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Time-Difference-of-Arrival (TDOA)-based Distributed Target Localization by A Robotic Network

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***Abstract*—Localization and tracking of a moving target arises in many different contexts and is of particular interest in the field of robotic networks. One important class of localization schemes exploits the time-difference-of-arrival (TDOA) of a signal emitted by the target and detected by multiple sensors. In this work, we propose a fully distributed approach to TDOA-based localization and tracking of a moving target in 3D space by a group of mobile robots. We utilize a networked extended Kalman filter to estimate the target's location in a distributed manner, and guarantee successful localization under fixed and time-varying undirected communication topologies if every agent is part of a network with a minimum of 4 connected, non-coplanar agents. Since localization performance under TDOA-based schemes degrades as the target moves away from the convex hull formed by the agents, it is important for the network to track the target as it moves in space. We thus further propose a movement control strategy based on the norm of the estimation covariance matrices, with a tuning parameter to balance the trade-off between estimation performance and the total distance traveled by the robots. A numerical example involving robots with simplified three-dimensional dynamics is provided to illustrate the performance of the proposed approach.**

***Index Terms*—Distributed localization, time-difference-of-arrival (TDOA), Networked extended Kalman filter, robotic networks, structural observability**

I. Introduction

OVER the past two decades, wireless sensor networks (WSNs), often enabled by mobile robots, have received increasing attention due to their potential application to a number of diverse areas [1], such as environmental monitoring, space exploration, military applications, target tracking, and health care. Time-difference-of-arrival (TDOA)-based algorithms are widely used for precise localization of a target, examples of which include wireless ranging radar systems, cellular positioning systems [2], and acoustic telemetry in fishery research [3]. This paper considers the problem of TDOA-based localization and tracking of a moving target with a robotic network in a distributed manner. We propose a networked extended Kalman filter, and derive the conditions for successful localization under fixed and time-varying communication topologies.

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A. Related Work

Generally speaking, TDOA algorithms rely on a target emitting a signal periodically, which is detected by special receivers deployed either at fixed locations or on mobile robots. If multiple receivers detect the same signal, it is possible to infer the target's location using the differences among detection times at these receivers.

Much of the work on TDOA-based localization in the literature adopts a centralized approach, in which a reference node is chosen and the times of arrival (TOA) of the emitted signal for all other nodes in the network are subtracted from the reference node's TOA, generating TDOA measurements at fusion hub. If the propagation speed of the signal is known, the TDOA measurements can be converted to range-difference measurements, which are then used to estimate the location of the target [4], [5]. This centralized approach has a long history and is widely used in aerospace systems [6]. Geometric treatment of the problem for a stationary target was considered in [7] and [8], where the target location is inferred from the geometric relations imposed by the TDOA measurements. When the target's location changes with time, dynamic approaches are generally used for localization, in which a filter is used to estimate the target's location. Examples of these methods include utilizing an Extended Kalman Filter (EKF) in [9], or an Unscented Kalman Filter and Particle Filters in [10].

Due to power and bandwidth constraints in WSNs or robotic networks, centralized information processing is often infeasible, particularly for a large-scale and unreliable networks. Moreover, some sensors cannot transmit their measurements to the reference node due to their limited communication ranges. These drawbacks motivated the investigation of distributed strategies for TDOA-based localization. In [11], decentralized source localization in multihop networks was considered, where a connected dominating set of nodes work as the network backbone to collect the measurements, and a leader node of that set is selected to estimate the target's location, essentially acting as a centralized estimator of the target's position. The need for a common reference node is alleviated in [12], where a network of paired sensors is utilized while requiring all such pairs to be able communicate with one another. As we discuss later in Section III, this decentralized approach can be improved upon by exchanging estimate information between nodes, allowing for successful estimation without requiring all agents in the network to be connected.

In this work, structural observability is used to investigate the network topology conditions for distributed localization of a moving target. Structural analysis deals with system properties that do not depend on the numerical values of the parameters, but only on the underlying structure (zeros and non-zeros) of the system matrices [13]–[15]. It turns out that if a structural property holds for one possible choice of non-zero elements as free parameters, it is true for almost all choices of non-zero elements and, therefore, is called a *generic property* of the system. Furthermore, it can be shown that those particular (non-admissible) choices for which the generic property does not hold, lie on some algebraic variety with zero Lebesgue measure [16]. While this work is similar to [14] and [17] in that it employs structural analysis on the system matrices, there is a significant difference. In particular, the results reported in [14] and [17] treat all non-zero elements as free parameters, which in turn disguises the importance of the *number* of TDOA measurements used in this localization scheme. In Section III, we explicitly consider the role played by using more TDOA measurements, and prove that the system can be rendered generically observable when a sufficient number of TDOA measurements are used.

B. Contributions

The main contribution of this paper is the introduction of a fully distributed solution to target localization under fixed and time-varying undirected communication topologies. This approach does not require a common reference node or data fusion center, nor does it require each agent to be heavily connected to estimate the target's location on its own. Instead, we show that every agent in the network can successfully localize the target if it is part of a network that has a minimum of 4 connected, non-coplanar agents.

The TDOA-based estimation performance relies on the relative positions between individual robots and the moving target. As the target moves away from the convex hull formed by the agents, estimation performance begins to degrade and it becomes paramount to track the target as it moves through space. Rather than continuously tracking the target, we further propose an adaptive movement strategy, where the robotic network moves only when the norm of the estimation covariance matrix exceeds a certain limit, to balance the trade-off between estimation performance and distance traveled by the entire mobile network.

While preliminary versions of some results of this work were presented at conferences [18]–[20], this paper presents an integrated framework of TDOA-based target localization and tracking control for mobile robots and represents several significant improvements over [18]–[20]. First, [18] was focused on a particular distributed extended Kalman filtering-based estimation scheme with TDOA, the convergence of which relies on assumptions that are difficult to verify. In contrast, this work treats a more general class of distributed estimators and exploits structural observability to derive explicit convergence conditions in terms of the network topology. Second, [19] dealt with the formation-based target-tracking control in the two-dimensional (2D) space, while in this paper both localization and tracking control (including the simulation results) are treated in the 3D space. Compared to [20], this work offers more integrative treatment of the problem by incorporating target tracking, extension to time-varying topologies, and presentation of more extensive simulation results.

C. Organization of this paper

The remainder of this paper is organized as follows. We first present the target movement model and the TDOA measurement model in Section II. In Section III, we discuss the problem of distributed localization and derive the convergence condition in terms of network topology via structural observability analysis. Section IV is focused on tracking control to balance the estimation performance and traveled distance. Finally, we provide a numerical simulation in Section V to illustrate these results before discussing some concluding remarks and future research directions in Section VI.

II. Problem Setup

We consider a moving target in the 3D space with $p(t) = \begin{bmatrix} p^x(t) & p^y(t) & p^z(t) \end{bmatrix}'$ denoting its coordinates at time t. The target moves randomly in space according to

$$\begin{bmatrix} \dot{p}(t) \\ \ddot{p}(t) \end{bmatrix} = \begin{bmatrix} 0 & I_3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p(t) \\ \dot{p}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ I_3 \end{bmatrix} w(t), \tag{1}$$

where I_3 is the 3×3 identity matrix and $w(t) \in \mathbb{R}^3$ is the process noise, which is assumed to be zero-mean, white Gaussian noise with covariance matrix Q.

The target emits a signal periodically that gets detected by a group of N robotic agents, or nodes, at different times depending on each agent's relative distance to the target. At each detection, agent i records the signal's TOA and acquires the TOAs of all other agents that can communicate their information to agent i. These agents form the set of *neighbors* of agent i, which is denoted as $\mathcal{N}_i$. Each agent then subtracts the TOAs of its neighbors from its own TOA, generating a list of time-difference-of-arrival (TDOA) measurements. Assuming that the propagation speed of the signal is known, the measurements available for each agent are given by

$$y_i(kT) = h_i(kT) + v_i(kT), \tag{2}$$

where

$$h_i(t) = \begin{bmatrix} h_{i,1}(t) \\ \vdots \\ h_{i,|\mathcal{N}_i|}(t) \end{bmatrix}, \tag{3}$$

with

$$h_{i,j}(t) = \|p(t) - p_i(t)\| - \|p(t) - p_{i,j}(t)\|. \tag{4}$$

Here, T is the period at which the signal is emitted, $v_i(t) \in \mathbb{R}^{|\mathcal{N}_i|}$ is the measurement noise, assumed to be zero-mean, white Gaussian noise with covariance matrix R_i, $p_i(t)$ is the position of agent i, and $p_{i,j}(t)$ is the position of the j-th neighbor of agent i.

Denoting the target's state as $x(t) = \begin{bmatrix} p'(t) & \dot{p}'(t) \end{bmatrix}'$, and discretizing the model in (1) with sampling time T, with slight abuse of notation, we can write the discrete-time model of the target as

$$x(k+1) = Ax(k) + Bw(k), \tag{5}$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 & T & 0 & 0 \\ 0 & 1 & 0 & 0 & T & 0 \\ 0 & 0 & 1 & 0 & 0 & T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{T^2}{2} & 0 & 0 \\ 0 & \frac{T^2}{2} & 0 \\ 0 & 0 & \frac{T^2}{2} \\ T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix}. \tag{6}$$

The time-varying measurement matrix $H_i(k)$ can be obtained from (2), where

$$H_i(k) = \begin{bmatrix} \frac{\partial h_{i,1}(k)}{\partial p^x(k)} & \frac{\partial h_{i,1}(k)}{\partial p^y(k)} & \frac{\partial h_{i,1}(k)}{\partial p^z(k)} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h_{i,|\mathcal{N}_i|}(k)}{\partial p^x(k)} & \frac{\partial h_{i,|\mathcal{N}_i|}(k)}{\partial p^y(k)} & \frac{\partial h_{i,|\mathcal{N}_i|}(k)}{\partial p^z(k)} & 0 & 0 & 0 \end{bmatrix}, \tag{7}$$

and

$$\frac{\partial h_{i,j}(k)}{\partial p^x(k)} = \frac{p^x(k) - p_i^x(k)}{\|p(k) - p_i(k)\|} - \frac{p^x(k) - p_{i,j}^x(k)}{\|p(k) - p_{i,j}(k)\|}, \tag{8}$$

$$\frac{\partial h_{i,j}(k)}{\partial p^y(k)} = \frac{p^y(k) - p_i^y(k)}{\|p(k) - p_i(k)\|} - \frac{p^y(k) - p_{i,j}^y(k)}{\|p(k) - p_{i,j}(k)\|}, \tag{9}$$

$$\frac{\partial h_{i,j}(k)}{\partial p^z(k)} = \frac{p^z(k) - p_i^z(k)}{\|p(k) - p_i(k)\|} - \frac{p^z(k) - p_{i,j}^z(k)}{\|p(k) - p_{i,j}(k)\|}. \tag{10}$$

III. Distributed Estimation

In this section, we look into the problem of distributed localization of a moving target using TDOA measurements. The goal is for every agent to estimate the target's position without requiring a central node to collect all measurements and propagate an estimate to all agents in the network. To that end, we first discuss a decentralized estimation scheme, where structural observability analysis is conducted to derive the minimum number of TDOA measurements required for an agent to estimate the target state on its own. We will then discuss the distributed estimation scheme where agents exchange estimate information, and present the necessary and sufficient condition in terms of network topology for achieving stable estimates.

A. Decentralized Estimation

In this approach, each agent runs its own filter using its own TDOA measurements. Here, agents exchange only their locations and TOA values to generate TDOA measurements without exchanging any other pieces of information. Each node implements an extended Kalman filter (EKF) to estimate the target's state

$$\hat{x}_i(k|k-1) = A\hat{x}_i(k-1|k-1), \tag{11}$$

$$\hat{x}_i(k|k) = \hat{x}_i(k|k-1) + K_i(k)\big[y_i(k) - h_i(\hat{x}_i(k|k-1))\big], \tag{12}$$

where $\hat{x}_i(k|j)$ is the i-th node's estimate of the state at time k after the j-th measurement has been processed, and $K_i(k)$ is its filtering gain, which is computed according to

$$K_i(k) = P_i(k|k-1)H_i(k)' \times \big[H_i(k)P_i(k|k-1)H_i(k)' + R_i\big]^{-1}, \tag{13}$$

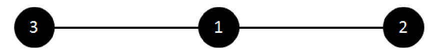

Fig. 1: Network of 3 agents monitoring the target with agent 1 as the reference node.

and

$$P_i(k|k-1) = AP_i(k-1|k-1)A' + BQB', \tag{14}$$

$$P_i(k|k) = \big[I - K_i(k)H_i(k)\big]P_i(k|k-1)\big[I - K_i(k)H_i(k)\big]' + K_i(k)R_iK_i(k)', \tag{15}$$

where $P_i(k|j)$ is the i-th agent's error covariance matrix at time k after the j-th measurement has been processed.

It is well-known (see [21] and [22]) that the estimation error for agent i under this scheme, which propagates as follows,

$$\tilde{x}_i(k+1) = A(I - K_i(k)H_i(k))\tilde{x}_i(k) + \eta_i(k), \tag{16}$$

is stable if and only if the pair $(A, H_i(k))$ is observable, where $\tilde{x}_i(k) \triangleq x(k) - \hat{x}_i(k|k)$ is the estimation error for agent i and the vector $\eta_i(k)$ collects the terms independent of $\tilde{x}_i(k)$. In the following, we will show that the pair $(A, H_i(k))$ is unobservable when agent i has less than 3 TDOA measurements. To avoid clutter, we will consider only agent 1 of the network, and drop the i subscript from the following analysis.

As discussed earlier, if a structural property is true for one *admissible* choice of non-zero elements, it is true for *almost all* choices of non-zero elements. Additionally, it can be shown that the choices of parameters for which the generic property does not hold, lie on a hypersurface (see Definition 1) in the free parameter space with zero Lebesgue measure [16]. Due to the fixed structure of our system matrix A in (6) and the time-varying measurement matrix $H(k)$ in (7), it is beneficial to utilize structural analysis when examining the observability of our system. In the following, we employ a structural approach to establish the minimum number of TDOA measurements needed to render the process generically observable.

Definition 1: Let $f = f(x_1, \ldots, x_n)$ be a polynomial in the n variables $x_1, \ldots, x_n$ with coefficients in $\mathbb{R}$. Then the point $\bar{x} = (\bar{x}_1, \ldots, \bar{x}_n)$ in $\mathbb{R}^n$ is called a zero of f if $f(\bar{x}_1, \ldots, \bar{x}_n) = 0$. The set of zeros of f is called the locus of f. A subset V of $\mathbb{R}^n$ is called a hypersurface in $\mathbb{R}^n$ if it is the locus of a nonconstant polynomial.

First, we consider the case where agent 1 only has two neighbors and, therefore, only two TDOA measurements as shown in Figure 1. The measurement matrix $H(k)$, in this case, admits the following structure

$$H_\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 \\ \lambda_4 & \lambda_5 & \lambda_6 & 0 & 0 & 0 \end{bmatrix}. \tag{17}$$

The system is said to be generically observable if the pair (A, H_λ) is observable for almost all values of $T, \lambda_1, \ldots, \lambda_6$. In other words, the system is generically observable if and only if the observability matrix $\mathcal{O}$ is full rank for almost all values of $T, \lambda_1, \ldots, \lambda_6$, where

$$\mathcal{O} = \big[H_\lambda' \quad (H_\lambda A)' \quad (H_\lambda A^2)' \quad \cdots \quad (H_\lambda A^5)'\big]'. \tag{18}$$

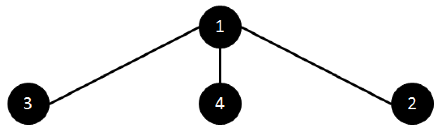

Fig. 2: Network of 4 agents monitoring the target with agent 1 as the reference node.

It is well known that rank$(\mathcal{O}) < 6$ if and only if all 6×6 minors of $\mathcal{O}$ are zero [16]. We can easily verify that all 6×6 minors of $\mathcal{O}$ in (18) are zero, regardless of the values of $T, \lambda_1, \ldots, \lambda_6$. This implies that the process in (5) with measurement matrix (7) is unobservable when the node has two or less TDOA measurements[1].

Now we consider the case where agent 1 has three neighbors and, therefore, three TDOA measurements as shown in Figure 2. The measurement matrix $H(k)$, in this case, admits the following structure

$$H_\lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 \\ \lambda_4 & \lambda_5 & \lambda_6 & 0 & 0 & 0 \\ \lambda_7 & \lambda_8 & \lambda_9 & 0 & 0 & 0 \end{bmatrix}. \tag{19}$$

Checking all 6×6 minors of $\mathcal{O}$, we observe that some minors of $\mathcal{O}$ are not identically zero and are all of the form

$$\begin{aligned} \alpha T^3 (\lambda_3 (\lambda_5\lambda_7 - \lambda_4\lambda_8) + \lambda_2 (\lambda_4\lambda_9 - \lambda_6\lambda_7) \\ + \lambda_1 (\lambda_6\lambda_8 - \lambda_5\lambda_9))^2, \end{aligned} \tag{20}$$

for some $\alpha \in \mathbb{R}$. Therefore, we conclude that rank$(\mathcal{O}) = 6$ for almost all values of T and $\lambda_1, \ldots, \lambda_9$, and that the pair $(A, H(k))$ is generically observable if the agent has a minimum of 3 TDOA measurements[2]. Furthermore, the set of values that render the pair unobservable is a hypersurface in the free parameter space where the expression in (20) is zero. Interestingly, this means that the process is generically observable except when:

- The sampling time used for discretization of the system in (5) is 0, or
- All of the points that satisfy

$$\det\left(\begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_4 & \lambda_5 & \lambda_6 \\ \lambda_7 & \lambda_8 & \lambda_9 \end{bmatrix}\right) = 0,$$

  i.e., when the 3 TDOA measurements are linearly dependent, and the 4 agents are coplanar.

For all agents in the network to be able to estimate the target's position under this decentralized scheme, we would require that all such pairs $(A, H_1(k))$,$(A, H_2(k))$,…,$(A, H_N(k))$ to be observable, i.e., we would require the pair $(I_N \otimes A, D_H)$ to be observable, where $\otimes$ denotes the Kronecker product, and

$$D_H(k) \triangleq \begin{bmatrix} H_1(k) & & 0 \\ & \ddots & \\ 0 & & H_N(k) \end{bmatrix}. \tag{21}$$

[1]If the node has only one TDOA measurement, and therefore only one neighbor, then there is only one 6×6 minor of $\mathcal{O}$ and it is det$(\mathcal{O})$.

[2]If the node has more than 3 TDOA measurements, it can be verified that rank$(\mathcal{O}) = 6$ if rank$(H_\lambda) \geq 3$.

Let $\underline{\hat{x}}(k|k) = \left[\hat{x}_1(k|k)' \;\; \ldots \;\; \hat{x}_N(k|k)'\right]'$ denote the network-wide estimate of the network-wide state $\underline{x}(k) = \left[x(k)' \;\; \ldots \;\; x(k)'\right]' = 1_N \otimes x(k)$, where $1_N \in \mathbb{R}^N$ is the column vector whose entries are all 1. The dynamics of this network-wide state can be derived as follows

$$\begin{aligned} \underline{x}(k+1) =& 1_N \otimes (Ax(k) + Bw(k)) \\ =& (I_N \otimes A) (1_N \otimes x(k)) \\ &+ (I_N \otimes B)(1_N \otimes w(k)) \\ =& (I_N \otimes A) \underline{x}(k) + (I_N \otimes B)\underline{w}(k), \end{aligned} \tag{22}$$

with $\underline{w}(k) \triangleq 1_N \otimes w(k)$ representing the network-wide process noise. Denoting the i-th agent's estimation error by $\tilde{x}_i(k) \triangleq x(k) - \hat{x}_i(k|k)$, and the network-wide estimation error $\underline{\tilde{x}}(k) \triangleq \left[\tilde{x}_1(k)' \;\; \ldots \;\; \tilde{x}_N(k)'\right]'$, the dynamics of $\underline{\tilde{x}}(k)$ are given by

$$\begin{aligned} \underline{\tilde{x}}(k+1) = (I_N \otimes A) \left(I_{6N} - K(k) D_H(k)\right) \underline{\tilde{x}}(k) \\ + \underline{\eta}(k), \end{aligned} \tag{23}$$

where $K(k)$ is a block-diagonal matrix of the filter gains $K_1(k) \ldots K_N(k)$, and the vector $\underline{\eta}(k)$ collects the terms independent of $\underline{\tilde{x}}(k)$. This network-wide estimation error can be stabilized if the pair $(I_N \otimes A, D_H(k))$ is generically observable, where each agent needs to have a sufficient number of neighbors to estimate the process using only its own TDOA measurements.

Under this formulation, each agent can estimate the target's location when it has a minimum of 3 TDOA measurements, corresponding to each agent having a minimum of 3 neighbors. This decentralized approach requires each agent to be heavily connected such that the target system is observable using each agent's own measurements. Next, we discuss how the number of required communication links can be reduced, and argue that it is possible to estimate the target's location without the need for heavily connecting the agents.

B. Distributed Estimation

Consider the dynamical system in (22), and noting that for a stochastic matrix $W \in \mathbb{R}^{N \times N}$, $W 1_N = 1_N$, we can rewrite (22) as

$$\begin{aligned} \underline{x}(k+1) =& 1_N \otimes (Ax(k) + Bw(k)) \\ =& W 1_N \otimes Ax(k) + 1_N \otimes Bw(k) \\ =& (W \otimes A) \underline{x}(k) + (I_N \otimes B)\underline{w}(k). \end{aligned} \tag{24}$$

For this modified dynamical system in (24), a centralized filter can be designed with estimation error dynamics that can be expressed as

$$\begin{aligned} \underline{\tilde{x}}(k+1) = (W \otimes A) \left(I_{6N} - K_c(k) D_H(k)\right) \underline{\tilde{x}}(k) \\ + \underline{\eta}(k), \end{aligned} \tag{25}$$

where $K_c(k)$ is the filter gain, which can stabilize the error dynamics if the pair $(W \otimes A, D_H(k))$ is generically observable. In the following, we will show that it is possible to obtain a network-wide estimation error with dynamics similar to (25) by averaging the estimates among neighboring agents.

Let $W \in \mathbb{R}^{N \times N}$ be a stochastic matrix with entries $w_{ij} > 0$ if $i = j$ or if agents i and j can exchange information;

otherwise $w_{ij} = 0$. We assume here that every agent has access to its own information (i.e., $w_{ii} > 0$), and that the communication links are bidirectional, namely, if agent j can send its information to agent i, then the reverse is also true, $w_{ij}, w_{ji} > 0$. Every agent in the network implements a filtering scheme similar to the decentralized case, but followed by updating its estimate by averaging the estimates from neighbors and itself. The filter implemented by each agent in the network is then given by

$$\begin{aligned} \hat{x}_i(k|k-1) &= A\bar{x}_i(k-1), && (26)\\ \hat{x}_i(k|k) &= \hat{x}_i(k|k-1) \\ &\quad + K_i(k)\big[y_i(k) - h_i(\hat{x}_i(k|k-1))\big], && (27)\\ \bar{x}_i(k) &= \sum_{j=1}^{N} w_{ij}\hat{x}_j(k|k). && (28) \end{aligned}$$

Denoting $\hat{x}_i(k|k-1)$ by $\hat{x}_i(k)$, and substituting (27) and (28) into (26), we can express a one-step formulation of agent i's estimate can as

$$\hat{x}_i(k+1) = \sum_{j=1}^{N} w_{ij}\Big[A\hat{x}_j(k) + AK_j(k)\Big(y_j(k) - h_j(\hat{x}_j(k))\Big)\Big]. \tag{29}$$

The i-th agent's estimation error is then given by

$$\tilde{x}_i(k+1) = \sum_{j=1}^{N} w_{ij}\left[A(I - K_j(k)H_j(k))\tilde{x}_j(k) + \eta_j(k)\right]. \tag{30}$$

Denoting the network-wide estimation error by $\underline{\tilde{x}}(k) \triangleq \big[\tilde{x}_1(k)' \quad \ldots \quad \tilde{x}_N(k)'\big]'$, then

$$\underline{\tilde{x}}(k+1) = (W \otimes A)\left(I_{6N} - K(k)D_H(k)\right)\underline{\tilde{x}}(k) + \underline{\eta}(k), \tag{31}$$

which is similar to (25), except that here the gain matrix $K(k)$ is restricted to be block-diagonal. As explained in [14], computing such a constrained gain is possible via an iterative cone-complementary optimization algorithm; see [23] and [24] for details. In [25], the authors derived a suboptimal filtering gain inspired by the Markovian jump linear system filtering problem, where

$$K_i(k) = P_i(k|k-1)H_i(k)' \times \left[H_i(k)P_i(k|k-1)H_i(k)' + R_i\right]^{-1}, \tag{32}$$

and

$$\begin{aligned} P_i(k|k-1) &= A\bar{P}_i(k-1)A' + BQB', && (33)\\ P_i(k|k) &= \big[I_6 - K_i(k)H_i(k)\big]P_i(k|k-1) \\ &\quad \times \big[I_6 - K_i(k)H_i(k)\big]' \\ &\quad + K_i(k)R_iK_i(k)', && (34)\\ \bar{P}_i(k) &= \sum_{j=1}^{N} w_{ij}P_j(k|k). && (35) \end{aligned}$$

Therefore, to ensure the convergence of the networked filter, one has to ensure that the networked system is observable [14]. To that end, we investigate the conditions on the matrix W and, therefore, the topology of the undirected communication graph among agents, that would render the pair $(W \otimes A, D_H)$ observable, and therefore ensure the convergence of the networked filter. We first note the following property regarding the powers of the matrix W from [26].

Lemma 1: Let $[W^l]_{ij}$ denote the (i,j) element of the matrix W^l, where W is the stochastic matrix representing the communication topology with $w_{ii} > 0$. Then, $[W^l]_{ij} > 0$ if there is a path between agents i and j of length less than or equal to l; otherwise $[W^l]_{ij} = 0$.

The pair $(W \otimes A, D_H)$ is observable if and only if $\mathrm{rank}(\mathcal{O}) = 6N$. Here,

$$\mathcal{O} = \begin{bmatrix} D_H(k) \\ D_H(k)(W \otimes A) \\ D_H(k)(W \otimes A)^2 \\ \vdots \\ D_H(k)(W \otimes A)^p \end{bmatrix} = \begin{bmatrix} D_H(k) \\ D_H(k)(W \otimes A) \\ D_H(k)(W^2 \otimes A^2) \\ \vdots \\ D_H(k)(W^p \otimes A^p), \end{bmatrix} \tag{36}$$

where $p = 6N - 1$. Equivalently, denoting $\mathcal{O}_i$ as the block column representing agent i's subsystem, we can write $\mathcal{O} = \left[\ \mathcal{O}_1 \mid \ldots \mid \mathcal{O}_N\ \right]$. From the structure of D_H, it is easy to see that $\mathrm{rank}(\mathcal{O}) = \sum_{i=1}^{N}\mathrm{rank}(\mathcal{O}_i)$.

We are now ready to present the main results in this paper.

Theorem 1: For a network of agents with time-invariant and undirected topology, the system under the proposed distributed TDOA-based localization is observable if and only if every agent is part of a (sub)network that has at least 4 connected, non-coplanar agents.

Proof: Sufficiency: We consider the case where agent i is part of a network that has only 4 connected agents, and note that the following results can be easily extended to the cases where the network has more agents. Since it is always possible to renumber the agents, we will only consider the subsystem corresponding to agent 1, and write

$$\mathcal{O}_1 = \begin{bmatrix} H_1(k) \\ \vdots \\ \left[W^{23}\right]_{41} H_4(k)A^{23} \end{bmatrix}. \tag{37}$$

This agent can be connected to the network in 4 possible ways that are shown in Fig. 3[3]. For the ease of presentation, in the following, we examine *Case (d)*, where the agent has only one neighbor, and show that $\mathrm{rank}(\mathcal{O}_1) = 6$. The other three cases are discussed in Appendix A.

[3]The graphs shown in Fig. 3 represent the minimum number of links required for each graph to be connected. It is possible to add more edges among agents in these graphs and adding more links will only help in terms of observability.

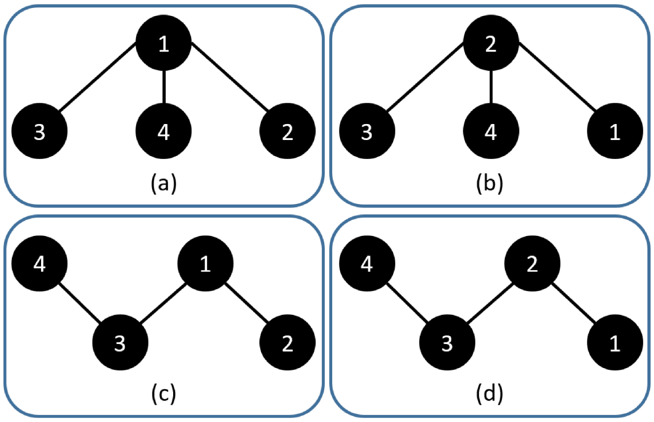

Fig. 3: The 4 possible configurations for node 1 to be connected to the network using the minimum number of edges.

Case (d): The structure of $H_i(k)$ for $i = 1, \dots, 4$ is as

$$H_1(\lambda) = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 \end{bmatrix},$$
$$H_2(\lambda) = \begin{bmatrix} -\lambda_1 & -\lambda_2 & -\lambda_3 & 0 & 0 & 0 \\ \lambda_4 & \lambda_5 & \lambda_6 & 0 & 0 & 0 \end{bmatrix},$$
$$H_3(\lambda) = \begin{bmatrix} -\lambda_4 & -\lambda_5 & -\lambda_6 & 0 & 0 & 0 \\ \lambda_7 & \lambda_8 & \lambda_9 & 0 & 0 & 0 \end{bmatrix},$$
$$H_4(\lambda) = \begin{bmatrix} -\lambda_7 & -\lambda_8 & -\lambda_9 & 0 & 0 & 0 \end{bmatrix}.$$

Recalling that

$$\text{rank}\left(\begin{bmatrix} C \\ \vdots \\ CA^{n-1} \end{bmatrix}\right) = \text{rank}\left(\begin{bmatrix} CA^k \\ \vdots \\ CA^{k+n-1} \end{bmatrix}\right),$$

it can be shown that the pair

$$\left(A, \begin{bmatrix} H_1(\lambda) \\ H_3(\lambda) \\ H_4(\lambda) \end{bmatrix}\right)$$

is generically observable, implying that

$$\text{rank}\left(\begin{bmatrix} H_1(\lambda) \\ H_3(\lambda) \\ H_4(\lambda) \\ \vdots \\ H_1(\lambda)A^6 \\ H_3(\lambda)A^6 \\ H_4(\lambda)A^6 \end{bmatrix}\right) = 6.$$

for almost all values of T and λ. Additionally, $\text{rank}(\mathcal{O}_1) = 6$ for almost all values of T and λ. Specifically, this generic property holds for all values of T and λ, except for when the 4 agents are coplanar. Since it is always possible to renumber the agents, then if every agent is connected to the network that has a minimum of 4 connected, non-coplanar agents, then the pair $(W \otimes A, D_H(k))$ is generically observable. The proof for the other cases follows a similar approach, and is presented in Appendix A.

Necessity: If an agent is disconnected, or is part of a network that has less than 4 agents, then there are not enough pieces of information to estimate the target's position centrally, let alone distributively, and the error dynamics in (25) – and therefore (31) – cannot be stabilized. ∎

For a time-varying, undirected communication graph, Theorem 1 can be extended to offer a scalable approach that is somewhat robust to communication link dropout.

Corollary 1: For a time-varying undirected network, if every agent remains part of a network that has a minimum of 4 connected, non-coplanar agents then the networked system under the proposed distributed estimation scheme is observable.

Proof: As the network connectivity changes, if every agent remains part of a network that has a minimum of 4 connected, non-coplanar agents, then it can be shown that $\text{rank}(\mathcal{O}_i) = 6$ for every agent in the network. This, in turn, ensures that the networked system is observable by Theorem 1. ∎

IV. Target Tracking with Coordinated Robotic Network Movement

In this section we look into the second part of the problem. We begin the discussion by investigating the agent formation needed for optimum localization of the target.

A. Optimal Formation

For a network of N agents, there is a total number of $N(N-1)/2$ possible agent pairs. Let $\mathcal{I}_0 = \{(i,j) | 1 \le j < i \le N\}$ denote the set of all agent pairs and $\mathcal{I} = \{(i,j) | j < i, w_{ij} > 0\}$, a subset of $\mathcal{I}_0$, represent the set of agent pairs used for estimation. From the definition of w_{ij}, it is clear that $\mathcal{I}$ depends on the communication topology among agents.

The Cramer-Rao bound (CRB) is a lower bound for the covariance matrix of unbiased estimators and is given by the inverse of the Fisher information matrix [27]. With $y \triangleq \begin{bmatrix} y_1' & \dots & y_N' \end{bmatrix}$, the Fisher information matrix is given by

$$J = E\left[\left(\frac{\partial}{\partial p} \ln f(y|p)\right)\left(\frac{\partial}{\partial p} \ln f(y|p)\right)'\right], \quad (38)$$

where $f(y|p)$ is the probability density function (PDF) of y given p and $E[\cdot]$ denotes the expectation on y. Note that the measurement noise v_i is assumed to be Gaussian with zero mean. Assuming that $R_i = \sigma_v^2 I_{|\mathcal{N}_i|}$, one arrives at a 3×3 Fisher information matrix by Chan and Ho[4] [28]

$$J = \frac{1}{\sigma_v^2} G G', \quad (39)$$

where

$$G = \begin{bmatrix} g_{ij} & \dots \end{bmatrix}_{(i,j) \in \mathcal{I}}, \quad (40)$$
$$g_{ij} = g_i - g_j, \quad (41)$$
$$g_i = \frac{p - p_i}{\|p - p_i\|}. \quad (42)$$

Clearly, g_i is a unit-length vector pointing from agent i to the target, and the matrix G depends on the target position, agents' positions, and the set $\mathcal{I}$ of agent pairs used for localization.

[4]Here, the propagation speed of the transmitted signal is normalized to one.

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Since the CRB is a square matrix, we seek a formation of agents that minimizes of the trace of the CRB,

$$\mathrm{tr}(J^{-1}) = \sigma_v^2 \mathrm{tr}\left([GG']^{-1}\right), \tag{43}$$

which is a lower bound for the sum of variances of unbiased estimators for all elements of the target's position p. In order to obtain the lowest possible CRB, and therefore a better performance by the estimator, all agent pairs in the network need to be considered, and $\mathcal{I} = \mathcal{I}_0$, requiring a fully-connected network. The necessary and sufficient conditions to achieve a minimum CRB are presented in [27]. It is known that the optimum 2D formation is that of a *uniform angular array* where all agents are equally distributed around a circle of radius R, for some arbitrary $R > 0$, centered around the target [27]. Similarly, in the 3D case, the optimal configurations for a complete network are the 3D equivalent of uniform angular arrays, known as the Platonic solids.

Remark 1: The biggest drawback in achieving optimal estimation is that it requires the complete connectivity of the corresponding agent network. This requirement can be relaxed if one is interested in sub-optimal performance. We note that since the performance depends on the shape of the formation, one can search the shape space of the agent formation space to arrive at topology-specific shapes that minimize (at least locally), the associated CRB. In this work, we specify the formation to be that of a Platonic solid, even when the network is not completely connected.

The goal of tracking the moving target by the network is to ensure adequate localization using the distributed estimator. However, it is not necessary to continuously move the robots with the target. In order to balance the trade-off between the cumulative distance traveled and the estimation performance, each agent can utilize the norm of the error covariance matrix, $\|P_i\|$, and only apply the tracking control if the $\|P_i\| > \underline{b}$ for some constant $\underline{b} > 0$. Note that $\underline{b}$ is a design parameter a user can set depending on the specific problem. In particular, this parameter allows the user to mediate between two extreme cases, 1) minimizing energy and remaining stationary ($\underline{b} = \infty$), and (2) maximizing estimation performance by continuously tracking the moving target ($\underline{b} = 0$). While this switching control strategy is not guaranteed to drive each agent to its corresponding desired location, it can greatly reduce the total distance traveled by all agents, as is illustrated in the following section.

In order to guarantee that all agents move together, or keep still at the same time, we exploit the average-based consensus scheme in (28) and (35) between TDOA measurements. Namely, every $\tau << T$ seconds, each robotic agent i in the network communicates and averages its estimate and covariance matrix with its neighbors $\mathcal{N}_i$. This ensures that all agents agree on their covariance matrices and coordinate their movements[5].

[5]Exchanging information on a much quicker time scale is only needed to ensure that the agents coordinate their movement, and is not a requirement for the distributed estimator.

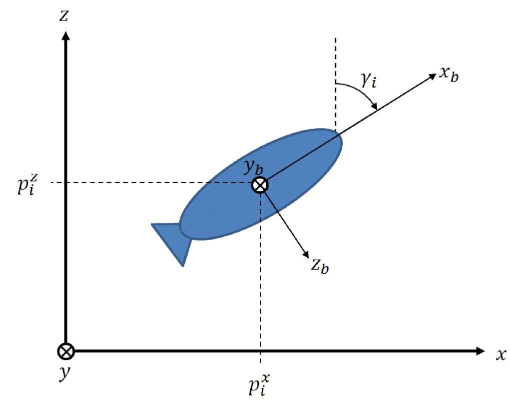

Fig. 4: Illustration of the simplified model of a propelled underwater robot with steering control, with a view on the sagittal plane. The robot has similar yaw control in the horizontal plane.

V. SIMULATION RESULTS

In this work, we use each robot's estimate of the target's state to arrive at the desired position of each robot. To facilitate the simulation, we consider a simplified model of a propelled underwater vehicle with steering control in yaw and pitch. The model used for each agent in the network is given by:

$$\begin{bmatrix} \dot{p}_i^x \\ \dot{p}_i^y \\ \dot{p}_i^z \\ \dot{v}_i \\ \dot{\phi}_i \\ \dot{\gamma}_i \\ \dot{\omega}_{i,\phi} \\ \dot{\omega}_{i,\gamma} \end{bmatrix} = \begin{bmatrix} v_i \cos(\phi_i)\cos(\gamma_i) \\ v_i \sin(\phi_i)\cos(\gamma_i) \\ v_i \sin(\gamma_i) \\ a_i \\ \omega_{i,\phi} \\ \omega_{i,\gamma} \\ u_{i,2} \\ u_{i,3} \end{bmatrix} \tag{44}$$

where $p_i = \begin{bmatrix} p_i^x & p_i^y & p_i^z \end{bmatrix}$ denotes the position of agent i in the inertial frame, v_i represents the agent's linear velocity along its body-fixed x axis, which stretches from the robot center to the front of the robot. The angle γ_i is the angle between the inertial z axis and agent i's body-fixed x axis, while ϕ_i is the robot's yaw angle measured as the angle between the inertial x axis and the projection of the body-fixed x-axis onto the inertial $x-y$ plane. The terms $\omega_{i,\phi}$ and $\omega_{i,\gamma}$ are the rates of change of the angles ϕ_i and γ_i, while $a_i, u_{i,2}$, and $u_{i,3}$ represent the control inputs for the robot. Figure 4 illustrates the the robot position and relevant angle in the sagittal $(x-z)$ plane.

The estimated position of the moving target is fed into a feedback linearization control strategy to drive each agent to its desired position. The desired state for agent i can be obtained from its estimate of target's state:

$$p_i^\star = \quad \hat{p}_i + d_i \tag{45}$$

$$v_i^\star = \quad \|\hat{\dot{p}}_i\| \tag{46}$$

$$\phi_i^\star = \quad \tan^{-1}\left(\frac{\hat{\dot{p}}_i^y}{\hat{\dot{p}}_i^x}\right) \tag{47}$$

$$\gamma_i^\star = \quad \sin^{-1}\left(\frac{\hat{\dot{p}}_i^z}{\|\hat{\dot{p}}_i\|}\right) \tag{48}$$

$$\tag{49}$$

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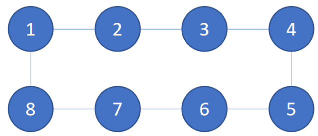

Fig. 5: The fixed communication topology among a network of 8 agents. It is clear that no agent in the network has a sufficient number of neighbors (and therefore TDOA measurements) to estimate the process on its own.

where d_i is a constant vector that defines the position of agent i relative to the target.

The proposed approach was simulated for a network of $N = 8$ agents with initial positions:

$$
\begin{aligned}
p_1 &= \begin{bmatrix} -7 & 3 & 5 \end{bmatrix}' \\
p_2 &= \begin{bmatrix} 1 & -2 & -5 \end{bmatrix}' \\
p_3 &= \begin{bmatrix} 5 & -1 & 4 \end{bmatrix}' \\
p_4 &= \begin{bmatrix} 7 & 4 & -2 \end{bmatrix}' \\
p_5 &= \begin{bmatrix} -5 & 2 & -2 \end{bmatrix}' \\
p_6 &= \begin{bmatrix} 0 & -6 & 7 \end{bmatrix}' \\
p_7 &= \begin{bmatrix} 7 & -5 & 1 \end{bmatrix}' \\
p_8 &= \begin{bmatrix} -1 & 2 & -1 \end{bmatrix}'
\end{aligned}
$$

as shown in Figure 6. The measurement noise covariance matrix for each agent was set to $R_i = I_{|\mathcal{N}_i|}$. Estimators of each agent were initialized with $\hat{x}_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}'$ and $P_i = 100 I_6$. The initial position of the moving target was set to $\begin{bmatrix} 17 & -5 & -15 \end{bmatrix}'$, with a process noise covariance matrix $Q = 0.01 I_3$. The emitted signal from the source was assumed to have a period of $T = 1$ s, while the average-based consensus was carried every $\tau = 0.1$ s. The desired formation for all agents was set to that of a cube, i.e. a Platonic solid with 8 vertices, with a the radius of the underlying sphere set to 10.

First we utilized the fixed communication topology shown in Fig. 5. Figs. 7(a) and 7(b) show the final network configuration for the two extreme cases $\underline{b} = 0$ and $\underline{b} = \infty$, respectively. When $\underline{b} = 0$, the network prioritizes minimizing the covariance norms over the total traveled distance, causing the network to continuously track the moving target with its desired formation. On the other hand, when $\underline{b} = \infty$, the robots attempt to hold their positions to minimize the overall traveled distance without any consideration to the covariance norms. Figs. 8 and 9 show the estimation performance (estimation error and covariance in (31) and (35), respectively) for different values of $\underline{b}$, while Fig. 10 depicts the corresponding cumulative distance traveled by all agents in the network. From these figures, it can be seen that as $\underline{b}$ increases, the overall traveled distance decreases while the covariance norms increase, causing the network to remain stationary more often in exchange for less reliable estimation, as is also captured in Fig. 9.

For the case of a time-varying communication network, we implemented a *distance* rule where a communication link exists between any two agents if the distance between

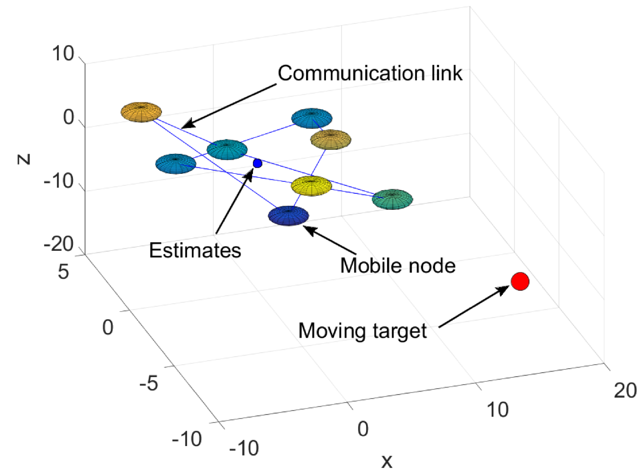

Fig. 6: Initial setup for simulation environment with a fixed communication topology. The big circle represents the moving target, while the small circles (overlapping one another at the origin) represent the initial estimates of the target's location for each robot. The ellipsoids represent the mobile robots, while the thin lines connecting them represent the communication links.

them is strictly less than 21. To ensure that no agent has a sufficient number of measurements to estimate the process on its own, we deformed the desired tracking formation so that the distance between some agents would be large enough. Fig. 11 shows the initial and final setup as the agents constantly track the target position (i.e., $\underline{b} = 0$). Initially the communication graph is fully connected as all agents are close enough. As the distance between the robots increases, some of the communication links are lost. Finally, it is clear from Fig. 12 that successful estimation is achieved even as the communication graph changes and some of the communication links are dropped.

VI. Conclusion

In this paper, we investigated the problem of distributed localization of a moving target by a network of agents using TDOA measurements from first observability principles. Structural observability principles were utilized to highlight the importance of having enough measurements to accurately localize the moving target. We showed that a decentralized approach without exchanging estimates requires every agent in the network to be heavily connected to localize the target on its own. On the other hand, under the distributed approach, we showed that when the agents exchange information and fuse their estimates, then it is indeed possible for every agent in the network to estimate the target's position without needing to be heavily connected. Specifically, we showed that if every agent is connected to a network with a minimum of 4 connected, non-coplanar agents, then the process is rendered generically observable and each agent can accurately localize the target. This work can be extended to time-varying communication topologies that are commonly encountered when the communi-

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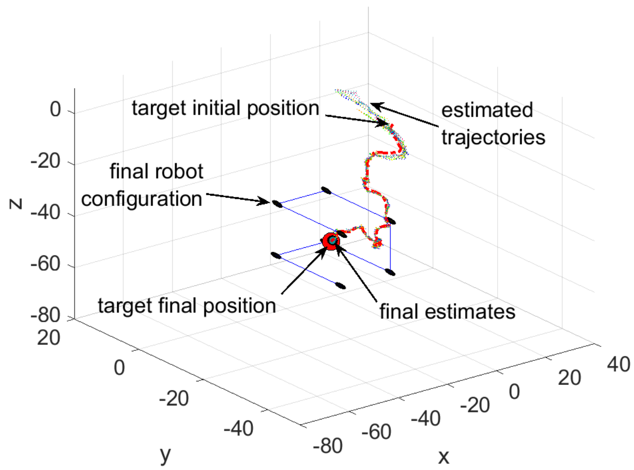
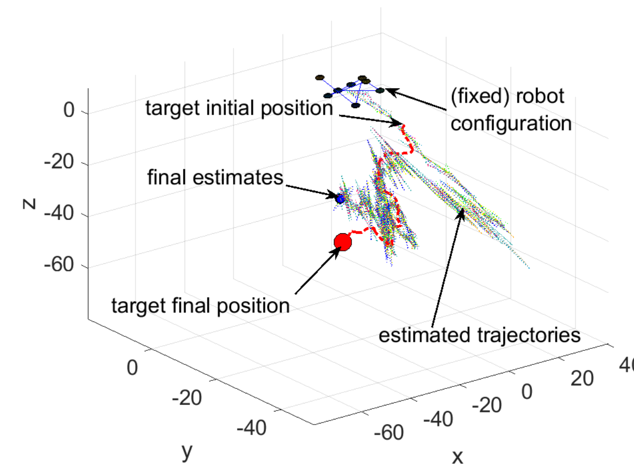
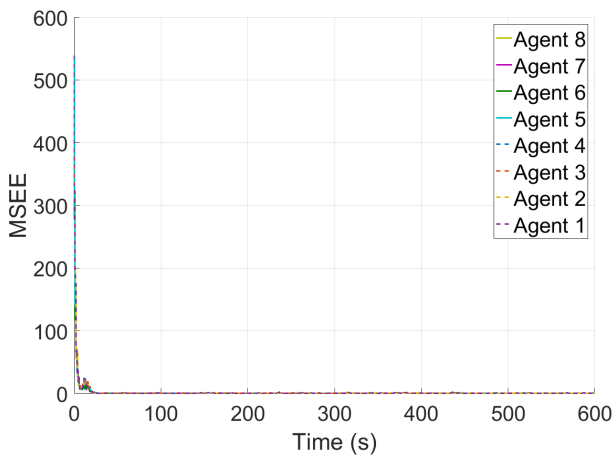


(a)

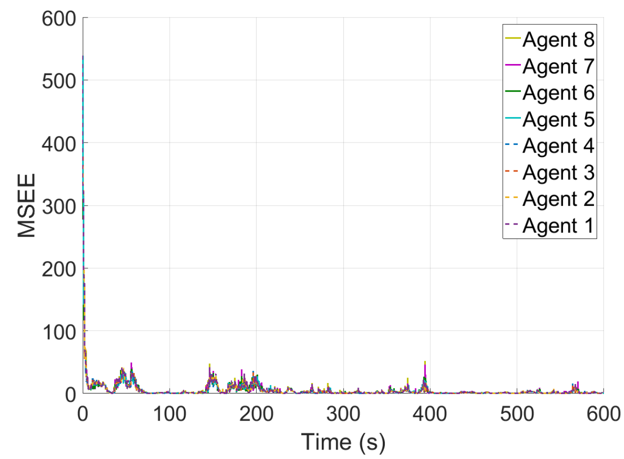


(b)

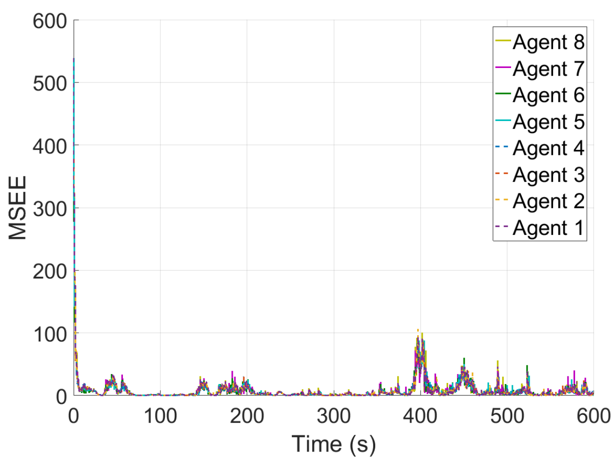
Fig. 7: Simulation results of the robots for the two extreme values of $\underline{b}$ under a fixed communication topology where (a) the robots are constantly moving with target ($\underline{b} = 0$), and (b) the robots are staying put ($\underline{b} = \infty$).



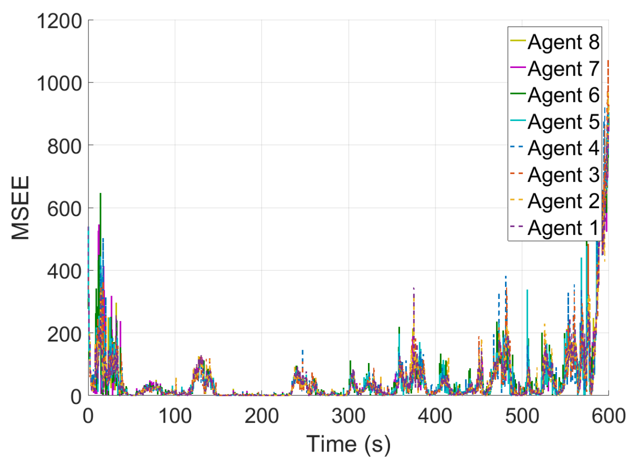
(a) $\underline{b} = 0$.



(b) $\underline{b} = 30$



(c) $\underline{b} = 100$



(d) $\underline{b} = \infty$

Fig. 8: Comparison of the Mean Squared Estimation Error (MSEE) of for each agent in the network for different values of $\underline{b}$ under a fixed communication topology.

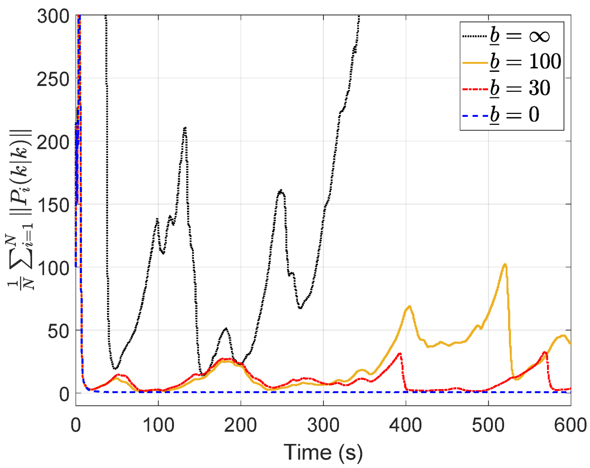


Fig. 9: Comparison of average covariance norms for different $\underline{b}$ values under a fixed communication topology.

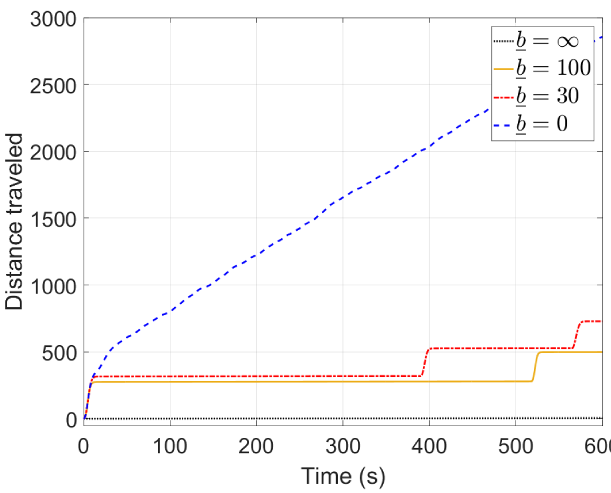


Fig. 10: Comparison of the collective distance traveled by the entire network for different $\underline{b}$ values under a fixed communication topology.

cation links depend on the distances between the robots as they move. The distributed filtering approach offers a degree of robustness to communication link dropouts, and has been shown to perform well if the minimum connectivity condition holds. Furthermore, we proposed a movement control strategy that aims to balance the trade-off between estimation performance and total distance traveled by the network. A single parameter, $\underline{b}$, is specified to tune the network performance between the two extreme cases of continuously tracking the target (best estimation performance and high energy consumption), or remaining stationary (maximum energy conservation at the expense of accurate localization). Through simulation, we notice a graceful degradation in estimation performance for larger values of $\underline{b}$ in order to reduce energy expenditure.

This work can be expanded in several ways, including the analysis of directed communication graphs. Additionally, different distributed control strategies can be introduced for formation control and tracking of the target with obstacle and collision avoidance. Other interesting problems include online-tuning of the parameter $\underline{b}$ to change the network objective and react to changes in the environment (e.g., to reduce energy expenditure further when the robots are low on battery). Ultimately, we plan to experimentally validate these algorithms in TDOA-based-tracking of tagged fish using a group of gliding robotic fish [29].

Appendix A
Cases (a),(b), and (c) in Theorem 1

We show that $\text{rank}(\mathcal{O}_1) = 6$ for the remaining three cases in Fig. 3.

Case (a): This case is the easiest to examine, as agent 1 has a sufficient number of neighbors. The measurement matrix $H_1(k)$ has the following structure

$$H_1(\lambda) = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 \\ \lambda_4 & \lambda_5 & \lambda_6 & 0 & 0 & 0 \\ \lambda_7 & \lambda_8 & \lambda_9 & 0 & 0 & 0 \end{bmatrix}, \tag{50}$$

and from the discussion on decentralized approach, the pair $(A, H_1(\lambda))$ is generically observable. It immediately follows that $\text{rank}(\mathcal{O}_1) = 6$, since

$$\text{rank}\left(\mathcal{O}_1\right) = \text{rank}\left(\begin{bmatrix} [W]_{11} H_1(\lambda)A \\ [W^2]_{11} H_1(\lambda)A^2 \\ \vdots \\ [W^7]_{11} H_1(\lambda)A^7 \end{bmatrix}\right) = 6,$$

for almost all values of T and λ.

Case (b): The structure of $H_i(k)$ for $i = 1, \dots, 4$ is

$$\begin{aligned} H_1(\lambda) &= \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 \end{bmatrix}, \\ H_2(\lambda) &= \begin{bmatrix} -\lambda_1 & -\lambda_2 & -\lambda_3 & 0 & 0 & 0 \\ \lambda_4 & \lambda_5 & \lambda_6 & 0 & 0 & 0 \\ \lambda_7 & \lambda_8 & \lambda_9 & 0 & 0 & 0 \end{bmatrix}, \\ H_3(\lambda) &= \begin{bmatrix} -\lambda_4 & -\lambda_5 & -\lambda_6 & 0 & 0 & 0 \end{bmatrix}, \\ H_4(\lambda) &= \begin{bmatrix} -\lambda_7 & -\lambda_8 & -\lambda_9 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

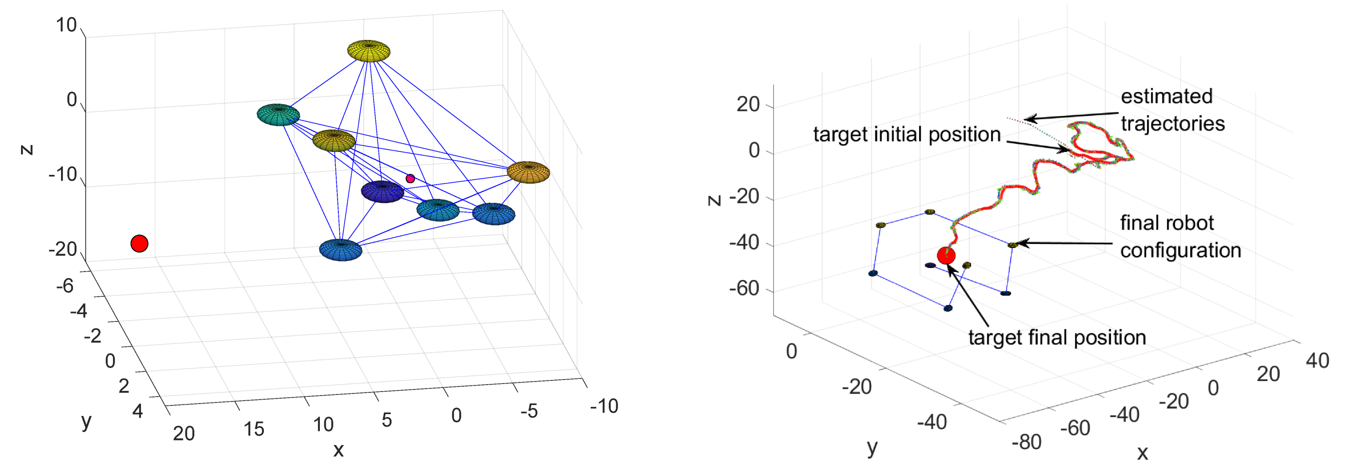
We first note that the pair

$$\left(A, \begin{bmatrix} H_1(\lambda) \\ H_2(\lambda) \end{bmatrix}\right),$$

is generically observable, which can be shown following the same analysis in the discussion for the decentralized approach. This in turn, ensures that

$$\text{rank}\left(\begin{bmatrix} H_1(\lambda) \\ H_2(\lambda) \\ \vdots \\ H_1(\lambda)A^6 \\ H_2(\lambda)A^6 \end{bmatrix}\right) = 6,$$

for almost all values of T and λ, and it immediately follows that $\text{rank}(\mathcal{O}_1) = 6$.



(a) Initial setup for distance-based communication links.

(b) Final setup for distance-based communication links.

Fig. 11: Evolution of the communication topology as the agents move in space to track the robot at different times.

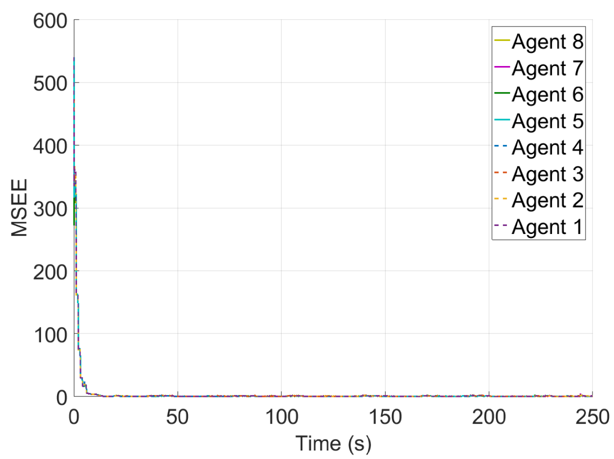


Fig. 12: Mean Squared Estimation Error (MSEE) under a time-varying communication graph with continuous tracking ($\underline{b} = 0$) .

Case (c): The structure of $H_i(k)$ for $i = 1, \ldots, 4$ follows

$$
\begin{aligned}
H_1(\lambda) &= \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & 0 & 0 & 0 \\ \lambda_4 & \lambda_5 & \lambda_6 & 0 & 0 & 0 \end{bmatrix}, \\
H_2(\lambda) &= \begin{bmatrix} -\lambda_1 & -\lambda_2 & -\lambda_3 & 0 & 0 & 0 \end{bmatrix}, \\
H_3(\lambda) &= \begin{bmatrix} -\lambda_4 & -\lambda_5 & -\lambda_6 & 0 & 0 & 0 \\ \lambda_7 & \lambda_8 & \lambda_9 & 0 & 0 & 0 \end{bmatrix}, \\
H_4(\lambda) &= \begin{bmatrix} -\lambda_7 & -\lambda_8 & -\lambda_9 & 0 & 0 & 0 \end{bmatrix}.
\end{aligned}
$$

Following similar procedures to those presented in the decentralized approach discussion, it can be shown that the pair

$$
\left(A, \begin{bmatrix} H_1(\lambda) \\ H_3(\lambda) \\ H_4(\lambda) \end{bmatrix} \right),
$$

is generically observable, and that

$$
\operatorname{rank}\left(\begin{bmatrix} H_1(\lambda) \\ H_2(\lambda) \\ H_3(\lambda) \\ \vdots \\ H_1(\lambda)A^6 \\ H_2(\lambda)A^6 \\ H_3(\lambda)A^6 \end{bmatrix} \right) = 6,
$$

for almost all values of T and λ. Therefore, $\operatorname{rank}(\mathcal{O}_1) = 6$ for almost all values of T and λ.

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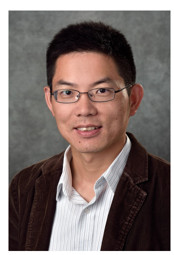

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