

Valley Hall In-Plane Edge States as Building Blocks for Elastodynamic Logic Circuits

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In this work, we investigate theoretically and demonstrate experimentally the existence of valley Hall edge states in the in-plane dynamics of honeycomb lattices with bivalued strut thickness. We exploit these states to achieve nontrivial waveguiding of optical modes that is immune to backscattering from sharp corners. We also present how different types of interfaces can be combined into multibranch junctions to form complex waveguide paths and realize a variety of structural logic designs with unconventional wave-transport capabilities. We illustrate this potential with two applications. The first is a direction-selective energy-splitting waveguide tree featuring a pronounced asymmetric wave-transport behavior. The second is an internal waveguide loop along which the energy can be temporarily trapped and periodically released, effectively working as a signal delay. The modal complexity of in-plane elasticity has important consequences for the regime of manifestation of the edge states, as the availability of viable total band gaps is shifted to higher frequencies compared to the out-of-plane counterpart problem. It also poses additional experimental challenges, associated with proper acquisition and deciphering of the in-plane modes, the solution of which requires a systematic use of in-plane laser vibrometry.

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I. INTRODUCTION

Acoustoelastic metamaterials and phononic crystals are architected structures that are artificially designed to control and manipulate mechanical wave propagation. Some of their most well-known properties are mechanical filtering and wave directionality [1–4], waveguiding (conventional and subwavelength) [5–7], and energy trapping [8–10]. In recent years, topological insulators (TI) endowed with topologically protected edge states (TPES) have paved the way to new strategies for wave manipulation in quantum and electronic systems [11–16]. Inspired by these condensed matter problems, researchers have been able to adapt the working principle of TI to acoustic and elastic systems to achieve analogous wave control capabilities in both the static [17–21] and dynamic regimes [22–30].

One of the first quantum phenomena to inspire a mechanical analog was the quantum Hall effect (QHE), which allows one-way, nonreciprocal wave propagation by creating chiral edge states that are robust against defects and disorders [23–26]. The QHE can be realized by breaking time-reversal symmetry through the application of an external (e.g., magnetic) field [31] that results in the

lifting of a degenerate Dirac cone in the band diagram of the lattice and the opening of a nontrivial bandgap. In a corresponding mechanical system, it is possible to resort to external active controls, such as spinning rotors [23,24] or circulating fluids [25,26], to achieve analogous unidirectionally propagating edge states. However, unlike in electronic systems, the complexity required to apply external mechanical fields partially hinders the practicality of QHE for applications at the device level. In light of these limitations, researchers started exploiting the mechanical analog of the quantum spin Hall effect (QSHE), for which time-reversal symmetry breaking is not required. The QSHE can be realized by manipulating sublattice configurations to create mechanical pseudospins and pseudo-spin-dependent effective edges, ultimately achieving topologically protected helical edge modes that are also immune to backscattering [27–29]. While enjoying the inherent advantages of being fully passive, QSHE requires carefully engineered configurations in order to obtain the doubly degenerate Dirac cones whose lifting is essential to establish helical edge wave conditions. As a result, its practical applicability is also met with difficulties. Mechanical systems involving topologically protected floppy edge modes [17–20,30] have also been proposed to obtain asymmetric wave propagation. Although these systems can be easily manufactured and indeed achieve

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asymmetric wave transport without requiring any active control, the asymmetric behavior only occurs at frequencies lower than the folding region of the acoustic modes [30].

The most recent efforts in this line of developments have been directed toward creating mechanical analogs of the quantum valley Hall effect (QVHE), whose only requirement is the breaking of space-inversion symmetry (SIS), which can be achieved in a relatively agile fashion in many acoustic and mechanical systems. In acoustics, this effect has been demonstrated for sonic crystals obtained from arrays of rodlike scatterers [32,33]. In elastodynamics, the concept was first realized in honeycombs with masses added at lattice nodes [34,35] and then extended to lattices with curved struts [36,37] and thin plates endowed with arrays of surface masses [38–42]. In hexagonal lattices with a C_{6v} symmetry, Dirac points, or valleys, are observed at the high-symmetry corners of the Brillouin zone. SIS breaking, which preserves C_{3v} symmetry, results in band gap opening and in the lifting of the cones. This process leaves behind two adjacent valleys, $\mathbf{K}$ and $\mathbf{K}'$, that are largely separated in the reciprocal space and feature opposite values of the locally defined topological index known as the valley Chern number. The Chern number dichotomy between the valleys implies low modal compatibility between phonons associated with the $\mathbf{K}$ and $\mathbf{K}'$ points (which can be interpreted as waves traveling in opposite directions in the lattice), which ultimately manifests as a low degree of backscattering. A complete and lucid description of the working principle of QVHE is given in Ref. [37]. The SIS breaking requirement is relatively simple to achieve through a mechanical modulation of the unit cell parameters, making this approach much more amenable to manufacturing and therefore applicable to elastic systems. Most efforts on QVHE to date (with the notable exception of one theoretical work on in-plane elasticity in discrete systems [43]) have been devoted to the analysis of the out-of-plane flexural (antisymmetric Lamb wave) response of thin elastic plates.

In this work, we shift our attention to the in-plane dynamics of thin lattices in plane-stress conditions. Realizing viable QVHE conditions in in-plane systems features some additional challenges compared to their out-of-plane counterparts. In out-of-plane problems, a total band gap can be easily obtained at low frequencies [32–35]. This allows a straightforward establishment of edge waves that are not contaminated by any of the bulk modes (as a side note, it is worth noting that edge modes have also been successfully achieved in systems and frequency regimes where SIS breaking only induces partial band-gap opening [36,40]; this result can be explained by invoking the low modal compatibility and the large difference in the density of states between the edge and bulk modes at the frequencies of interest, which makes the establishment of edge modes more favorable). In contrast, in-plane mechanics are

characterized by additional modal complexity associated with the coexistence of longitudinal and shear modes even at low frequencies. As a result, identifying total band gaps that naturally allow for spectrally isolated edge modes inevitably requires exploring the high-frequency optical regime.

Our structure of choice is a modified hexagonal lattice consisting of beamlike struts with bivalued thickness. This configuration is characterized by extreme geometric simplicity, which makes it easy to manufacture using conventional cutting or printing techniques. While this structure features multiple band gaps that are amenable to QVHE, here we focus our attention on the high-frequency regime, where a wide total band gap is available. Under these conditions, we study numerically and demonstrate experimentally the establishment of nontrivial waveguides and their robustness against backscattering from sharp corners. Moreover, we leverage this platform to realize complicated paths involving multibranch junctions and different types of domain walls to achieve a variety of special energy channeling and routing effects. As a first example, we design a direction-selective energy-splitting waveguide tree that allows one-way wave propagation at high frequencies. We also design a triangular waveguide loop that works as a signal delay: here, a portion of the energy injected in the system is temporarily trapped in the loop and then released periodically at an output boundary with programmable delays.

II. TOPOLOGICALLY PROTECTED EDGE MODES

To obtain a lattice with a C_{3v} symmetry after breaking the SIS, we start with a regular hexagonal lattice with a C_{6v} symmetry made of 2-cm-long struts with a 7.5:1 in-plane aspect ratio [Fig. 1(a)]. The geometric and material properties are chosen to match those of the specimen that we intend to test experimentally, which is manufactured via water-jet cutting from a sheet of acrylonitrile butadiene styrene (ABS): length of the struts $l = 2$ cm, out-of-plane sheet thickness $d = 3.2$ mm, Young's modulus $E = 2.14$ GPa, Poisson's ratio $\nu = 0.35$, and density $\rho = 1040$ kg/m³. The phonon dispersion relation, obtained via Bloch unit cell analysis conducted via finite-element analysis (FEA), is shown in Fig. 1(b). The struts are modeled as shear-deformable beams according to Timoshenko beam theory to expedite the calculation with respect to a 2D elasticity model, without compromising the accuracy. In the frequency range of interest (below 1.0×10^5 Hz), dictated by the bandwidth of our actuation equipment, we identify two Dirac points at the $\mathbf{K}$ point. A detailed calculation of the Berry phase confirming the Dirac-like nature of these points is reported in Sec. I of the Supplemental Material (SM) [44].

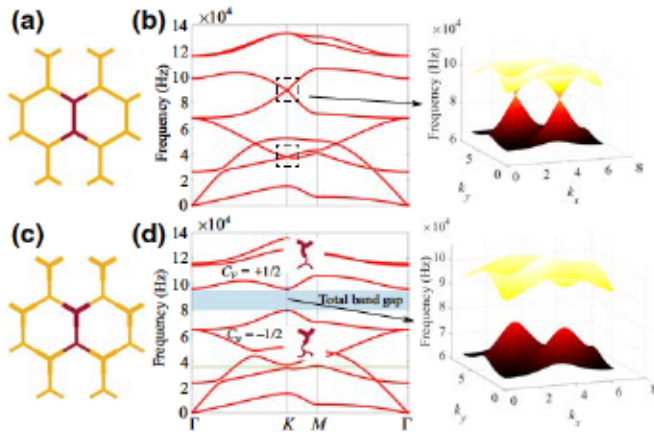


FIG. 1. (a) Unit cell (maroon) of a regular hexagonal lattice with uniform strut thickness and (b) its corresponding band diagram. The crossing points in the dashed squares are Dirac points. The surface plot on the right provides a three-dimensional (3D) visualization of the Dirac cones at the K point. (c) Unit cell (maroon) of a hexagonal lattice with modulated strut thickness. The thickness of the thick (thin) parts is increased (decreased) by 20% with respect to the reference value in (a). (d) Band diagram for the bivalued hexagonal lattice. The green and blue shaded areas denote partial and total band gaps, respectively. The two inserted displaced cells represent the eigendisplacements at the K points for the fifth and the sixth bands.

To break the SIS, we modulate the strut thickness so that the aspect ratio of the upper (lower) half of the unit cell is increased (decreased) by 20%, as shown in Fig. 1(c). The asymmetry of the lattice opens the Dirac cones and creates band gaps of 1.684×10^4 Hz and 2.720×10^3 Hz for the upper and lower cones, respectively; see Fig. 1(d). We decide to focus our analysis on the upper gap, which is wider and more tractable. To describe the topology of this gap, we calculate the valley Chern number C_v at the valley K (K') point and obtain the value $+(-)0.48$, which approximately tends to $+(-)\frac{1}{2}$ of classical QVHE theory.

We now proceed to consider a lattice consisting of two domains (termed lattice type A and B) separated by an interface. The cells above the interface have thin (thick) struts in the top half of the cell and thick (thin) struts in the bottom half, while those below the interface are mirror symmetric, as shown in Figs. 2(a) and 2(b). To confirm the existence of interface states, we conduct a supercell analysis. Each supercell consists of a finite strip of 12 cells: the top six cells belong to type A (B), while the bottom six to type B (A), as shown in Figs. 2(a) and 2(b). In this way, the lattice resulting from the repetition of the supercell along the x axis displays a horizontal interface along the zigzag direction. The asymmetry of the unit cell allows two possible types of interface: we name an interface where two thin struts meet a “thin interface” [Fig. 2(a)] and the other type a “thick interface” [Fig. 2(b)]. We apply 1D Bloch conditions along the x axis while leaving the top and bottom

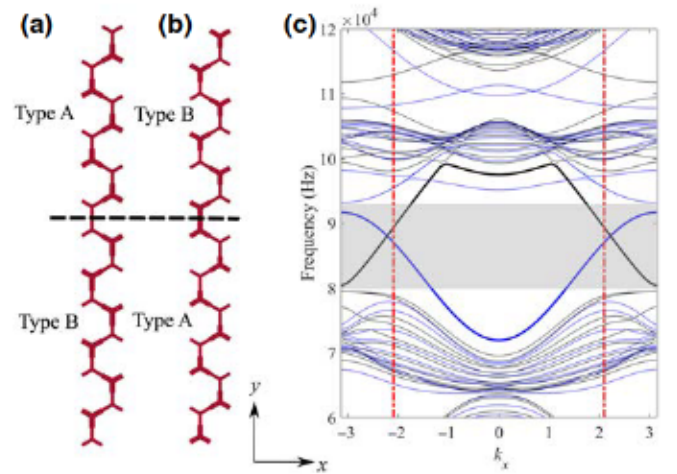


FIG. 2. (a),(b) Supercells featuring the two types of nontrivial interfaces along the zigzag directions, with (a) thin and (b) thick interfaces connecting the two lattice types. (c) Band diagram of the supercells. Blue and black curves refer to the thin and thick interfaces shown in (a) and (b), respectively. The thick curves denote the interface modes. The gray area marks the non-trivial band gap. The dashed red lines indicate the location of the K' (left) and K (right) points.

boundaries free to mimic a finite-width, infinitely long horizontal strip. From the supercell band diagrams obtained for the two different interfaces, superimposed in Fig. 2(c), we detect the emergence of two new bands within the band gap. The mode shapes calculated at the valley K point ($k_x = 2\pi/3$) on these new bands, reported in Sec. I of the SM [44], indicate that the emerging bands are indeed interface modes.

III. EXPERIMENTAL EVIDENCE OF TOPOLOGICALLY PROTECTED IN-PLANE WAVEGUIDING

An important feature of a TPES is that it allows waves to propagate along sharp corners with negligible backscattering. Effective rationales for this phenomenon have been proposed in the literature in the context of flexural waves. In Sec. II of the SM [44], we recall and extend these considerations, providing additional insight in the process that is germane to our specific problem of in-plane wave propagation.

Here, we focus our attention on verifying this phenomenon, both numerically and experimentally, for in-plane phonons in our lattice of choice. To document the robustness against backscattering, we model a Z-shaped thin interface with two 60° corners, as shown in Fig. 3(a). The interface splits the lattice into two subdomains with mirror symmetry about the interface. The dynamic response is computed via full-scale dynamic FEA. A burst excitation with frequency $\omega = 8.707 \times 10^4$ Hz with 35 cycles is applied in the form of a horizontal

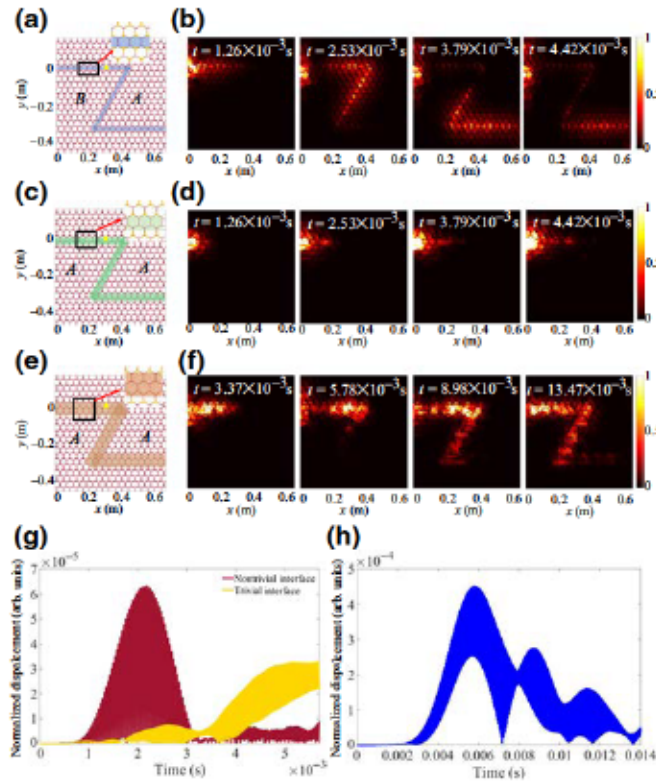


FIG. 3. Full-scale simulations of lattices with (a) nontrivial (blue shade), (c) narrow trivial (green shade), and (e) wide trivial (red shade) interfaces. The two regions, A and B, separated by the nontrivial interface in (a) present a mirror symmetry about the interface. For trivial interfaces in (c) and (e), the two regions have the same lattice type. (b), (d), (f) Wavefields for the (a) nontrivial, (c) narrow, and (e) wide trivial interfaces, respectively. (g) Magnitude of displacement at the locations marked by a star along the narrow trivial (yellow) and nontrivial (maroon) interfaces. (h) Magnitude of displacement along the wide trivial (blue) interface. The displacements in (g) and (h) are normalized by the largest measured values.

in-plane force at the left edge of the interface. As is evident from Fig. 3(b), the wave indeed propagates along the interface without significant backscattering until it reaches the right boundary. The backscattering-immune behavior can also be confirmed by plotting the displacement time history of an arbitrary point located along the interface, as shown in Fig. 3(g). The presence of a single dominant peak, associated with the incoming wave, indicates that no appreciable reflections are generated at the corners. Since the excitation frequency is within the bulk band gap, no leakage into the bulk is observed.

An alternative strategy to realize a similarly shaped waveguide is by constructing a trivial interface. This can be done starting from a uniform lattice (say, type A) and replacing the cells featuring thickness modulation with regular hexagonal cells along the interface. Let us consider two trivial Z-shaped domain walls with interface widths

spanning over one and two cells, as shown in Figs. 3(c) and 3(e), respectively. Interestingly, when the trivial interface is one cell in width, as in Fig. 3(c), the wave remains mostly localized around the excitation point, with little energy propagating along the interface, as shown in Fig. 3(d). Therefore, a trivial interface with one-cell thickness is not sufficient to effectively close the band gap and establish efficient waveguide transport conditions. By sufficiently widening the interface, as in the case of Fig. 3(e), involving a two-cell interface, waves can eventually propagate along the trivial waveguide, as shown in Fig. 3(f). However, strong backscattering is observed at the corners, significantly weakening the signal transmitted and rendering this a very inefficient waveguide. This is best seen by looking at Fig. 3(h), which again shows the time history of the response sampled at a point along the first segment of the waveguide. A second peak at around 9×10^{-3} s is clear evidence of a strong reflected wave. In conclusion, only the topologically protected interface guarantees efficient and scattering-free waveguiding.

We then move on to perform a series of experiments to confirm the topological attributes of the nontrivial waveguide. To acquire the in-plane wavefields, we scan the lattice with a 3D scanning laser Doppler vibrometer (SLDV, Polytec PSV-400-3D) using one scan point per lattice node. The specimen is a water-jet-cut ABS lattice, comprising 34×20 unit cells with the same dimensions and parameters used in simulations; see Fig. 4(a). The green-shaded area denotes the Z-shaped interface. For convenience of setup, we rotate the lattice by 90° counterclockwise and frame it using clamps at the two sides, leaving the top and bottom boundaries free. The excitation is prescribed as an in-plane normal point force applied on the boundary at the opening of the topological interface. The force is exerted using an electrodynamic shaker (Bruel & Kjaer Type 4809, powered by a Bruel & Kjaer Type 2718 amplifier) placed at the bottom of the lattice, as shown in Fig. 4(b). We prescribe a burst excitation at $\omega = 8.707 \times 10^4$ Hz with 35 cycles in order to activate a very narrow frequency interval falling entirely on the topological edge mode branch within the band gap, approximately at $k_x = 2\pi/3$. From visual inspection of the wavefield, we observe that the wave indeed propagates along the interface without appreciable backscattering [see Fig. 4(c)], consistent with the numerical prediction in Fig. 3(b). Note that the magnitude is here attenuated along the propagation path due to unavoidable damping in the physical specimen. The robustness against backscattering is eloquently captured by the spectral plane representation of Fig. 4(d), obtained by applying a discrete Fourier transform (DFT) to the experimental data sampled along the interface. Indeed, the spectral magnitude map presents a strong asymmetry between the $+k_x$ and $-k_x$ directions, suggesting negligible reflection from the corner. It can also be appreciated that the spectral response features excellent agreement

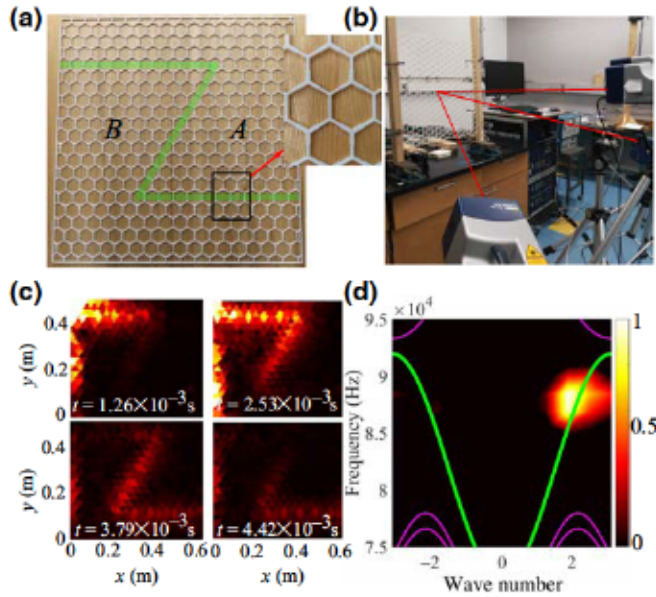


FIG. 4. (a) Experimental specimen cut from a thin ABS sheet. A Z-shaped thin interface with two 60° corners highlighted in green. (b) Experimental setup showing the 3D SLDV heads and the specimen rotated by 90° counterclockwise in order to facilitate excitation through the shaker. (c) Snapshots of the wavefield induced through burst excitation at $\omega = 8.707 \times 10^4$ Hz. The point velocity magnitudes are normalized by the largest measured value. The magnitude is attenuated due to damping in the specimen. (d) DFT of the experimental data sampled along the interface, matching the interface mode (green) calculated from supercell analysis and revealing the absence of appreciable backscattering. The magenta curves are bulk bands.

with the edge mode branch obtained from supercell analysis.

IV. MULTI-INTERFACE JUNCTIONS FOR ENERGY TRANSPORT MANIPULATIONS

In Sec. II of the SM [44], we offer a rationale for how waves propagate around 60° and 120° corners with or without changes of interface type, and the conclusions align with what has been observed in out-of-plane counterpart problems [34–42]. These considerations can be fully leveraged for structural logic circuit designs with unconventional energy transport capabilities. Inspired by former theoretical studies of topological current splitters based on the valley Hall effect [45,46], we devote our attention to the wave manipulation and opportunities of multi-interface junctions in cellular metastructures. Recently, a study of topological junctions in phononic plates has documented the potential of junctions as energy splitters and filters for flexural (out-of-plane) waves, introducing the notion of tessellated mechanical domains as topological super-networks [42]. Here, we provide a systematic exploration of analog opportunities available in cellular configurations

undergoing in-plane elastic wave propagation using a suite of FEA and laser vibrometry experiments.

The objective is to exploit the energy-splitting properties of topological junctions to achieve a series of new asymmetric wave-transport and energy-trapping effects, thus expanding the gallery of technological potentials of valleytronic concepts in mechanical systems. We start by noting that, according to our previous discussion, the lattice types on the two sides of a topologically protected interface must present a mirror symmetry. Hence, to ensure that all the interfaces converging at a junction are topologically protected, it is necessary that the junction connect an even number of interfaces. Here, we illustrate this idea through two examples.

The first design is a direction-selective elastic energy splitter intended to allow strongly asymmetric wave transport at high frequencies. The FEA model and experimental specimen are shown in Figs. 5(a) and 5(b), respectively. Three thin and one thick interface (according to the terminology introduced before) are connected at point o , resulting in three ports on the left side of the lattice and one on the right. The angle between the three thin interfaces is 60° , while the one between the thin and thick interfaces is 120° . A burst excitation with carrier frequency falling inside the band gap is applied at port a on the left, and the simulated and experimentally measured wavefields are shown in Figs. 5(c) and 5(d), respectively. It can be seen that the wave prefers to split and steer at the junction and propagate back along ob and oc , eventually localizing at the left edge of the lattice without ever reaching the opposite edge. In contrast, the simulated and experimental wavefields obtained by exciting at port d on the right are shown in Figs. 5(e) and 5(f), respectively. As we can see, the wave propagates along the thick interface from right to left and, at junction o , splits (with negligible reflections) into two transmitted signals traveling to ports b and c . The characteristics of the interfaces coalescing at junction o ensure that the wave propagates along the two thin interfaces ob and oc that form a 120° angle with the thick interface od . As a result, transmission from right to left is achieved efficiently. Note that opposite group velocities between the thin oa and the thick od interfaces prevent the wave from propagating straight through the junction, despite the perfect alignment between oa and od .

De facto, the lattice endowed with the internal multipath junction behaves dynamically as a medium with effective asymmetrical wave-transport characteristics. It works as an isolator for excitations prescribed at the left edge and as a conductor for excitations prescribed at the right edge. In this respect, it achieves a functionality that is a high-frequency analog of what has been recently shown experimentally for topological Maxwell structural lattices (with ligaments with finite bending stiffness); see Ref. [30]. In such systems, we observe a similar behavior whereby excitations prescribed at the floppy edge remain localized

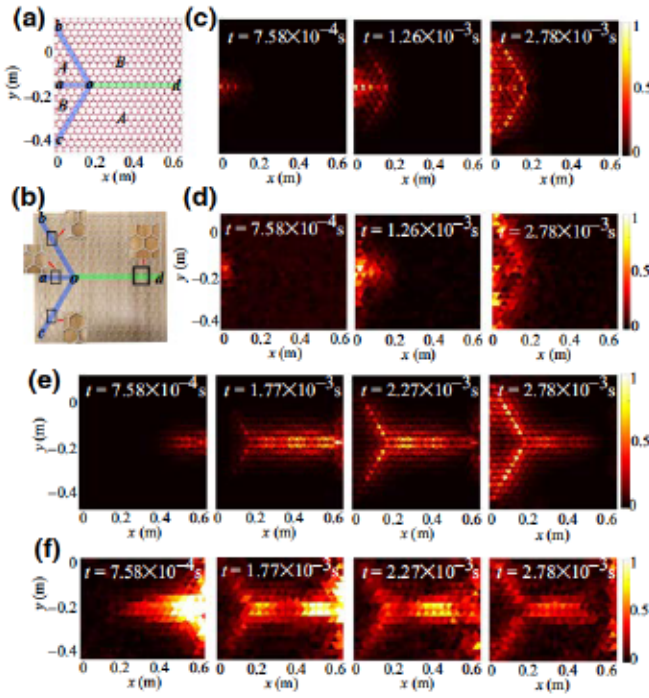


FIG. 5. (a) FEA model and (b) experimental specimen of a one-way elastic demultiplexer. *a–d* are the four ports on the edges. Three thin interfaces, *ao*, *ob*, and *oc*, forming 60° angles, are marked by blue shades. A thick interface *od* connecting the right boundary to junction *o* and forming a 120° angle with two of the thin interfaces is denoted by a green shade. (c),(d) Snapshots of wavefields obtained from (c) simulation and (d) experiment with burst excitation at $\omega = 8.707 \times 10^4$ Hz applied at port *a*. (e),(f) Snapshots of wavefields obtained from (e) simulation and (f) experiment with excitation at port *d*. The velocity magnitudes are normalized by the largest measured value. In the experimental case, the magnitude is attenuated due to damping.

close to the excitation, while those prescribed at the stiff boundary are transmitted across the sample. Interestingly, despite the fact that the two effects are based on different mechanisms, they are both edge phenomena that can be seen as manifestation of the bulk-boundary correspondence, they both enjoy topological protection, and they are both described by topological invariants (the valley Chern number here and the topological polarization vector in Ref. [30]). From an application standpoint, the asymmetrical transport, more specifically, the emergence of an isolating edge, results in high transmission efficiency, as the signal that has traveled across the sample is prevented from traveling back toward the input edge, even after scattering from the output edge. Because of the trapping path formed by segments *ob*, *ba*, *ac*, and *co*, retrotransmission is not allowed not only along the waveguide, but also along any other propagation path.

For our second example, we arrange three thin and one thick interfaces as shown in Fig. 6(a) to realize a triangular loop embedded in the lattice domain. The idea is to create

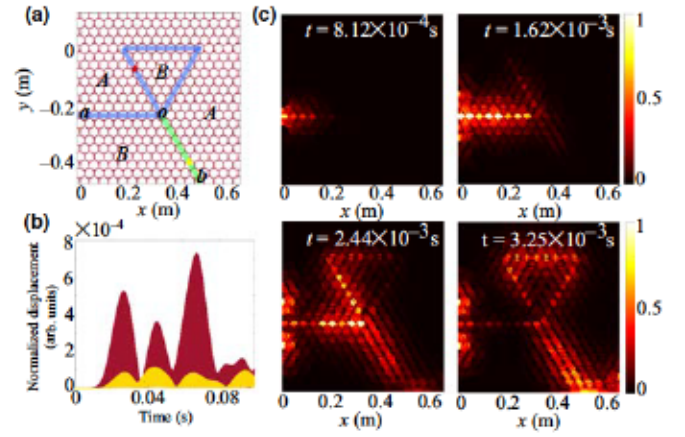


FIG. 6. (a) Full-scale simulation of a lattice with a triangular loop realized with topological interfaces. Thin and thick interfaces are marked in blue and green, respectively. (b) Magnitudes of displacements computed at the points marked by maroon and yellow stars along the thin and thick interfaces, respectively, in (a). (c) Snapshots of a wavefield induced through a burst excitation at $\omega = 8.707 \times 10^4$ Hz applied at port *a*. The displacement magnitudes are normalized by the largest value measured in the wavefield.

an energy sink that locally traps the energy injected into the lattice from an edge and delays its release at another edge. When a wave traveling from port *a* impinges onto junction *o*, the energy splits, with a portion propagating along the upper thin interface (which forms a 60° angle with *oa*), and the other following the lower thick interface at a 120° angle. The energy propagating upward circulates clockwise in the triangular loop, eventually coming back to junction *o*. Here, the energy splits again into a packet that enters the loop for a second cycle and a second packet that travels along the thick interface *ob* and eventually reaches the lower edge after some delay; see Fig. 6(c). This process, which is repeated at every cycle, breaks down the burst of energy and releases it at port *b* after finite intervals, with a period that depends on the geometric characteristics of the loop. As a result, in contrast with the input signal, which consists of a single packet, the output signal is broken down into multiple packets that are transmitted over an extended time window, as shown in Fig. 6(b).

The ability to create arbitrarily shaped and long waveguiding loops has implications for the design of efficient energy-harvesting systems. Energy harvesting from mechanical vibrations requires the activation of transducers embedded in mechanical systems that convert mechanical energy into electrical energy to provide an electrical (e.g., voltage) output that can be harnessed and stored. Placing harvesters must be carried out without compromising efficiency and parsimony: on one hand, one wants to maximize the output of the system by deploying many harvesters; on the other hand, too many harvesters can be an impractical and expensive solution, in addition to

detrimentally affecting the mechanical performance of the host system. Moreover, in a conventional medium subjected to a point excitation, the energy travels outward from the excitation point with amplitude decreasing with the distance from the source. Therefore, even if harvesters were distributed densely in the domain, the output of those located far from the source would become progressively negligible. In contrast, one of the by-products of waveguides is the ability to predict precisely where the energy from an excitation propagates. As a result, one could deploy harvesters along the waveguide path, thus making sure that they are all going to be engaged by the excitation and that the signal they experience overall maintains its amplitude (in the absence of significant damping and ignoring some inevitable dispersion-induced packet attenuation). In this respect, if the objective were to maximize the harvesting output, one would want to design long waveguides, whose total length may exceed the sample size, which would therefore require tortuous paths with corners and bends. The ability offered by QVHE to manage backscattering at these points and, more specifically, the possibility to create internal loops becomes then an invaluable asset. It is important to note that, for this kind of application, a closed loop confined to the interior of the domain would be ideal, as it would force the energy to remain confined along the path (until the inevitable damping destroys the signal), thus allowing multiple harvesting events per transducer. The implementation of our junctions, however, requires the existence of an extra leg (*ob*), along which part of the signal is released in each period, and thus, from a strictly internal energy-harvesting perspective, lost.

V. CONCLUSION

In this work, we experimentally demonstrate the in-plane dynamics of topologically protected interface modes in honeycomb lattices with a bivalued strut thickness pattern that breaks the space-inversion symmetry. We document similarities and differences between topological interfaces formed by connecting thin and thick portions of the unit cell. We show the robustness against backscattering of wave propagation about 60° corners with identical types of interfaces and about 120° with mixed interfaces. Exploiting this knowledge, we design multi-interface junctions and use them to realize different structural logic circuits and elastic ports featuring unconventional wave manipulation effects at optical frequencies. Our results extend the applicability of valley Hall effects to in-plane elastic dynamics and enrich the current understanding of these phenomena in mechanical systems by providing a new interpretation angle that blends topological and mechanistic arguments. The presented modular interface design philosophy can provide inspiration for a broad range of applications for energy management, trapping, routing, and harvesting.

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