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# Joint optimization of budget allocation and maintenance planning of multi-facility transportation infrastructure systems

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ABSTRACT

Transportation infrastructure, such as pavements and bridges, is critical to a nation's economy. However, a large number of transportation infrastructure is underperforming and structurally deficient and must be repaired or reconstructed. Maintenance of deteriorating transportation infrastructure often requires multiple types/levels of actions with complex effects. Maintenance management becomes more intriguing when considering facilities at the network level, which represents more challenges on modeling interdependencies among various facilities. This research considers an integrated budget allocation and preventive maintenance optimization problem for multi-facility deteriorating transportation infrastructure systems. We first develop a general integer programming formulation for this problem. In order to solve large-scale problems, we reformulate the problem and decompose it into multiple Markov decision process models. A priority-based two-stage method is developed to find optimal maintenance decisions. Computational studies are conducted to evaluate the performance of the proposed algorithms. Our results show that the proposed algorithms are efficient and effective in finding satisfactory maintenance decisions for multi-facility systems. We also investigate the properties of the optimal maintenance decisions and make several important observations, which provide helpful decision guidance for real-world problems.



## 1. Introduction

Transportation infrastructure, such as pavements and bridges, plays a critical role in a nation's economy, yet a large number of constructions are underperforming and structurally deficient, which must be repaired or reconstructed. As reported by the American Society of Civil Engineers (ASCE), one of every five miles of highway pavement is in poor conditions and the annual cost associated with the operation and maintenance of highways reaches $72.7 billion in 2014 (ASCE, 2017). However, according to the U.S. Department of Transportation's fiscal year budget report, the annual budget appropriated for the Federal Highway Administration (FHWA) to maintain and improve the safety, condition, and performance of the national highway system is less than $50 billion in the past decades. The sheer deterioration of transportation infrastructure and the multi-billion annual investment shortfalls demand an effective maintenance optimization model to ensure the reliability and serviceability of deteriorating transportation infrastructure (DTI) under limited budgets.

Maintenance management of multi-facility DTI systems is a challenging task. First, maintenance decisions are typically made by agencies responsible for an entire system consisting of multiple facilities. The system-level maintenance planning is often subject to limited budgets, leading to economic dependencies among facilities. Optimal maintenance decisions at the facility-level optimization may no longer be optimal or even feasible to the system-level problem. Second, multiple types/levels of maintenance actions are generally required in the maintenance operation of DTI systems. These maintenance actions can be generally classified into two categories: corrective maintenance (CM) and preventive maintenance (PM). CM, i.e., reconstruction, is in response to failures and restores a facility to an as-good-as-new state. PM preventively maintains a facility to avoid or delay failures and often includes multiple types of treatments, which usually have complex effects, such as instant improvement of performance index, deterioration suppression, and deterioration rate reduction (Neves & Frangopol, 2005). For example, there exist seven major types of PM actions in pavement maintenance operations: crack seal, chip seal, microsurfac-

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ing, slurry seal, fog seal, and thin hot/cold-mix overlay (Johnson, 2000). The impact of crack seal is the lowest among the seven treatments and can only suppress the deterioration for approximately three years, while thin hot-mix overlay not only improves the condition of pavements but also reduces the deterioration rate for five to eight years. It is difficult to incorporate the complex maintenance effects in maintenance optimization for multi-facility systems.

In this paper, we develop a joint budget allocation and maintenance optimization model for a DTI system consisting of heterogeneous facilities. Facilities can have different deterioration processes and maintenance cost profiles. We formulate the joint optimization problem as a general integer programming. Exact solutions to small-scale problems can be obtained using state-of-the-art solvers. To solve large-scale problems, we reformulate the problem by modeling the maintenance optimization of each individual facility in a finite time horizon as a Markov decision process (MDP) and optimizing the total maintenance cost of all facilities with budget constraints. A priority-based two-stage method is developed to solve the reformulated problem. Computational studies are conducted to assess the performance of the proposed method. Our results show that the proposed algorithms provide optimal solutions to small-scale problems considered in our computational studies and solve large-scale problems with satisfactory solutions (i.e., average percentage optimality gap less than 2%) efficiently. We further investigate the properties of the optimal maintenance decisions and make several important observations. The main contribution of this paper is threefold.

(1) Develop a general integer programming formulation that combines budget allocation and maintenance planning for multi-facility DTI systems, which provides exact solutions for small-scale problems to assess the performance of the proposed heuristic solution method.
(2) Model the facility's deterioration by a realistic deterioration process (i.e., non-stationary, stochastic) rather than using simplified deterioration profiles (e.g., deterministic, linear), and consider multi-level PM with complex effects: random condition improvement, random age reduction, and deterioration rate reduction.
(3) Develop a priority-based two-stage solution method to find satisfactory solutions. Computational studies show that the proposed algorithms are efficient and effective in solving large-scale problems.

The remainder of this paper is organized as follows. Section 2 reviews some relevant literature on maintenance optimization models for multi-facility infrastructure systems. Section 3 provides the model development for the joint optimization of budget allocation and multi-facility maintenance planning. The priority-based two-stage solution method is presented in Section 4. In Section 5, computational studies are conducted to evaluate the proposed method and properties of the optimal maintenance decisions are also investigated. Finally, concluding remarks and future extensions are outlined in Section 6.

## 2. Literature review

Maintenance optimization for systems with multiple subsystems/components has received much attention in the past decades. Most multi-component maintenance models consider economic dependencies among components due to setup costs. A setup cost is a fixed system-dependent cost due to mobilizing repair crew, disassembling machines, and downtime loss, which is incurred when any maintenance action is performed. Opportunistic maintenance (OM) is a commonly used approach for multi-component maintenance when setup costs exist. OM typically refers to a scheme in which PM is performed on various functioning components at the opportunity when one or more components are correctively maintained. The simultaneous implementation of multiple maintenance activities can reduce both system- and component-level risks to failure, and lead to potential cost savings (Ding & Tian, 2012; Laggoune, Chateauneuf, & Aissani, 2009). OM and its variants (Ding & Tian, 2012; Laggoune et al., 2009; Shafiee, Finkelstein, & Bérenguer, 2015; Tian, Jin, Wu, & Ding, 2011) have been extensively studied. Laggoune et al. (2009) develop age-based preventive maintenance models for individual components in a multi-component series system. They take CM as an opportunity to preventively replace a group of non-failed components. Ding and Tian (2012) develop opportunistic maintenance approaches for wind farms and propose the opportunistic maintenance policies by optimizing thresholds on components' ages. Other multi-component maintenance considering economic dependencies due to setup costs can be found in Tian and Liao (2011), Bouvard, Artus, Bérenguer, and Cocquempot (2011), and Keizer, Teunter, and Veldman (2016). However, limited budgets are ignored in the majority of multi-component maintenance models.

There is a large body of literature on maintenance optimization of infrastructure systems consisting of multiple facilities. Sathaye and Madanat (2011) categorize multi-facility optimization approaches as either top-down or bottom-up. The fundamental idea of the top-down approach (Golabi, Kulkarni, & Way, 1982; Kuhn & Madanat, 2005) is to optimize multi-facility maintenance problems from a system-level perspective. Kuhn and Madanat (2005) formulate the network-level infrastructure optimization problem as an MDP with consideration of budget constraints and solve it via linear programming. They assume that facilities are homogeneous in terms of facility deterioration and cost profiles. Therefore, facilities with the same condition state have the same maintenance decision. This assumption reduces the problem dimension, significantly simplifying the solution method. However, the top-down approach ignores heterogeneity commonly existed among multiple facilities.

Recently, the bottom-up approach (Durango-Cohen & Sarutipand, 2007; Furuya & Madanat, 2012; Ouyang, 2007; Pantelias & Zhang, 2009; Yeo, Yoon, & Madanat, 2013) has become more prevalent in the literature, because it allows the incorporation of heterogeneity among facilities (e.g., facility-specific deterioration and maintenance cost) into maintenance models. A commonly used bottom-up approach proceeds as follows. The optimal maintenance decision for each individual facility without considering their interactions is first obtained. Then, the solution from the system-level perspective is optimized based on the solutions to the facility-level problems. Yeo et al. (2013) consider a budget allocation problem with a finite planning horizon for heterogeneous infrastructure systems using the bottom-up approach. They first ignore the budget constraint and decompose the optimization problem into individual MDPs for individual facilities, where the optimal solution and several suboptimal solutions are obtained. They further use a heuristic algorithm (i.e., pattern search heuristic and evolutionary algorithm) to find the best maintenance combination from the solutions obtained previously with consideration of the budget binding. Furuya and Madanat (2012) extend the maintenance model in Yeo et al. (2013) by taking into account two additional constraints, economies of scale and capacity constraints caused by simultaneous maintenance activities on adjacent facilities. Monte-Carlo simulation is used in Furuya and Madanat (2012) to obtain average costs in a finite planning horizon. Ouyang (2007) incorporates travelers' route choices in the pavement resurfacing planning problem with budget constraints and formulates the problem as a deterministic dynamic programming with multidimensional continuous states. To reduce computational difficulties, a parametric approximation technique is applied to approximate the value functions

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and the policy iteration algorithm is used to solve the dynamic programming in Ouyang (2007). Durango-Cohen and Sarutipand (2007) present a quadratic programming formulation for maintenance optimization of multi-facility DTI systems with economic dependence. However, most aforementioned multi-facility maintenance models use rather simplified deterioration profiles (e.g., deterministic, linear) to reduce modeling and computational difficulties.

In addition to the interdependence among facilities, complex maintenance effects from multiple types of maintenance actions required for DTI systems further complicate multi-facility maintenance planning. The complex effects typically include instant improvement of performance index, deterioration suppression, and deterioration rate reduction during an effective period (Neves & Frangopol, 2005). However, many studies in the existing literature make simplified assumptions on maintenance effects (Alaswad, Cassady, Pohl, & Li, 2017; Hu, Jiang, & Liao, 2017; Mercier & Castro, 2019; Nguyen, Dijoux, & Fouladirad, 2017; Shen, Cui, & Ma, 2019; Yang, Ye, Lee, Yang, & Peng, 2019; Zhou, Kou, Xiao, Peng, & Alsaadi, 2020). For example, Nguyen et al. (2017) consider a virtual age model and assume that an imperfect maintenance action reduces the virtual age by a proportion of the age just before the maintenance. Mercier and Castro (2019) consider two imperfect maintenance models for a deteriorating system. One assumes that the imperfect maintenance reduces the deterioration of a system accumulated from the last maintenance action by a fixed proportion, and the other assumes that the age reduction caused by imperfect maintenance is proportional to the time elapsed since the last maintenance activity. Zhou et al. (2020) develop a sequential imperfect PM model for urban buses and model the effect of the imperfect PM by a failure intensity reduction/aggravation proportional to the difference of the failure intensity between the initial and latest failure observations. Only a handful of studies consider complex maintenance effects in single-facility maintenance planning problems (Neves, Frangopol, & Petcherdchoo, 2006; Neves & Frangopol, 2005; Okasha & Frangopol, 2010; Shi, Xiang, & Jin, 2019a; Shi, Xiang, & Li, 2019b). In particular, Shi et al. (2019b) develop a multi-level PM maintenance optimization model for a single-facility system and consider three types of effects for each level of PM: random condition improvement, deterioration rate reduction, and duration of the maintenance effects. Despite the significant impacts of these complex effects on maintenance decisions, limited maintenance models of multi-facility systems consider different maintenance outcomes of different maintenance actions (Furuya & Madanat, 2012; Kuhn & Madanat, 2005; Medury & Madanat, 2013; Yeo et al., 2013).

Our review shows that most existing studies on maintenance optimization of multi-facility DTI systems either ignore the complex effects of maintenance actions on DTI systems or make simplified assumptions on DTI's deterioration profiles. Moreover, there is a lack of efficient methods to solve large-scale multi-facility DTI maintenance planning problems.

## 3. Model development

### *3.1. System description*

Consider a DTI system consisting of $n$ heterogeneous facilities and denote the facility set by $N, N = \{1, 2, \dots, n\}$. Facilities can have different deterioration processes and maintenance costs. Since the deterioration rate typically increases as a facility ages, we model the deterioration process of each facility as an appropriate non-stationary stochastic process. It is assumed that all facilities deteriorate independently. We discretize the continuous deterioration level into several distinct intervals to represent different condition states. Assume that each facility has $\psi + 1$ condition states, i.e., $s \in \Psi = \{0, 1, \dots, \psi\}$, where a larger state represents a worse yet functioning facility condition, and states 0 and $\psi$ denote the as-good-as-new condition and the failure condition, respectively. Let the age state be represented by $\tau \in \Gamma = \{0, 1, \dots, \gamma\}$. The complete state is denoted by $\omega = (s, \tau)$, $\omega \in \Omega = \Psi \times \Gamma$, and is assumed to be observed through periodic inspections with a fixed inspection interval $\delta$. The actual age of each facility is thus $\tau\delta$. Given an appropriate stochastic deterioration process, the state $\omega$ forms a discrete-time Markov chain with age-dependent transition probabilities.

Upon each inspection, several actions are available for each facility. Let $A$ denote the action space, $A = \{0, 1, \dots, l, l+1\}$, where action 0 means do nothing, $l + 1$ denotes CM, and action $j$ represents the $j$th level PM, $j \in A \backslash \{0, l+1\}$. We assume that CM is the only action available to a failed facility and it restores the facility back to the as-good-as-new state (i.e., $\omega = (0, 0)$). Maintenance effects of each level of PM action include the random condition improvement and random age reduction. The reduced age consequently leads to the reduction in the deterioration rate. Many studies in the civil engineering literature consider the condition improvement and deterioration rate reduction to be independent for DTI systems (Neves et al., 2006; Neves & Frangopol, 2005). Therefore, we similarly assume that the condition improvement and age reduction are independent. A PM action with a higher level (i.e., larger $j$) means that it leads to more effective maintenance outcomes. In this study, it is assumed that both PM and CM take negligible time. This assumption can be justified when the time required to complete a PM or CM action is relatively short comparing with the operational time. Since DTI systems are typically in service for a long time, such an assumption is appropriate.

Next, we model state transitions. For notational convenience, we omit the index of each facility. Let $h_0(s'|s, \tau)$ denote the condition transition probability that a facility with a current condition state $s$ and an age state $\tau$ deteriorates to a condition state $s'$ without maintenance intervention in one period. We assume that the deterioration process is irreversible when no maintenance action is performed, and thus $h_0(s'|s, \tau) = 0$ if $s' < s$ and $\sum_{s' \in \Psi} h_0(s'|s, \tau) = 1$. The transition probabilities under PM are more complicated. Recall that imperfect PM improves the facility's condition and reduces its age simultaneously. Let $Q_j = [q_j(s'|s)]_{s, s' \in \Psi}$ represent the condition improvement probability matrix of the $j$th level PM, where $q_j(s'|s) = 0$ if $s' > s$ and $\sum_{s' \in \Psi} q_j(s'|s) = 1,\ j \in A \backslash \{0, l+1\}$. This indicates that PM improves a facility to a condition state that is no worse than its current condition state. Note that $q_j(\psi|\psi) = 1,\ j \in A \backslash \{0, l+1\}$, because the only action available at a failure state is CM. Let $\Delta\tau_j$ denote the age reduction caused by the $j$th level PM, which is assumed to be a discrete random variable with the probability mass function $g_j(\Delta\tau_j)$, $\Delta\tau_j \in \Gamma_j^{\Delta},\ j \in A \backslash \{0, l+1\}$. We similarly assume that the actual age reduction is also a multiple of the inspection interval $\delta$. The facility's age immediately following the application of the $j$th level of PM is $\tau_j' = \max\{\tau - \Delta\tau_j, 0\}$. The distribution of the random age reduction caused by each level of PM can be estimated based on historical deterioration data and maintenance data. Suppose that a non-stationary stochastic process with an age-dependent parameter is chosen to describe the facility deterioration; PM reduces the facility age by a random amount, which is modeled by an appropriate distribution. Given deterioration increment data and maintenance data, we can obtain the likelihood function and estimate the distribution of the random age reduction for each level of PM using the maximum likelihood estimation method. Alternatively, reasonable assumptions regarding the age reduction distribution can be made based on engineering domain knowledge. Since the two types of maintenance effects are independent, the transition probability $h_j(s'|s, \tau)$ when the $j$th level PM is performed is given by

$$h_j(s'|s,\tau) = \sum_{m\in\Psi} g_j(\Delta\tau_j) q_j(m|s) h_0(s'|m,\tau_j'), \tag{1}$$

where $\tau_j' = \max\{\tau - \Delta\tau_j, 0\}$ and $\sum_{\Delta\tau_j\in\Gamma_j^\Delta}\sum_{s'\in\Psi} h_j(s'|s,\tau) = 1$. Note that there may exist practical problems where the condition improvement and age reduction are dependent, and our model can be easily modified to accommodate that by using a joint probability matrix. It is also noted that the perfect PM is a special case of the imperfect maintenance actions considered in this study. Let $p(\omega'|\omega, a)$ denote the transition probability given the complete state $\omega$ and the action $a$, $\omega\in\Omega$, $a\in A$. Based on the transition probabilities $h_j(s'|s,\tau)$, $j\in A\backslash\{l+1\}$, we have

$$p(\omega'|\omega,a) = \begin{cases} h_0(s'|s,\tau), & \text{if } a = 0, \tau' = \tau+1 \\ h_a(s'|s,\tau), & \text{if } a \in A\backslash\{0,l+1\}, \tau' = \tau_j'+1 \\ h_0(s'|0,0), & \text{if } a = l+1, \tau' = 1 \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Limited maintenance budgets are considered in the maintenance planning for the multi-facility DTI system. It is assumed that maintenance budgets are allocated at the beginning of each decision period and the remaining budget during a period can not be rolled over to future planning periods.

### 3.2. Joint budget allocation and maintenance optimization model

We formulate the joint optimization of budget allocation and maintenance planning as an integer programming. Consider a finite planning horizon $T$. Maintenance decisions are made at times in the set $\mathscr{T} = \{0, 1, \dots, T-1\}$. Let maintenance decisions of all facilities over the decision making horizon be denoted by $X = \{x_{a_t}^{\omega_{i,t}} : \omega_{i,t}\in\Omega, a_t\in A, i\in N, t\in\mathscr{T}\}$. The decision variable $x_{a_t}^{\omega_{i,t}}$ represents the maintenance decision of facility $i$ in state $\omega_{i,t}$ at epoch $t$, which equals 1 if maintenance action $a_t$ is performed and 0 otherwise. Since only one action is chosen for each facility in any state at an epoch, we have $\sum_{a_t\in A} x_{a_t}^{\omega_{i,t}} = 1, \omega_{i,t}\in\Omega, i\in N, t\in\mathscr{T}$. We first derive the $t$-step transition probability for each facility. The subscript $i$ representing the index of each facility is omitted for notational convenience. Let $\phi_t(\omega_t|\omega_0, X)$ denote the $t$-step transition probability of a facility that transits to a state $\omega_t$ from its initial state $\omega_0$ after $t$ periods under maintenance decisions $X$. The 1-step transition probability is $\phi_1(\omega_1|\omega_0,X) = \sum_{a_0\in A} p(\omega_1|\omega_0,a_0)x_{a_0}^{\omega_0}$. If action $j$ is chosen for state $\omega_0$, i.e., $x_j^{\omega_0} = 1$, we have $\sum_{a_0\in A} p(\omega_1|\omega_0,a_0)x_{a_0}^{\omega_0} = p(\omega_1|\omega_0,j)$. Similarly, we derive the 2-step transition probability $\phi_2(\omega_2|\omega_0,X) = \sum_{a_1\in A}\sum_{\omega_1\in\Omega}\sum_{a_0\in A}(p(\omega_1|\omega_0,a_0)x_{a_0}^{\omega_0})(p(\omega_2|\omega_1,a_1)x_{a_1}^{\omega_1})$. The generic form of $\phi_t(\omega_t|\omega_0, X)$ is given by

$$\begin{aligned}
&\phi_t(\omega_t|\omega_0,X) \\
&= \begin{cases} \sum\limits_{a_0\in A} p(\omega_1|\omega_0,a_0)x_{a_0}^{\omega_0}, & t=1 \\ \sum\limits_{a_{t-1}\in A}\sum\limits_{\omega_{t-1}\in\Omega} \phi_{t-1}(\omega_{t-1}|\omega_0,X)p(\omega_t|\omega_{t-1},a)x_{a_{t-1}}^{\omega_{t-1}}, & t\in\{2,\dots,T\}. \end{cases} \\
&= \begin{cases} \sum\limits_{a_0\in A} p(\omega_1|\omega_0,a_0)x_{a_0}^{\omega_0}, \quad t=1 \\ \sum\limits_{\omega_{t-1}\in\Omega}\cdots\sum\limits_{\omega_2\in\Omega}\sum\limits_{\omega_1\in\Omega}\sum\limits_{a_{t-1}\in A}\cdots\sum\limits_{a_1\in A}\sum\limits_{a_0\in A}\prod\limits_{k=0}^{t-1} p_i(\omega_{k+1}|\omega_k,a_k)x_{a_k}^{\omega_k}, \quad t\in\{2,\dots,T\}. \end{cases}
\end{aligned} \tag{3}$$

We assume that the states of all facilities at epoch 0 are known, denoted by $\omega_0 = (\omega_{1,0},\dots,\omega_{n,0})$. Let $\theta_t(X;\ \omega_0)$ represent the expected maintenance cost incurred by all facilities as epoch $t$ given the initial system state $\omega_0$. Based on the $t$-step transition probabilities, we obtain $\theta_t(X;\ \omega_0)$ as follows

$$\begin{aligned}
&\theta_t(X;\omega_0) \\
&= \begin{cases} \sum\limits_{i\in N}\sum\limits_{a\in A} c_i(a)x_a^{\omega_{i,0}}, & t=0 \\ \sum\limits_{i\in N}\sum\limits_{a\in A}\sum\limits_{\omega_{i,t}\in\Omega} \phi_{i,t}(\omega_{i,t}|\omega_{i,0},X)c_i(a)x_a^{\omega_{i,t}}, & t\in\mathscr{T}\backslash\{0\}, \end{cases}
\end{aligned} \tag{4}$$

Let $b_t$ denote the maintenance budget for period $t$, $t\in\mathscr{T}$. We model the budget constraints as $\theta_t(X;\ \omega_0)\le b_t$, $t\in\mathscr{T}$, namely the expected maintenance cost incurred at each epoch can not exceed the allocated budget. Note that constraining the expected maintenance cost of each period does not guarantee that the budget constraint is satisfied in all possible scenarios. However, the number of possible scenarios grows exponentially as the number of facilities or the number of decision periods increases, and therefore it is computationally burdensome to find an optimal policy that satisfies the budget constraints in all scenarios. Thus, we consider the expected maintenance cost at each period in this study.

Let $f(X;\ \omega_0)$ represent the total expected maintenance cost of all facilities over the finite planning horizon given that the system is initially in state $\omega_0$. A discount factor is generally needed if a relatively long planning horizon is considered. The objective is to minimize the total expected maintenance cost subject to budget constraints. We formulate the problem of our interests as an integer programming. The analytical formulation is given by

(P):

$$\min_X f(X;\omega_0) = \sum_{t\in\mathscr{T}}\theta_t(X;\omega_0) + \sum_{i\in N}\sum_{\omega_{i,T}\in\Omega}\phi_{i,T}(\omega_{i,T}|\omega_{i,0},X)\eta_i(\omega_{i,T}) \tag{5}$$

s.t.

$$\eta_i(\omega_{i,T}) = \begin{cases} c_i(l+1), & \omega_{i,T}\in\Omega_\psi, i\in N \\ 0, & \omega_{i,T}\in\Omega\backslash\Omega_\psi, i\in N \end{cases} \tag{6}$$

$$\theta_t(X;\omega_0)\le b_t, t\in\mathscr{T} \tag{7}$$

$$\sum_{a\in A} x_a^{\omega_{i,t}} = 1, \omega_{i,t}\in\Omega, i\in N, t\in\mathscr{T} \tag{8}$$

$$\left(1 - x_{l+1}^{\omega_{i,t}}\right)s_{i,t} \le \psi - 1, \omega_{i,t}\in\Omega, i\in N, t\in\mathscr{T} \tag{9}$$

$$x_a^{\omega_{i,t}}\in\{0,1\}, \omega_{i,t}\in\Omega, a\in A, i\in N, t\in\mathscr{T} \tag{10}$$

Eq. (5) is the objective function representing the total expected maintenance cost of all facilities from epoch 0 to $T$ given that the system is initially in state $\omega_0$. Eq. (6) provides the maintenance cost of each facility in any state at the end of the planning horizon, where $\Omega_\psi$ denotes the failure state space, $\Omega_\psi = \{(\psi,\tau) : \tau\in\Gamma\}$. We assume that at the end of the planning horizon, i.e., epoch $T$, if a facility is in a failure state, CM is performed. Otherwise, we do nothing on the facility. As such, the maintenance cost of a failed facility at epoch $T$ is CM cost and the cost of a functioning facility is zero. Eq. (7) ensures that the expected maintenance cost incurred at each epoch can not exceed the allocated budget. Eq. (8) guarantees that only one action is performed on each facility in any state at a decision epoch. Eq. (9) ensures the implementation of CM on a failed facility at any epoch. Eq. (10) is the integrality constraint for each decision variable.

The formulated P is nonlinear because of the nonlinear term in the $t$-step transition probabilities (Eq. (3)). We use the standard linearization method (Rardin & Rardin, 1998) to linearize P. We introduce auxiliary binary variables $z_{a^t}^{\omega_i^t} = \prod_{k=0}^{t} x_{a_k}^{\omega_{i,k}}$, $z_{a^t}^{\omega_i^t}\in$

$\{0, 1\}$, where $a^t = (a_0, \ldots, a_t)$ and $\omega_i^t = (\omega_{i,0}, \ldots, \omega_{i,t})$, $t \in \mathscr{T}\backslash\{0\}$. The auxiliary variables are subject to the following constraints.

$$z_{a^t}^{\omega_i^t} \le x_{a_k}^{\omega_{i,k}}, \omega_{i,k} \in \Omega, a_k \in A, k \in \{0, \ldots, t\}, t \in \mathscr{T}\backslash\{0\}, \tag{11}$$

$$z_{a^t}^{\omega_i^t} \ge \sum_{k=0}^{t} x_{a_k}^{\omega_{i,k}} - t, \omega_{i,k} \in \Omega, a_k \in A, k \in \{0, \ldots, t\}, t \in \mathscr{T}\backslash\{0\} \tag{12}$$

The numbers of decision variables and constraints in P after linearization grow exponentially as the number of planning horizon increases. Our computational studies in Section 5 show that the state-of-the-art solver (i.e., CPLEX) can only solve small-scale problems. Therefore, we reformulate the problem and develop efficient algorithms to find satisfactory solutions for large-scale problems in the next sections.

### 3.3. Model reformulation

We reformulate the integer programming (P) as a sum of multiple MDP models with budget constraints. Let $\Pi$ denote the maintenance decisions of all facilities in any state over the entire planning horizon, $\Pi = \{\pi_{i,t}(\omega_{i,t}) \in A : \omega_{i,t} \in \Omega\backslash\Omega_\psi, i \in N, t \in \mathscr{T}\} \cup \{\pi_{i,t}(\omega_{i,t}) = l+1 : \omega_{i,t} \in \Omega_\psi, i \in N, t \in \mathscr{T}\}$. We denote the total expected maintenance cost from epoch 0 to $T$ given the initial system state $\omega_0$ by $g(\Pi; \omega_0)$. Problem P is reformulated as follows.

(P′):

$$\min_{\Pi} g(\Pi; \omega_0) = \sum_{i\in N} u_{i,0}(\omega_{i,0}; \Pi) \tag{13}$$

s.t.

$$u_{i,t}(\omega_{i,t}; \Pi) = \begin{cases} \sum_{\pi_{i,t}(\omega_{i,t})\in A} \begin{bmatrix} c_i(\pi_{i,t}(\omega_{i,t})) + \sum_{\omega_{i,t+1}\in\Omega} \\ p_i(\omega_{i,t+1}|\omega_{i,t}, \pi_{i,t}(\omega_{i,t}))u_{i,t+1}(\omega_{i,t+1}; \Pi) \end{bmatrix}, & \omega_{i,t} \in \Omega, i \in N, t \in \mathscr{T} \\ \eta_i(\omega_{i,T}), & \omega_{i,T} \in \Omega, i \in N, t = T \end{cases} \tag{14}$$

$$\theta_t'(\Pi; \omega_0) \le b_t, t \in \mathscr{T} \tag{15}$$

where

$$\theta_t'(\Pi; \omega_0) = \begin{cases} \sum_{i\in N} c_i(\pi_{i,0}(\omega_{i,0})), & t = 0 \\ \sum_{i\in N}\sum_{\omega_{i,t}\in\Omega} \phi_{i,t}'(\omega_{i,t}|\omega_{i,0}, \Pi)c_i(\pi_{i,t}(\omega_{i,t})), & t \in \mathscr{T}\backslash\{0\}, \end{cases} \tag{16}$$

and

$$\phi_{i,t}'(\omega_{i,t}|\omega_{i,0}, \Pi) = \begin{cases} p_i(\omega_{i,1}|\omega_{i,0}, \pi_{i,0}(\omega_{i,0})), & t = 1 \\ \sum_{\omega_{i,t-1}\in\Omega} \phi_{i,t-1}(\omega_{i,t-1}|\omega_{i,0}, \Pi)p_i(\omega_{i,t}|\omega_{i,t-1}, \pi_{i,t-1}(\omega_{i,t-1})), & t \in \{2, \ldots, T\}. \end{cases} \tag{17}$$

Eq. (13) is the objective function which is the sum of the expected maintenance costs of all facilities in state $\omega_0$ in the multiple MDP models. Eq. (14) represents the expected maintenance cost of facility $i$ in state $\omega_{i,t}$ from epoch $t$ to $T$ given maintenance decisions $\Pi$. Note that we have $u_{i,T}(\omega_{i,T}; \Pi) = \eta(\omega_{i,T})$ in Eq. (14), which is the maintenance cost at the end of the planning horizon. Eq. (15) ensures that the expected maintenance cost incurred at each epoch does not exceed the allocated budget.

The Lagrangian relaxation (L$\mu$) to the reformulated model (P′) is as follows.

(L$\mu$):

$$\min_{\Pi} g_\mu(\Pi; \omega_0) = \sum_{i\in N} u_{i,0}^\mu(\omega_{i,0}; \Pi) - \sum_{t\in\mathscr{T}} \mu_t b_t \tag{18}$$

s.t.

$$u_{i,t}^\mu(\omega_{i,t}; \Pi) = \begin{cases} \sum_{\pi_{i,t}(\omega_{i,t})\in A} \begin{bmatrix} c_i^\mu(\pi_{i,t}(\omega_{i,t})) + \sum_{\omega_{i,t+1}\in\Omega} \\ p_i(\omega_{i,t+1}|\omega_{i,t}, \pi_{i,t}(\omega_{i,t}))u_{i,t+1}^\mu(\omega_{i,t+1}; \Pi) \end{bmatrix}, & \omega_{i,t} \in \Omega, i \in N, t \in \mathscr{T} \\ \eta_i(\omega_{i,T}), & \omega_{i,T} \in \Omega, i \in N, t = T \end{cases} \tag{19}$$

where

$$c_i^\mu(\pi_{i,t}(\omega_{i,t})) = (1 + \mu_t)c_i(\pi_{i,t}(\omega_{i,t})), \omega_{i,t} \in \Omega, i \in N, t \in \mathscr{T} \tag{20}$$

Eq. (18) is the objective function representing the difference between the sum of the expected penalized maintenance costs of all facilities from epoch 0 to $T$ and the sum of the penalized budgets over the entire decision periods. Note that $\mu$ is the Lagrangian multipliers, $\mu = (\mu_0, \ldots, \mu_{T-1})$. Eq. (19) represents the expected penalized maintenance cost of facility $i$ in state $\omega_{i,t}$ incurred from epoch $t$ to $T$ under maintenance decisions $\Pi$. The penalized maintenance costs (i.e., $c_i^\mu(\pi_{i,t}(\omega_{i,t}))$) in Eq. (20) are distributed to all facilities in any states over the entire planning periods. Given values of the Lagrangian multipliers, the Lagrangian relaxation problem can be decomposed into $n$ individual MDP problems. We omit the subscript $i$ representing the index of the facility for notational convenience. Eq. (21) shows the optimality equation with the penalized maintenance cost for each facility.

$$v_t^\mu(\omega) = \begin{cases} \min_{a\in A} \left\{ c^\mu(a) + \sum_{\omega'\in\Omega} p(\omega'|\omega, a)v_{t+1}^\mu(\omega') \right\}, & \omega \in \Omega \\ c^\mu(l+1) + v_t^\mu(0, 0), & \omega \in \Omega_\psi. \end{cases} \tag{21}$$

where $v_t^\mu(\omega)$ represents the cost-to-go function of a facility in state $\omega$ from epoch $t$ to $T$ with the penalized maintenance cost $c^\mu(a)$. We have $\sum_{i\in N} v_{i,0}^\mu(\omega_{i,0}) - \sum_{t\in\mathscr{T}} \mu_t b_t = \min_\Pi g_\mu(\Pi; \omega_0)$.

Each individual MDP can be solved efficiently by the value iteration or the policy iteration algorithm. Upon solving $n$ MDP problems, we obtain the optimal solution to L$\mu$, denoted by $\Pi_\mu^*$. The optimal value of the Lagrangian relaxation problem provides a lower bound of the primal problem. To obtain the tightest lower bound, we need to solve the Lagrangian dual problem: $\max_{\mu\ge 0} F(L\mu)$, where $F(L\mu) = g_\mu(\Pi_\mu^*, \omega_0)$. An efficient solution procedure is developed to solve the Lagrangian dual problem in the next section.

## 4. Solution Procedure

In this section, we develop a priority-based two-stage method to solve the Lagrangian dual problem. The structure of the two-stage method is illustrated in Fig. 1. In Stage 1, given values of the Lagrangian multipliers $\mu$, we obtain the optimal solution $\Pi_\mu^*$ to the Lagrangian relaxation problem (L$\mu$) and a lower bound $L$ by solving multiple individual MDPs. If $\Pi_\mu^*$ is infeasible to the primal problem (P′), we use the proposed priority-based heuristic algorithm to modify the infeasible solution to a feasible one ($\hat{\Pi}$). The feasible solution provides an upper bound $U$. Given $\Pi_\mu^*$ and $U$, we use the subgradient optimization method to update the Lagrangian multipliers. The two stages are repeated iteratively until the difference between the upper and lower bounds satisfies a prespecified criterion or a predetermined maximum number of iterations is reached. The priority-based two-stage method proposed in this paper is an extension of the solution approach in Ohlmann and Bean (2009). The main difference between our method and the one in Ohlmann and Bean (2009) is how to modify the infeasible solution obtained in Stage 1 into a high-quality feasible one. Ohlmann and Bean (2009) randomly select a pair of a facility and a state, and modify the maintenance action for the selected pair. The alternative maintenance action can be any action

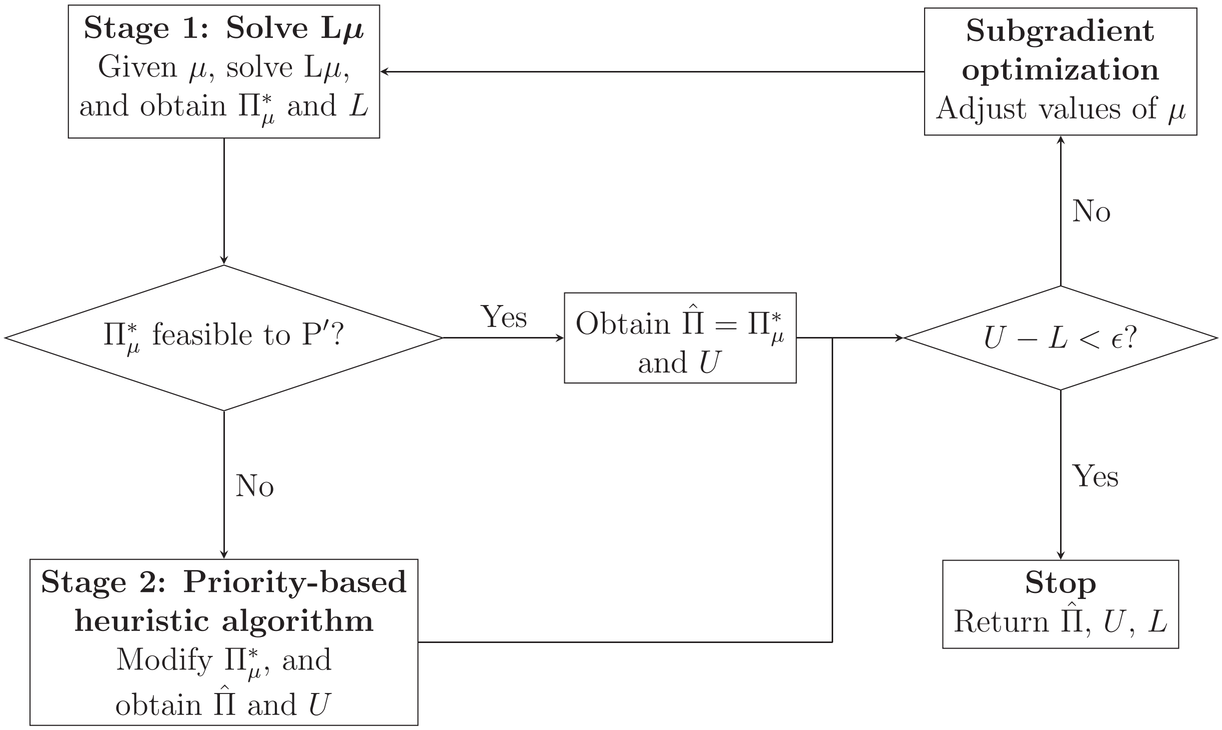


**Fig. 1.** Structure of the two-stage method.

available to that facility-state pair. Using the method in Ohlmann and Bean (2009), it is possible that a more expensive maintenance action is chosen as the alternative action, which further violates the budget constraint. In contrast to the random selection method in Ohlmann and Bean (2009), we select a facility-state pair based on certain priority and only consider less expensive maintenance actions for the selected pair so that the maintenance cost can be more efficiently and effectively improved to satisfy budget constraints.

## 4.1. Priority-based heuristic algorithm

Based on the solution $\Pi^*_\mu$ to the Lagrangian relaxation problem in Stage 1, we seek a high-quality feasible solution to the primal problem in Stage 2. If $\Pi^*_\mu$ is feasible to P′, we have obtained the feasible solution $\hat{\Pi} = \Pi^*_\mu$. The objective value of the primal problem $g(\hat{\Pi}, \omega_0)$ is computed and an upper bound $U$ is obtained. If $\Pi^*_\mu$ is infeasible to P′, the question becomes how to quickly find a good feasible solution. We propose a priority-based heuristic algorithm to efficiently modify the infeasible solution into a high-quality feasible one. We start with the decision epoch when we have the first budget violation and modify the maintenance actions at that epoch such that the expected maintenance cost incurred is below the allocated budget at the current epoch. We repeat this procedure epoch by epoch forward in time. Suppose the budget constraint is first violated at epoch $t$. Let $\chi_t$ denote the set of all possible facility-state pairs for action modification under the current solution $\hat{\Pi}$, $\chi_t = \{(i, \omega_{i,t}) : \phi'_{i,t}(\omega_{i,t}|\omega_{i,0}, \hat{\Pi}) > 0, \omega_{i,t} \in \Omega \backslash \Omega_\psi, i \in N\}$. Note that the $t$-step transition probabilities for a facility in some future states are zero because these states are not attainable by the facility at epoch $t$. For example, if we do nothing on a functioning facility with an initial state $(s, \tau)$ at epoch 0, the facility can not transit to a state $(s', \tau')$, $\tau' \neq \tau + 1$, at epoch 1. We consider only facility-state pairs with positive $t$-step transition probabilities at epoch $t$ in the set $\chi_t$. Facilities in failure states are also excluded in $\chi_t$, since CM is the only action available to them. Based on the current solution $\hat{\Pi}$, we construct an action set $\rho_t$ that includes the current decisions of all facility-state pairs in the set $\chi_t$, $\rho_t = \{\hat{\pi}_{i,t}(\omega_{i,t}) : (i, \omega_{i,t}) \in \chi_t\}$. We rank all facility-state pairs based on their priorities and modify their actions one by one. The essential task becomes how to select the facility-state pair and determine the alternative maintenance action for the selected pair so that the budget constraint at the current period is satisfied and the increase in the total maintenance cost is minimized. Different facility-state pairs may have different impacts on system operations and incur different maintenance costs. If we first change the current action of a facility in a worse state with a less effective yet cheaper one, the expected maintenance cost incurred in the current period can often be reduced significantly but it is more likely that the facility will fail soon and cause increases of maintenance costs in future periods. On the other hand, if we replace the action of a facility in a better state with a less expensive one first, there is less risk for the facility to fail; however, the cost decrease may not be sufficient to meet the budget constraint. In this paper, we explore three priority rules, represented by Rules 1, 2, and 3, respectively, to determine the priority of facility-state pairs.

**Rule 1: facility-state pair with the lowest-level action first**. In Rule 1, we prioritize facility-state pairs based on their current actions and rank these actions in an ascending order. A facility-state pair requiring a low-level action implies that it has less risk to fail. By selecting such a pair first, it is less likely that future maintenance costs will be penalized by facility failures. If there is a tie among multiple actions, we arbitrarily break the tie. We now present how to determine the alternative action for a selected facility-state pair. The alternative action is chosen such that the increase of the expected penalized maintenance cost of the selected facility is minimized. Denote the alternative action for a selected pair $(i, \omega_{i,t})$ by $\hat{\pi}'_{i,t}(\omega_{i,t})$, which is determined as follows.

$$\hat{\pi}'_{i,t}(\omega_{i,t}) = \arg \min_{a < \hat{\pi}_{i,t}(\omega_{i,t})} \left\{ \kappa^\mu_{i,t}(\omega_{i,t}; a) - \kappa^\mu_{i,t}\left(\omega_{i,t}; \hat{\pi}_{i,t}(\omega_{i,t})\right) \right\}, \quad (22)$$

where

$$\kappa_{i,t}^{\mu}(\omega_{i,t}; a) = c_i^{\mu}(a) + \sum_{\omega' \in \Omega} p_i(\omega'|\omega_{i,t}, a) v_{i,t+1}^{\mu}(\omega'). \tag{23}$$

Note that the feasible alternative actions are the ones with lower maintenance costs comparing to the current one so that the expected maintenance cost of the current period can be decreased to meet the budget constraint. We have $v_{i,t}^{\mu}(\omega_{i,t}) = \min_{a \in A} \kappa_{i,t}^{\mu}(\omega_{i,t}; a)$ and $\kappa_{i,t}^{\mu}(\omega_{i,t}; a)$ is the expected penalized maintenance cost of facility $i$ in state $\omega_{i,t}$ from epoch $t$ to $T$ if action $a$ is performed.

**Rule 2: facility-state pair with the highest-level action first**. Rule 2 ranks facility-state pairs based on their actions in a descending order so that the expected maintenance cost incurred in the current period can be effectively reduced to satisfy the budget constraint. The tie among multiple actions is also broken arbitrarily. The alternative action for a selected facility-state pair is similarly determined by Eq. (22).

**Rule 3: facility-state pair with the smallest increase in the expected penalized maintenance cost first**. Given the alternative actions of all facility-state pairs at the current epoch, we first select the facility-state pair in Rule 3 such that the increase of the expected penalized maintenance cost is minimized. Eq. (24) shows how we choose the facility-state pair.

$$(i^*, \hat{\omega}_{i^*,t}) = \arg \min_{(i, \omega_{i,t}) \in \chi_t} \left\{ \kappa_{i,t}^{\mu}(\omega_{i,t}; \hat{\pi}_{i,t}'(\omega_{i,t})) - \kappa_{i,t}^{\mu}\left(\omega_{i,t}; \hat{\pi}_{i,t}(\omega_{i,t})\right) \right\}. \tag{24}$$

The alternative action $\hat{\pi}_{i,t}'(\omega_{i,t})$ for each facility-state pair in Eq. (24) is determined using Eq. (22). We find the facility-state pair with the first priority among those in the set $\chi_t$ that minimizes the increase of the expected penalized maintenance cost. Note that Rule 3 guarantees the smallest impact of each action modification on the total maintenance cost.

Based on one of the three priority rules as stated previously, we first select a facility-state pair, e.g., $(i^*, \omega_{i^*,t})$, and replace its current action $\hat{\pi}_{i^*,t}(\omega_{i^*,t})$ with the alternative one $\hat{\pi}_{i^*,t}'(\omega_{i^*,t})$. We then update the action set $\rho_t$, the maintenance solution $\hat{\Pi}$ and the expected maintenance cost $\theta_t'(\hat{\Pi}; \omega_0)$. The action modification procedure is repeated iteratively until the budget constraint of the current period is satisfied or $\rho_t$ becomes empty. Algorithm 1 summarizes the priority-based heuristic in detail.

**Algorithm 1** Priority-based heuristic.

**Input:** Infeasible solution $\Pi_{\mu}^*$ to P′;
**Output:** High-quality feasible solution $\hat{\Pi}$ to P′ and the objective value $g(\hat{\Pi}, \omega_0)$;

```
Initialize Π̂ ← Π*_μ
for t = 0 : T − 1 do
    Construct ρ_t
    while θ'_t(Π̂; ω_0) − b_t > 0 and ρ_t ≠ ∅ do
        Select π̂_{i*,t}(ω_{i*,t}) from ρ_t based on Rule 1, 2, or 3;
        Obtain π̂'_{i*,t}(ω_{i*,t}) based on Eq. (22);
        π̂_{i*,t}(ω_{i*,t}) ← π̂'_{i*,t}(ω_{i*,t});
        Update ρ_t;
        Update Π̂;
        Update θ'_t(Π̂; ω_0);
    end while
    if θ'_t(Π̂; ω_0) − b_t > 0 then
        Stop procedure: Π̂ is infeasible;
        return
    end if
    Update φ'_{i,t+1}(ω_{i,t+1}|ω_{i,0}, Π̂), ∀ω_{i,t+1} ∈ Ω;
    Update θ'_{t+1}(Π̂; ω_0);
end for
```

### 4.2. Subgradient optimization

Subgradient optimization is a commonly used method for the Lagrangian dual problem because of its easy implementation and effective performance (Fisher, 1981). Given initial values of the Lagrangian multipliers $\mu^0$, a sequence of $\{\mu^k\}$ is generated by the following rule,

$$\mu_t^{k+1} = \max\left\{0, \mu_t^k + \xi_k\left(\theta_t'(\Pi_{\mu^k}; \omega) - b_t\right)\right\}, \forall t \in \mathscr{T}, \tag{25}$$

where $\Pi_{\mu^k}^*$ is the optimal solution to $\mathrm{L}\mu^k$ and $\xi_k$ is a positive scalar step size at the $k$th iteration. The step size $\xi_k$ at each iteration is computed as

$$\xi_k = \frac{\sigma\left(U^k - F(\mathrm{L}\mu^k)\right)}{\sum_{t \in \mathscr{T}} \left[\theta_t'(\Pi_{\mu^k}; \omega_0) - b_t\right]^2} \tag{26}$$

where $0 \le \sigma \le 2$ and $U^k$ is the upper bound of P′ generated in Stage 2. Note that $\sigma$ is reduced if the lower bound has not been updated for a certain number of iterations. The algorithm stops if the difference between the upper bound and the lower bound satisfies a prespecified criterion or a maximum number of iterations is reached. Algorithm 2 summarizes the standard subgradient optimization method.

**Algorithm 2** Subgradient method Beasley (1993).

**Input:** Instance of $\mathrm{L}\mu$ and initial states $\omega_0$;
**Output:** Lower bound $L$ and upper bound $U$ of P′;

```
Initialize ε, σ, countL, maxIter, ν ← 0, k ← 0, μ_t^0 ← 0, ∀t ∈ 𝒯,
    L ← 0, and U ← ∞;
while stopping criteria not satisfied do
    Obtain Π*_{μ^k} and F(Lμ^k) by solving n MDPs;
    if F(Lμ^k) > L then
        L ← F(Lμ^k);
        ν ← 0;
    else
        ν ← ν + 1;
    end if
    if Π*_{μ^k} is infeasible then
        Repair Π*_{μ^k} to Π̂^k and obtain g(Π̂^k, ω_0) via Algorithm 1;
        if g(Π̂^k, ω_0) < U and θ'_t(Π̂^k, ω_0) − b_t ≤ 0, ∀t ∈ 𝒯 then
            U ← g(Π̂^k, ω_0);
        end if
    end if
    k ← k + 1;
    Update μ^k based on Eqs. (25) and (26);
    if ν > countL then
        σ ← σ/2;
        ν ← 0;
    end if
    if U − L < ε or k > maxIter then
        break;
    end if
end while
```

## 5. Computational studies

In this section, we first compare the computational times of solving the reformulation model (P′) using the proposed two-stage method with those of solving the integer programming (P) using CPLEX for small-scale problems. Next, we examine the performance of the three priority rules in our priority-based two-stage

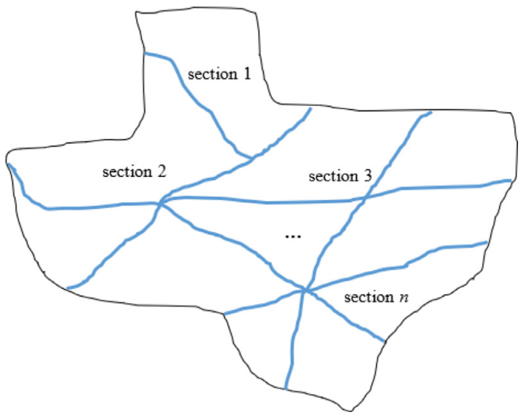


**Fig. 2.** A pavement system of multiple pavement sections.

**Table 1**
Parameter settings in the computational studies.

| | Parameter | Value |
|---|---|---|
| Scale parameter | $\beta$ | $U(0.2, 0.8)$ |
| Condition improvement probability matrix | $Q_1^{k_1}$ | $k_1 \in U\{1, 2\}$ |
| | $Q_2^{k_2}$ | $k_2 \in U\{1, 2\}$ |
| | $Q_3^{k_3}$ | $k_3 \in U\{1, 2\}$ |
| Age reduction set | $\Gamma_1^{\Delta}$ | $\{U\{1, 2\}, U\{2, 4\}, U\{3, 6\}\}$ |
| | $\Gamma_2^{\Delta}$ | $\{U\{2, 4\}, U\{4, 8\}, U\{6, 12\}\}$ |
| | $\Gamma_3^{\Delta}$ | $\{U\{3, 6\}, U\{6, 12\}, U\{9, 18\}\}$ |
| Maintenance cost | $c(1)$ | $U(1, 2)$ |
| | $c(2)$ | $U(4, 8)$ |
| | $c(3)$ | $U(8, 16)$ |
| | $c(4)$ | $U(16, 32)$ |

method for large-scale problems. We also investigate the properties of the optimal maintenance decisions by comparing the optimal maintenance decisions with and without budget constraints.

Consider a pavement system of multiple pavement sections (illustrated in Fig. 2). The state of each pavement section is inspected at the beginning of each year, i.e., $\delta = 1$. We use the international roughness index (IRI) to characterize the deterioration of each pavement section. The IRI is reported in units of inches per mile (in/mi), which is scaled with zero being the smoothest and infinity being the upper limit. Pavements with the IRI greater than 170 are considered to have “unacceptable” ride quality (FHWA & FTA, 2017). Thus we have the failure threshold of each pavement section $\zeta = 170$. Since the Crack, Rut, and Ride indices are all measured on 0–10 scales, we follow the convention and discretize the continuous IRI into 11 intervals, representing 11 condition states, i.e., $\Psi = \{0, 1, \ldots, 10\}$. The number of age states is considered to be 51, i.e., $\Gamma = \{0, 1, \ldots, 50\}$. We model the deterioration process of each pavement section using a non-stationary gamma process in the computational studies, which has a property of independent increments so that the state of each facility can be modeled as a Markov chain. The gamma process has an age-dependent shape parameter $\alpha(t) = ct^b$ and a scale parameter $\beta$. The transition probability without maintenance intervention $h_0(s'|s, \tau)$ is derived from the cumulative distribution function of the gamma process. We consider a PM strategy with three levels for the pavement system and the action space is $A = \{0, 1, 2, 3, 4\}$. Each level of the PM action has its own condition improvement probability matrix $Q_j$ and age reduction set $\Gamma_j^{\Delta}$, $j = 1, 2, 3$. We assume that the random age reduction caused by each level of PM has three possible values with equal probabilities, i.e., $\Gamma_j^{\Delta} = \{\Delta\tau_j^1, \Delta\tau_j^2, \Delta\tau_j^3\}$ and $g_j(\Delta\tau_j^1) = g_j(\Delta\tau_j^2) = g_j(\Delta\tau_j^3) = 1/3$, $j = 1, 2, 3$. To model the facility heterogeneity in the computational studies, we consider different deterioration processes, maintenance effects, and maintenance costs for different pavement sections. We use $\alpha(t) = 5.62t^{1.5}$ as the shape parameter function, which is determined in Shi et al. (2019b) using the IRI data of multiple road sections over several years in the state of Florida (FDOT, 2015). We assume that the time-varying shape parameter is the same for all pavement sections. Other model parameters that vary among multiple pavement sections are summarized in Table 1, where $U(\cdot, \cdot)$ and $U\{\cdot, \cdot\}$ represent the continuous uniform distribution and the discrete uniform distribution, respectively. Details of the condition improvement probability matrices are presented in Appendix A.1.

### 5.1. Performance of the proposed method on small-scale problems

We consider a two-period problem in this computational study because the number of decision variables and the number of constraints in P after linearization are more than a million even for a two-facility system with three planning periods due to the large state space (i.e., $|\Omega| = 561$). We solve P′ with two-periods using our priority-based two-stage method and compare the computational results with those of P solved using CPLEX. Different numbers of pavement sections are considered in the computational study. Table 2 presents the results for different cases. NA is reported when the computational time is longer than four hours. From Table 2, we can see that our priority-based algorithms have zero optimality gaps for all problem scales considered. Note that the optimality gap is determined as the difference between the optimal value from CPLEX and the upper bound from our proposed method for $n \le 23$. For $n \ge 24$ when NA is reported in CPLEX, the optimality gap is measured by the difference between the upper and lower bounds obtained using our algorithms. We also observe that the computational time of solving P using CPLEX increases dramatically as the number of facilities grows, and it is beyond the prespecified time limit when the number of facilities is relatively large (i.e., $n \ge 24$). The computational times of our proposed algorithms are significantly less than those of CPLEX.

### 5.2. Performance comparison of the three priority rules

We now compare the computational performance of the three priority rules (i.e., Rules 1, 2, and 3) in the proposed priority-based two-stage method for solving multi-period problems. We consider two different planning horizons, $T \in \{5, 10\}$ and 16 different numbers ($n$) of pavement sections with the smallest one being 10 and the largest one being 40. For each problem scale (i.e., each $T$ and each $n$), we examine ten problem instances, and obtain the average computational time (avg.time) and the average percentage optimality gap (avg.gap), i.e., $\left[\sum_{m=1}^{10}(U_m - L_m)/U_m\right]/10$, where $U_m$ and $L_m$ represent the upper and lower bounds of instance $m$, respectively. Table 3 summarizes the results of the three priority rules for multi-period problems. From Table 3, we can see that the average times of the three priority rules grow reasonably as the number of pavement sections and the number of planning horizon increase. Rule 2 outperforms Rules 1 and 3 in most cases (21 out of 32 cases) in terms of computational time. This is because Rule 2 modifies the action of a facility-state pair with the highest-level maintenance action first, and generally reduces the expected maintenance cost incurred at an epoch more sufficiently than the other two rules. Rule 3 shows advantages of obtaining smaller optimality gaps than the other two rules, though it requires some extra computational efforts. This is because Rule 3 guarantees the smallest impact on the total maintenance cost so that the objective value obtained using Rule 3 is closer to the optimal objective value. We also observe that the average percentage optimality gaps of the three priority rules are below 2% for all problem scales considered, which indicates that our proposed methods provide highly satisfactory solutions to large-scale problems.

**Table 2**
Computational times (in seconds), optimal values, and optimality gaps.

| *n* | solver | | our algorithm | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | Rule 1 | | Rule 2 | | Rule 3 | |
| Time | Optimal value | Time | Optimality gap | Time | Optimality gap | Time | Optimality gap | |
| 2 | 27 | 19.8 | 0.8 | 0 | 0.9 | 0 | 1.0 | 0 |
| 3 | 76 | 35.8 | 4.1 | 0 | 4.1 | 0 | 3.9 | 0 |
| 4 | 112 | 51.7 | 3.1 | 0 | 3.1 | 0 | 3.0 | 0 |
| 5 | 238 | 67.7 | 8.7 | 0 | 8.9 | 0 | 9.1 | 0 |
| 6 | 250 | 83.7 | 4.3 | 0 | 4.4 | 0 | 4.4 | 0 |
| 7 | 522 | 99.7 | 11 | 0 | 10 | 0 | 11 | 0 |
| 8 | 2016 | 99.7 | 12 | 0 | 12 | 0 | 12 | 0 |
| 9 | 3023 | 115.7 | 24 | 0 | 24 | 0 | 24 | 0 |
| 10 | 3189 | 129.1 | 17 | 0 | 17 | 0 | 17 | 0 |
| 11 | 3253 | 145.1 | 32 | 0 | 31 | 0 | 31 | 0 |
| 12 | 3750 | 161.1 | 19 | 0 | 20 | 0 | 20 | 0 |
| 13 | 5093 | 177.1 | 38 | 0 | 38 | 0 | 34 | 0 |
| 14 | 6264 | 193.1 | 22 | 0 | 23 | 0 | 24 | 0 |
| 15 | 7304 | 209.1 | 43 | 0 | 43 | 0 | 41 | 0 |
| 16 | 8202 | 212.9 | 38 | 0 | 38 | 0 | 39 | 0 |
| 17 | 8982 | 228.9 | 63 | 0 | 66 | 0 | 65 | 0 |
| 18 | 10,530 | 244.8 | 20 | 0 | 20 | 0 | 20 | 0 |
| 19 | 10,908 | 260.8 | 29 | 0 | 29 | 0 | 29 | 0 |
| 20 | 10,392 | 276.8 | 22 | 0 | 22 | 0 | 21 | 0 |
| 21 | 10,750 | 292.8 | 31 | 0 | 31 | 0 | 32 | 0 |
| 22 | 11,643 | 292.8 | 36 | 0 | 36 | 0 | 37 | 0 |
| 23 | 14,089 | 308.8 | 52 | 0 | 45 | 0 | 45 | 0 |
| 24 | NA | | 36 | 0 | 37 | 0 | 37 | 0 |
| 25 | NA | | 50 | 0 | 50 | 0 | 50 | 0 |
| 26 | NA | | 39 | 0 | 39 | 0 | 41 | 0 |
| 27 | NA | | 53 | 0 | 52 | 0 | 53 | 0 |
| 28 | NA | | 41 | 0 | 41 | 0 | 41 | 0 |
| 29 | NA | | 55 | 0 | 55 | 0 | 56 | 0 |
| 30 | NA | | 55 | 0 | 55 | 0 | 54 | 0 |

**Table 3**
Average and maximum computational times (in seconds) of solving P using the three different priority rules.

| n | T | Rule 1 | | Rule 2 | | Rule 3 | |
|---|---|---|---|---|---|---|---|
| | | avg.time | avg.gap (%) | avg.time | avg.gap (%) | avg.time | avg.gap (%) |
| 10 | 5 | 72 | 0.00 | 54 | 0.00 | 49* | 0.00 |
| 10 | 10 | 107 | 0.00 | 82 | 0.00 | 81* | 0.00 |
| 12 | 5 | 56 | 0.00 | 52* | 0.00 | 61 | 0.00 |
| 12 | 10 | 220 | 0.00 | 207* | 0.00 | 247 | 0.00 |
| 14 | 5 | 1193 | 0.29 | 1144* | 0.29 | 1344 | 0.29 |
| 14 | 10 | 3553 | 0.95 | 3439* | 0.95 | 4025 | 0.66** |
| 16 | 5 | 184 | 0.00 | 175* | 0.00 | 202 | 0.00 |
| 16 | 10 | 2562 | 0.65 | 2468 | 0.21 | 581* | 0.00** |
| 18 | 5 | 1213 | 0.15** | 1111* | 0.15** | 1302 | 0.20 |
| 18 | 10 | 1855 | 0.00 | 1783* | 0.00 | 2071 | 0.00 |
| 20 | 5 | 311* | 0.00 | 313 | 0.00 | 319 | 0.00 |
| 20 | 10 | 3808* | 1.22 | 3819 | 1.22 | 3867 | 1.22 |
| 22 | 5 | 2402 | 0.87 | 2286* | 0.87 | 2610 | 0.49** |
| 22 | 10 | 5394 | 0.11** | 5219* | 0.11** | 6212 | 0.13 |
| 24 | 5 | 2162 | 0.45** | 2033* | 0.45** | 2475 | 0.46 |
| 24 | 10 | 3869 | 0.53 | 3593* | 0.53 | 4438 | 0.53 |
| 26 | 5 | 2277 | 0.50 | 2066* | 0.50 | 2499 | 0.43** |
| 26 | 10 | 7609 | 0.17** | 7100* | 0.17** | 8643 | 0.18 |
| 28 | 5 | 1621 | 0.04 | 1515 | 0.04 | 556* | 0.00** |
| 28 | 10 | 8488 | 0.44 | 7873* | 0.44 | 9687 | 0.41** |
| 30 | 5 | 871 | 0.00 | 597* | 0.00 | 641 | 0.00 |
| 30 | 10 | 5955 | 0.01** | 5506 | 0.01** | 3663* | 0.39 |
| 32 | 5 | 3770 | 0.61 | 3509* | 0.61 | 4020 | 0.04** |
| 32 | 10 | 4913 | 0.38 | 586* | 0.00** | 5331 | 0.00** |
| 34 | 5 | 3874 | 0.17 | 3715 | 0.17 | 3369* | 0.01** |
| 34 | 10 | 7551 | 1.17** | 7131* | 1.17** | 8369 | 1.24 |
| 36 | 5 | 2356 | 0.06 | 2288* | 0.06 | 2767 | 0.06 |
| 36 | 10 | 4806 | 0.05 | 4563* | 0.05 | 5436 | 0.05 |
| 38 | 5 | 944 | 0.00 | 870* | 0.00 | 1024 | 0.00 |
| 38 | 10 | 7722* | 1.17** | 8393 | 1.32 | 8467 | 1.17** |
| 40 | 5 | 3179 | 0.28 | 3160* | 0.28 | 3466 | 0.22** |
| 40 | 10 | 10878* | 0.87 | 11092 | 0.87 | 11460 | 0.59** |

Note: * represents the minimal avg.time of the three priority rules for each problem scale; ** represents the minimal avg.gap of the three priority rules when they are not the same for each problem scale

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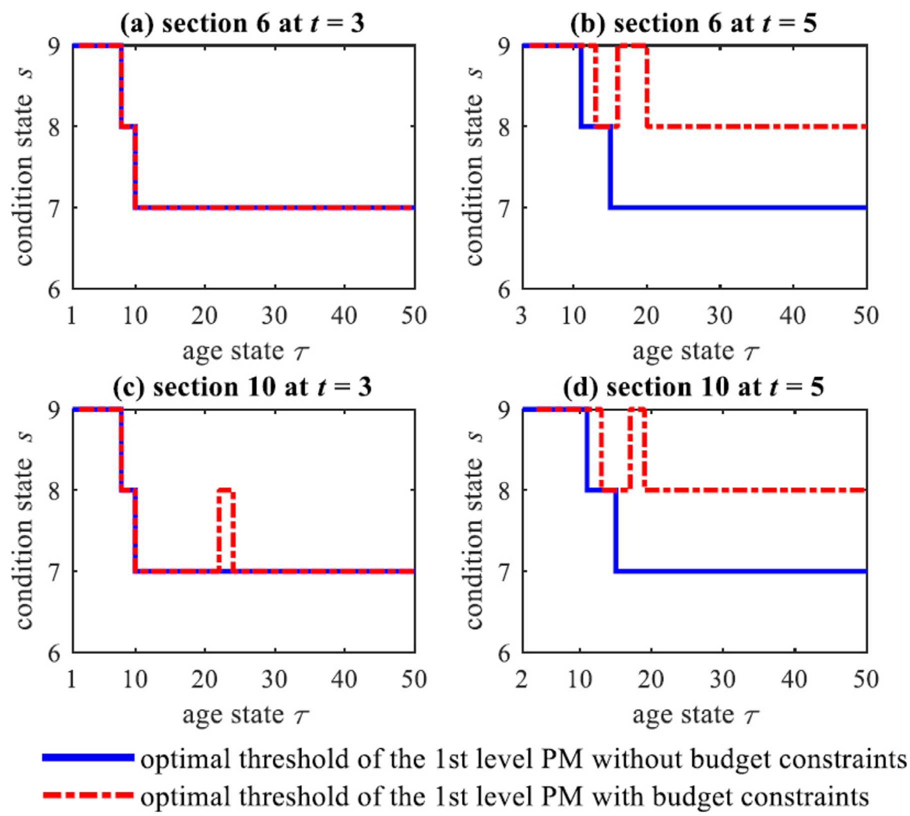


**Fig. 3.** Optimal PM thresholds for Sections 6 and 10 at epochs 3 and 5 (with budget violations).

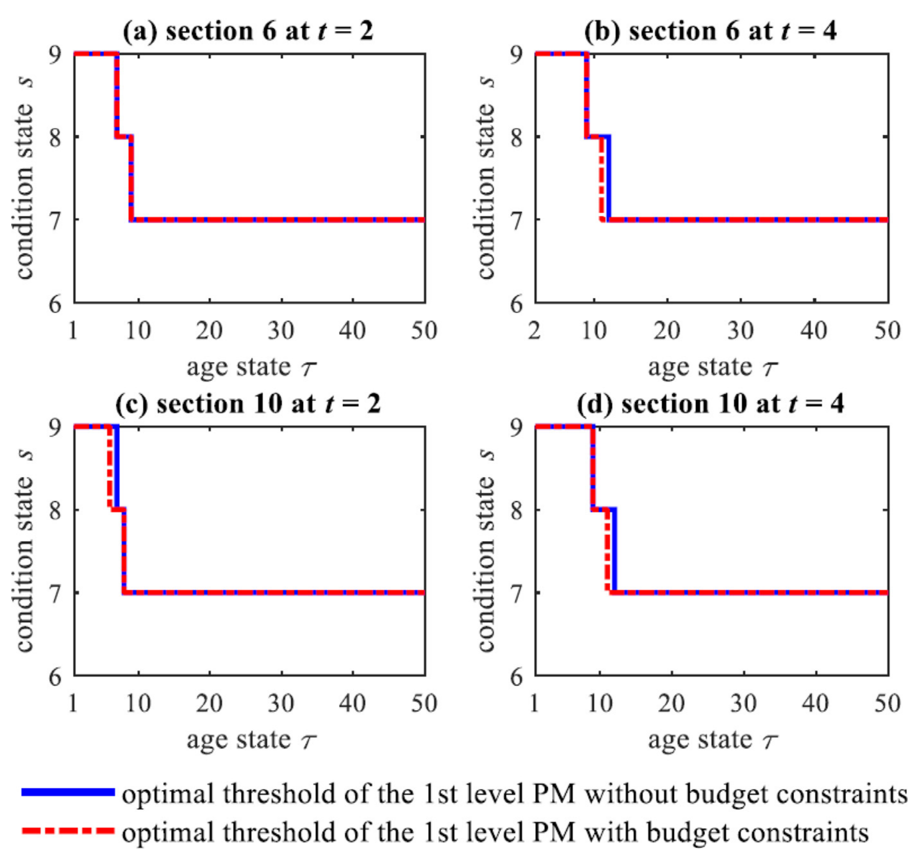


**Fig. 4.** Optimal PM thresholds for Sections 6 and 10 at epochs 2 and 4 (with no budget violations).

### *5.3. Managerial insights*

In this section, we investigate the properties of optimal maintenance decisions for multi-facility DTI systems by comparing the optimal maintenance decisions with and without budget constraints. We arbitrarily choose one problem instance considered in Section 5.2 for the analysis. This problem instance considers the maintenance optimization of 10 pavement sections ($n = 10$) over 10 planning periods ($T = 10$). When ignoring the budget constraints, the expected maintenance costs incurred at epochs 3 and 5 exceed the budgets considered in this instance. We first compare the optimal PM thresholds when the budget violation occurs. Two pavement sections (i.e., sections 6 and 10) are arbitrarily chosen for illustration purposes. Fig. 3 shows the optimal PM thresholds with and without budget constraints for the two sections at epochs 3 and 5. We also compare the optimal PM thresholds at decision epochs immediately before the violation occurs (i.e., $t = 2$ and 4), as shown in Fig. 4. From Figs. 3 and 4, we make the following important observations.

**Observation 1:** At the decision epoch when the budget violation occurs, the optimal PM thresholds with budget constraints are greater than or equal to their respective counterparts without considering budget constraints. For example, for age states that are greater than 15 at epoch 5, the optimal threshold of the 1st level PM for section 6 is $s = 8$ or 9 with the budget constraints and is $s = 7$ without the budget constraints (shown in Fig. 3(b)). A similar result is also observed for the other eight pavement sections. This implies that less effective but cheaper maintenance decisions should be considered when the budget violation occurs. In addition, we can see that the monotonically non-decreasing structure of the optimal decisions, which is typically seen in similar problem settings with no budget constraints (Shi et al., 2019a), does not exist when budget constraints present.

**Observation 2:** If the budget is sufficient in the period immediately before the decision epoch when a violation occurs, the optimal PM thresholds in that period can be lowered to encourage preventive maintenance actions in order to reduce the maintenance cost to be incurred in the next period. For example, as shown in Fig. 4 (b), when $\tau = 11$, the optimal threshold of the 1st level PM for sections 6 at epoch 4 is $s = 7$ with budget constraints and is $s = 8$ without budget constraints. The maintenance decisions of the other eight pavement sections also have a similar pattern. This implies that performing more effective maintenance actions at earlier decision epochs when budgets are sufficient can help reduce maintenance costs in future periods and satisfy budget constraints.

We also compare the optimal PM thresholds at the other six epochs and find that there is no difference of the optimal PM thresholds. Our conjecture is that the budget constraints mainly impact the decisions at epochs when the budgets are violated and earlier epochs close to when the violations occur.

## 6. Conclusion

In this paper, we consider a joint optimization problem of budget allocation and multi-level preventive maintenance planning with complex effects for deteriorating transportation infrastructure (DTI) consisting of multiple facilities. We first formulate a general integer programming for the joint optimization problem over a finite planning horizon. Optimal maintenance decisions to small-scale problems can be obtained using state-of-the-art solvers. In order to solve large-scale problems, we further reformulate the problem by modeling the maintenance optimization as a finite-horizon Markov decision process (MDP) for each individual facility and optimizing the total maintenance cost of all facilities subject to budget constraints. We decompose the reformulated problem into multiple MDP models and develop an efficient priority-based two-stage method to find high-quality solutions. The effectiveness of the proposed priority-based two-stage method is evaluated on a pavement system with multiple sections. Our numerical results show that the proposed algorithms are efficient and effective in finding satisfactory maintenance decisions for large-scale problems. We also investigate the properties of the optimal maintenance decisions and make several important observations, which provide helpful decision guidance for real-world problems.

Because multiple facilities in a DTI system are typically subject to common environmental conditions, we will extend our model by considering dependent facility deterioration in future work rather than assuming that facilities degrade independently as considered in this paper. For example, we will investigate the impacts of natural disasters, such as earthquake, hurricane, Tsunami, on the maintenance management of DTI systems. Another worthy extension is to use the chance constraint to ensure that the probability of satisfying the budget constraint (i.e., the percentage of scenarios that satisfy the budget constraint) is greater than a prespecified threshold. In addition to budget constraints, it is interesting to incorporate other types of interactions among facilities, such as traffic capacity, travelers' route choices, in multi-facility maintenance planning.

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## Appendix A

### *A1. Condition improvement probability matrices*

$$Q_1^1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$Q_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.7 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$Q_2^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$Q_3^1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$Q_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

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