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Reliable facility location design with round-trip transportation under imperfect information Part I: A discrete model

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ABSTRACT

In this paper, we propose a discrete model to investigate reliable location design with round-trip transportation under imperfect information. The unique feature of this problem is that under each disruption scenario, a customer's outbound and inbound trips are different when she travels to obtain a service. The discrete model is formulated as a compact non-linear integer programming problem and solved efficiently by a customized Lagrangian relaxation algorithm. Numerical experiments find that the discrete model performs well for small and medium-scale problem instances. Sensitivity analyses reveal the impacts of several parameters on both the cost components and the optimal facility layouts.

1. Introduction

In the wake of recent anthropogenic (e.g., terrorist attacks, strike, etc. (D'Amico, 2002; Schewe, 2004; Hirsch et al., 2015)) and natural (e.g., hurricanes, earthquakes, etc. (Godoy, 2007; Sharkey et al., 2015; Sheppard and Landry, 2016)) disasters, people have realized the importance of planning redundant facilities in infrastructure systems to enhance system resilience against unexpected disruptive events. A series of models on reliable facility problems (Snyder and Daskin, 2005) were developed to optimize facility locations for both primary and backup services. These models aim to balance the tradeoff between planning-stage investment and expected operation cost, incorporating all possible facility disruption scenarios. Most of these reliable facility location models assume that perfect information about all facility functioning states is available to customers in real time and thus they always know a proper facility to visit (e.g., the closest operating facility). However, because of institutional barriers (Birenbaum, 2009), technical constraints (Baillieul and Antsaklis, 2007), and customer unpreparedness (Berman et al., 2009), this assumption might not always reflect reality. Furthermore, because of likely communication and infrastructure disruptions, it is even more difficult to access real-time facility states in unexpected emergency scenarios (Hasan and Folient, 2015).

Under imperfect information, customers do not know real-time states of facilities, thus they have to visit a series of pre-assigned facilities until they either find a functioning facility to get the service or abort the attempt, receiving a penalty instead. Only a few studies have investigated this type of location problems under imperfect information (Berman et al., 2009; Yun et al., 2015, 2017). These studies contributed to modeling reliable location design with imperfect information with a focus on outbound transportation cost (for a customer to reach the service from her home). Although differentiating outbound and inbound (i.e., for this customer to return home after obtaining the service) transportation costs is not necessary in problems under perfect information because they are

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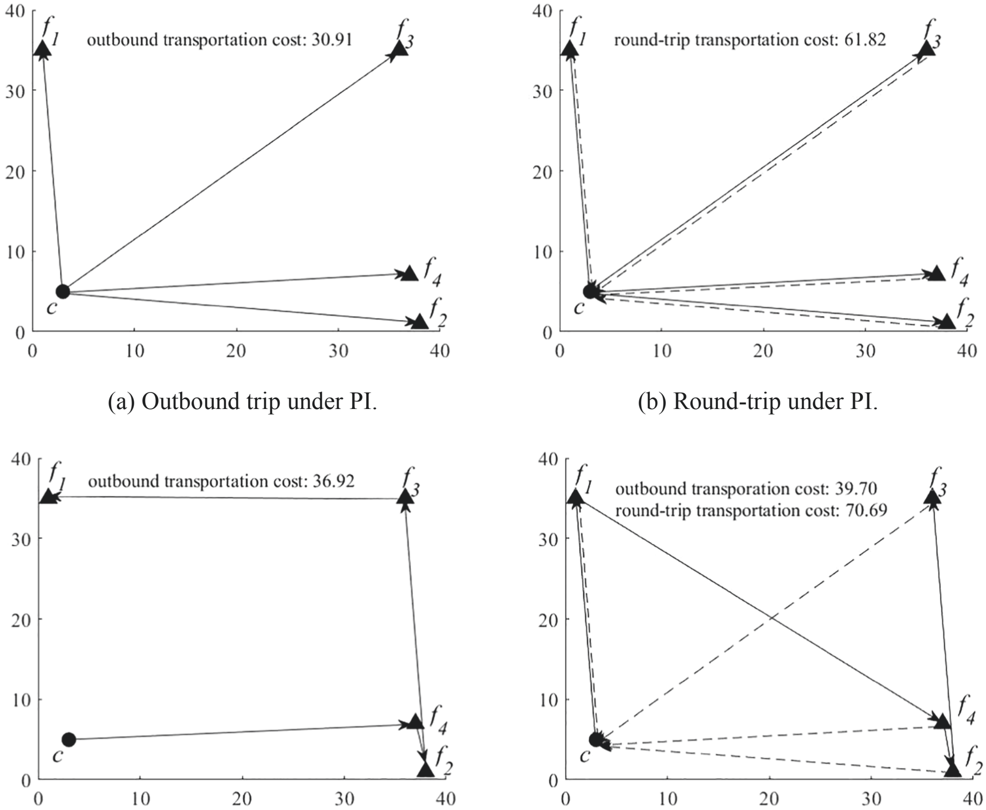


(a) Outbound trip under PI. (b) Round-trip under PI.

(c) Outbound trip under II. (d) Round-trip under II.

Fig. 1. Motivating examples under perfect information (PI) and imperfect information (II).

simply identical, these two costs are quite different in the imperfect information context. Whereas a customer may visit a series of facilities consecutively during the outbound trip, the inbound trip involves simply returning from her last stop directly to her home. This phenomenon is not uncommon in the real world even in everyday life. One example is looking for charging stations, people driving electric vehicles need to go to charging stations from their homes to recharge their vehicles. If the charging station is disrupted (e.g., being occupied by other vehicles, or experiencing a technical problem), they will try other stations until finding one available or giving up trying. After that, they will go back to their homes. While location of charging stations has been intensively studied (Li et al., 2016; Liu and Wang, 2017; Yıldız et al., 2019), their possible disruptions have been seldom investigated. The same repeated trying behavior is also frequently observed when one circulates across busy parking lots looking for a parking spot (which is common on university campuses such as the University of South Florida). Therefore, it is necessary to consider the inbound trip in calculating the transportation cost. Further, we provide the following motivating example shown in Fig. 1, to explain this difference in a quantitative way. This example investigates a rectangle space $[0, 40] \times [0, 40]$. The dot represents a customer at location $(3, 5)$, as denoted by c. The triangles are four service facilities, such as recharging stations, denoted by f_1 at $(1, 35)$, f_2 at $(38, 1)$, f_3 at $(36, 35)$ and f_4 at $(37, 7)$. We assume that all the facilities are disrupted independently with the same probability, 20%. Assume that this customer always travels in a straight line between two consecutive nodes, and thus the transportation cost is equal to the straight-line distance that the customer has traveled. Fig. 1(a) and Fig. 1(b) show the optimal solution under perfect information (PI) for considering the outbound transportation cost only and the round-trip transportation cost, respectively. We see that they both have the same facility visiting priorities, $f_1 \to f_4 \to f_2 \to f_3$, although the round-trip transportation cost (61.82) is obviously double the outbound transportation cost (30.91). Therefore, the round-trip transportation cost in the perfect information context can be trivially incorporated in the existing models by doubling the expected transportation cost. Fig. 1(c) shows the optimal visiting sequence under imperfect information (II) when only the outbound transportation cost is considered. It yields an expected transportation cost of 36.92. Fig. 1(d) shows the optimal visiting sequence under imperfect information when both outbound and inbound transportation costs are considered. This case yields an expected transportation cost of 70.69. Comparing these two figures, we find that the optimal visiting sequences in Fig. 1(c) and Fig. 1(d) are different, and the round-trip transportation cost is no longer a trivial multiple of the one-way cost. Comparing the outbound transportation cost in both cases, there is an obvious difference. This highlights the need to consider inbound and outbound transportation costs as separate cost components for location design problems under imperfect information.

To fill these gaps, this paper aims to study reliable location design under imperfect information considering round-trip transportation costs with a discrete model. This study extends the work in Yun et al. (2017) by considering round-trip transportation costs.

We formulate a non-linear integer programing model to describe the discrete version of the investigated problem. In this model, customers use a "trial-and-error" strategy to obtain the service because of imperfect information. In this strategy, a customer visits the pre-assigned facilities successively to obtain the service. She may abort the attempt when all pre-assigned facilities are disrupted or it is not worth visiting more facilities. Regardless of whether she obtains or gives up the service, she will return to her home location. A penalty cost is incurred when she does not obtain the service. In our model, we assume that each facility is disrupted with a site-dependent probability. It is not necessary for customers to be aware of accurate disruption probabilities. Customers always follow pre-assigned sequences when visiting facilities one-by-one. However, planners need to determine disruption probabilities by using historical data and evaluation tools to better design facility locations. This model describes an NP-hard problem and cannot be efficiently solved by commercial solvers. Therefore, we develop a customized Lagrangian relaxation (LR) algorithm for solving the discrete model. Numerical experiments are constructed to compare efficiency and solution quality of the customized algorithm and to draw managerial insights. Overall, the model proposed in this paper can efficiently solve reasonable-sized instances of the investigated problem. The numerical results also find how the system costs and the optimal facility layouts are impacted by several relevant factors, such as imperfect information, consideration of round-trip cost and backup facilities.

We want to note that this Part I paper only focuses on developing a discrete model for the investigated problem and discussing its properties. This development is complemented by a continuum approximation model for even larger scale problem instances and analytical insights, as presented in the Part II paper (Yun et al., 2019). The discrete model and the continuous model will be compared in the Part II paper.

The remainder of the paper is organized as follows. Section 2 reviews the literature relevant to reliable facility location. Section 3 formulates the discrete problem and proposes a customized solution algorithm. Section 4 uses the proposed model to solve numerical examples and draws insights into the impacts of inbound trip and imperfect information. Section 5 concludes the paper and suggests future research directions.

2. Literature review

Facility location problems have been studied for several decades (Nair and Miller-Hooks, 2014; Benedyk et al., 2016; Gong and Du, 2016; Zheng et al., 2017; Faturechi et al., 2018; Chauhan et al., 2019). Recently, reliability in location problems has become a focus (Snyder and Daskin, 2006; Snyder et al., 2007; Qi et al., 2010; Chen et al., 2011; Peng et al., 2011; An et al., 2013; Bai et al., 2015; Xie et al., 2016; Yu et al., 2017; Tran et al., 2017; Marufuzzaman and Eksioglu, 2017; Mohammadi et al., 2019). Reliable facility location problems aim to optimize facility locations with considering backup service in face of the natural and anthropogenic disasters. The concept of using backup services to enhance facility system resilience stems from Daskin's early work (Daskin, 1983) on expected coverage problems. Snyder and Daskin (2005) extended this concept to the uncapacitated fixed charge location problem (UFLP) that considers facilities being disrupted independently with an identical probability. This studied problem is formulated as a mixed integer programming problem and solved by a customized LR algorithm. Cui et al. (2010) further considered site-dependent disruption probabilities and proposed both discrete and continuous formulations to solve this problem. Li et al. (2013) developed a supporting model to address correlated disruptions due to physical or virtual connections between facilities. Xie et al. (2015) extended this effort by incorporating general correlation patterns, including negative correlations. Based on the innovation of reliable uncapacitated fixed charge location (RUFL) problems, numerous models have been proposed for reliable location design in specific systems. Li and Ouyang (2011) proposed a reliable model to optimize the sensor deployment for network traffic surveillance considering sensor failures. An et al. (2015) presented a stochastic mixed-integer non-linear program (MINLP) model to solve an emergency service facility location problem. Lu et al. (2015) proposed a model to optimize the expected cost under the worst-case scenario with given correlated disruption probabilities in a robust manner. Xie et al. (2016) proposed an integer programming model for a reliable location-routing problem where facilities may be disrupted with an identical probability. Zhang et al. (2016) proposed a mixed non-linear integer programming model for the location problem considering both inventory risk-pooling effects and economies of scale. In these studies, the customers can obtain the perfect information about every facility state and visit the most suitable facility directly.

Later, people realized that the information is actually not always perfect in many systems due to technology limitations, institutional barriers, etc. Under imperfect information, a customer has a new travel pattern to obtain the service or bear the penalty cost. Some researchers proposed a few models to deal with these reliable facility location problems. Berman et al. (2009) investigated the median location problem. This study considers that a customer always tries facilities according to their distances from this customer in any facility disruption scenario. Later, Berman et al. (2011) proposed a forward dynamic programming procedure to optimize a customer's search path when the states of facilities are unknown. Berman et al. (2013) studied several location models with Median and Center objectives under complete and incomplete information where facility failures may be correlated. Yun et al. (2015) proposed a discrete model to formulate the RUFL problem under imperfect information. This study assumes that a customer may not always visit the closest facility in each local step but instead focuses on global optimality. Yun et al. (2017) further extended this study into site-dependent disruption probabilities.

All the above mentioned studies focus on the outbound trip in facility location design to formulate the transportation cost. However, the inbound trip is also important for deciding the customer's visiting sequence under imperfect information. Note that considering the inbound trip may yield drastically different results than not considering it; e.g., Customers may choose to visit facilities not far from their homes instead if the cost of the inbound trip (which is a direct trip to the home different from the outbound route visiting a sequence of facilities) is of concern. To address this issue, this paper studies a reliable facility location problem considering round-trip transportation under imperfect information. The proposed model is a non-linear integer programing model considering site-dependent facility disruptions. Because it is an NP-hard problem, a customized LR algorithm is proposed for solving the proposed model.

3. Discrete model

This section proposes a discrete reliability model to optimize the facility location layouts under imperfect information with round-trip transportation costs. Section 3.1 formulates this problem as a mixed non-linear integer programming model in a compact form. Because it is difficult to solve this model with standard algorithms or commercial solvers, Section 3.2 proposes a customized LR algorithm to solve it efficiently. To facilitate solution of the proposed model by commercial solvers for comparison purposes, Section 3.2 also discusses a technique of linearizing the proposed model in the end.

3.1. Model formulation

For the readers' convenience, the definitions of the key symbols are summarized in Appendix A. We denote the set of customer locations by $\mathscr{I}$ and the set of candidate facility locations by $\mathscr{J}$. A customer at location i (or customer i for short) generates demand λ_i. Building a facility at location j (or facility j) requires an opening cost f_j, and this facility has a site-dependent disruption probability q_j.

Due to imperfect information, a customer must visit the relevant facilities one-by-one according to their pre-assigned ranks indexed by $r \in \{0, 1, 2, ..., R\}$. R is the maximum rank. Let c_{ij} denote the transportation cost per unit demand for customer i to visit facility j from her initial location (e.g., the product of a transportation rate α and the distance between customer i and facility j) as rank $r = 0$ assignment. Let $c_{ij_i^{r-1}j_i^r r}$ denote the transportation cost per unit demand when customer i visits facility j_i^r from facility j_i^{r-1} at rank $r \geqslant 1$ (e.g., the product of α and the distance between facilities j_i^{r-1} and j_i^r). If customer i eventually does not obtain the service because of facility disruptions, she will bear a penalty cost φ_i per unit demand. Customers may abandon the service in advance when some pre-assigned facilities are still operational because it is not worth to visit those facilities. To formulate the model in a compact model that incorporates all loss of service scenarios, we introduce a dummy facility j_0 with a zero disruption probability, i.e., $q_{j_0} = 0$, to capture the penalty cost. If customers reach the dummy facility, they can only go to another dummy facility in the next rank, and the transportation cost is $c_{ij_0j_0r} = 0,\ \forall\ r = 1, 2, ..., R + 1$. The visiting sequence for any customer i is shown in Fig. 2.

In any facility failure scenario, customer i must return to her home location regardless of whether the last visited facility is functioning or disrupted. Thus, the formula of the expected inbound transportation cost returned from the last visited facility is different from that returned from another previously visited facility. The expected inbound transportation cost returned from the last visited facility includes two components. The first component is the expected inbound transportation cost conditioning on that the last visited facility is functioning, which has a similar formula structure as that back from another previously visited facility. The second component is the expected inbound transportation cost conditioning on that the last visited facility is disrupted. To formulate the expected cost in all possible visiting sequences in a unified form, we equivalently represent the second cost component as the expected outbound transportation cost from the last visited facility to the first dummy facility. Fig. 3(b) clearly shows how to make this transformation. Owing to the introduction of the dummy facility, we should divide the penalty cost into the inbound and outbound costs. Fig. 3 shows this division under the extreme and general cases. The extreme case in which customer i simply gives up obtaining the service without attempting any facility is equivalently represented as customer i directly visiting dummy facility j_0 and then returning to her home location. Note that the corresponding expected transportation cost is equal to $c_{ij_0} + (1 - q_{j_0})c_{ij_0} = \varphi_i$. Since q_{j_0} is zero, c_{ij_0} is equal to $\varphi_i/2$. In the other loss of service cases when customer i aborts the attempt after trying some facilities, equivalently, customer i goes to facility j_0 from facility j_i^r that was visited last. Note that penalty cost φ_i is equal to $c_{j_i^r j_0} + (1 - q_{j_0})c_{ij_0}$. Then, since q_{j_0} is zero and c_{ij_0} is $\varphi_i/2$, $c_{j_i^r j_0}$ is equal to $\varphi_i/2$. It is obvious that this setting correctly counts the expected penalty cost in these cases.

For notation convenience, we define

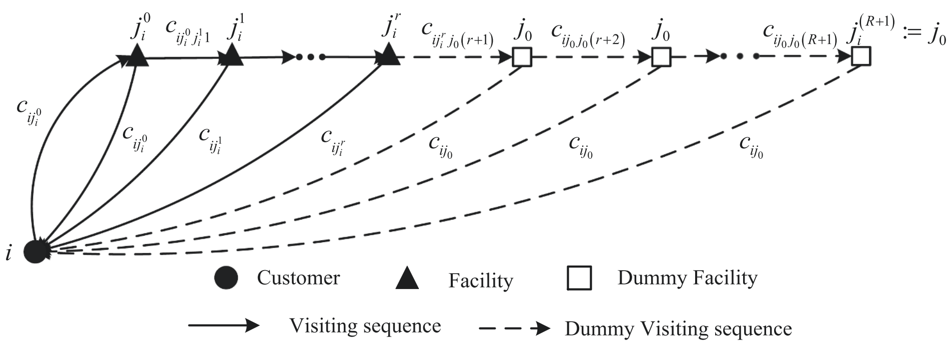
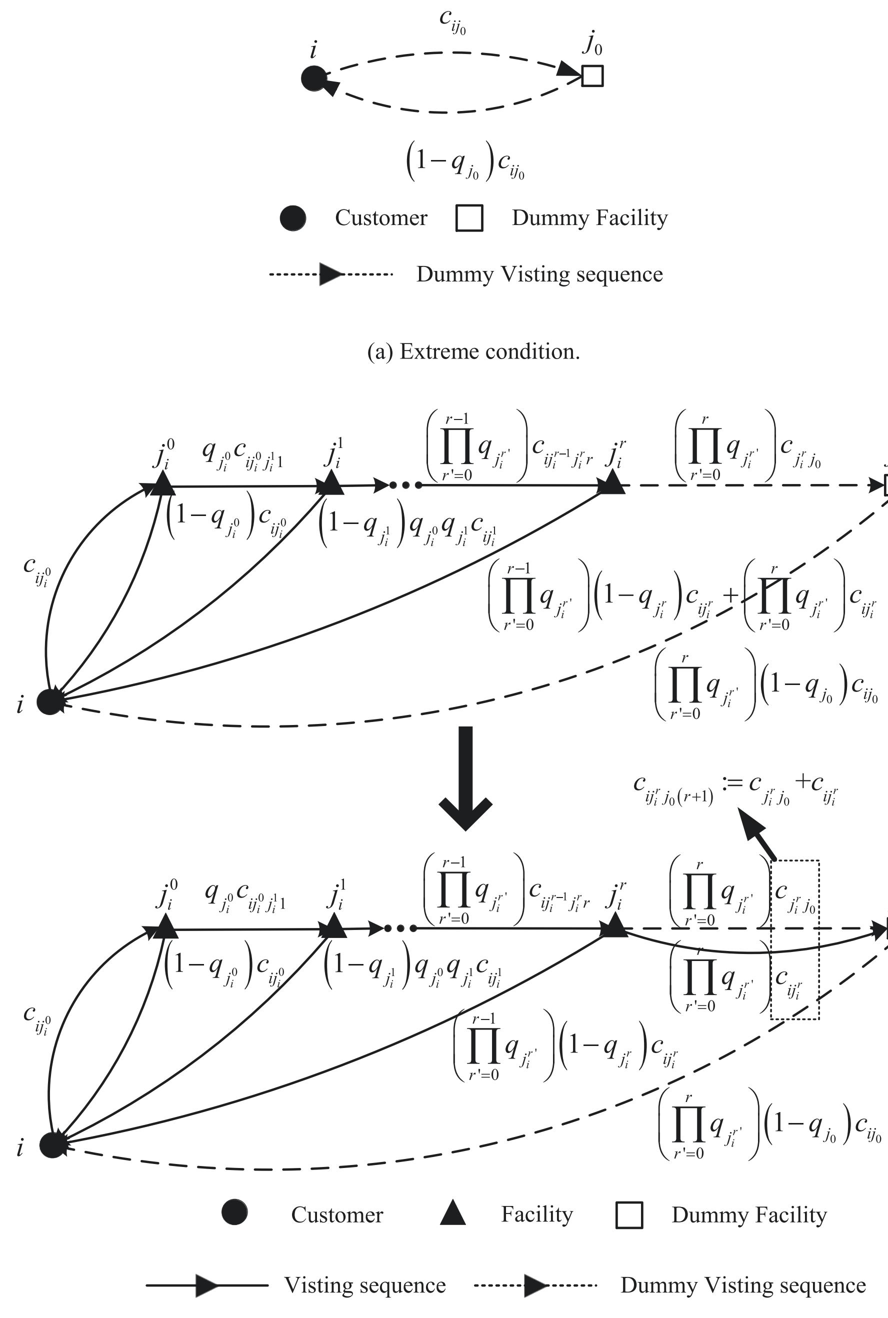


Fig. 2. Visiting sequence for any customer i including pre-assigned facilities and dummy facility.



(a) Extreme condition.

(b) General condition.

Fig. 3. Illustration of expected transportation cost setting for the dummy facility.

$$\bar{\mathscr{J}} := \mathscr{J} \cup \{j_0\},\ \mathscr{J}_j^+ := \begin{cases} \bar{\mathscr{J}} & j = j_0; \\ \mathscr{J}/\{j\} & j \neq j_0 \end{cases},\ \mathscr{J}_j^- := \begin{cases} j_0 & j = j_0; \\ \bar{\mathscr{J}}/\{j\} & j \neq j_0 \end{cases},\ \forall\, j \in \bar{\mathscr{J}} \tag{1}$$

where $\mathscr{J}_j^+$ and $\mathscr{J}_j^-$ denote the candidate facility locations that the customer can visit before and after visiting facility j, respectively.

To construct the objective for the discrete model, we introduce two auxiliary variables: $Y = \{y_j\}_{j \in \mathscr{J}}$ for location decisions and $X = \{x_{ij}\}_{i \in \mathscr{I}, j \in \bar{\mathscr{J}}}$, $X' = \{x_{ijj'r}\}_{i \in \mathscr{I}, j \in \bar{\mathscr{J}}, j' \in \mathscr{J}_j^-, r = 1, 2, \cdots, R+1}$ for customer-to-facility assignments, as follows:

$$y_j = \begin{cases} 1, & \text{if facility } j \text{ is open;} \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

$$x_{ij}=\begin{cases}1, & \text{if customer } i \text{ is assigned to facility } j \text{ at rank } 0;\\ 0, & \text{otherwise,}\end{cases} \tag{3}$$

$$x_{ijj'r}=\begin{cases}1, & \text{if } i \text{ is assigned to } j \text{ at rank } r-1 \text{ and to } j' \text{ at rank } r, \ \forall\, r=1,2,\cdots,R+1;\\ 0, & \text{otherwise.}\end{cases} \tag{4}$$

We also define the probability variables $P=\{p_{ijj'r}\}_{i\in\mathscr{I},j\in\overline{\mathscr{J}},j\in\mathscr{J}_j^-,r=1,2,\cdots,R+1}$ as follows:

$$p_{ijj'r}=\begin{cases} q_j, & r=1\\ q_j\sum_{j'\in\mathscr{J}_j^+}p_{ij'j(r-1)}x_{ij'j(r-1)}, & r>1\end{cases} \tag{5}$$

Here, $p_{ijj'r}$ denotes the probability for customer i to go to facility j' at rank r from facility j.

With this, the total facility fixed opening cost is formulated as follows:

$$\sum_{j\in\mathscr{J}} f_j y_j. \tag{6}$$

Because the dummy facility has been introduced, the expected transportation and penalty costs can be calculated together, and we refer to their summation as the expected operational cost. Note that the probability that customer i returns home after visiting facility j at rank 0 is $1-q_j$; then, the corresponding operation cost is equal to

$$\sum_{j\in\overline{\mathscr{J}}}\lambda_i(2-q_j)c_{ij}x_{ij}. \tag{7}$$

Furthermore, the probability that customer i returns home after visiting facility j at rank $1\leqslant r\leqslant(R+1)$ is the probability that all facilities under rank $r-1$ are disrupted, yet facility j is functioning. Then, the corresponding operation cost is equal to

$$\sum_{j\in\overline{\mathscr{J}}}\lambda_i\left(\sum_{j'\in\mathscr{J}_j^-}p_{ijj'r}(c_{ijj'r}+(1-q_{j'})c_{ij'})x_{ijj'r}\right). \tag{8}$$

According to Eqs. (7) and (8), the total operation cost for all customers at all ranks is equal to

$$\sum_{i\in\mathscr{I}}\left(\sum_{j\in\overline{\mathscr{J}}}\lambda_i(2-q_j)c_{ij}x_{ij}+\sum_{r=1}^{R+1}\sum_{j\in\overline{\mathscr{J}}}\lambda_i\left(\sum_{j'\in\mathscr{J}_j^-}p_{ijj'r}(c_{ijj'r}+(1-q_{j'})c_{ij'})x_{ijj'r}\right)\right) \tag{9}$$

With Eqs. (6) and (9), we can formulate the discrete reliable location problem under imperfect information with round-trip (DRLP-IIRT) as follows:

$$\min_{X,X',Y,P}\sum_{j\in\mathscr{J}}f_jy_j+\sum_{i\in\mathscr{I}}\lambda_i\sum_{j\in\overline{\mathscr{J}}}\left((2-q_j)c_{ij}x_{ij}+\sum_{j'\in\mathscr{J}_j^-}\sum_{r=1}^{R+1}p_{ijj'r}(c_{ijj'r}+(1-q_{j'})c_{ij'})x_{ijj'r}\right) \tag{10}$$

$$\sum_{j\in\overline{\mathscr{J}}}x_{ij}=1,\ \forall\, i\in\mathscr{I} \tag{11}$$

$$x_{ij}+\sum_{r=1}^{R+1}\sum_{j'\in\mathscr{J}_j^+}x_{ij'jr}\leqslant y_j,\ \ \forall\, i\in\mathscr{I}, j\in\mathscr{J} \tag{12}$$

$$x_{ij}=\sum_{j'\in\mathscr{J}_j^-}x_{ijj'1},\ \forall\, i\in\mathscr{I}, j\in\overline{\mathscr{J}} \tag{13}$$

$$\sum_{j'\in\mathscr{J}_j^+}x_{ij'j(r-1)}=\sum_{j'\in\mathscr{J}_j^-}x_{ijj'r},\ \ \forall\, i\in\mathscr{I}, j\in\overline{\mathscr{J}}, r=2,\ \ldots, R+1 \tag{14}$$

$$\sum_{j\in\overline{\mathscr{J}}}x_{ijj_0(R+1)}=1,\ \forall\, i\in\mathscr{I} \tag{15}$$

$$p_{ijj'1}=q_j,\ \forall\, i\in\mathscr{I}, j\in\overline{\mathscr{J}}, j'\in\mathscr{J}_j^- \tag{16}$$

$$p_{ijj'r}=q_j\sum_{j'\in\mathscr{J}_j^+}p_{ij'j(r-1)}x_{ij'j(r-1)},\ \forall\, i\in\mathscr{I}, j\in\overline{\mathscr{J}}, j'\in\mathscr{J}_j^-, r=2,\ \ldots, R+1 \tag{17}$$

$$x_{ij}\in\{0,1\},\ \forall\, i\in\mathscr{I}, j\in\overline{\mathscr{J}} \tag{18}$$

$$x_{ijj'r} \in \{0, 1\}, \forall\, i \in \mathscr{I}, j \in \overline{\mathscr{J}}, j' \in \mathscr{J}_j^-, r = 1, \ldots, R + 1 \tag{19}$$

$$y_j \in \{0, 1\}, \forall\, j \in \mathscr{J} \tag{20}$$

Objective (10) decides the optimal facility locations and the best customer visit sequences that minimize the total system cost. Constraint (11) requires that a customer must visit one facility at rank 0. Constraint (12) ensures that a customer cannot visit an unbuilt facility or visit the same facility repeatedly at multiple ranks. Constraints (13) and (14) ensure that each customer must visit the pre-assigned facilities successively according to their ranks. Constraint (15) guarantees that each customer must go to the dummy facility finally so that the penalty cost can be correctly formulated. Constraints (16) and (17) illustrate the iteration relationship between facility visiting probabilities at adjacent ranks. Constraints (18)-(20) postulate binary decision variables.

3.2. Solution algorithm

The DRLP-IIRT model is an NP-hard integer programming model, and it is difficult to solve it directly, particularly for large-scale problem instances. Yun et al. (2015) shows a similar problem that cannot be efficiently solved by off-the-shelf solvers. Therefore, we develop a customized LR algorithm to solve the DRLP-IIRT model for a near-optimum solution.

In the customized LR algorithm, we first select constraint (12) to relax with Lagrangian multipliers $\mu := \{\mu_{ij} \geqslant 0\}_{i\in\mathscr{I}, j\in\mathscr{J}}$ and add it to objective (10), which yields the following relaxed problem (RP):

$$Z(\mu) := \min_{X,X',Y} \sum_{j\in\mathscr{J}} \left(f_j - \sum_{i\in\mathscr{I}} \mu_{ij}\right) y_j + \sum_{i\in\mathscr{I}} \left(\begin{array}{l} 2\lambda_i c_{ij_0} x_{ij_0} + \sum_{j\in\mathscr{J}} (\lambda_i (2 - q_j) c_{ij} + \mu_{ij}) x_{ij} \\ + \sum_{j\in\mathscr{J}} \sum_{r=1}^{R+1} \left(2\lambda_i p_{ijj_0 r} c_{ijj_0 r} x_{ijj_0 r} + \sum_{j'\in\mathscr{J}\setminus\{j\}} (\lambda_i p_{ijj'r} (c_{ijj'r} + (1 - q_{j'}) c_{ij'}) + \mu_{ij'}) x_{ijj'r} \right) \end{array} \right) \tag{21}$$

subject to (11) and (13)–(20).

Solving RP for any given Lagrangian multipliers μ yields a lower bound of the DRLP-IIRT model. Because of the relaxation of constraint (12), we can decompose RP into two independent sub-problems: problem Sub-1 is

$$Z^Y(\mu) = \min_Y \sum_{j\in\mathscr{J}} \left(f_j - \sum_{i\in\mathscr{I}} \mu_{ij}\right) y_j \tag{22}$$

subject to binary constraint (20); problem Sub-2 is

$$Z^{X,X'}(\mu) = \min_{X,X'} \sum_{i\in\mathscr{I}} \left(\begin{array}{l} 2\lambda_i c_{ij_0} x_{ij_0} + \sum_{j\in\mathscr{J}} (\lambda_i (2 - q_j) c_{ij} + \mu_{ij}) x_{ij} + \\ \sum_{j\in\mathscr{J}} \sum_{r=1}^{R+1} \left(\begin{array}{l} 2\lambda_i p_{ijj_0 r} c_{ijj_0 r} x_{ijj_0 r} + \\ \sum_{j'\in\mathscr{J}\setminus\{j\}} (\lambda_i p_{ijj'r} (c_{ijj'r} + (1 - q_{j'}) c_{ij'}) + \mu_{ij'}) x_{ijj'r} \end{array} \right) \end{array} \right) \tag{23}$$

subject to (11) and (13)–(19).

For the given μ, we can solve Sub-1 easily: if $f_j - \sum_{i\in\mathscr{I}} \mu_{ij} < 0$, set $y_i = 1$; otherwise, set $y_i = 0$. Sub-2 can be solved by Dijkstra's algorithm as a shortest path problem, which is interpreted in Appendix B.1.

If the solution from RP is feasible in the DRLP-IIRT model and the objective value of RP is equal to that of the DRLP-IIRT model, this is the optimal solution to the DRLP-IIRT model. Otherwise, we will construct a feasible solution to DRLP-IIRT as an upper bound. We first fix the solution to Sub-1 that indicates the actual installed facilities as $\mathscr{J}^* = \{j \mid y_j = 1, \forall\, j \in \mathscr{J}\}$. Then, based on these installed facilities, we obtain the other variables by solving the problem $\bar{Z}^{X,X'}(\mu)$, which is adapted from Sub-2 as follows:

$$\bar{Z}^{X,X'}(\mu) = \min_{X,X',j\in\mathscr{J}^*} \sum_{i\in\mathscr{I}} \lambda_i \sum_{j\in\bar{\mathscr{J}}} \left((2 - q_j) c_{ij} x_{ij} + \sum_{j'\in\mathscr{J}_j^-} \sum_{r=1}^{R+1} p_{ijj'r} (c_{ijj'r} + (1 - q_{j'}) c_{ij'}) x_{ijj'r} \right) \tag{24}$$

subject to (11)–(19).

We use a heuristic algorithm, which is interpreted in Appendix B.2 to solve the problem $\bar{Z}^{X,X'}(\mu)$. Then we obtain a feasible solution $\{X, X', Y\}$, which will be plugged into objective to obtain an upper bound to the DRLP-IIRT model.

If the lower and upper bounds are identical, the feasible solution is guaranteed to be the optimal solution to the DRLP-IIRT model. Otherwise, we update Lagrangian multipliers μ by a sub-gradient method to improve the upper bound (or the best feasible solution so far) and further tighten the lower bound.

The sub-gradient method iteratively updates μ, and $\mu^k = \left\{\mu_{ij}^k\right\}$ denotes its value at each step k. Initialize multipliers

$\{\mu_{ij}^1 = [f_j/|\mathscr{I}| + 0.5]\}_{i\in\mathscr{I},j\in\mathscr{J}}$. At each iteration step $k \geqslant 1$, we update multipliers μ^k to μ^{k+1} for the next iteration as follows:

$$\mu_{ij}^{k+1} = \mu_{ij}^k + t_k\left(x_{ij} + \sum_{r=1}^{R}\sum_{j'\in\mathscr{J}\backslash j} x_{ij'jr} - y_j\right), \forall\ i \in \mathscr{I}, j \in \mathscr{J} \tag{25}$$

where t_k denotes a step size formulated as

$$t_k = \frac{u_k(UB_k - LB_k)}{\sum_{i\in\mathscr{I}}\sum_{j\in\mathscr{J}}\left|x_{ij} + \sum_{r=1}^{R}\sum_{j'\in\mathscr{J}\backslash j} x_{ij'jr} - y_j\right|} \tag{26}$$

In Eq. (26), UB_k denotes the best upper bound up to iteration step k. LB_k denotes the lower bound solved at iteration step k. Parameter u_k denotes a constant at iteration step k, and its initial value is set in interval $(0, 2]$. u_k will be updated by $u_k = u_k/\theta$ if LB_k does not increase for $K = 6$ consecutive iterations, where θ is the scaling factor. Note that the term in the denominator is set to the absolute value because it yields better convergence than the squared term in the classic LR algorithm.

The termination criteria of the customize LR algorithm are shown as follows:

(1) $(UB_k - LB_k)/UB_k \leqslant \xi$, where $\xi = 0.005$ is the error tolerance.
(2) $k \geqslant k_{\max}$, where $k_{\max} = 10^6$ is a maximum iteration number.
(3) $u_k \leqslant u_{\min}$, where $u_{\min} = 10^{-3}$ is a minimum value.
(4) The solution time reaches time limit $T = 1800$ sec.

To clear the process of LR algorithm, Fig. 4 shows the flowchart of our customized algorithm.

To highlight the efficiency of our LR algorithm, we also solve the proposed model by using a widely used commercial solver. Commercial solvers are usually more efficient in solving a linear programming model than in solving a non-linear programming model. Therefore, a linearization method is used to transform our proposed model. In the linearization method, a new variable $w_{ijj'r}$ is introduced to replace $p_{ijj'r}x_{ijj'r}$. Then, we add a set of new constraints formulated below

$$w_{ijj'r} \leqslant p_{ijj'r}, \forall\ i \in \mathscr{I}, j \in \overline{\mathscr{J}}, j' \in \mathscr{J}_j^-, r = 1, 2, \cdots, R + 1 \tag{27}$$

$$w_{ijj'r} \leqslant x_{ijj'r}, \forall\ i \in \mathscr{I}, j \in \overline{\mathscr{J}}, j' \in \mathscr{J}_j^-, r = 1, 2, \cdots, R + 1 \tag{28}$$

$$w_{ijj'r} \geqslant 0, \forall\ i \in \mathscr{I}, j \in \overline{\mathscr{J}}, j' \in \mathscr{J}_j^-, r = 1, 2, \cdots, R + 1 \tag{29}$$

$$w_{ijj'r} \geqslant p_{ijj'r} + x_{ijj'r} - 1, \forall\ i \in \mathscr{I}, j \in \overline{\mathscr{J}}, j' \in \mathscr{J}_j^-, r = 1, 2, \cdots, R + 1 \tag{30}$$

Finally, a commercial solver (e.g., Gurobi) can be applied to the DRLP-IIRT model with the linearized formulation.

4. Case studies

This section first describes how to design the problem instances. Then, we present a series of case studies to test the performance of the proposed model and shed insights into the optimal facility location layouts. Next, we conduct a sensitivity analysis to the effects of several key parameters on the optimal solution. Finally, we conduct a case study with real world data to illustrate the application of our proposed model.

4.1. Experimental design

The proposed model is tested on a network including 49, 88 or 150 nodes that are located in the continental United States (Daskin, 1995). The 49-node set is consist of Washington, DC and 48 state capitals of the US. The 88-node set contains the 49-node set and the 50 most populous cities in the US, minus duplicates. Both 49-node and 88-node sets are derived from 1990 census data. The 150-node set includes the 150 largest cities of USA derived from the 2010 census data. The data is processed to obtain the model parameter values as follows. The nodes in these sets represent the locations for both customers and candidate facilities. Customer demands $\{\lambda_i\}_{i\in\mathscr{I}}$ are set to the corresponding state population divided by 10^5 for the 49-node set and the corresponding city population divided by 10^4 for the other two node sets. The median home value in each city denotes the fixed opening cost f_j. To capture detours in roadway networks, we calculate the distance between two locations by multiplying a coefficient of 1.2 to the great circle distance (Qureshi et al., 2002). The facility disruption probability q_j is set as $\rho e^{-f_j/200000}$, $\forall\ j \in \mathscr{J}$. Coefficient ρ dictates the magnitude of disruption probabilities. The penalty costs per unit demand for all customers are the same and set as $\varphi = 10000$. The default values of parameters in the DRLP-IIRT model are set as follows: $\rho = 0.05$, $\alpha = 1$, and $R = 3$.

We code LR in the Java language. The program runs at a personal computer with a 3.40 GHz CPU, 16.0 GB RAM and the OS of Windows 7-x64. The parameters in LR have the following values: $u_k = 2$, $\theta = 1 + 0.1\rho$, $K = 6$, $\xi = 0.5\%$, $k_{\max} = 10^6$, $u_{\min} = 10^{-3}$ and $T = 1800$ sec.

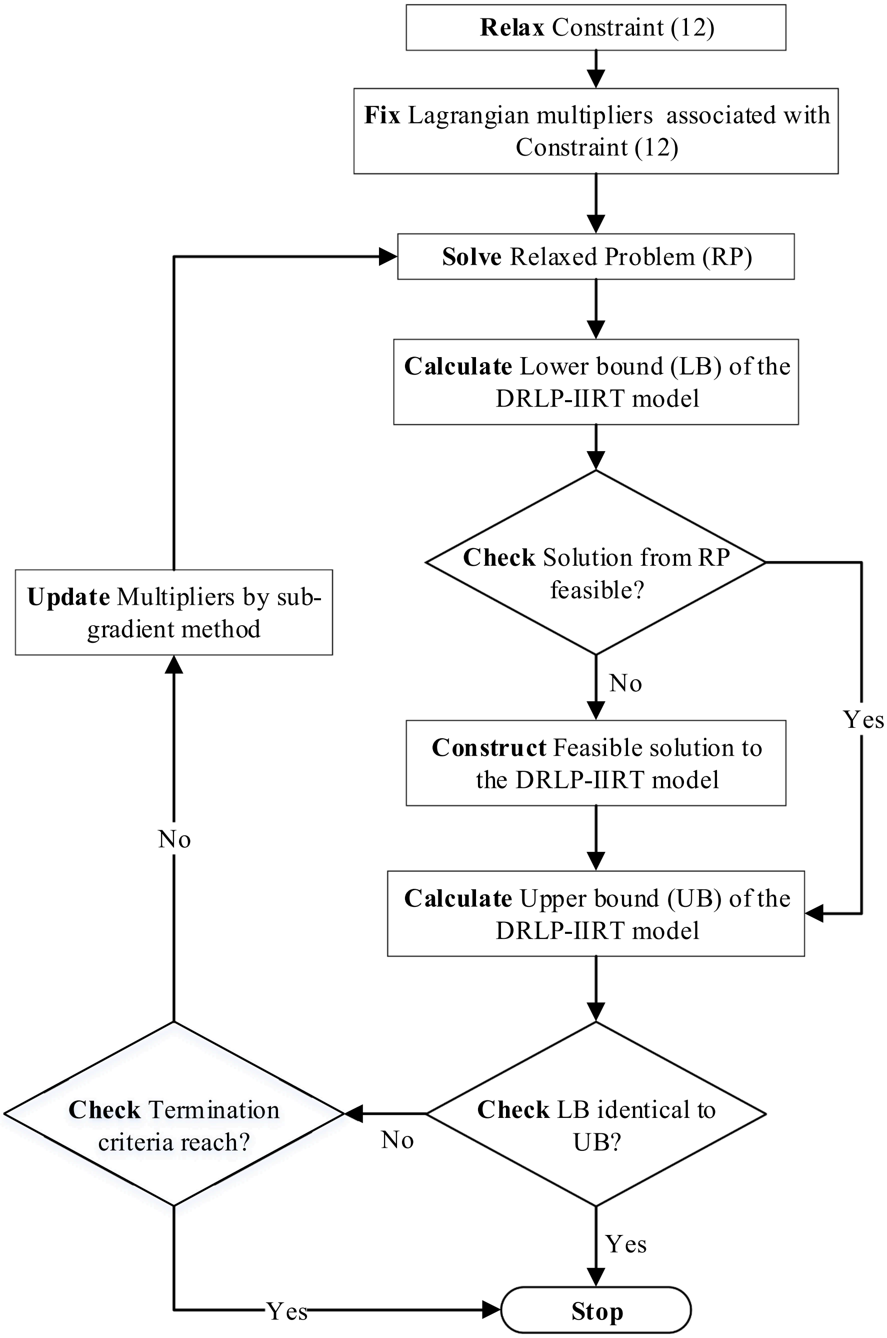


Fig. 4. Flowchart of the customized LR algorithm.

4.2. Model performance

Gurobi (version 7.0.2) is a widely used commercial solver for solving the mathematic programming model. Thus, we test the performance of our LR algorithm by comparing with that of Gurobi. The gap and time limit of Gurobi are set as 0.5% and 1800 sec, respectively, corresponding to the settings in the LR algorithm. The other options of Gurobi are set to their default values. The results are shown in Table 1. In the last line of each node set, it shows the average values of each performance index. This table shows that all the instances can be solved by LR within the time limit, and the gap is less than 5%, which is acceptable for the majority of engineering practices. However, the solver, Gurobi, solved the 49-node set instances with a longer solution time, indicating that the performance of our proposed algorithm is superior. For the instance of the 49-node set with $\rho = 0.05$, we see that the best objective solved by LR is less than that of Gurobi but their gaps are reversed. This is because the two solution methods obtain different lower

Table 1
Performance comparison for different instances.

Node	ρ	Best objective (dollar)		Gap		Solution time (sec)		Facility locations	
		LR	Gurobi	LR	Gurobi	LR	Gurobi	LR	Gurobi
49	0.05	1460350	1463504	0.50%	0.36%	4.17	661.65	1–7, 29–31	1–7, 29–31
	0.1	1529502	1532142	0.50%	1.09%	8.21	1801.05	1–8, 29–31	1–8, 29–31
	0.2	1693779	1695295	0.50%	4.53%	8.76	1800.87	1–8,12, 22,29–31	1–3,5–8,12, 22,29–31,35
	0.4	2206490	2529934	0.89%	30.07%	17.70	1800.60	1–7,12,14, 22,29–31, 35,39	1–8,12,22, 26,28–31
	Average	1722530	1805219	0.60%	9.01%	9.71	1516.04		
88	0.05	2160780	NA	0.50%	NA	281.03	> 1800	3,4,7, 10,12,15, 18,28,30, 32,33,46, 67,72	NA
	0.1	2255482	NA	0.62%	NA	398.19	> 1800	3,4,7, 10,12,15, 18,23,28, 30,32,33, 46,67,72	NA
	0.2	2475358	NA	1.22%	NA	235.79	> 1800	3,4,5,7, 10,15,18, 23,28,30, 32,33,46, 47,67,72	NA
	0.4	3149047	NA	0.60%	NA	117.53	> 1800	1,3,4,5,7, 10,12,15, 18,28,30, 32,33,40, 46,48,55, 67	NA
	Average	2510167	NA	0.73%	NA	258.13	> 1800		
150	0.05	3361146	NA	0.58%	NA	1800.12	> 1800	81,94,106, 120,123, 126,131, 138,141, 142,144, 148,150	NA
	0.1	3511968	NA	1.01%	NA	1800.14	> 1800	81,94,106, 120,123, 126,131, 138,141, 142,144, 148,150	NA
	0.2	3835379	NA	2.16%	NA	1116.03	> 1800	81,84,106, 120,123, 126,131, 132,138, 141,142, 144,148, 150	NA
	0.4	4612649	NA	4.14%	NA	604.36	> 1800	65,68,81, 94,113, 120,126, 132,138, 141–144, 148–150	NA
	Average	3830286	NA	1.98%	NA	1330.16	> 1800		

bounds. In this particular instance, LR underestimates the lower bound in comparison to Gurobi. From this table, we also observe that the best objective is increasing with the increase of ρ, which means that the larger disruption probability incurs a higher total system cost, probably to counter-act higher uncertainties. The number of facilities also increases with ρ to guarantee that customers can obtain the service. In all instances associated with each node set, a subset of facilities is always picked for installation in the optimal solution for all ρ values, which means that these facilities play a key role for the system's reliability. Further, we observe that as the instance scale increases, the solution time increases super-linearly and the optimality gap is drastically widened. Considering this

Table 2
Benefit of backup facilities.

Node	R	ρ = 0.05		ρ = 0.2	
		Total system cost (dollar)	Reduced percentage	Total system cost (dollar)	Reduced percentage
49	0	2264571		4680632	
	1	1488520	34%	2133546	54%
	2	1461380	35%	1749751	62%
	3	1460350	36%	1693779	63%
	4	1460310	36%	1684056	64%
	5	1460309	36%	1683458	64%
	6	1460309	36%	1683251	64%
	7	1460309	36%	1683222	64%
	Average	1564507	35%	2123962	62%
88	0	3463728		7097625	
	1	2210796	36%	3133928	56%
	2	2162679	38%	2572508	64%
	3	2160780	38%	2475358	65%
	4	2160714	38%	2455459	65%
	5	2160711	38%	2453386	65%
	6	2160711	38%	2452462	65%
	7	2160711	38%	2456320	65%
	Average	2330104	37%	3137131	64%
150	0	5020247		7008604	
	1	3410832	32%	4545670	35%
	2	3362587	33%	3914321	44%
	3	3361146	33%	3835379	45%
	4	3361106	33%	3817436	45%
	5	3361104	33%	3826397	45%
	6	3361104	33%	3824863	45%
	7	3361104	33%	3822804	45%
	Average	3574904	33%	4324434	44%

trend, we can speculate that if the scale of instances keeps increasing, our model and algorithm will take long time to obtain solutions or may not even give meaningful feasible solutions at some point. This implies that the discrete model and LR algorithm will probably have difficulty in addressing large-scale of instances. For the instances of the 150-node set, the solution time is much smaller when $\rho = 0.4$, compared with smaller values. This result indicates that the LR method finds a local optimal solution easily for the large-scale instances with large facility disruption probabilities. Once the local optimal solution is found, the best bound is often unchanged for many iterations, which causes the fast decrease of u_k. This leads the LR algorithm to meet the termination criteria $u_k \leqslant u_{\min}$ in a short time.

It is important to provide backup service to customers for reducing the penalty cost (as well as the total system cost) in the reliable location design context. Therefore, we study the benefit of backup facilities for DRLP-IIRT, as shown in Table 2 and Fig. 5. In this table, $R = 0$ indicates that only one primary facility is provided to each customer without any backup facility. When $R = 1$, one additional backup facility is provided to each customer, which leads to the total system cost decrease by 34% (54%) for the 49-node instance with $\rho = 0.05$ ($\rho = 0.2$), 36% (56%) for the 88-node instance with $\rho = 0.05$ ($\rho = 0.2$) and 32% (35%) for the 150-node instance with $\rho = 0.05$ ($\rho = 0.2$) compared with the corresponding instances of $R = 0$. These cost decreases can be interpreted as the marginal benefits of one additional backup facility. As R increases, the marginal benefits of backup facilities still exist but start to

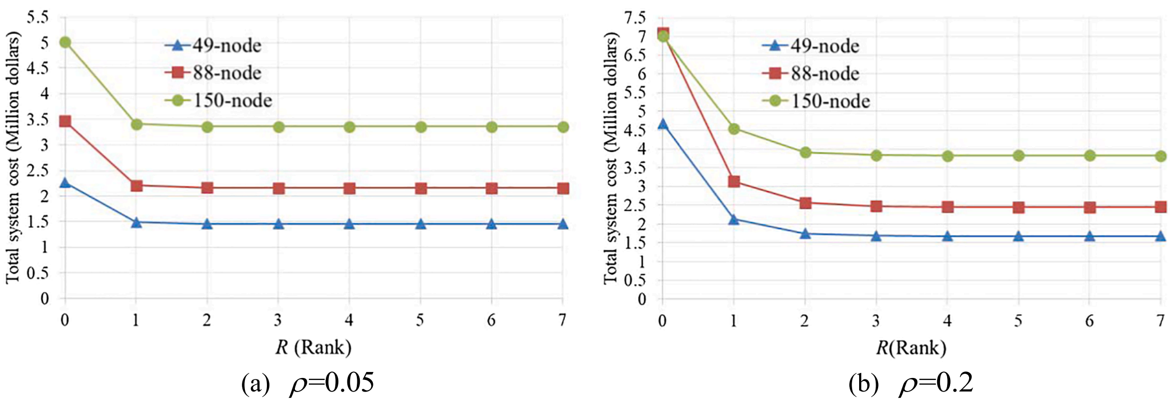
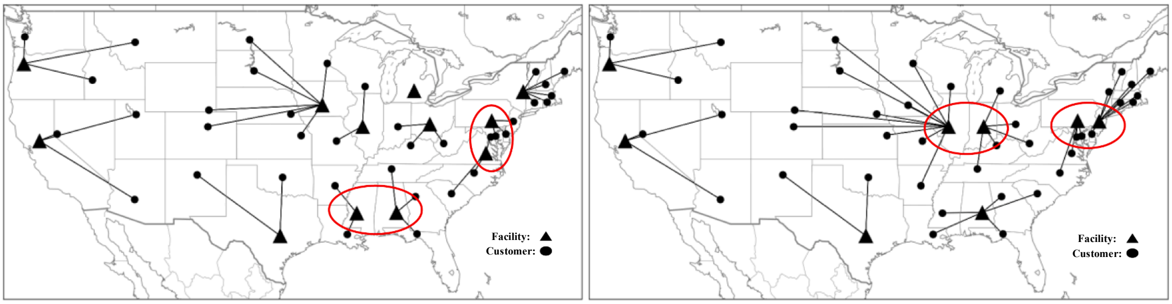
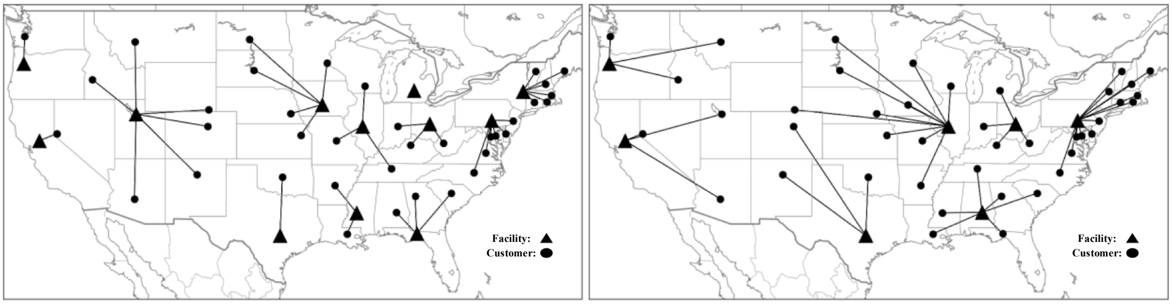


Fig. 5. Effect of R on total system cost.



(a) Imperfect information with round-trip. (b) Imperfect information with outbound trip only.



(c) Perfect information with round-trip. (d) Perfect information with outbound trip only.

Fig. 6. Optimal facility layouts under different transportation costs and information schemes.

flatten out. When provided with sufficient backup facilities, the total cost is reduced to 64% (36%) of that when $R = 0$ for the 49-node instance, 62% (35%) for the 88-node instance and 67% (55%) for the 150-node instance. With an increase in ρ, the marginal benefits of backup facilities also increase, indicating that considering backup facilities is important for unreliable situations. We observe that when $R = 3$ the maximum cost reductions are obtained, while larger R does not significantly increase the marginal benefits. We also show the average values for each performance index in Table 2. Fig. 5 shows this trend clearly. Therefore, in the following analysis we only consider instances in which $R \leqslant 3$. This is why we set $R = 3$ in the majority of our case studies.

We discuss the optimal location layouts under different parameter settings. Fig. 6 compares the optimal facility location layouts of the 49-node instances under four situations with $\rho = 0.2$: (a) imperfect information with round-trip; (b) imperfect information with outbound trip only; (c) perfect information with round-trip and (d) perfect information with outbound trip only. For simplification, we just show the connection between customers and their primary facilities in Fig. 6. Comparing Fig. 6(a) and Fig. 6(b), we see that more facilities are built when customers consider round-trip transportation to shorten travel distances and reduce transportation costs. Although the outbound trip and the inbound trip are the same under perfect information, omitting the inbound trip will reduce the proportion of the transportation cost to the total cost, which leads to the facility layout being suboptimal, as shown in Fig. 6(c) and Fig. 6(d). Comparing Fig. 6(a) and Fig. 6(c), we see that some facilities layout under imperfect information is more aggregated than that under perfect information shown in the red circles. The same observation exists when comparing Fig. 6(b) and Fig. 6(d). This is because customers have to visit the pre-assigned facilities individually under imperfect information, and more aggregated facility layouts are beneficial for customers to obtain the service with lower transportation cost. Comparing Fig. 6(a) and Fig. 6(c), we observe that some facility layouts under imperfect information are more aggregated than those under perfect information shown in the red circles. The same observation exists when comparing Fig. 6(b) and Fig. 6(d). This is because customers with imperfect information have to visit the pre-assigned facilities individually, and more aggregated facility layouts are beneficial for customers to obtain the service at lower transportation cost. This aggregated facility layout is common in the area where customer demands are intense. Assuming customer demands are dispersed in an area, e.g. in the western part of the US, the facility layout may not be aggregated. Comparing Fig. 6(b) and Fig. 6(d), we notice that the number of facilities in Fig. 6(b) more than that of the facilities in Fig. 6(d). This is because the expected travel distances for customers with imperfect information are longer than the distances for those with perfect information. To ensure customers can be served by facilities in reasonable distances and avoid the penalty cost as much as possible, more facilities may be built to shorten the travel distance under imperfect information.

Now, we compare the total system costs with the round-trip and the outbound trip only to highlight the importance of considering the inbound trip. The optimal system cost with the outbound trip is denoted by C^*_{OT}. We use C_{RT} to express the actual total system cost with the inbound trip when the optimal facility layouts with outbound trip only is implemented. $\varepsilon_{OT} = (C^* - C^*_{OT})/C^*$ denotes the difference in total system cost with the round-trip and outbound trip only, whereas $\varepsilon_{RT} = (C_{RT} - C^*)/C^*$ denotes the actual system cost deviation after applying the “wrong” facility location design. Table 3 shows the total system costs of several problem instances of the 49-node set with the round-trip and outbound trip only. The total optimal cost for a system considering outbound trips alone is

Table 3
Total system cost estimation with the round-trip and outbound trip only.

ρ	C^*	C^*_{OT}	C_{RT}	ε_{OT}(%)	ε_{RT}(%)
0.05	1,460,350	1,018,129	1,614,236	30.28%	11%
0.1	1,529,502	1,076,203	1,651,829	29.64%	8%
0.2	1,693,779	1,194,551	1,801,682	29.47%	6%
0.4	2,206,490	1,548,738	2,352,850	29.81%	7%

lower than that with the round-trip in all instances. Additionally, ε_{OT} values around 30% indicate that the inbound trip cost accounts for one-third of the total system cost. However, for systems where round trips do take place (e.g., in an electric vehicle community charging system where one needs to return home after finding a charging spot or giving up the searching), omitting the inbound trip may lead to sub-optimal facility layouts. Table 3 shows that the actual total system cost C_{RT} under the suboptimal facility location design (due to ignorance of the inbound trip cost) is higher than the optimal system cost C^*. ε_{RT} reflects the percentage of this increase and its value is > 6%. These results indicate that facility location design should consider both bounds if customers do have inbound trips following outbound trips. Otherwise, the total system cost will increase under the suboptimal facility location design due to the ignorance of the inbound trip costs. Therefore, the correct routing scheme is important for facility location problems and we will discuss it next.

Further, we discuss how important it is to use the correct locations and the correct information/routing scheme. Table 4 shows the ratios of total system costs under different locations and information/routing schemes for the instances of the 49-node set with $\rho = 0.1$. Taking ratio $= 1.080$ for example, the numerator of this ratio is equal to the sum of the facility costs corresponding to the locations under imperfect/outbound, and then we find the optimal operation cost given those locations under imperfect/round. The denominator of the ratio is equal to the optimal system cost under imperfect/round. In this table, we see that all the ratios are greater than or equal to 1. This result indicates that the manager correctly obtains the optimal system cost only when he or she gets the correct information/routing scheme. Otherwise, he or she will underestimate the total system cost. However, these four numbers are actually no greater than 1.005 in Table 4. It means that if the facility locations are obtained by an incorrect information scheme, the actual total system cost is not that different from that with the correct information scheme. This indicates that mistaking the scheme would not be matter much in some problem settings. However, for managers and planners, incorrect information scheme may lead to a wrongly estimated system cost much off from the actual total system cost. This inconsistency will cause them to fail in calculating financial indicators accurately. Further, we observe that the ratios under misestimating the routing scheme are greater than that for misestimating the information scheme. This means that the correct routing scheme is more important than the correct information scheme for the facility locations design. However, both information and routing schemes influence facility location and customer routing design, and they should be correctly considered.

Finally, we compare the performance of the discrete model and continuous model proposed by Yun et al. (2019).Since the areas of the nodes in the 88-node set or 150-node set are not contiguous areas, we choose the 49-node set and use the method proposed by Peng et al. (2014) to adaptively convert discrete spatial data into continuous functions in a metric space for dealing 49-node set instance by the continuous model. Table 5 shows the comparison results between the discrete and continuous models. In this table, we see that the system costs obtained by the discrete model and continuous model are close. It indicates that the continuous model can also better solve the reliable facility location problems. The facility number of the continuous model is less than that of the discrete model. Correspondingly, the construction cost of the continuous model is also less than that of the discrete model. Fewer facilities and construction cost lead that the transportation cost obtained by the continuous model is greater. The highlight of the continuous model is the extremely short solution time. These results indicate that the continuous model can solve our studied facility location problem in a very short time with a comparable solution.

4.3. Sensitivity analysis

This section reports on sensitivity analyses to parameters ρ, α and φ on the optimal solutions of the 49-node instance with $R = 3$. Fig. 7 shows how parameters affect the optimal solutions. The default values of parameters are set as $\rho = 0.05$, $\alpha = 1$, and $\varphi = 10000$, and we select one parameter to vary at a time.

Fig. 7(a) shows the influence of the facility disruption probability coefficient ρ on all cost components. As ρ increases, all cost

Table 4
Ratios of total system costs under different locations and information/routing schemes.

Status of information/routing	Facility locations determined from			
	Imperfect/round	Imperfect/outbound	Perfect/round	Perfect/outbound
Imperfect/round	1.000	1.080	1.001	1.085
Imperfect/outbound	1.062	1.000	1.041	1.001
Perfect/round	1.001	1.076	1.000	1.077
Perfect/outbound	1.077	1.004	1.053	1.000

Table 5
Performance comparison between the discrete model and continuous model.

	Discrete model	Continuous model
Total system cost (dollar)	1,460,350	1,463,971
Construction cost (dollar)	690,600	652,748
Transportation cost (dollar)	769,702	811,198
Penalty cost (dollar)	48	25
Facility number	10	9
Solution time (sec)	4.17	0.03

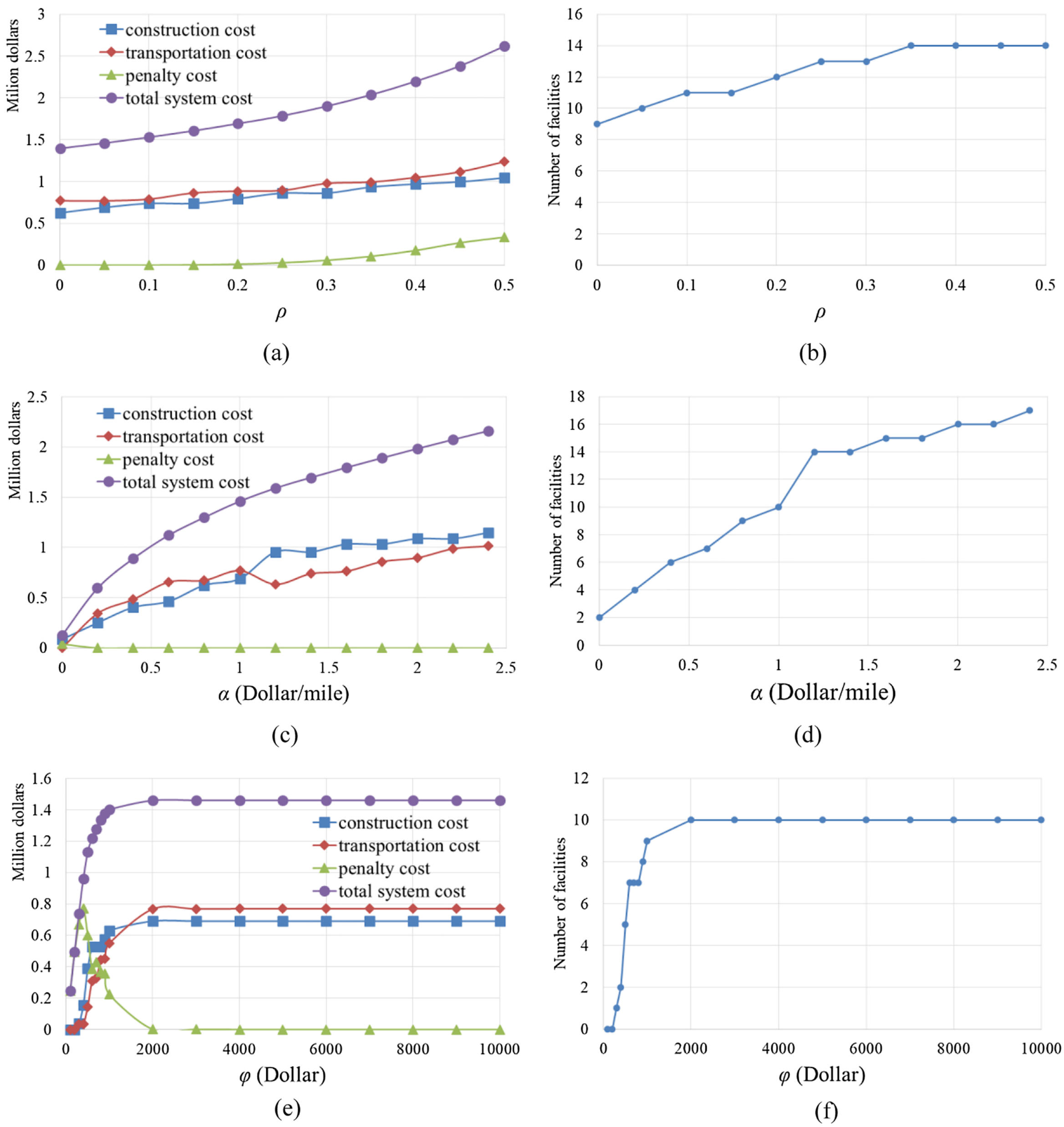


Fig. 7. Sensitivity analysis.

components increase. The total system cost increases slowly when ρ is less than 0.25. However, as ρ becomes larger, it increases rapidly and becomes very large. This is because the penalty cost becomes the dominating component of the total system cost increase as ρ becomes larger. Fig. 7(b) shows that the number of facilities increases as ρ increases. Five additional facilities are added to the total number of facilities as ρ increases from 0 to 0.5. Although the number of facilities with different ρ values might be the same, the optimal facility location designs are different due to different construction costs.

Fig. 7(c) and (d) illustrate how all cost components and the number of facilities change as the transportation rate α grows. We observe that the number of facilities and all cost components except the penalty cost increase significantly as α increases. It is obvious that the increase of the transportation cost rate will cause the transportation cost to increase. Additional facilities are built to reduce the customers' travel distance that reduces the growth rate of the transportation cost. However, this causes an increase in the construction cost. This is a tradeoff between the transportation and construction costs. Since there are a sufficient number of existing facilities to be assigned to customers of all ranks, customers merely bear the penalty cost with very small different probabilities. Therefore, the penalty cost does not change.

Fig. 7(e) and (f) present the impacts of increasing the penalty rate φ. When φ is less than 200, there is no gain from building new facilities, leading to no construction cost, and thus the total system cost and the penalty cost are identical. As φ increases from 300 to 2000, we see that the number of facilities and all cost components except the penalty cost are increasing quickly which soon reduces the penalty cost to a very low level. As φ continues to increase, the number of facilities and all cost components have either no change or a very small increase. This indicates that the system performance, e.g., facility locations and costs, remains unchanged when the penalty rate φ exceeds the critical value. These results provide useful information on how reliable system design confines variations of the penalty rate.

5. Conclusion

This paper proposes a discrete model for the RUFL problem with round-trip transportation under imperfect information. A counterpart continuous model to address large-scale RUFL problem instances will be presented in the Part II paper (Yun et al., 2019). In this study, we assume each facility may be disrupted with a site-dependent probability. However, a customer cannot obtain the real-time information about facility states and always attempts to visit the pre-assigned facilities consecutively to minimize her expected operational cost. This customer will obtain the service at the first operating facility (if available) within her reach; otherwise, she will give up the service. Then, the customer will return to her home location. A reliable facility location model is proposed to formulate this reliable facility location problem. Since the investigated problem is an NP-hard problem, we develop a customized LR algorithm to solve it to near-optimum solutions. Case studies indicate that the proposed model has a good performance on small and medium-scale instances. We also observe that the marginal benefit of backup facilities will decrease when more backup facilities are set. The round-trip needs to be considered in reliable facility location problems particularly with imperfect information. Further, we also point out that the correct information/routing scheme is important for obtaining the correct facility location. The results of the sensitivity analysis for key parameters indicate that the proposed model has a robust performance.

In the future research, we can further relax the assumption of independent disruption probabilities and allow facilities to have general correlated disruption patterns. Since the customer demands may be uncertain in many real-world applications, considering this uncertainty to this model framework will be also interesting. If relevant system level data are available, it will be helpful to apply the model to more realistic cases to improve the system preparedness.

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Appendix A. Notation list

Model set	Description
$\mathcal{I}, i \in \mathcal{I}$	Set of customer locations, indexed by i
$\mathcal{J}, j \in \mathcal{J}$	Set of candidate facility locations, indexed by j
$\overline{\mathcal{J}}$	Set of candidate facility locations including the dummy facility
$\mathcal{J}_{\mathrm{j}}^{+}$	Set of candidate facility locations that can be visited before visiting facility j
$\mathcal{J}_{\mathrm{j}}^{-}$	Set of candidate facility locations that can be visited after visiting facility j
Model parameter	Description
λ_i	Demand of customer i
f_j	Fixed opening cost of facility j
φ_i	Penalty cost when customer i gives up on the service
q_j	Disruption probability of facility j
α	Unit-distance transportation rate
r	Facility rank for a customer, the maximum facility rank is R
$c_{ij}(c_{ij'})$	Unit-demand transportation cost from customer i to facility j (j')
j_i^r	Facility for customer i at rank r
$c_{ijj'r}(c_{ij_i^{r-1}j_i^r r})$	Unit-demand transportation cost from facility j (j_i^{r-1}) to facility j' (j_i^r) for customer i at rank r

ρ	Coefficient to control the overall magnitude of the disruption probabilities
Decision variable	Description
$y_j \in Y$	Facility location decisions: $y_j = 1$ denotes facility j is constructed, $y_j = 0$ otherwise
$x_{ij} \in X$	Customer assignment decisions: $x_{ij} = 1$ denotes customer i is assigned to facility j at rank 0; $x_{ij} = 0$ otherwise
$x_{ijj'r} \in X'$	Customer assignment decisions: $x_{ijj'r} = 1$ denotes customer i is assigned to facility j at rank $r-1$ and to facility j' at rank r; $x_{ijj'r} = 1$ otherwise
$p_{ijj'r}$	Probability that customer i visits facility j' at rank r after visiting facility j
$w_{ijj'r}$	Auxiliary decision variable identical to $p_{ijj'r}x_{ijj'r}$
LR parameter	Description
μ	Lagrangian multipliers
k	Iteration step, the maximum iteration step is $k_{\max}$
t_k	Step size at k formulated by function (26)
u_k	Constant at k, the minimum value is $u_{\min}$
ξ	Error tolerance
T	Time limitation
UB_k	The best upper bound up to iteration step k
LB_k	The lower bound solved at iteration step k

Appendix B. Algorithm descriptions

B.1. Shortest path algorithm to problem $Z^{X,X'}(\mu)$

For customer i, we first construct a graph with $R+1$ ranks (staring from rank 0) for problem $Z_i^{X,X'}(\mu)$, as illustrated in Fig. 8.

The root node is associated with customer i, and the nodes at each rank $r = 0, \cdots, R+1$ represent this customer's rank-r assignment. At each rank $r = 0, \cdots, R$, there are $|\overline{\mathscr{J}}|$ nodes, and a node (j, r) corresponds to that facility j that is assigned to this customer at rank r. Rank $R+1$ only contains one node $(j_0, R+1)$, denoting that a customer must visit the dummy facility at rank $R+1$. Customer node i is connected to all nodes at rank 1 through directed arcs, indicating that the customer is allowed to visit any location in $\overline{\mathscr{J}}$ at assignment rank 0. The arc cost from customer i to each node $(j \in \mathscr{J}, 0)$ is $(\lambda_i(2-q_i)c_{ij} + \mu_{ij})$, and the arc cost from customer i to the dummy node $(j_0, 1)$ is $2\lambda_i c_{ij_0}$. For each $r = 1, \cdots, R+1$, node $(j \in \overline{\mathscr{J}}, r-1)$ is connected to nodes $(j' \in \mathscr{J}_j^-, r)$ (or node $(j_0, R+1)$ if $r = R+1$), representing that the customer can visit a location in $\mathscr{J}_j^-$ at rank r after visiting j at rank r-1. The arc cost from node $(j, r-1)$ to $(j' \in \mathscr{J} \backslash \{j\}, r)$ is $(\lambda_i(c_{ijj'r} + (1-q_{j'})c_{ij'}) + \mu_{ij'})$, and the arc cost from $(j, r-1)$ to (j_0, r) is $2\lambda_i p_{ijj_0r} c_{ijj_0r}$.

We denote the nodes at rank r by set N_r, $\forall\ r = 0, \cdots, R+1$ and the arcs from rank r-1 (or customer i) to r by A_r, $\forall\ r = 0, \cdots, R+1$. Let N denote the set of all nodes in this graph, i.e., $N = \{N_r\}_{r=0,\cdots,R+1}$, and A denote the set of all arcs in this graph by $A = \{A_r\}_{r=0,\cdots,R+1}$. For each node $n \in N_r$, $\forall\ r = 1, \cdots, R+1$, let N^{n+} denote the set of preceding nodes connected to node n at rank r-1. For each node $n \in N_0$, the preceding node connected to node n is just the root node. For each arc $(n', n) \in A$, we denote its corresponding arc cost by $a_{n'n}$. Then, problem $Z_i^{X,X'}(\mu)$ can be solved by a special version of Dijkstra's algorithm. Basically, at each rank $r = 0, \cdots, R+1$, for each node $n \in N_r$, calculate $v_n := \min_{n' \in N^{n+}}\{v_{n'} + a_{n'n}\}$ and let $p_n := \mathrm{argmin}_{n' \in N^{n+}}\{v_{n'} + a_{n'n}\}$ (a tie can be broken arbitrarily). Then, the optimal objective of $Z_i^{X,X'}(\mu)$ is identical to v_{j_0} at rank $R+1$, and variables X, X' can be solved by iteratively tracing back the shortest path in the following steps.

Step T1: Set $n := (j_0, R+1)$ and $r := R+1$.

Step T2: If $r = R+1$, we break $p_n = (j^*, r-1)$, then set $x_{ij^*j_0r} = 1$ and $x_{ijj_0r} = 0$, $\forall\ j \in \overline{\mathscr{J}} \backslash \{j^*\}$. Otherwise, we break $n = (j'^*, r)$ and $p_n = (j^*, r-1)$, and then set $x_{ij^*j'^*r} = 1$ and $x_{ijj'r} = 0$, $\forall\ j \in \overline{\mathscr{J}} \backslash \{j^*\}$, $j' \in \mathscr{J}_j^- \backslash \{j'^*\}$.

Step T3: If $r = 1$, set $x_{ij^*} = 1$ and $x_{ij} = 0$, $\forall\ j \in \overline{\mathscr{J}} \backslash \{j^*\}$, then return X, X'. Otherwise, update $n := p_n$, $r := r-1$, and go to step 2.

In the end, we obtain the optimal objective of $Z^{X,X'}(\mu)$ by adding $Z_i^{X,X'}(\mu)$ for all customers.

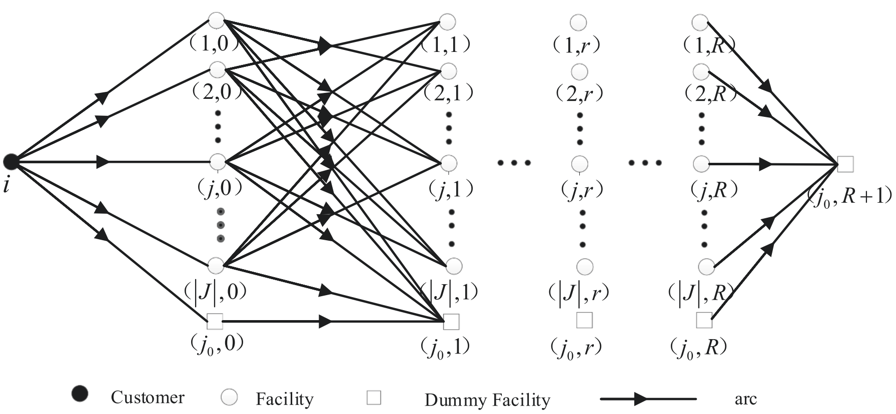

Fig. 8. The graph for the shortest algorithm.

B.2. Heuristic for problem $\overline{Z}^{X,X'}(\mu)$

This heuristic for each problem $\bar{Z}_i^{X,X'}(\mu)$, $i \in \mathscr{I}$ is built on a graph with the same topology as that in Fig. 8 except that only nodes with built facilities, i.e., those associated with $\overline{\mathscr{J}}^*: =\mathscr{J}^* \cup \{j_0\}$, are preserved (whereas other nodes without built facilities, i.e., those associated with $\mathscr{J} \backslash \mathscr{J}^*$, are removed from the graph). Another difference is that the arc cost from customer i to every node $(j \in \overline{\mathscr{J}}^*, 1)$ is $\lambda_i(2 - q_j)c_{ij}$, and the arc cost from $(j, r-1)$ to $(j' \in \mathscr{J}_j^-, r)$ is $\lambda_i(c_{ijj'r} + (1 - q_{j'})c_{ij'})$. We inherit the notation in the previous section. Furthermore, let S_n denote the set of all nodes that the customer i had visited before visiting node n. We initialize $S_n = \varnothing$ for each node $n \in N$. Then, a near-optimum solution to problem $\overline{Z}_i^{X,X'}(\mu)$ can be solved by the following heuristic algorithm adapted from Dijkstra's algorithm.

Step SP1: Set $n: =i$, $v_n: =0$, and $r: =0$.

Step SP2: Denote node (j_0, r) by j_0^r for the simplicity of notation. For each node $n \in N_r \backslash \{j_0^r\}$, we calculate $v_n: =\min_{n' \in N^{n+}}\{v_{n'} + a_{n'n} | \ n \notin S_{n'}\}$ and $p_n: =\operatorname{argmin}_{n' \in N^{n+}}\{v_{n'} + a_{n'n} | \ n \notin S_{n'}\}$ (a tie can be broken arbitrarily). Add the node p_n, and set $S_n: =S_{p_n} \cup \{p_n\}$. For node j_0^r, we again obtain $v_{j_0^r}: =\min_{n' \in N^{j_0^r+}}\{v_{n'} + a_{n'j_0^r}\}$ and $p_{j_0^r}: =\operatorname{argmin}_{n' \in N^{j_0^r+}}\{v_{n'} + a_{n'j_0^r}\}$ (a tie can be broken arbitrarily). Update $r: =r + 1$.

Step SP3: If $r = R + 1$, return $v_{(j_0,R)}$ as the near-optimum objective of $\overline{Z}_i^{X,X'}(\mu)$. Otherwise, go to step 3.

Then, similarly, variables X and X' can be solved by the tracking algorithm T1-T3 proposed in the previous section.

In the end, we obtain the near-optimum objective of $\overline{Z}^{X,X'}(\mu)$ by adding $\overline{Z}_i^{X,X'}(\mu)$ for all the customers.

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