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A Robust State Estimator for Multi-Agent Systems Under Impulsive Noise and Missing Measurements

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Abstract— **We address the problem of estimating the state of a Multi-Agent System (MAS) with dynamics subjected to impulsive disturbances, based on measurements that are corrupted with impulsive noise and are sometimes missing. To facilitate the online implementation of the proposed state estimator for MAS, a graph formulation is proposed first. Then, making use of the Huber Loss, the estimator adopts a general cost function that addresses missing measurements and is robust to impulsive noise and disturbances. The solution is validated under a synthetic scenario, where a team of UAVs equipped with onboard video cameras, inertial sensors, transceivers, and GPS, cooperatively geolocate and track a ground moving target agent. Comparison results with respect to three different state-of-the-art estimators are provided to show the superior performance and benefits of the proposed robust estimator.**

I. INTRODUCTION

State estimation plays a major role in Multi-Agent Systems (MAS) applications. At present, MAS commonly rely on measurements from global positioning systems (GPS) to determine the position and orientation of each one of its individual agents. Unfortunately, interferences, jamming signals, or lack of GPS coverage may generate situations when some agents lack geospatial information, either permanently or temporarily. In this difficult situation, the MAS can still generate information about the agents' positions using alternative sensor modalities, such as vision based-sensors, RF sensors, and acoustic sensors. Unfortunately, these sensor modalities are prone to impulsive noise: vision-based sensors provide large errors when a visual landmark is temporarily obstructed or misinterpreted, and RF/acoustic sensors can report false measurements due to multi-path reflections. Furthermore, impulsive disturbances in MAS scenarios emerge for agents that most of the time move along smooth paths, but occasionally engage in sharp turns or evasive maneuvers.

One of the most popular strategies to estimate the state of dynamical systems from noisy measurements is the Kalman Filter (KF) [1]. This algorithm is appealing in that it allows the incorporation of statistical information about measurement noise and the process dynamics into the estimation processes, enabling the construction of good estimators with scarce measurement data. In addition, it reduces the estimation problem to a simple least-squares optimization, which can be efficiently solved. The KF however has a disadvantage which in fact is shared with other algorithms that minimize sums of squared residuals: sensitiveness to impulsive noise [2]. This limitation is due to the fact that the KF is designed under the assumption of Gaussian noise. Under this assumption, large disturbances are unlikely and the filter forces the estimates to be much smoother than what they should be. To address non-Gaussian measurement noise, ℓ_1-norm based estimators have been proposed [3], [4], which are shown to outperform the KF. However, they did not consider impulsive disturbances in the process model.

There are a few studies that explored the estimation of system state under large disturbances using the KF, by modeling disturbances as a mixture of two different processes [5], [6]. A more recent study [7] uses a combination of ℓ_1 and ℓ_2-norm criteria to estimate the system state under abrupt disturbances, but it does not consider impulsive noise in the measurement data. In our previous study [8], we developed a maximum likelihood estimator to address impulsive noise and disturbances, by modeling noise and disturbances as a mixture of Gaussian and Laplacian terms. Built upon [8], we seek a more robust estimator in this study and further consider the practical issue of missing measurements.

The main contributions of this paper include (i) the development of a robust state estimator that captures the statistical information effectively about impulsive noise/disturbances in MAS based on the Huber Loss; (ii) the development of a general cost function to address missing measurements in MAS; (iii) the representation of the proposed MAS state estimation problem as a graph that effectively encodes the state of the estimator over time and facilitates the online implementation of the estimator; and (iv) the extensive comparison studies with existing state-of-the-art estimation algorithms that show the superior performance of our approach. For demonstration purpose, the scenario where a team of Unmanned Aerial Vehicles (UAVs) cooperate to track and geolocate a target moving on the ground is investigated.

The rest of the paper is organized as follows. Section II introduces the mathematical formulation of the MAS state estimation problem. The main result, which consists of a robust estimator for MAS, is presented in Section III. The estimator is validated under two synthetic MAS target tracking scenarios of increasing complexities in Section IV. Finally,

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conclusions and future work are introduced in Section V.

II. PROBLEM FORMULATION

Consider a MAS that consists of N UAVs and a ground moving target. The UAVs are tasked to cooperatively track the target, whose dynamics are unknown. Each UAV is equipped with an onboard video camera, RF transceivers, and navigation sensors including GPS and Inertial Measurement Unit (IMU). To estimate the state of the target, i.e., the target *geolocation* [9], each UAV makes use of its own relative measurements of the target captured by means of its onboard vision system and fuses this information with its own GPS-based location. In addition, making use of the RF transceivers, each UAV incorporates into its own estimation, the measurements captured by and received from other UAVs. Under this scenario, multiple challenges may be encountered, e.g., (i) different sensors may generate measurements at different sampling rates and with different accuracies, (ii) the vision systems and the GPS may produce erroneous measurements (outliers), (iii) measurements will not be produced if the target is out of the UAVs' sensing range, and (iv) measurements may be lost during the transmission of data among the UAVs.

To mathematically formulate this problem, we introduce $x_T[k]$ to represent the state of the target at time step k. Notice that $x_T[k]$ is the unknown variable we aim to estimate. The true dynamics of the target can be described by $x_T[k+1] = f(x_T[k], u[k], d[k])$, where $f(\cdot)$ is the dynamic function of the system, $u[k]$ is the control input at time k, and $d[k]$ is the uncertain disturbance that affects system dynamics. As $f(\cdot)$, $u[k]$ and $d[k]$ are unknown and unmeasured, we approximate the dynamics of the target using a modified white acceleration model [10] as follows:

$$x_T[k+1] = Fx_T[k] + Gw[k] \tag{1}$$

where the state $x_T[k]$ contains the target's position and velocity, i.e., $x_T[k] = [p_x[k], \dot{p}_x[k], p_y[k], \dot{p}_y[k]]^\top$. (p_x, p_y) is the 2D position of the target, $(\dot{p}_x, \dot{p}_y)$ defines the velocity of the target, and the superscript $\top$ denotes transposition. $F = \text{diag}[F_c, F_c]$, $F_c = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$, and Δt is the sampling interval. $G = \text{diag}[G_c, G_c]$, $G_c = \begin{bmatrix} \frac{\Delta t^2}{2} & 0 \\ 0 & \Delta t \end{bmatrix}$. The term $w[k]$ captures the modeling errors and uncertain disturbances, whose covariance matrix R is assumed stationary over time.

Remark 1.- At this point, it is worth mentioning that more accurate and appropriate models can be adopted to approximate the target's dynamics if adequate knowledge of the target is available, which can be perceived through object detection and classification techniques.

Two types of measurements are used to estimate the state of the target: (i) the absolute position of a UAV provided by its onboard GPS sensor described by $y_i[k] = C_i x_i[k] + v_i[k]$ and (ii) the relative position of the target with respective to the UAV described by $y_{iT}[k] = C_i x_i[k] - Cx_T[k] + v_{iT}[k]$, which is captured by means of the onboard UAV's vision

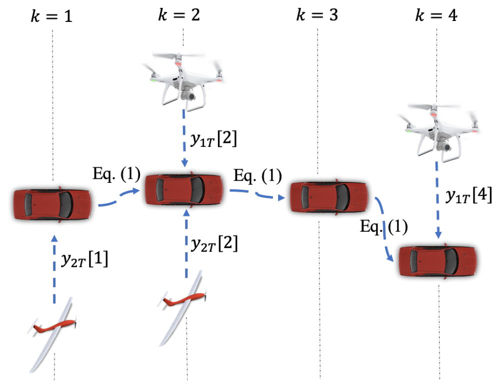

Fig. 1. MAS estimation state problem in a multi-UAV tracking scenario.

system. $C_i = C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$. $x_i[k]$ is the state of UAV$_i$, $i \in \{1, 2, \ldots, N\}$. $v_i[k]$ and $v_{iT}[k]$ are GPS induced and vision-system induced measurement errors, respectively. $v_{iT}[k]$ is relatively small most of the time, but can be large occasionally due to false detection of the vision-system. As the precision of the GPS and IMU is fairly good, the state of the UAV $x_i[k]$ is directly obtained from these sensors without considering system dynamics. However, it is worth mentioning that GPS may also generate large errors (outliers) due to interference and signal jamming.

Denote the derived state of UAV$_i$ as $\hat{x}_i[k]$. Then, the relative position of the target can be described by

$$y_{iT}[k] = C_i \hat{x}_i[k] - Cx_T[k] + e_i[k] \tag{2}$$

where the term $e_i[k]$ accounts for both GPS and vision-system induced measurement noise. The covariance matrix of $e_i[k]$, denoted as Q, is assumed constant.

A. Graph Representation of the MAS State Estimation Problem

The aforementioned multi-UAV tracking problem can be efficiently described by a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, where each node in the node set $\mathscr{V}$ represents the state of a UAV or target at some instant of time k, and each edge in the edge set $\mathscr{E}$ represents the constraint between two nodes that is imposed by measurements or by the target's dynamics. In particular, each edge (n_s, n_e) that starts from node n_s and ends at node n_e is described by the following equation

$$z = An_s - Bn_e + \varepsilon \tag{3}$$

As illustrated in Figure 1, for UAV-to-target edges (measurement constraint), the edge equation (3) matches with equation (2) through the following associations

$$\begin{aligned} z &= y_{iT}[k], \quad A = C_i, \quad n_s = \hat{x}_i[k], \\ B &= C, \quad n_e = x_T[k], \quad \varepsilon = e_i[k]. \end{aligned} \tag{4}$$

For target-to-target edges (motion constraint), the edge equation (3) matches with equation (1) through the following associations

$$\begin{aligned} z &= O, \quad A = G^{-1}F, \quad n_s = x_T[k], \\ B &= G^{-1}, \quad n_e = x_T[k+1], \quad \varepsilon = w[k]. \end{aligned} \tag{5}$$

where $O = [0,0,0,0]^\top$. In both cases, the terms z, A, and B are known. The terms n_s and n_e correspond to the states of the UAV or target. The term ε is the measurement noise or unmeasured disturbance. The state $\hat{x}_i[k]$ associated with UAV$_i$ is directly obtained from its onboard GPS and IMU sensors. The state of the target, i.e., $x_T[k]$, is unknown and needs to be estimated by considering both the motion constraint in equation (1) and the measurement constraint in equation (2). By stacking all the unknown node variables in a vector $\boldsymbol{x}$, all the $\varepsilon = e_i[k]$ in equation (4) in vector $\boldsymbol{\varepsilon}_{ei}$, and all the $\varepsilon = w[k]$ vectors in equation (5) in vector $\boldsymbol{\varepsilon}_w$, all the UAV-to-target edges associated with UAV$_i$ can be combined and represented in a compact form as $\boldsymbol{z}_{ei} = H_{ei}\boldsymbol{x} + \boldsymbol{\varepsilon}_{ei}$, and all target-to-target edges can be represented as $\boldsymbol{z}_w = H_w\boldsymbol{x} + \boldsymbol{\varepsilon}_w$.

This graph-based representation of the MAS state estimation problem is especially efficient when used for a real-time implementation. At each new time step k, a node representing the target's state and an edge representing the motion constraint is inserted into the graph. If new measurements are generated at this time step, nodes representing the associated UAVs' states and edges representing the new measurements are also inserted into the graph. Note that the insertion of new nodes and edges into the graph correspond to the addition of new rows (and columns) to $\boldsymbol{z}_{ei}$, H_{ei}, $\boldsymbol{z}_w$, or H_w. As these changes do not impact the remaining graph, a good data structure that facilitates the state estimation is maintained. In practice, due to computation and memory limitations, we limit the size of the graph by removing nodes and edges that correspond to old data as new nodes and edges are inserted, which correspond to the removal of rows (and columns) from $\boldsymbol{z}_{ei}$, H_{ei}, $\boldsymbol{z}_w$, or H_w.

III. Main Result: A Robust Estimator for MAS

In this section, we introduce a novel robust estimator to solve the MAS state estimation problem formulated in the previous section.

A. Objective Functions with Missing Measurements

Consider the state-space model shown in equation (1). Assume Gaussian noise and $N = 1$. Under the Bayesian Maximum Likelihood method, the unknown state of the target can be estimated by

$$\hat{x}_T[k] = \arg\min \Big((x_T[k] - \hat{x}_T^*[k])^\top P[k]^{-1} (x_T[k] - \hat{x}_T^*[k]) + (y_T[k] - Cx_T[k])^\top Q^{-1} (y_T[k] - Cx_T[k]) \Big), \tag{6}$$

where $\hat{x}_T^*[k] = F\hat{x}_T[k-1]$ is the estimate of $x_T[k]$ given the previous state estimate and the dynamics, and $P[k]$ is the covariance matrix of $(x_T[k] - \hat{x}_T^*[k])$. $y_T[k] = y_i[k] - y_{iT}[k]$ is the position of the target measured by the UAV, where $i = 1$.

If the model and the initial values $\hat{x}_T^*[0]$ and $P[0]$ are accurate and no measurements are missing, the covariance matrix $P[k]$ can be updated by $P[k] = \text{cov}(x_T[k] - \hat{x}_T^*[k]) = FP^*[k-1]F^\top + GRG^\top$, where $P^*[k] = \text{cov}(x_T[k] - \hat{x}_T[k])$. $P^*[k]$ can be calculated recursively via the Kalman filter recursive algorithm [1]: $P^*[k] = P[k] - P[k]C^T(CP[k]C^T + Q)^{-1}CP[k]$.

Suppose now that $m \geq 1$ successive measurements are missing before the current measurement at time k. Recursively , $\hat{x}_T^*[k] = F\hat{x}_T[k-1] = F^m\hat{x}_T[k-m]$. The first available measurement before the current measurement at time k is thus the measurement generated at time $k-m$, and the covariance matrix $P[k]$ should be updated by $P[k]^{[m]} = \text{cov}(x_T[k] - F^m\hat{x}_T[k-m])$, where the superscript $[m]$ indicates the presence of m successive missing measurements before the time instance under evaluation. Then, the *one step* dynamics described in equation (1) become the m-steps dynamics, described as $x_T[k] = F^m x_T[k-m] + \sum_{j=1}^m F^{m-j}Gw[k-m+j]$. Via some calculations, we can derive $F^m = \text{diag}[F_c^m, F_c^m]$ with $F_c^m = \begin{bmatrix} 1 & m\Delta t \\ 0 & 1 \end{bmatrix}$, and $\sum_{j=1}^m F^{m-j}Gw[k-m+j] = G^{[m]}w^*[k]$ with $G^{[m]} = \text{diag}[G_c^{[m]}, G_c^{[m]}]$, $G_c^{[m]} = \begin{bmatrix} \frac{(m\Delta t)^2}{2} & 0 \\ 0 & m\Delta t \end{bmatrix}$, where $w^*[k]$ has the same distribution as $w[k]$. Therefore, the m-step dynamics have the exactly same formula as the one-step dynamics described in equation (1), only with Δt replaced by $m\Delta t$. The corresponding recursive formula for $P^{[m]}[k]$ is now expressed by $P^{[m]}[k] = F^m P^*[k-1](F^m)^\top + G^{[m]}R(G^{[m]})^\top$.

If the latest M UAV-to-target measurements are used estimate the current state of the target, following the maximum likelihood method with Gaussian assumptions on the errors, the objective function is adjusted to

$$\hat{x}_T[k] = \arg\min \Bigg(\sum_{j=k-k_M}^{k} (x_T[j] - \hat{x}_T^*[j])^\top (P^{[m_j]}[j])^{-1} (x_T[j] - \hat{x}_T^*[j]) + \sum_{j=k-k_M}^{k} \delta_j (y_T[j] - Cx_T[j])^\top Q^{-1} (y_T[j] - Cx_T[j]) \Bigg), \tag{7}$$

where m_j denotes the number of successive missing measurements before time j, k_M represents the time when the first among the M measurements is generated. The term $\delta_j = 0$ if there is no measurement generated at time j, and $\delta_j = 1$ otherwise. Note that for any instant j with no observation, $\hat{x}_T[j] = \hat{x}_T^*[j]$ is the result of minimizing the cost function in equation (7).

When the graph representation described in the previous section is applied, the estimates of the target's states can be obtained by solving $\underset{\boldsymbol{x}}{\text{minimize}} \ \ \|A_w(H_w\boldsymbol{x} - \boldsymbol{z}_w)\|_2^2 + \|A_{e1}(H_{e1}\boldsymbol{x} - \boldsymbol{z}_{e1})\|_2^2$, where A_w and A_{e1} are diagonal matrices with $\left(P^{[m_j]}[j]\right)^{-\frac{1}{2}} G$, $\forall j \in \{k-k_M, \ldots, k-1, k\}$ and $Q^{-\frac{1}{2}}$, $\forall \delta_j = 1$, $j \in \{k-k_M, \ldots, k-1, k\}$, as their diagonal elements, respectively. $\|\cdot\|_2$ denotes the l_2 norm.

In cases when multiple UAVs are tasked to track the target cooperatively, we propose a weighted cost function to fuse all measurements, which considers heterogeneous MAS with UAVs carrying cameras of different precision. In particular, let σ_i be the standard deviation of a measurement generated by the UAV$_i$'s camera, indicating its precision, for $i = 1, 2, \cdots, N$. The target's states can then be estimated by

$$\underset{\boldsymbol{x}}{\text{minimize}} \ \ \|A_w(H_w\boldsymbol{x} - \boldsymbol{z}_w)\|_2^2 + \sum_{i=1}^{N} \frac{1}{\sigma_i} \|A_{ei}(H_{ei}\boldsymbol{x} - \boldsymbol{z}_{ei})\|_2^2 \tag{8}$$

Here for each element of A_w, i.e., $\left(P[j]^{[m_j]}\right)^{-\frac{1}{2}} G$, m_j now denotes the number of successive time instances before time j when no measurements are generated.

B. Robust Loss Function

A *loss function* measures how well an algorithm or model fits the outcomes. Commonly used loss functions for a variable x are the Squared Loss $\rho_\infty(x) = x^2$, the Absolute Loss $\rho_0(x) = |x|$, and the Huber Loss, defined as $\rho_a(x) = \begin{cases} \frac{1}{2}x^2, & \text{for } |x| \le a \\ a(|x| - \frac{1}{2}a), & \textit{otherwise} \end{cases}$, where a is a positive number. The Huber Loss function $\rho_a(x)$ was first introduced in [11]. Diverse control problems require state estimation techniques which, in turn, require robustness in such a way that the obtained results are less influenced by outliers. The Squared Loss approach is highly sensitive to outliers because it is punished very heavily by the squaring of the error. The Absolute Loss approach avoids the problem of weighting outliers too much by scaling the loss only linearly. The Huber Loss is a compromise between Squared Loss and Absolute Loss. In particular, the Huber Loss approaches the Squared Loss when $a \to \infty$ and Absolute Loss when $a \to 0$.

In the optimization function (8), the Squared Loss is applied for both estimation and measurement error components. However,both components may involve impulsive noise (outliers), which can be addressed by robust loss functions. In this study, we use the Huber loss for both the estimation and measurement errors to improve the performance. A more general form of equation (8) that allows the selection of loss functions is given below

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad \rho_{a_w}\left(A_w(H_w\boldsymbol{x} - z_w)\right) + \sum_{i=1}^{N} \frac{1}{\sigma_i} \rho_{a_i}\left(A_{ei}(H_{ei}\boldsymbol{x} - z_{ei})\right) \quad (9)$$

where a_w and a_i, $i \in \{1, 2, \ldots, N\}$ are zeros if the Absolute Loss is chosen, infinities if the Squared Loss is chosen and positive numbers if the Huber Loss is chosen. To solve problem (9), standard methods of convex optimization can be used, such as CVX [12] and YALMIP [13]. In the following simulation studies, we set $a_w = a_i = 2$, $\forall i \in \{1, 2, \ldots, N\}$, and we use CVX to solve problem (9).

IV. SIMULATION RESULTS

In this section, we conduct extensive simulation studies to evaluate the performance of the proposed robust estimator for MAS. Two scenarios with increasing levels of complexities are considered.

	1% (uniform)	10% (uniform)	20% (uniform)	$v = 1$ (t)	$v = 1.5$ (t)
measurement error	10.8188m	25.1747m	35.6328m	29.7823m	11.3826m
KF estimation error	8.0046m	15.2958m	22.7106m	18.8719m	7.5597m
BLS estimation error	7.8278m	14.9382m	22.2341m	19.0643m	7.3467m
sum-of-norms estimation error	7.0772m	6.4094m	10.4175m	7.4666m	6.0241m
robust estimator error	**5.3048m**	**5.5084m**	**8.0318m**	**6.2184m**	**5.7279m**

TABLE I

COMPARISON OF THE RMS ESTIMATION ERRORS OF DIFFERENT ESTIMATORS UNDER OUTLIERS OF DIFFERENT CHARACTERISTICS.

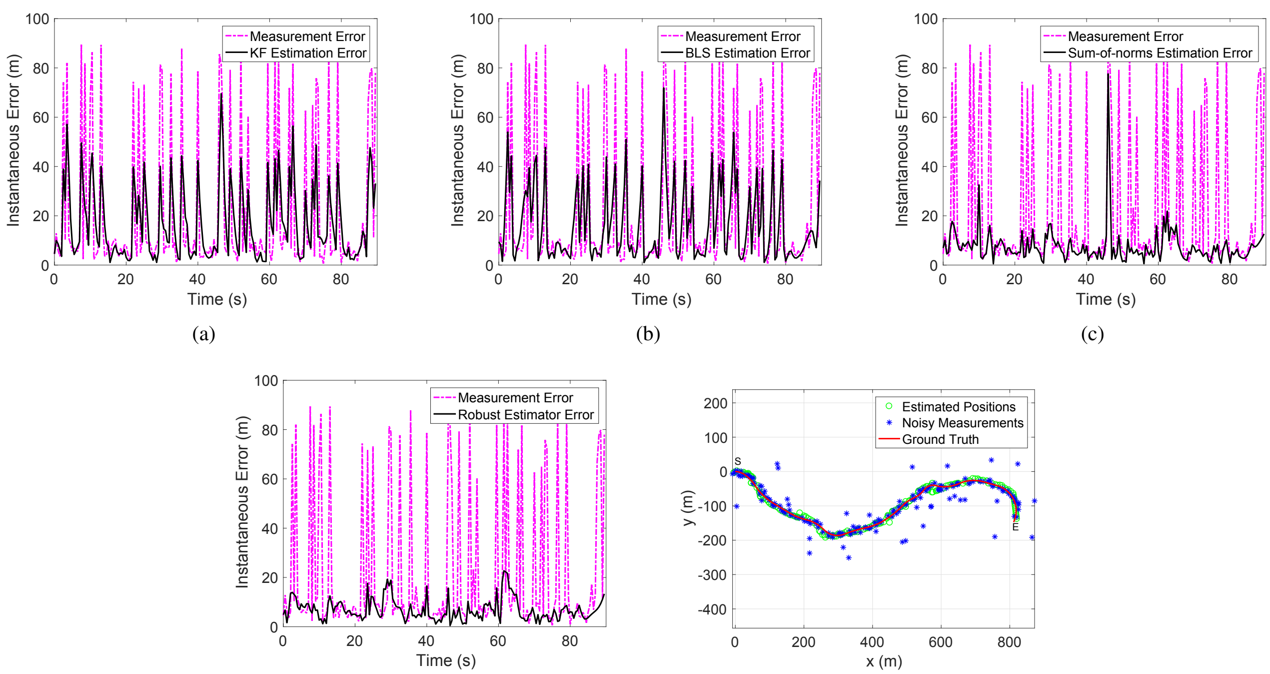

Fig. 2. Impact of Outliers: The instantaneous estimation performance of the following algorithms: a) Kalman Filter, b) BLS estimator, c) sum-of-norms estimator, and d) proposed robust estimator. e) The target's trajectory estimated by the proposed robust estimator. 20% uniformly distributed outliers were introduced.

A. First Scenario: A Single UAV Tracks a Target

We start from studying the simplest case where a single UAV ($N = 1$) is tasked to geolocate and track a ground moving target. To simulate the moving target, we adopt the kinematic uni-cycle model [14]. The UAV determines the relative position of the target using the geolocation approach described in [9]. The GPS/IMU onboard of the UAV operates at 20Hz, and the sampling interval $\Delta t = 0.05s$. To estimate the state of the target, the latest $M = 15$ UAV-to-target relative measurements are used to construct a graph like the one shown in Figure 1. The other parameters are configured as follows. $R = P[0] = I_{4\times4}$ and $Q = I_{2\times2}$, where $I_{n\times n} \in R^{n\times n}$ is a unit matrix. The first measurement $y_T[0]$ is used to initialize the position of the target.

To demonstrate the performance and benefits of the proposed robust estimator, we compare it with three state-of-the-art estimators: (i) Kalman Filter (KF), (ii) the batched least square (BLS) estimator that also uses the latest 15 UAV-to-target measurements, and (iii) the sum-of-norms estimator introduced in [8]. In this and following simulations, the weight parameter λ of the sum-of-norms estimator, which characterizes the relative weights of impulsive (Laplacian) and Gaussian components in the noise, is tuned using a range of values as suggested in [8] to achieve a good performance.

1) Impact of Outliers: In order to evaluate the resilience of the proposed robust estimator against outliers of different characteristics, two different models were implemented to simulate the outliers. The first approach is similar to the one used in [8], which corrupts the measurements with uniformly distributed noise ranging from 40m to 70m. In our simulations, we change the amount of outliers introduced by varying the percentages with respect to the total number of measurements. The second approach corrupts the measurements with noise generated from the Student's t-distribution, which has a similar shape as the normal distribution but with a heavier tail. The parameter of the t-distribution, the *degrees of freedom* ν, is varied in simulations to model outliers of different characteristics. Note that a smaller ν leads to a heavier tail. The simulation results are summarized in Table I, where the root-mean-square (RMS) estimation error is used as the performance evaluation metric. Figure 2 shows the instantaneous estimation errors generated by different estimators and the target's trajectory estimated by the proposed robust estimator, when 20% uniformly distributed outliers are present. These results demonstrate the promising performance of the proposed robust estimator,

	1% missing	10% missing	20% missing	40% missing
measurement error	35.8215m	35.4122m	34.7541m	37.7812m
KF estimation error	22.9701m	23.1594m	23.7785m	33.5473m
BLS estimation error	22.5718m	22.8851m	25.1728m	31.2794m
sum-of-norms estimation error	10.9128m	10.4728m	15.5712m	19.2022m
robust estimator error	**8.1266m**	**8.4694m**	**8.8115m**	**10.8259m**

TABLE II

COMPARISON OF RMS ESTIMATION ERRORS OF DIFFERENT ESTIMATORS UNDER DIFFERENT PERCENTAGES OF MISSING MEASUREMENTS.

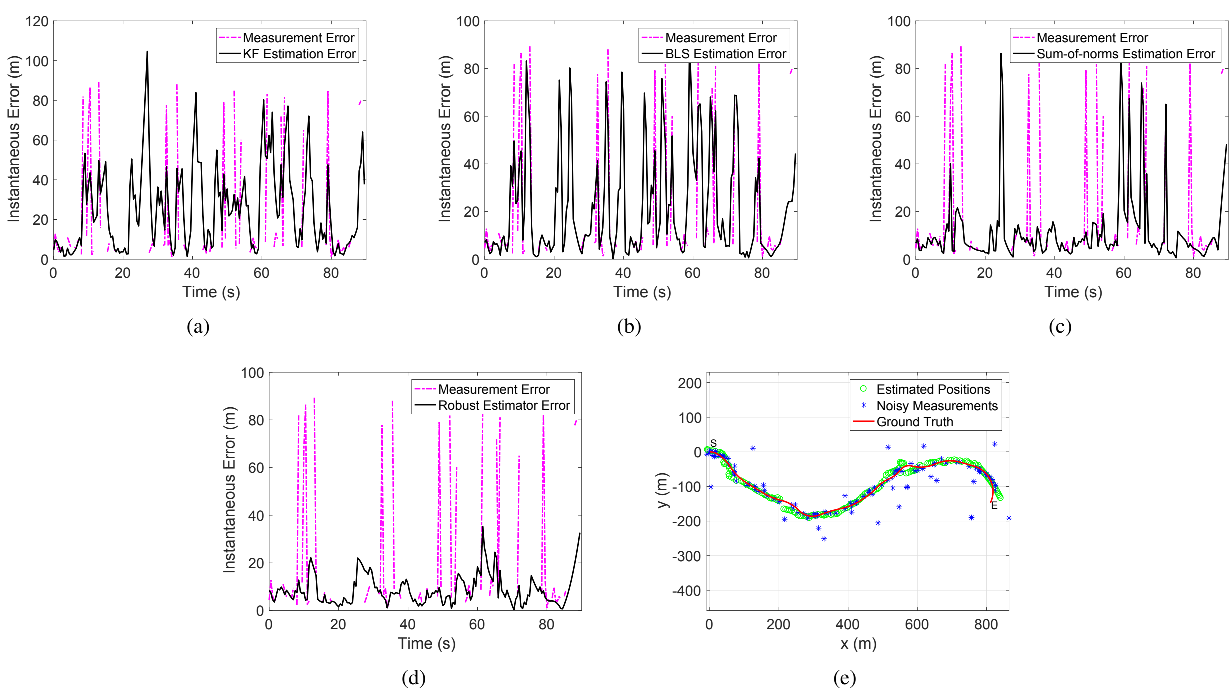

Fig. 3. Impact of Missing Measurements: The estimation performance of the following algorithms: a) Kalman Filter, b) BLS estimator, c) sum-of-norms estimator, and d) proposed robust estimator. e) The target's trajectory estimated by the proposed robust estimator. The total number of measurements missing is 40%.

which outperforms the other methods especially when the outliers are prominent.

2) Impact of Missing Measurements: To evaluate the performance of the proposed robust estimator under missing measurements, we used the dataset created in the previous experiment which includes 20% uniformly distributed outliers. Then, we randomly removed different percentages of the measurements. The results of this simulation study are provided in Table II. Additionally, the instantaneous estimation errors of different estimators and the target's trajectory estimated by the proposed robust estimator under 40% measurement missing rate are visualized in Figure 3. Notice that the proposed robust estimator is resilient to missing measurements and it outperforms the other three estimators significantly, especially when a large portion of the measurements are missing.

B. Second Scenario: Two UAVs Cooperatively Track a Target

We consider now a more complicated scenario, where a MAS consisting of two UAVs are tasked to track a ground moving target cooperatively. We adopt the same experimental settings described in the previous section to configure this experiment, but with an additional UAV included in the formulation. The transmission delays between the two UAVs are ignored. To simulate outliers and cameras of different precisions, we corrupt 20% of all measurements generated by the first UAV with uniformly distributed noise ranging from 40m to 70m, and 10% of all measurements generated by the second UAV with uniformly distributed noise ranging from 30m to 60m.

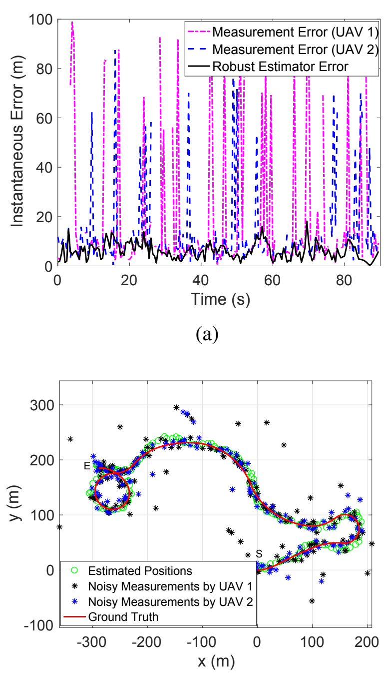

Fig. 4. a) The instantaneous estimation performance of the proposed robust estimator for measurements collected by two UAVs and b) the estimated trajectory of the target.

As measurements may be lost during the transmission, we randomly remove 20% of the total number of measurements produced by each UAV. The estimation performance of the proposed robust estimator is shown in Figure 4(a), which has an RME estimation error of 6.6577m. Compared with the measurement error of 36.7993m for the first UAV and 22.6096m for the second UAV, our MAS estimator improves the precision by around 240%. Figure 4(b) visualizes the trajectory of the target estimated by our robust MAS estimator.

V. CONCLUSION

In this paper, we developed a robust state estimator for MAS. This estimator has a general cost function that addresses missing measurements in MAS and is robust to impulsive noise and disturbances by adopting the Huber Loss. To facilitate online implementation of the estimation algorithm, a graph representation of the MAS state estimation problem was also developed. Simulations of two MAS scenarios with different complexities validate the proposed method. Comparisons with state-of-the-art estimation algorithms further demonstrate the promising performance of the proposed robust estimator. In the future, we will conduct systematic analysis on the parameters' impact, and implement the robust estimator in a real MAS.

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