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Identifying Critical Links in Transportation Network Design Problems for Maximizing Network Accessibility

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Abstract
A significant amount of research has been performed on network accessibility evaluation, but studies on incorporating accessibility maximization into network design problems have been relatively scarce. This study aimed to bridge the gap by proposing an integer programming model that explicitly maximizes the number of accessible opportunities within a given travel time budget. We adopted the Lagrangian relaxation method for decomposing the main problem into three subproblems that can be solved more efficiently using dynamic programming. The proposed method was applied to several case studies, which identified critical links for maximizing network accessibility with limited construction budget, and also illustrated the accuracy and efficiency of the algorithm. This method is promisingly scalable as a solution algorithm for large-scale accessibility-oriented network design problems.

The concept of *accessibility* in the context of transportation planning formed more than half a century ago, yet it did not receive sufficient emphasis and understanding until recent decades (*1*). First formally defined as "the potential of opportunities for interaction" by Hansen, accessibility measures the ability or ease for travelers to reach destinations, activities, and services (*1–4*). Its relationship to mobility, the ability to move between places, has been well illustrated in the literature as well, and the two are neither mutually incompatible nor promotive. Having acknowledged the importance of accessibility as a vital metric for evaluating the network performance on how many opportunities a network configuration brings to travelers, many metropolitan planning organizations (MPOs) have highlighted in their plans the goal to improve accessibility, as well as mobility. It is pointed out, however, that traditional planning practice exercised in the United States is more mobility-oriented, which plans for building roads to accommodate traffic and may, in turn, induce more traffic (*3*). Unwisely guided by the sole mobility perspective, addressing accessibility issues in transportation networks can be challenging and demands more attention in theory and practice. In addition, lacking methods has been identified as another obstacle in improving network accessibility (*4*). To this end, in this paper, we study an accessibility maximizing problem supporting the network planning by identifying critical links:

(a) Considers both the travel time budget (TTB) and road construction cost budget (CCB);
(b) Maximizes the number of origin-destination (OD) pairs for which the travel time is below TTB in the network by opening critical links.

The results identify critical links and routes in the network that are indispensable to make travel to destinations possible and below the desired travel time given limited resources.

Literature Review

As was stated earlier, accessibility has been acknowledged as an important measure for evaluating network performance. Although not as prevalent as evaluating traffic congestion, which is in many respects an inverse

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indicator of mobility, measuring accessibility and therefore evaluating transportation network have already gained abundant research interests. El-Geneidy and Levinson discussed several accessibility measuring methods that are commonly adopted in accessibility evaluation studies (*1*). The cumulative opportunity measure, reflecting the total number of potentials at the destination, is one of the most basic measures adopted in related studies (*2, 5–9*). In annual reports released by the Accessibility Observatory at the University of Minnesota, for example, this metric was used to rank the network accessibility for major MPOs in the United States (*10*, *11*). The primary interest that drove researchers was probably examining urban networks to detect areas with low accessibility levels and gain insights into system improvement. Efforts have been made in developing better approaches for reflecting the accessibility levels of the networks. These approaches examine the network accessibility from both the aggregated level, which measures potentials within given travel time budgets (TTBs) and the individual level that considers the person-based space and time constraints (*12–16*). The application to transit systems is another rich literature body. A summary of methods for evaluating transit accessibility was provided by Lei and Church (*17*). Taking advantage of the General Transit Feed Specification (GTFS), researchers were able to measure and evaluate the accessibility of the transit network in a more detailed fashion with plentiful foci that vary from the mode share to equity issues (*10, 17–21*).

Evaluating accessibility stimulates research into how a transportation network can be designed with better accessibility in the first place, rather than mitigating accessibility issues afterward. This leads researchers to network design problems (NDPs) with accessibility considerations. The first formulation that bears a resemblance to the network design problem (NDP) traces back to 1963, which was proposed by Dantzig for good transshipment (*22*). Later Magnanti and Wong provided a more generic form of the NDP, the mixed integer linear program (MILP) (*23*). The basic version of the problem is to select a subset of arcs in the network so as to minimize the cost for flowing all commodities produced and consumed at the vertices in the network and the construction cost of the selected arcs. A full spectrum of variations was developed on this base and special cases of the NDP are also well-known and studied problems. The minimum spanning tree (MST) and the Steiner tree problem are two important problem instances among these and plenty of solution algorithms exist. The former and its directed version, the arborescence problem, is used to determine a tree configuration so that all vertices are connected through a spanning tree with the minimum total cost at the edges. In the accessibility context, the solution to a MST problem makes all the rest vertices accessible from one vertex. The latter, in contrast, only requires a subset of the vertices be connected. This leaves some nodes isolated in the network. A connection between the two classic problems and NDP exists, but the nature of the problem is not generic enough to account for the accessibility features that often raise concerns in a transportation network (*24*, *25*).

The most relevant NDP is probably the fixed-cost transportation problem (FCTP) (*23*, *26*). The problem is typically formulated as a MILP with the following objective: $\sum_k \sum_{ij} c_{ij}x_{ij} + \sum_{ij} f_{ij}y_{ij}$, in which x_{ij} and y_{ij} are continuous and binary variables representing the volume and construction decisions, respectively. The objective is to minimize flow-based transport variable costs and design incurred fixed costs. A single commodity version of the problem was studied by Ortega and Wolsey (*27*). The uncapacitated problem was proposed to be solved by a Branch-and-Cut (BC) algorithm. The Lagrangian relaxation-based heuristic algorithm was proposed by Aguado to solve a capacitated problem (*28*). Accounting for the chronological sequence for building roads and streets in reality, Kennington and Nicholson extended the problem by adding a time-space index to the problem (*29*). In addition to the comprehensive problem review mentioned earlier, Crainic presented the problem in the context of freight logistics and discussed potential budget constraints including financial resource budget constraints and capacity constraints (*30*). By fixing the design variables, as Crainic pointed out, an uncapacitated FCTP becomes an uncapacitated multicommodity flow problem and can be further decomposed into the shortest path problems.

Incorporating other objectives into the network design process, including accessibility, has attracted greater research interest in recent years. For example, problems such as planning an equitable transportation network to avoid population bias, planning routes for emergency evacuation that covers a maximum number of people, and designing transit routes for maximizing service value have been studied (*31–33*). The problem has also been modeled as mathematical programming with an equilibrium constraint (MPEC) that incorporates the user equilibrium conditions in optimizing the network configuration (*34*).

We note that a few studies on NDP aim to maximize the network accessibility, which are very limited in the literature, as is pointed out in the related work reported by Di et al. (*35*). Antunes et al. modeled a city center accessibility maximization problem in which accessibility is measured as a weighted metric of the population, center sizes, and travel costs between them (*36*). Local search and simulated annealing algorithms were proposed as solution algorithms. However, travel cost changes owing to design decisions were not explicitly modeled in the model. Aboolian et al. studied a facility network design problem that maximized the number of people gaining benefits from facilities, in which the accessibility is defined as the time needed for receiving services from a facility (*37*). Murawski and Church studied an accessibility problem in identifying critical links between rural areas and health facilities (*38*). The accessibility was related to the population that can be covered within the maximal acceptable distance. The problem was formulated as an integer optimization problem and solved using CPLEX. Tong et al., focused on maximizing an individual's accessibility, integrated space-time prism into the transportation network design framework (*4*). They modeled the problem for minimizing the number of inaccessible destinations given both the individual's time budget and limited financial resources. The time-space NDP was solved by the Lagrangian relaxation (LR) algorithm. Along the same lines, Di et al. developed a flow-based accessibility maximization problem that incorporates user equilibrium (UE) and system optimal (SO) principles in the bi-level optimization problem with a travel time budget. Deterministic and stochastic demand cases were both examined and they proposed an approximation algorithm for solving them (*35*).

Motivation and Contribution

We perceived that there have been fruitful studies on evaluating transportation network using all sorts of accessibility metrics, but relatively few place the accessibility improvement as the primary goal in the network design problems. Moreover, the overview of the accessibility-oriented NDP literature suggests that a very important motive, identification of potential construction of roads that will bring in more reachable destinations which would not be possible without them, has not been well articulated. To fill the perceived gap between the accessibility measurement and the network design problem with an emphasis on the increasing accessibility, while at the same time complying with budget constraints, we proposed an optimization problem for identifying critical links for accessibility improvement. This study extends this line of research in the following three aspects:

- Explicitly maximizes the accessibility of the network with regards to the weighted or unweighted number of destinations/opportunities reachable within a given TTB;
- Identifying links that are critical to inaccessible destinations instead of further improving travel time to destinations already reachable within a given TTB;
- Proposes an integer program (IP) formulation of the problem and a Lagrangian relaxation-based solution framework which renders a natural decomposition of the problem. Three decomposed sub-problems lend themselves to efficient solutions (algorithms) and together contribute a scalable algorithm for network planning problems in real-world size.

The remainder of the paper is structured as follows. The next section provides the mathematical formulation of the network design problem followed by the model limitations and discussion of the problem complexity. Following that, we present the Lagrangian relaxation-based solution algorithm. Case studies on both small and large networks are presented and discussion of the model are provided in the penultimate section. The paper concludes with some future work directions.

Mathematical Formulation

In this section, we propose an IP formulation of the NDP that is constrained by both the TTB and CCB. The formulation minimizes the total number of OD pairs for which the travel time is above the given travel time budget in the designed network. To guarantee a feasible model, a simple network transformation is needed in some cases, which is briefly discussed before presentation of the mathematical formulation. The limitation and complexity of the formulation are briefly discussed at the end of the section.

Problem Description

In accessibility studies, a fundamental question that is frequently asked is given a certain travel time threshold or budget, how many destinations, activities, and services can travelers reach from their origins. For example, El-Geneidy and Levinson measured the number of jobs accessible within various TTB values (e.g., 15 min, 20 min, 60 min and etc.) by automobile and transit for the Twin Cities metropolitan area network (*1*). The network design process has the aim of maximizing the network accessibility, therefore, targets are identified at potential transportation infrastructures (roads, transit routes, ferries, and etc.) that are to be built on the existing network to bring more opportunities accessible under the TTB. In practice, however, financial resources are often case very limited and it is not realistic for an MPO to perform a network redesign for the sole purpose of maximizing accessibility. Therefore, constrained by limited resources, where to build new infrastructure to improve accessibility for a given TTB becomes the core question. We focus on the road network improvement by utilizing limited financial resources for road construction. A set of potential links that can be constructed are given and the solution to the formulated optimization problem determines an optimal subset that maximizes the accessibility.

Some assumptions that we made in this study are as follows:

- Link travel time is known a priori and is constant, and one can use either congested or uncongested travel time for different planning purposes;
- Potential OD pairs for planning for accessibility are given, and opportunities measured at destinations are given and remain constant regardless of their accessibility status.

Network Transformation and an Example

The motive for making a network transformation is to ensure the feasibility. A transformation is needed if there are some origin-destinations (ODs) that are not physically connected under the existing network. The transformation is as follows: for a potential OD pair that is not connected in the existing network, an artificial link (path) is created by directly connecting the origin and the destination. The travel cost and construction expenditure of these artificially created links are chosen carefully to be slightly higher than the TTB and zero, respectively. An example network is presented in Figure 1. The network was adapted from a paper published by Tong et al. (*4*).

In the example presented in Figure 1, there are nine existing links shown by the solid line and four candidate links shown by the dashed line. There are three OD pairs to be considered for accessibility improvement. These are (A, D), (D, A) and (B, C). If in addition to these three pairs we also want to plan for (E, D), then a direct artificial link connecting E and D should be created. This guarantees that the model is always feasible no matter whether E and D are connected or not at the optimal solution. In other words, whether link FD is built or not, the pair ED should always be connected by either an existing physical route or an artificial one.

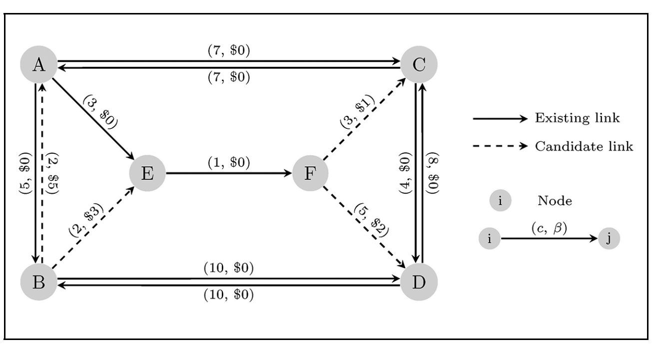

Figure 1. A small transportation network with candidate links for construction.

Notation and Formulation

Notations and parameters used throughout the paper are documented in Table 1.

The accessibility maximization problem is formulated as a minimization problem as below.

$$Min \sum_{k \in K} d_k z_k \tag{1}$$

$$s.t. \sum_{ij \in A \cup \bar{A}} x_{ij}^k - \sum_{ji \in A \cup \bar{A}} x_{ji}^k = b_j^k, \quad \forall j \in N, k \in K \tag{2}$$

$$T_u z_k \geqslant \sum_{ij \in A \cup \bar{A}} c_{ij} x_{ij}^k - T, \quad \forall k \in K \tag{3}$$

$$x_{ij}^k \leqslant y_{ij}, \quad \forall ij \in \bar{A}, \forall k \in K \tag{4}$$

$$\sum_{ij \in \bar{A}} \beta_{ij} y_{ij} \leqslant B \tag{5}$$

$$x_{ij}^k \in \{0, 1\}, \quad \forall ij \in A \cup \bar{A}, k \in K \tag{6}$$

$$y_{ij} \in \{0, 1\}, \quad \forall ij \in \bar{A} \tag{7}$$

$$z^k \in \{0, 1\}, \quad \forall k \in K \tag{8}$$

The formulation presented above explicitly minimizes the number of inaccessible OD pairs in the network under a certain TTB. For a better illustration of this, we introduced the accessibility indicator variable z_k into the formulation. The relative importance of an OD pair can be measured by the parameter, d_k, which specifically reflects some feature of this OD pair. For instance, it can be the number of people, number of jobs at the destinations, or simply taking the value of 1 representing the case in which all OD pairs are treated equally. Therefore, the objective of the program is equivalent to maximizing the number of opportunities reachable within the TTB for all origins in the network.

Constraints (2) are the standard flow conservation constraints. If for the OD pair k, node j is the origin node then b_j^k is equal to negative one, and if it is the destination node, it equals one, and for the intermediate nodes, it takes the value zero. Constraint set (3) defines the indicator variable: with a big-M type of parameter, T_u, the indicator is enforced to be 1 if the travel time between the corresponding OD pair is greater than the travel time budget, which indicates a inaccessible OD. Note that in the constraint, for an OD pair k, as long as there is a path with less than the TTB, the OD pair is accessible. In other words, the accessibility planning emphasis is given to those currently inaccessible OD pairs. Further

Table 1. Notations and Parameters Used in the Study

Notation	Description
	Indices and sets
N	Set of nodes in a transportation network
A	Set of links representing existing roads in a transportation network
$\bar{A}$	Set of candidate links representing potential roads to be constructed for improving accessibility
$\bar{A}'$	Set of selected candidate links, $\bar{A}' \subset \bar{A}$
$G(N, A)$	A graph consisting of the node set N and the link set A
K	Set of origin-destination (OD) pairs to be planned for accessibility
i, j	Indices for nodes, $i, j \in N$
(i, j)	Indices for links, $(i, j) \in A \cup \bar{A}$
k	Indices for OD pairs, $k \in K$
$O(k), D(k)$	Origin and destination of the OD pair k, $O(k), D(k) \in N$
	Parameters and decision variables
d_k	Parameter reflecting a certain feature of the OD pair k, for example, travel demand, number of jobs at the destination, population at the origin
β_{ij}	Construction cost for link (i, j), $\beta_{ij} \in R_{++}$
c_{ij}	Constant travel time on link (i, j), $c_{ij} \in R_{++}$
B	Construction cost budget parameter, $B \in R_{++}$
T	Travel time budget parameter usually in minutes, $T \in R_{++}$
T_u	The estimated longest travel time in the network, $T_u \in R_{++}$
x_{ij}^k	Decision variables, whether link (i, j) is used by the path connecting OD pair k, $x_{ij}^k \in \{0, 1\}$, whose vector form is $\boldsymbol{x}$
y_{ij}	Decision variables, whether link (i, j) should be built, $y_{ij} \in \{0, 1\}$, whose vector form is $\mathbf{y}$
z_k	Decision variables, take the value 1 if the OD pair k is inaccessible and otherwise 0; $z_k \in \{0, 1\}$
μ_{ij}^k	Lagrangian multipliers (LMs) whose vector form is $\boldsymbol{\mu}$
λ_k	Lagrangian multipliers (LMs) whose vector form is $\boldsymbol{\lambda}$
α_1, α_2	Step-size used in the sub-gradient updating step
l	Iteration index used in the solution algorithm
g	Duality gap threshold

reducing the travel time for the already accessible OD pairs is not favored for the solution of the model, which is consistent with the goal of the study: identifying critical links that make more ODs accessible. Constraints (4) stipulate that if for any OD pair link ij is on the path selected to the destination at the optimal solution, then the candidate link ij has to be constructed. If a candidate should be built, variable y_{ij} takes the value one and otherwise zero. The construction cost budget is constrained by the inequality (5). It is because of this constraint that the problem is of practical significance which allocates the financial resource to links that best contribute to the system with regard to maximizing the accessibility. Lastly, as the three sets of the decision variables are all binary, the formulated problem is an IP.

Model Limitation

Link travel time in this formulation is flow-independent. In other words, regardless of whether a destination is accessible within a given time budget, link travel times remain invariant. From the perspective of travel demand modeling, this may not be realistic as demand is sensitive to travel time as opposed to travel distance. Destinations made accessible, or shortened travel time to a destination, may encourage travel, and as a function of the flow link travel time varies with the flow. However, from the accessibility perspective, as is practiced in some related work, travel time variance and congestion effects owing to accessibility change are not the core of these studies. What is more important, instead, is whether the opportunities and/or potentials at a particular destination are physically accessible within a time budget, and if not, what improvement can be made so that the destination is reachable to travelers. Therefore, we chose not to model congestion in the current formulation, which allows a relatively simple model that renders an efficient solution algorithm making it applicable to large networks. To support long-term network planning decisions, however, one can use congested or uncongested link travel time for different modeling purposes.

Problem Complexity

Before moving on to the solution algorithm section, we will briefly discuss the complexity of the formulation with regards to the problem size and NP-completeness.

Problem Size. The problem can be viewed as a multicommodity flow and network design problem. The flow conservation constraints are written for each node in the network and more importantly for each commodity, and in our case is the set of OD pairs. Even though the formulation does not seem to be complex, there are in effect many constraints and variables even for a medium-size transportation network. Considering the network of the Twin Cities metro area which has more than 47,000 links and 20,000 nodes. The number of OD pairs with positive demand ($|K|$) that are typically planned for by the MPO is more than 540,000. This makes the cardinality of the constraints huge. Constraint set (2) contains $|N| \times |K| = 1.08 \times 10^{10}$ equalities. Constraint set (4) contains $|\bar{A}| \times |K|$ inequalities, and depending on the number of potentials the number of these inequalities is at least 10^5. In all, considering both the constraint and variable number the above problem considering the complete set of OD pairs is a big problem. It is by no means of tractable size for commercial optimization problem solvers. Some decomposition of the problem seems natural considering the size of the problem. An illustrative network example is presented in the following section and computational time comparison with a solver is provided for comparison.

Time Complexity. The formulated problem and its more general case, the NDP and multi-commodity network flow problem, are combinatorial optimization problems. The complexity of the problem is well-studied in the literature, usually by showing its relationship to the knapsack problem. It is NP-hard and more precisely NP-complete (*32*, *39*). There is not a known polynomial solution algorithm for this type of problem. In the solution algorithm section, we adopted the Lagrangian relaxation framework to approach this problem, which is always in realistic cases a huge problem. The adoption of the LR allows us to ease the complexity of the problem by decomposing it into three relatively easier problems, the shortest path problem, the knapsack problem, and an unconstrained minimization problem, all of which can be solved more efficiently.

Solution Algorithm

Lagrangian relaxation has been recognized as an efficient decomposition algorithm for converting a complex problem into more tractable ones and has been applied in many transportation fields, such as reliable shortest paths or network design problems (*4*, *28*, *40*). In this section, the overview of the proposed LR approach together with decomposed sub-problems and the multiplier updating method is presented first. Next, a numerical example of an illustrative network employed to show the accuracy and efficiency of the proposed algorithm is presented.

Lagrangian Relaxation

The idea of the Lagrangian relaxation-based decomposition idea is simple: moving some "complicating" constraints to the objective function which are penalized if constraints are not satisfied or rewarded otherwise. By replacing the constraints, it is usually possible to have the original problem decomposed into sub-problems that are easier to solve. The framework works in the same fashion as the primal-dual algorithm that improves the gap between the primal value and dual problem value.

First, we will construct the Lagrangian function by introducing a set of Lagrangian multipliers:

$$\begin{aligned} L(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = & \sum_{k \in K} d_k z_k \\ & + \sum_{k \in K} \sum_{ij \in \bar{A}} \mu_{ij}^k \left(x_{ij}^k - y_{ij} \right) \\ & + \sum_{k \in K} \lambda_k \left(\frac{1}{T} \sum_{ij \in A \cup \bar{A}} c_{ij} x_{ij}^k - 1 - \frac{T_u}{T} z_k \right). \end{aligned} \tag{9}$$

The Lagrangian dual problem is to minimize the Lagrangian function with respect to the LMs.

$$\begin{aligned} L(\boldsymbol{\mu}) &= \inf_{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}} L(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{\mu}, \boldsymbol{\lambda}) \\ &= \inf_{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}} \left\{ \sum_{k \in K} d_k \sum_{ij \in A \cup \bar{A} \cup \bar{A}} c_{ij} x_{ij}^k + \sum_{k \in K} \sum_{ij \in \bar{A}} \mu_{ij}^k \left(x_{ij}^k - y_{ij} \right) + \sum_{k \in K} \lambda_k \left(\frac{1}{T} \sum_{ij \in A \cup \bar{A}} c_{ij} x_{ij}^k - 1 - \frac{T_u}{T} z_k \right) \right\} \\ & s.t. \ (2), (5), (6) \end{aligned} \tag{10}$$

The problem defined in Equation 10 can be further decomposed into three sub-problems by regrouping the variables which are presented below.

Sub-problem of $\boldsymbol{x}$ – Paralleled Shortest Path Problems. The first sub-problem is related to the decision variables $\boldsymbol{x}$. The problem presented in Equation 11 is, in fact, a set of shortest path problems on the graph $G(N, A \cup \bar{A})$. Given the fixed value of the Lagrangian multipliers (LMs), one can simply update the generalized cost for each link and solve the paralleled shortest path problems efficiently using network algorithms such as label correcting. Note that the generalized travel cost on the graph G is origin-destination based as the LM λ_k is OD specific which results in different link costs even for the same link. Furthermore, the computation of the generalized cost of existing links and candidate links are slightly different.

Having the cost updating step implemented correctly, the sub-problem can be addressed.

$$L_{\boldsymbol{x}}(\boldsymbol{\mu},\boldsymbol{\lambda}) = \min_{\boldsymbol{x}} \sum_{k\in K}\left(\sum_{ij\in A}\frac{\lambda_k}{T}c_{ij}x_{ij}^k + \sum_{ij\in\bar{A}}\left(\frac{\lambda_k}{T}c_{ij} + \mu_{ij}^k\right)x_{ij}^k\right)$$
$$s.t. \sum_{ij\in A\cup\bar{A}\cup\tilde{A}} x_{ij}^k - \sum_{ji\in A\cup\bar{A}\cup\tilde{A}} x_{ji}^k = b_j^k, \quad \forall j\in N$$
$$x_{ij}^k \in \{0,1\}, \ \forall ij\in A\cup\bar{A}, \quad \forall k\in K \tag{11}$$

The time complexity of the above shortest path problem is, if taking the label setting algorithm with some priority queue structure as the fundamental tool for solving the shortest path problem, $O(|K|(|A| + |N|log|N|))$.

Sub-problem of y – Knapsack Problem. The second decomposed sub-problem relates to decision variables $\boldsymbol{y}$, which takes the following form:

$$L_y(\boldsymbol{\mu}) = \min_{y} -\sum_{k\in K}\sum_{ij\in\bar{A}}\mu_{ij}^k y_{ij}$$
$$s.t.\sum_{ij\in\bar{A}}\beta_{ij}y_{ij} \leq B \tag{12}$$
$$y_{ij}\in\{0,1\}, \quad \forall ij\in\bar{A}$$

The problem presented in Equation 12 is a knapsack problem. The analogy to the knapsack problem is selecting candidate links for a construction that maximizes the benefits that are measured using the LMs given the limited capacity of the knapsack. The authors want to remark that as the LMs μ_{ij}^k are defined for each constraint in the set (4), the "value" of an item—a physical candidate link—is actually the sum of the LMs for all OD pairs. That is, if we define μ_{ij} to be the "value", it is computed as:

$$\mu_{ij} = \sum_{k\in K}\mu_{ij}^k. \tag{13}$$

As is well known, the knapsack problem is NP-hard as well, but the dynamic programming approach can solve the problem in pseudo-polynomial time.

Sub-problem of z – Unconstrained Minimization Problem. The last sub-problem is related to the accessibility indicator decision variable. The problem is fairly easy as there are no additional constraints on z_k besides the definition constraints. The problem takes the following form:

$$L_z(\lambda) = \min_{z}\sum_{k\in K}\left(d_k - \lambda_k\frac{T_u}{T}\right)z_k$$
$$s.t.\ z_k\in\{0,1\}, \forall k\in K. \tag{14}$$

The solution to the problem is obvious: if the component coefficient $d_k - \lambda_k\frac{T_u}{T}$ is evaluated to be a positive number, z_k takes 0 and otherwise, it takes 1.

By solving the three sub-problems separately and then adding up the optimal values of the three problems and an extra term, we can find the lower bound (LB) of the original problem. This can be defined as:

$$LB = L_{\boldsymbol{x}}(\boldsymbol{\mu},\boldsymbol{\lambda})^* + L_{\boldsymbol{y}}(\boldsymbol{\mu})^* + L_{\boldsymbol{z}}(\boldsymbol{\lambda})^* - \sum\nolimits_{k\in K}\lambda_k, \tag{15}$$

In which $L_x(\boldsymbol{\mu},\boldsymbol{\lambda})^*$, $L_y(\boldsymbol{\mu})^*$, and $L_z(\boldsymbol{\lambda})^*$ represents the optimal value at the current iteration obtained by solving the sub-problems of $\boldsymbol{x},\boldsymbol{y},$ and $\boldsymbol{z}$ for the given $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$. The last term in Equation 15 should be computed using the incumbent value of the LMs.

The upper bound (UB) of the problem can be calculated by evaluating the number of inaccessible OD pairs on the given graph $G\left(N, A\cup\bar{A}'\right)$, in which the set $\bar{A}'$ denotes the selected candidate links by the optimal solution of the sub-problem $\boldsymbol{y}$. It is a valid UB because the solution for the knapsack problem is always feasible for the original problem and moreover if the algorithm converges the UB value is the optimal value of the original problem. One can apply the shortest path algorithm on the graph and compare the travel time with TTB to obtain the UB.

In the framework of LR, after computing the UB and LB of the problem, we aim to decrease the gap between the two. To decrease the duality gap and update the sub-problems, we adopted the sub-gradient method. In the literature, the sub-gradient method has been widely used to find the direction for updating the LMs (*4*). The Lagrangian multiplier (LM) updating strategy is presented as follows:

$$\mu_{ij}^{k(l+1)} = \mu_{ij}^{k(l)} + \alpha_1^{(l)}\left(x_{ij}^{k*} - y_{ij}^*\right), \forall ij\in\bar{A}, k\in K, \tag{16}$$

$$\lambda_k^{(l+1)} = \lambda_k^l + \alpha_2^{(l)}\left(\frac{1}{T}\sum\nolimits_{ij\in A\cup\bar{A}}c_{ij}x_{ij}^{k*} - 1 - \frac{T_u}{T}z_k^*\right), k\in K, \tag{17}$$

In which x_{ij}^{k*}, y_{ij}^*, and z_k^* are the optimal solutions of the corresponding sub-problems respectively, and $\alpha_1^{(l)}$ and $\alpha_2^{(l)}$ are the step-size parameters that usually take on a positive number between 0 and 2, as suggested by the literature (*40*). Suggestions from the literature and our prior tests found that it is good practice to include the gap measure in the adjustment of the step-size. In this paper, we multiplied the second term in Equations 16 and 17 with the ratio of the difference of the incumbent UB and LB to l_2 -norm of the sub-gradient. If we compact the coefficient for travel time updating as a vector c, the two ratios are $\frac{UB-LB}{\|x^*-y^*\|_2}$ and $\frac{UB-LB}{\left\|c^tx^*-1-\frac{T_u}{T}z^*\right\|_2}$ respectively.

To summarize, the LR-based solution algorithm converts a complex problem, which is also possibly very large in real transportation networks, into three much easier problems: a set of parallel shortest path problems on a given graph, a knapsack problem, and an unconstrained minimization problem. All of the sub-problems can be addressed relatively fast and this significantly reduces the computational time. A comparison of the proposed algorithm with a commercial solver is provided in the next sub-section and the case study section to cast light on the efficiency and accuracy of the LR-based algorithm. The complete steps of the LR-based algorithm are summarized below.

Step 0 (Initialize):	Initialize LMs $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ with non-negative numbers; set duality gap threshold g; set l to 0
Step 1 (Solve sub-problems):	Update travel cost and network topology; Solve shortest path problems in (11), the knapsack problem in (12) with Equation 13 using dynamic programming, and the minimization problem in (14); Denote their optimal solutions as $\boldsymbol{x}^*$, $\boldsymbol{y}^*$, and $\boldsymbol{z}^*$.
Step 2 (Update bounds):	Compute incumbent *LB* using Equation 15, update the *LB* if the incumbent is larger; Compute *UB* using $\boldsymbol{y}^*$, update the *UB* if the incumbent is smaller.
Step 3 (Update LM, gap)	Update the LMs using Equations 16 and 17; Update the gap by $(UB - LB)/UB$
Step 4: (Termination)	If $\frac{UB-LB}{UB} \leq g$, terminate the algorithm; Otherwise, go back to step 1, increment iteration number l by 1.

Illustrative Example

To gain further insights into the solution algorithm, we solved the illustrative example shown in Figure 1 using the proposed LR algorithm. The travel time, construction cost, and OD pairs are shown in the figure. For these tests, the d_k values are all set to 1.0. We tested out the following travel time budget and construction budget combinations: $T = 10, B = 5$, and $T = 10, B = 8$, and $T = 12, B = 12$. Here, we will refer to this case using the abbreviation T_B_ with T and B followed by the selected TTB and CCB values. These problems were also solved using the commercial solver, GUROBI 8.1.1, which was coded in Python and carried out on a personal computer with an i5-4590s Intel processor and 16GB RAM.

There are four candidate links in the illustrative network and it costs $11 in total to build them all. In the existing network, the shortest travel time for the OD pairs (A, D), (D, A), and (B, C) are 11, 15, and 18, respectively. Given a TTB of 10 and CCB of 5, the system can improve the accessibility by constructing links BE and FC which bring the travel time for (B, C) below TTB. This results in a total travel time of 32 which would have been 44 without any candidate links being constructed. Two out of three OD pairs are still inaccessible in this case. Increasing the construction budget, the algorithm outputs an additional link, FD, to build, which reduces the travel time for (A, D) to 9 and improves the accessibility of network. This results in a total travel time of 30. Increasing the construction budget will not improve the system anymore as the TTB becomes critical. Therefore, in the third test, we increased the TTB to 12, which is the shortest possible travel time for (D, A). Then, the program determined that all candidate links should be built to maximize the accessibility under the TTB of 12, which results in a total travel time of 27. Considering the results across the scenarios, the model can indeed improve the accessibility of the network by selecting the optimal links to open and the byproduct of this is the reduced total travel time.

We compared our results with the solutions obtained using Gurobi. The solutions (optimal value, selected candidate links) provided by the LR method are exactly the same as those produced using Gurobi for each case. The comparison of the two showed the accuracy of the proposed LR algorithm. In addition, the decomposition algorithm solves the problem faster than the commercial solver with an average computational time of only one-tenth of that used by Gurobi. This is encouraging as the network design problem in practice can be very large and the computational time can be a problem if resorting to a commercial solver.

In Figure 2, we plotted the convergence paths of the three tests. By updating the LMs and therefore the three sub-problems, each iteration rendered improved upper or lower bounds. This ultimately leads to a reduction of the duality gap and a converged solution. For these tests, the algorithm tends to find the optimal solution at the each stage of the process. The optimal values were obtained in the second iteration and the duality gap improved in the rest iterations until convergence.

Case Study

In this section, we provide two case studies for the network design problem—one is the Sioux Falls network and the other is the Twin Cities metro area network. The purpose of the two examples is twofold: (1) validating the

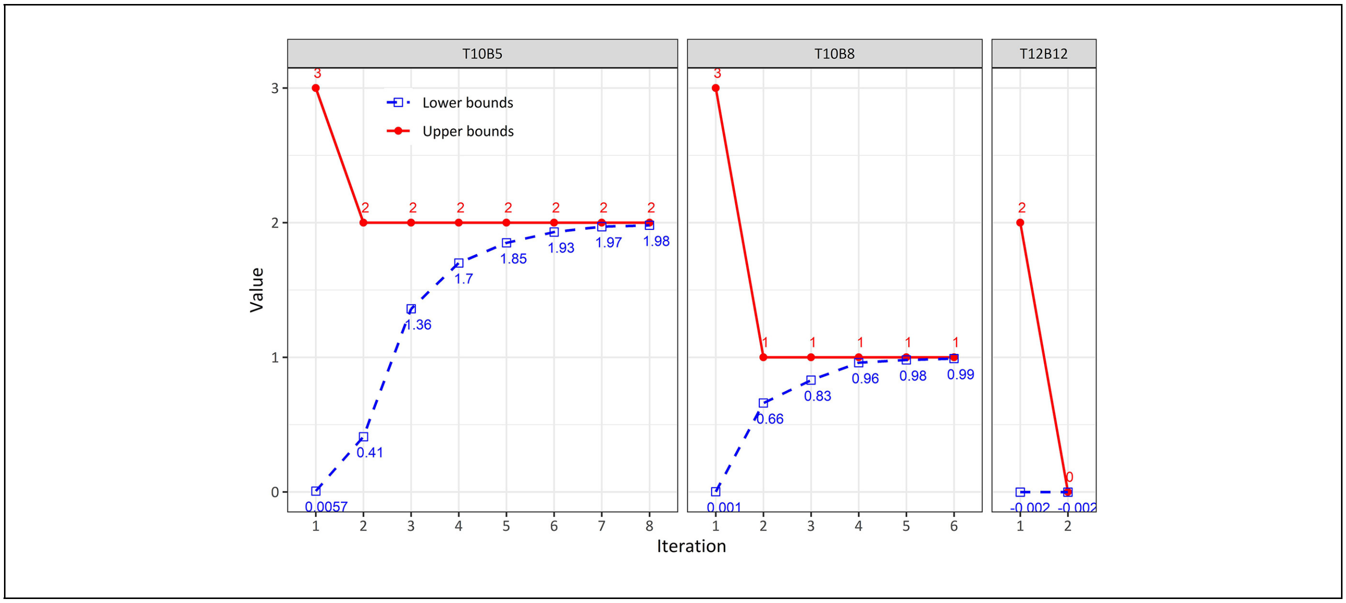

Figure 2. Convergence paths of the LR algorithm for various combinations of TTB and CCB.

proposed solution algorithm; and (2) showing the practical applications of the proposed model.

Sioux Falls Network

Sioux Falls network is comprised of 24 nodes and 76 links. We created seven pairs of candidate links (14 directional links in total) on the existing network. Their free-flow travel time (FFTT) was estimated from the known free-flow travel times for the existing links. We extracted 528 OD pairs with a positive demand between them and tried to maximize the number of OD pairs that can be traveled from/to within the given TTBs. The construction cost was generated approximately proportionally to their length. Parameter T_u takes the value of 40. Details of the candidate links are presented in Figure 3.

Tests were carried out to reflect the impacts of both the financial budget and TTB. By fixing the TTB to 15, we tested how the financial budget affects the accessibility. A piece of useful background information is: if the construction budget is 0, there are 144 inaccessible OD pairs. Given a budget of 50 units, 2 links out of 14 can be built and this makes 38 more OD pairs accessible. Increasing the budget to 100 units and 150 units, 4 and 6 links are selected and the improvement is 44 and 50 relative to the base case. If the budget is enlarged to 200 units, 8 links should be built and this enables the travel time for 3 more ODs to be below 15 minutes. Please refer to Table 2 for solution details. On the other hand, to demonstrate how TTB affects the results, for each budget level described above we increased the TTB to 20 minutes. Increased accessible pairs were found as expected. The other conclusion that we can draw from this comparison is that the critical links can be different for different TTB settings. Note that the number of links to open for T20B150 and T20B200 is 1 fewer than their corresponding case in T15, which indicates that some links that are critical for T15 are no longer critical for T20. To better present the results for T equal to 20 and to show how the financial budget can change the construction decision, graphs of the selected links are shown in Figure 3.

We recorded the optimal values (and solutions) and computational time for the same tests using Gurobi in Table 2. The optimal solutions obtained using the proposed algorithm are identical to those obtained using the optimization solver. For the Sioux Falls network, the computational time of two is rather comparable. Generally, we found that a larger TTB case requires less computational time. Although we cannot draw any general conclusion about the computational time sensitivity on the TTB, we discussed the issue in the following section. To show the convergence trajectory, in Figure 4 we plotted iteration-by-iteration duality gaps for these tests for 50 iterations. The duality gap is measured by $(UB - LB)/UB$. In cases in which TTB is 20, the gap decreases drastically in the few iterations at the beginning and the final optimal link solution is in effect found at the very beginning as well. In the figure, one can observe that a 1.0% gap can be reached within just a few iterations. In contrast, in the case of T15, the optimal solutions tend to be found relatively later, which results in a slower convergence speed. For example, in T15B200, the optimal solution was found at the 28th iteration and for

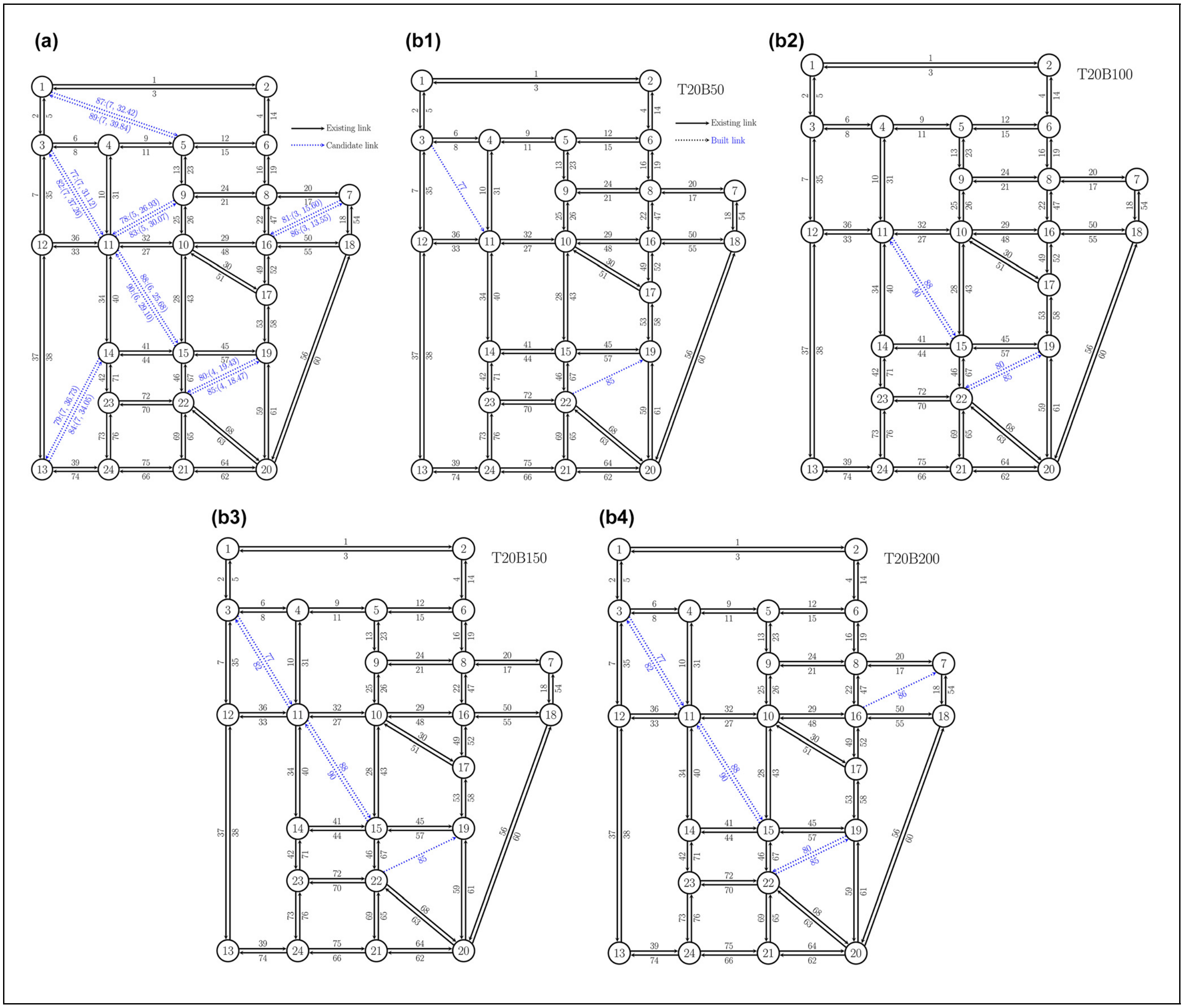

Figure 3. Sioux Falls network with candidate links and optimal network design results: (*a*) candidate links are marked with indices and FFTT (in minutes) and construction costs and (*b*) the subfigure shows the existing links and optimal links to build for given TTB and CCB. In subfigures (*b*), the CCBs are 50, 100, 150, and 200, respectively and all with the same TTB 20 min.

Table 2. Comparison of Solution Details Between the LR Algorithm and Gurobi

Network	Case		# Opened links	# Accessible destinations	Optimal value	Computation time (s)	Gurobi optimal value	Gurobi computation time (s)
Sioux Falls	T15	B50	2	422/528	106.0	10.30	106.0	6.48
		B100	4	428/528	100.0	5.46	100.0	5.92
		B150	6	434/528	94.0	10.11	94.0	7.00
		B200	8	437/528	91.0	9.16	91.0	6.63
	T20	B50	2	520/528	8.0	4.68	8.0	5.31
		B100	4	522/528	6.0	1.39	6.0	4.59
		B150	5	523/528	5.0	6.56	5.0	3.70
		B200	7	524/528	4.0	9.74	4.0	3.87
Twin Cities	T15	B50	2	4386/9090	4704	7800	–	–
		B100	3	4434/9090	4656	7672	–	–
	T30	B50	2	5708/9090	3382	4216	–	–
		B100	3	6978/9090	2112	3944	–	–

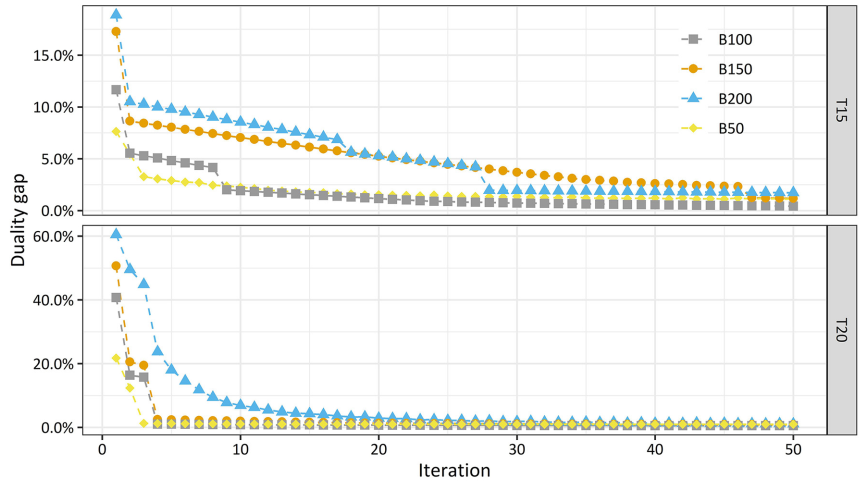

Figure 4. Iteration-by-iteration duality gap for various cases of the Sioux Falls network design problem.

T15B150 at the 46th. This explains the longer computation time for T15 cases. However, by tuning the initial LMs, as our tests suggested, the convergence speed can be improved. For the two sets of tests, the LMs were set to be the same.

Twin Cities Network

The case study on the Twin Cities metro area network is designed to show the potential applications of the proposed model and the solution algorithm in practice. The network is comprised of 20,784 nodes, 47,393 links (not including the centroid connectors), and 3,030 traffic analysis zones (TAZs). The east campus of the University of Minnesota was used as the test subject, which is connected to the west campus by bridges across the Mississippi River. The question we want to answer through tests is which bridges are critical to the students, faculty, and staff working in the university with regards to providing them the largest accessibility to other destinations. The three TAZs (shown in dark red in Figure 5) that the campus consists of translate to 9,090 ODs and a collection of 11 bridges that are surrounding the campus were considered. The problem is of practical relevance for network accessibility evaluation as well, for example, evaluating road closures for special events. T_u in the model was chosen to be 120 (in minutes), which provides an UB for the travel time in the Twin Cites network.

Results and Analysis. Four tests were performed with combinations of the 15- and 30-minute TTB and 50- and 100-unit CCB. Figure 5 on each panel shows four accessibility zones: zones already accessible within the given TTB (in blue), zones becoming accessible with construction of the selected bridges (in light red), zones becoming accessible with more selected bridges owing to a larger CCB (in yellow), and zones inaccessible in any case (in white).

When the travel time budget was 15 minutes, without building any bridge the accessible zones are mostly distributed in the east bank of the river, which indicates that trips across the river are not possible because of the lack of connecting infrastructure. If two bridges that are very close to the campus are included in the network, the accessible region largely extends to the west, which was not reachable before, and this increases the number of accessible zones by 150% from 598 to 1,495. If there is more budget, however, even though one more bridge is to be included in the network, the number of accessible zones increases by less than 20. Compared with the previous increment, this is not significant. It alludes to the accessibility analysis, and the existence of some critical links which can greatly boost the number of accessible zones making other links seemingly less significant. This phenomenon was also revealed in Murawski and Church's work, which found that "even a modest level of road improvement" can lead to a substantial increase in accessibility (*38*). It also warns us that one may ignore the capacity and congestion issue when solely focusing on the accessibility. Again, accessibility is only one aspect of the network performance and there is more beyond that that is well worth planning for.

When the travel time budget is 30 minutes, an increase of 20% in accessibility from 1945 to 2330 ODs was observed. It is a positive increase, but a less significant increase compared with a 15-minute TTB. One can see that even without those bridges many zones in the west area are already accessible. This indicates that for

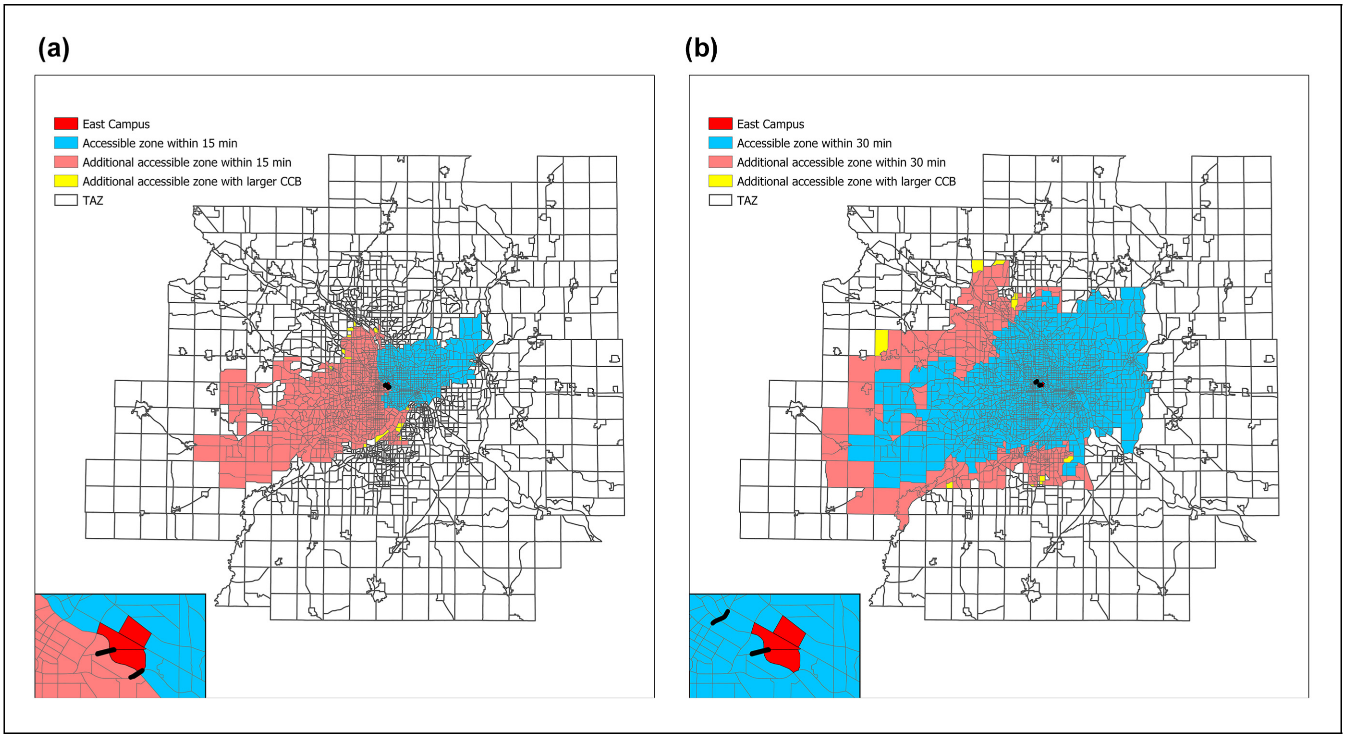

Figure 5. TAZs that are accessible for different TTB and CCB combinations. A zoomed-in view of the campus region and selected bridges to build is shown in the lower-left corner for B50 case: (*a*) accessible zones within a 15-minute TTB and (*b*) accessible zones within a 30-minute TTB.

different TTBs critical links can be different, and some links that are critical under a smaller TTB may not be important for a larger TTB. This was also supported by the optimal bridges found using the solution algorithm. In the case of T30B10, two different bridges were selected for accessibility maximization compared with T15B10. It can be seen that between T30 and T15 the same issue is found, that increasing the budget does not significantly help the accessibility improvement. The phenomenon can be related to the limited number of candidate links. In short, building two or three critical bridges can greatly enhance the accessibility level for the campus.

Computational Time Discussion. The authors implemented the proposed algorithm using C ++ programming language and the above tests were carried out on the same personal computer as mentioned before. For all of the tests recorded in Table 2, the duality gap threshold was set to 3.0% and on average it took 67 iterations for convergence. It was realized that in the proposed algorithm the most computationally expensive component is the shortest path computation—solving the sub-problem of x. For the above case, in each iteration of the algorithm, more than 9,000 shortest path problem solutions are needed in the worst case. It should be noted that the problem cannot be solved for each origin, but needs to be solved for each OD pair, because of the varied OD-specific LMs used for updating the link cost. One technique, however, is to pre-process the network: remove those OD pairs for which travel time is already below the preset TTB in the existing network. The objective value, the number of inaccessible ODs, will not be altered without these OD pairs because building more links will only reduce the travel time between them, rather than increasing them. By conducting this, we can reduce the number of OD pairs by 19% and 63% for T15 and T30, respectively. This is the why, in the tests for the Twin Cities network, the computational time for T30 cases is significantly smaller than for the T15 cases, as shown in Table 2. Taking advantage of the independent shortest path sub-problem, one can further decrease the computational time by parallel computing, which could be a future working direction for this study.

Conclusion

Aiming to bridge the gap between network accessibility evaluation studies and NDPs, this paper studied the accessibility maximization NDP. The objective of the proposed integer programming model is consistent with the prevalent accessibility measuring metric and the model further articulates the fundamental concern for the accessibility maximization problem: making more opportunities reachable within a given TTB rather than reducing the travel time for getting the opportunities already accessible. We proposed to decompose the OD-specific constrained problem into three sub-problems including a set of independent shortest path problems, a knapsack problem, and an unconstrained minimization problem. Thanks to the LR decomposition technique, we can deal with each one of the sub-problems more efficiently, with the former two being addressed using dynamic programming approach, and the latter one rendering an obvious solution. Various tests have been conducted on small to large-scale networks. We compared our solution with the commercial solver's optimal solution and proved the accuracy of the solution algorithm and showed the computational time was comparable or even faster than that of the solver. The method was also successfully applied to the Twin Cities metropolitan area network for identifying critical bridges that cross the Mississippi River which are capable of maximizing the number of destinations for people on campus. The paper extends the existing literature in the following aspects: (1) incorporating the TTB into the NDP for measuring accessibility and explicitly maximizing the number of reachable destinations within a given TTB; (2) provided a promisingly scalable solution framework for the problem which is likely to be large in real applications.

On the efficiency of the model, it is worth noting that the problem by definition is origin- (or destination-) specific instead of OD-specific. An origin-specific type of formulation can significantly reduce the computational time as we found that the shortest path computation was the most time-consuming component compared with the others. In addition, in real applications, the OD pairs are often in the millions, whereas making it O- or D-specific reduces the size to a few thousand. If incorporated with parallel computing, a significant enhancement in the efficiency of the algorithm is expected. The problem can also be extended to study a network design problem with the objective of minimizing the travel time for all OD pairs. It would also be interesting to compare the results of the two models with different objectives and see how the objective affects the network performance with respect to the accessibility level for a varied TTB.

Author Contributions

The authors confirm contribution to the paper as follows: study conception and design: Yufeng Zhang, and Alireza Khani; data collection: Yufeng Zhang; analysis and interpretation of results: Yufeng Zhang and Alireza Khani; draft manuscript preparation: Yufeng Zhang. All authors reviewed the results and approved the final version of the manuscript.

Declaration of Conflicting Interests

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