consisting of either one or two bifurcations). Each step is related to the curvature change of a laminate edge. Also, at some loading positions, snap-through is unattainable regardless of the input magnitude. This study can help us better understand the transient behaviors of asymmetric composite laminates as well as other pre-stressed, bistable shells. Thus, it can provide many crucial insights for their applications for morphing.

In what follows, section 2 details the numerical simulation and experimental setup for the bistable composite laminate under asymmetric point force. Section 3 presents and discusses the results regarding the transient deformation during the snap-through. Section 4 concludes this paper with a summary.

2. METHODS AND ANALYSIS

This study investigates the transient snap-through deformations of a 2-ply, [0°/90°], 100×100mm square laminate subjected to a point force at different locations (AS4 8552 CFRP prepregs). We also record the corresponding critical forces and critical displacements for achieving snap-throughs. A square laminate is selected here as a case study because it constitutes the most elementary component for many multi-stable and multi-patched adaptive composites.[13]

We choose the point load location carefully in order to capture a comprehensive understanding. With the center fixed, the four quadrants of the composite laminate are identical to each other due to symmetry. So we apply point load only in one quadrant for this study, and Figure 1 (b) summarizes the six application locations.

We first conducted finite element simulations to calculate the force-displacement relationships corresponding to different point force locations and then validate these results by experimental measurements. With the validated finite model, we can further investigate the transient deformation during snap-through. The next two subsections detail the finite element and experiment setup.

2.1 Finite Element Simulation

We first model the composite laminate in the "part module" in the finite element software (ABAQUS 6.14, static structural solver), and then assign the [0°/90°] fiber layout using the composite layup option. Table 1 summarizes the material properties used for this simulation. The finite element simulation consists of two steps. The first step simulates the curing process and the development of internal stress. In this step, we first constrain

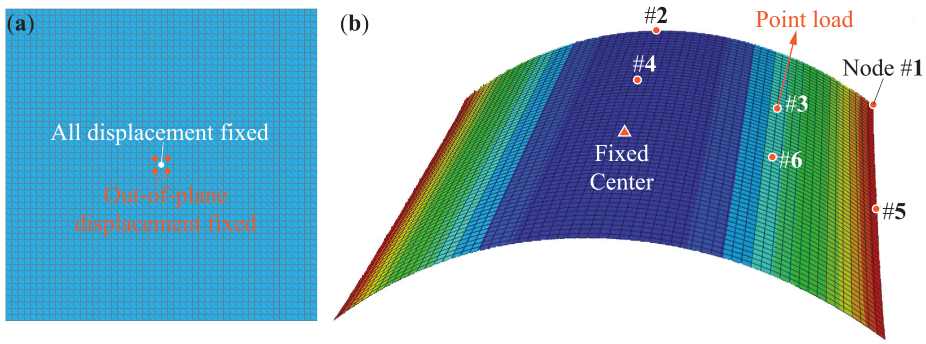

Figure 1. Finite Element simulation setup for this study. a) Meshing of composite laminate sample and the boundary conditions applied to the center nodes. b) Selected point force application locations. Node # 1 is at the corner. Node # 2 and #5 are at the center of two patch edges. Node # 3 is the mid-point between the patch center and corner node #1. Node # 4 and #6 are at the mid-point between patch center and edge nodes #2 and #5, respectively.

Table 1. Constituent Material Properties of AS4 8552 carbon composite prepregs. E_i and G_{ij} are the elastic modulus (unit of GPA). ν is the Poisson's ratio. α_{ij} are the thermal coefficients of expansion (unit of $^\circ C^{-1}$).

Property	E_1	E_2	ν_{12}	G_{12}	G_{13}	G_{23}	α_{11}	α_{22}	α_{33}
Value	135	9.5	0.3	5	7.17	3.97	-2×10^{-8}	3×10^{-5}	3×10^{-5}

the laminate at its original flat configuration (aka. no nodal displacements allowed) and heat it to 135°C. Then, the laminate cools to room temperature of 20°C. We then release the fixed nodal displacement constraints so that the laminate takes the post-cured shape, which is an unstable saddle shape as the laminate is a perfect square. We need to deform the laminate to one of the stable cylindrical shapes. In order to do so, the central node is fixed for all translations and in rotation about out-of-plane axis, and a sufficient displacement is given to the right-side edge. Therefore, the laminate sample settles into one of its stable states as shown in the figure 1 (b).

The second step of the finite element study simulates the snap-through under point force. In this step, we fix the in-plane displacements of the center node and constraint the out-of-plane displacements of the four adjacent nodes (Figure 1(a)). This setup eliminates any rigid-body motions and closely emulates the experimental setup (as we detail in the next sub-section). Then, we apply a sufficiently large out-of-plane displacement at the appropriate node point (aka. Node #1 to #6 in Figure 1(b)) until snap-through completes, and measure the corresponding reaction force.

2.2 Experimental Setup

We fabricate five composite laminate samples using unidirectional AS4 8552 CFRP prepregs (Supplier: Adhesive Prepregs for Composite manufacturers (APCM, LLC.)). The detailed fabrication procedure could be referred in our previous paper.[14] These laminates were measured for their force-displacement relationship on a universal tester machine (ADMIT eXpert with S-type 500 series load cell, Figure2).

A custom-designed, 3D-printed fixture is attached to the base plate of the tester machine so that we can fix the laminate at its center to the fixture via a bolt, and then apply the point load at the selected nodes using a string and an eyebolt (Figure 2). We use controlled displacement of the tester overhead to deform the laminate sample until the snap-through is complete (0.5 mm/sec overhead speed). It is worth noting that moisture ingression can significantly weaken the bi-stability and change the structural behavior.[15] Therefore, every sample is baked at 135 °C two hours before the experiment to ensure consistent measurements.

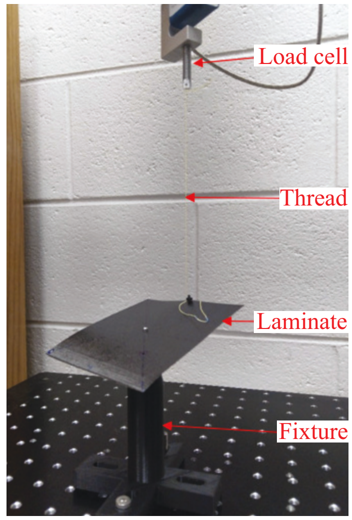

Figure 2. Experimental setup of the laminate snap-through test under point load.

3. RESULTS AND DISCUSSION

Figure 3 summarizes the relationships between total reaction force and total nodal displacement based on finite element simulations and experimental tests. Overall, the finite element simulation predictions agree with the test results well in terms of the deformation characteristics and reaction force magnitude. Some discrepancies exist and they probably originate from the fabrication imperfections as well as the sensitive nature of elastic instability. Regardless, they do not hinder us from examining the transient laminate deformations during snap-through and uncovering the effects of point load location.

3.1 Observations Based On The Response Curves

Our results indicate that, as we change the location of asymmetric point load, the transient behaviors of the composite laminate can change fundamentally *during* the snap-through, even though its two stable states are unaffected *before and after* snap-through.

If the point load is at the corner node #1, snap-through occurs in two distinctive steps (as indicated by the two sharp drops in the force-displacement curves in Figure 3(a)). By carefully observing the laminate deformation in finite element simulation (Figure 4), we find that the first step occurs when the initially straight edge of the laminate snaps to acquire a curved shape. This marks the onset of shifting from one state to the other. The second step occurs when the adjacent, initially curved edge snaps to an almost straight shape. It is worth noting that if we remove the point load before the second step, the laminate will return to its original stable state, so the snap-through is not complete until the second step occurs. Once the second step occurs, the laminate will settle into the other stable state if the point load is removed. Based on the simulation, the critical force required for snap-through is 0.70 N (maximum reaction force), and the critical displacement is 29.0 mm (at the occurrence of the second step).

However, if the point load is at the node #2 (aka., at the center of initially curved edge), the snap-through occurs in just one step. In this case, the initially curved edge at which node #2 locates and the adjacent straight edges change their shapes simultaneously. Hence the snap-through deformation occurs almost symmetrically about the axis defined by the fixed point at the laminate center and node #2 (Figure 5). The corresponding critical force required for completing the snap-through is 4.1 N, and the critical displacement is 16.7 mm.

If the point load is at node #3, snap-through also involves two steps, but they are tightly coupled. That is, the distance between these two steps in terms of nodal displacement is minimal. However, careful observation of the finite element simulation and experiment result can still reveal similar transient deformation patterns as those in the case of node #1.

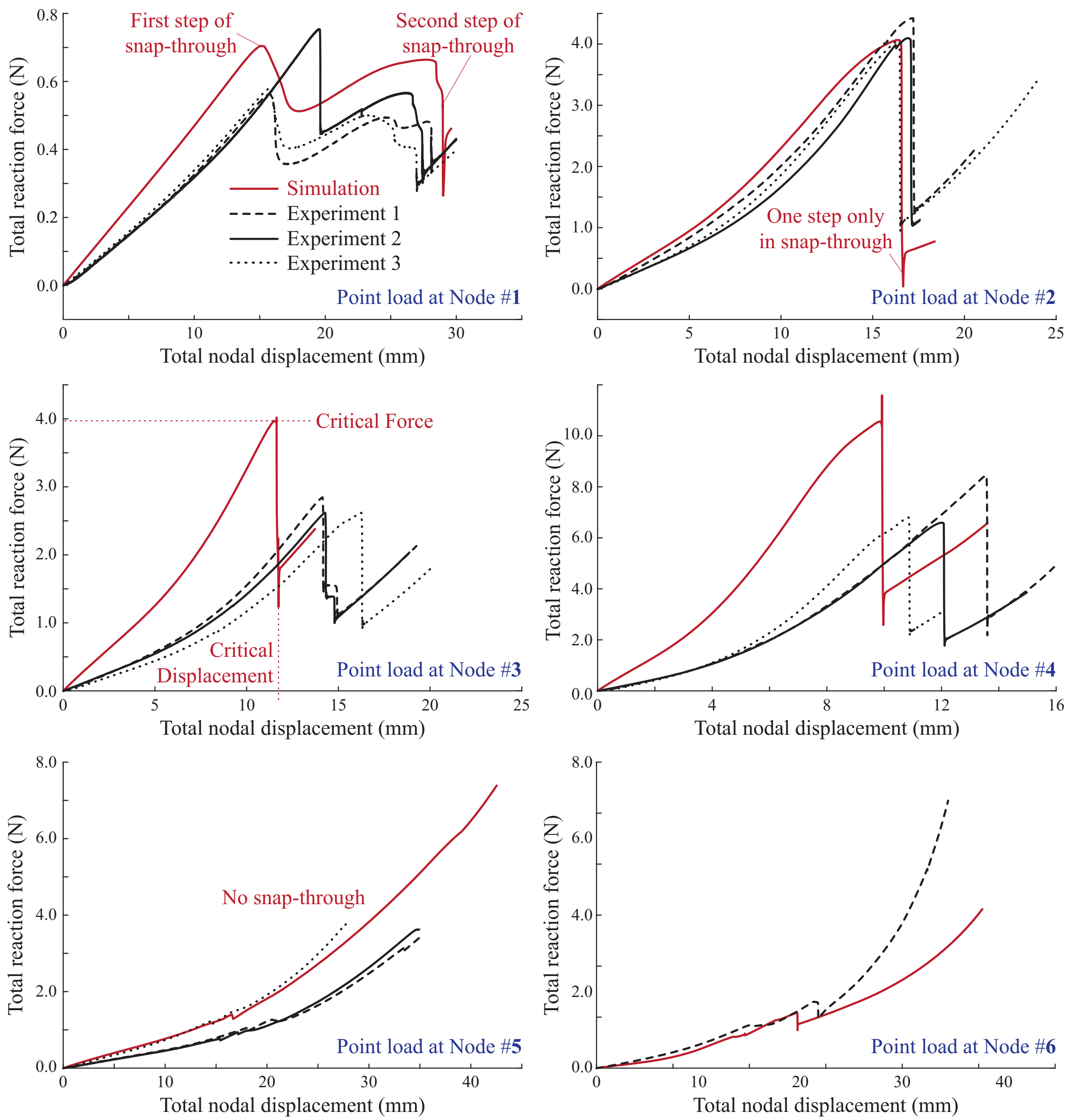

Figure 3. Reaction force-nodal displacement relationships of the square laminate when the point load is applied at different locations.

If the point load is at node #4, the transient snap-through deformation is similar to the response of node #2. However, the critical force required for snap-through increases significantly, whereas the critical displacement reduces. These differences indicate that the laminate gives more resistance as the point load moves closer to the fixed center. For clarity, the critical force and displacement corresponding to the six different point load locations are summarized in Table 2.

Finally, if the point load is at node #5 or node #6, snap-through never occurs regardless of the load amplitude. This unique observation has never been reported by previous studies. Indeed, any point load located on the axis defined by nodes #5 and #6 can not switch the laminate to the other stable state under the current setup.

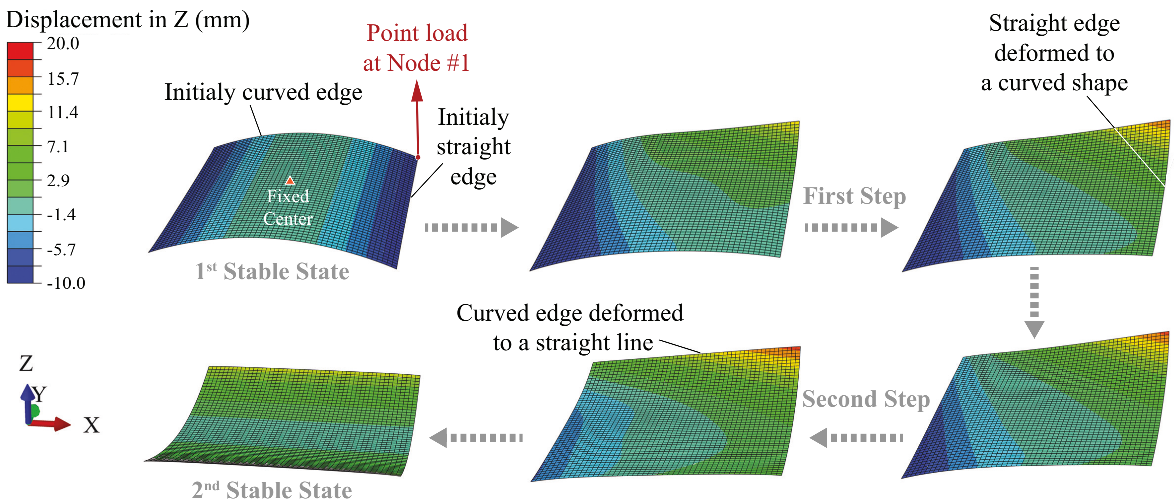

Figure 4. The different stages of laminate snap-through when the point load is at Node #1.

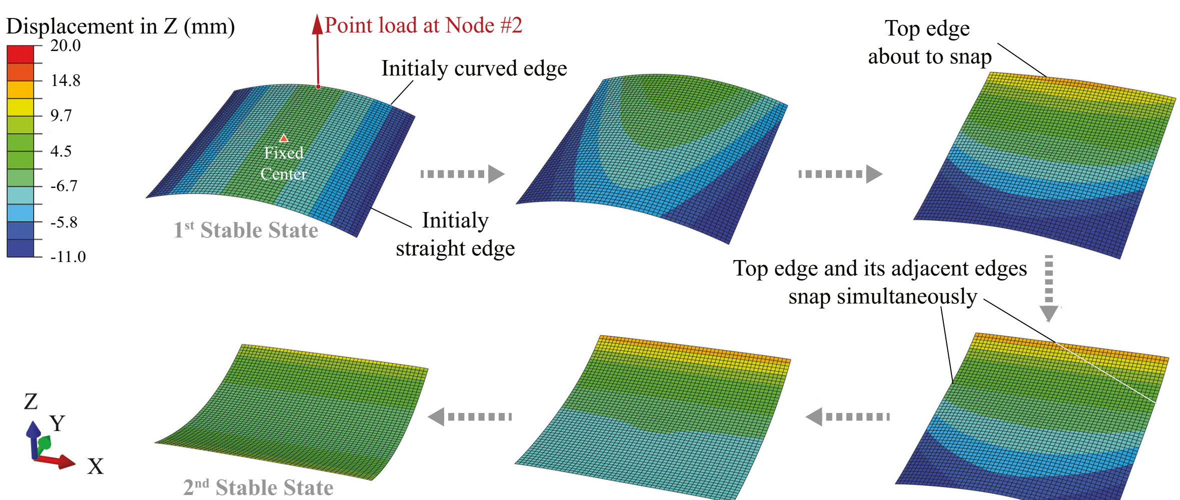

Figure 5. The different stages of laminate snap-through when the point load is at Node #2.

3.2 Further Discussions

Based on experimental and simulation results, we observe three distinct transient snap-through behaviors in the bistable laminate when it is subjected to asymmetric point load. These are two-step snap-through (node #1 and #3), one-step snap-through (node #2 and #4), and no snap-through (node #5 and #6).

Therefore, we can deduce that the snap-through of a bi-stable composite laminate completes only after a specific set of edges snap to a different shape (aka. from a straight line to a curved shape, and vice versa). As discussed earlier, there is a considerable lag between two steps in the Node #1 case. This lag depends on the location between the point load and its respective distance from the *active* adjacent edges. By *active*, we mean the ones which take part in the snap-through process; for Node #1, the *active* edges would be the top curved edge and the straight right-side edge. These two edges guide the whole snap-through process of the laminate. Whereas, there is no lag between the edge snaps for the Node #2 and Node #4 cases. It is because there are three *active* edges, the left-side straight edge, the right-side straight edge, and the top curved edge. All of them simultaneously shift to different shapes.

There is a very complex relationship between the position of the node, the critical displacement and critical

Table 2. Critical Displacement and Reaction forces for point loading at different nodes

Node	Method	Critical Force	Critical Displacement
1	Simulation	0.70N	29.0mm
	Experiment 1	0.56N	28.0mm
	Experiment 2	0.58N	26.9mm
	Experiment 3	0.75N	27.4mm
2	Simulation	4.07N	16.7mm
	Experiment 1	4.42N	17.3mm
	Experiment 2	4.06N	16.5mm
	Experiment 3	4.10N	17.1mm
3	Simulation	3.97N	11.7mm
	Experiment 1	2.85N	15.0mm
	Experiment 2	2.60N	14.8mm
	Experiment 3	2.59N	16.3mm
4	Simulation	10.56N	9.9mm
	Experiment 1	8.44N	13.6mm
	Experiment 2	6.55N	12.1mm
	Experiment 3	6.74N	10.9mm

force. Indeed, we further deduce that if the loading point (Node) is on the axis defined by node #2 and laminate center, the corresponding snap-through involves only one step. If the point load is on the axis defined by node #5 and laminate center, no snap-through will occur. Otherwise, the snap-through involves two steps.

We note another possibly insightful observation regarding the magnitude of input energy to the laminate during snap-through. The difference in strain potential energy of a laminate between its two stable states is a constant, irrespective of how it switches between these states (i.e., the deformation path it takes to switch from one state to the other). However, the total input energy via the point load for completing a snap-through is different depending on its location. When the snap-through to the other stable states occurs, any excess energy in the laminate is released as vibrational energy, and as a result, we can hear a "snap" sound. The intensity of this sound depends on the point load location. It is the least intensive when the point load is at Node #1, and most intensive at Node #4. This difference indicates that the required mechanical energy input for completing a snap-through is highest if the point force is at Node #4, and lowest at Node #1.

4. CONCLUSION

The paper aims to gain an insight into the transient behavior of bistable CFRP laminate during its snap-through process. As a case study, we selected a 2-ply [0°/90°] laminate and applied minimal boundary conditions (fixed at the center) and sufficient displacement at different locations to create a snap-through. Force-displacement (response) curves are generated for the simulated result and are validated by experimental measurements. The simulated model captured the behavior of laminate satisfactorily and showed different modes of snap-through, i.e. one-step, two-step or no snap at all. When the displacement is applied at Node #2 and Node #4, we observe one-step snapping, whereas, a two-step snapping is observed for Node #1 and Node #3. No-snap occurred for both Node #5 and Node #6.

The above study gives us a qualitative analysis of the transient snap-through response of CFRP laminates and some key observations which were unaccounted in the previous studies. These results can help in extending our knowledge about CFRP laminates, maybe also about bistable shells in general, regarding the snapping points associated with snap-through. And the characteristics of these snapping points vary with the position and the type of boundary conditions.

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