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The determinants of bank loan recovery rates in good times and bad – New evidence☆

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ABSTRACT

We find that factors explaining bank loan recovery rates differ depending on the state of an underlying credit cycle. Our modelling approach incorporates a two-state Markov switching mechanism to capture underlying economic conditions. This latent credit cycle variable helps to explain differences in observed recovery rates over time. Using US bank default loan data from Moody's Ultimate Recovery Database and covering the pre-and post-global financial crisis (GFC) period, the paper develops and implements a dynamic model for bank loan recovery rates. We accommodate the distinctive empirical features of the recovery rate data, while incorporating a large number of possible determinants. We find that certain loan-specific and other variables hold different explanatory power with respect to recovery rates in 'good' versus 'bad' times in the credit cycle, i.e. depending on underlying credit market conditions. Our findings demonstrate the importance of accounting for counter-cyclical expected recovery rates when determining capital retention levels.

1. Introduction

A bank's ability to accurately assess loan default probabilities and model the potential for recovery not only affects corporate loan pricing but also directly impacts their capacity to acquire and/or retain market share and, subsequently, to earn profits (Altman et al., 2004). Over the period 1987 through year-end 2015, Moody's reported 1100 US companies, with more than $50 million in outstanding debt who defaulted. Their total aggregate outstanding liabilities exceeded $1 trillion. The incidence of corporate loan defaults is exacerbated during recessions and is less pronounced during more benign or booming economic cycles (Bruche and Gonzalez-Aguado, 2010). The evidence on the determinants of recovery rates (RRs, hearafter) has largely been investigated without accounting for the observable temporal variation in RRs in relation to economic conditions, and the evidence remains mixed about the primary determinants. To address these issues, this paper utilzes data

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from Moody's Ultimate Recovery Database, focusing on defaulted bank loans over 1987 to 2015 in the US and investigates the differential impact of a dynamic credit market environment on RR determinants allowing for conditions.

Using the set of RR determinants proposed by Khieu et al. (2012), our methodological approach builds on the mixture regression framework proposed by Altman and Kalotay (2014). To proxy for the apparent but occasional shift in default occurrence regimes, we introduce latent binary Markov-switching variables representing economic conditions being in either a 'good' or 'bad' state at a given time. Augmentation of a model through the inclusions of a hidden Markov-switching process dates back to Goldfeld and Quandt (1973), and has been shown to be beneficial in many economic and financial contexts (see, e.g. Hamilton, 2010). We also incorporate a least absolute shrinkage and selection operator (LASSO) prior for the predictive regression coefficients to cater for potential correlation between the available RR determinants (Park and Casella, 2008). Estimation of the dynamic latent credit cycle and regime-specific RR determinant coefficients is achieved using a Bayesian inferential framework through the development of a posterior simulation algorithm along the lines of Kim and Nelson (1999). To validate the resulting estimated latent credit cycle, a correlation analysis is undertaken to compare the estimate again other available credit market indicators.[1] A formal Bayesian model evaluation framework is also used to assess the empirical support for the dynamic model over a corresponding static version.

Our empirical analysis produces some new insights about some of the available but mixed results resulting from the static models (consistent with prior work), and thus contributes further to the literature. We find evidence of the impact of the credit environment on RRs, with the most notable effects felt during 2002/03 and 2008/09, corresponding to the periods of the dot-com bubble burst and the global financial crisis (GFC), respectively. Importantly, we find evidence to support interpretation of the estimated credit cycle when benchmarked against alternative credit market indicators, and also when formally compared against a static model. The RR determinants that appear most critical to the RR irrespective of cycle are loan characteristics such as the size and type of the loan, with firm size and its interaction with loan type being important. Other factors such as having assets available to securitize the loan, how long it takes to emerge from bankruptcy proceedings and the prevailing health of the relevant firm industry just prior to default are found to be important, as is the credit spread at time of loan origination. However, there are many differences regarding the impact of RR determinants with respect to the credit cycle. In most cases, stronger effects are found in 'good' regimes, the larger component of the sample period. However, firm asset tangibility and whether the firm's industry is in distress have different directional effects under the two regimes, with greater impact apparent in bad regimes.

Hence, overall our results suggest that loan characteristics are more effective in predicting RRs during a good cycle. Conversely during a bad cycle only certain firm characteristics and collateral determinants appear to be strongly related to recovery. This finding reinforces the notion that only some of a firm's assets or attributes facilitate a full loan recovery – with inventory and accounts receivable more likely to be significantly predictive of the RR irrespective of the underlying cycle. This has consequences for 'haircuts' that banks may tolerate and on securities offered as collateral. Our results suggest that the size of the haircut should not only depend on the riskiness and liquidity of the security being offered, but also on the current and potential future states of the credit.

While Khieu et al. (2012) report increases in recoveries for term loans by larger firms, after controlling for cycles, we find that irrespective of loan type, firm size tends to moderate recoveries during bad times but has a relatively positive effect in good times. This finding suggests banks should give greater consideration to the impact of idiosyncratic risk emanating from large corporate borrowers, particularly on the impact to tail risk and the loan loss distribution. During bad times, banks with exposures to large corporate borrowers need to increase their tail-risk forecasts, in an attempt to mitigate their unexpected losses through the allocation of further reserve capital.

Overall, the implications of such findings have a direct impact on a bank's loan loss distribution, which is a vital tool for regulatory and economic capital allocation. The default loan recovery process suggests loan, recovery, borrower and economic aspects need to be considered with respect to changing economic conditions for banks to optimally allocate capital across cycles. Clearly, corporate loan recovery is time varying and the potential risk of not accurately addressing such a phenomenon effectively distorts estimates of the credit risk and on loan pricing, incorrectly under/over-providing for expected and/or unexpected loan losses.

These findings support the Basel III framework's recommendations for the use of counter-cyclical buffers, seeking to create an environment where the banking sector is subsequently protected from unintended consequences of periods of excessive aggregate credit growth. Under such arrangements, appropriate capital buffers will moderate the effects of risk from such a build-up phase and strengthen the bank's balance sheet as a going concern when recovery rates underperform. Conversely, during bad times, capital buffers are essential, as the supply of credit may be curtailed by regulatory capital requirements. Furthermore, throughout bad times, the banking system may also experience further unexpected loan losses emanating from lower recoveries, and at a time when shareholders are least likely to allocate further equity capital.

Therefore, if recoveries are more accurately anticipated by allowing for the time-varying effects of different cycles, it allows adjustment of capital buffers to address the counter-cyclical nature of the economy. This will allow improved management of unexpected credit losses in a complex environment when asset values fall as a results of weaker than expected cashflows (Varma and Cantor, 2004). Furthermore, it has important implications on the pro-cyclical effects of credit risk models, particularly for the larger banks using more complicated models for capital allocation (advanced internal rate based

[1] We are grateful for this suggestion made by an anonymous reviewer.

approach). However, systems and processes need to be sufficiently flexible to adjust in anticipation of impending credit cyclical changes. Overall this will enable the banking sector to implement appropriate prudential practices, including maintaining adequacy of capital requirements and controls, so as to remain viable within and across cycles.

The remainder of our paper proceeds as follows. In Section 2, we review the literature regarding the determinants of RRs and the proposed econometric specifications for the model calibrated with and without the latent credit cycle variable. Section 3 contains a description of the Moody's dataset used for the empirical analysis, with full details of the variables provided in an appendix. The Bayesian inferential framework and corresponding Markov Chain Monte Carlo (MCMC) simulation algorithms are then described in Section 4, with further algorithmic details given in two additional appendices. This is followed by the results of our research, a benchmarking analysis to check the interpretation of our estimated latent credit cycle and evaluation of our proposed models together with their predictive capability in Section 5. Section 6 concludes with a discussion and suggested directions for future research.

2. Literature review

Financial market participants including bank regulators are increasingly concerned with the management of risky assets under changing conditions, particularly bank loans. Not only are risk factors and market conditions of concern at the time of placing investments in loans, but gaining an understanding how these factors vary over time is increasingly viewed as being of critical importance when making lending decisions (Nazemi et al., 2017).

It is of particular importance to be mindful of the probability of default and subsequent loan recovery prospects during variable or extreme conditions as this will be critical to achieving expected returns to finance providers, such as banks, who typically deal with risky investments. The consequences of not considering this state dependent risk can be critical to banks as was demonstrated in the 2008/09 financial crisis by the inadequacy of their reserve capital. This significant shortcoming in prudential regulation, and corporate governance, attributable to poor operational risk management practices (Financial Crisis Inquiry Commission, 2011), underscores the importance of understanding financial risk, and in particular credit risk, when pricing corporate loan contracts.

When a corporate borrower defaults, the lender endeavors to recover all of the outstanding debt using all available collateral and liquidity mechanisms. Default alone, while not ideal for the borrower in terms of their ability to establish or maintain a strong credit rating, does not necessarily imply that the outstanding balance of the loan cannot ultimately be fully recovered during the post default period. In the vast majority of cases, lenders may recover their full entitlement as long as sufficient collateral is available. However, there are many occasions when none or only a part of the outstanding indebted balance is recovered (Kalotay and Altman, 2017). Not surprisingly, the projected recoveries in the event of default is one of the key inputs when determining the price of any credit related financial contract, including the value of the fundamental investment itself. Hence, it is important for lenders to understand the factors that affect the actual recovery, so that appropriate decisions, including loan terms, can be made (Jankowitsch et al., 2014).

There are three main variables determining the risk of a credit based financial contract: (i) the probability of default; (ii) the recovery in the event of default; and (iii) exposure at default which is the total value that the lending institution is exposed to. Altman et al. (2004) points out that while significant attention has been paid to probability of default, the recoveries and its apparent inverse relation with probability of default, has attracted less attention. Notably, the recovery is often treated as a constant variable, independent of probability of default. Existing studies have documented some empirical irregularities in the observed recovery distribution. Schuermann (2004) finds that the concept of average recovery, a quantity often reported by rating agencies, is potentially very misleading as the recovery distribution is restricted to exist over the unit interval. This restriction implies that the lender cannot lose or recover more than the outstanding amount at the time of default. Specifically, the observed recovery distribution is typically U-shaped, with the largest relative frequency occurring near or at unity, a non-negligible mass around zero, and a spread of recoveries observed across the interval itself. A flexible nonlinear model is more appropriate to reflect the relation of this recovery distribution to number of loan or firm characteristics. Additionally, a range of econometric methods have been previously used to study recoveries, including ordinary least squares (OLS), beta regression, and a quasi-maximum likelihood estimation (QMLE) and a fractional regression methodology, as explored by Gupton et al. (2002); Acharya et al. (2007) and Khieu et al. (2012), respectively.

Such analyses provide further insight into the possible determinants of a loan's expected recovery, but nevertheless most approaches have some shortcomings. Standard OLS ignores the unique distributional aspects of the observed recoveries, despite the fact that the resulting recovery values predicted from the model need not be bounded between zero and one. It also assumes constant marginal effects for each of the explanatory variables – a feature that is also unlikely given the constrained recovery distribution. Furthermore, while the beta distributions underpinning a beta regression framework covers some variation of distributional densities over the unit interval, it cannot simultaneously accommodate a relative frequency mass in the middle of the unit interval with the relative large frequency masses observed around zero and one (De Servigny and Renault, 2004). Finally, while the model underlying the QMLE based approach accommodates the constraints of the observed recovery values, it does so at the expense of a coherent distributional model, fitting as it does a model for (binary) Bernoulli observations when in fact the recovery values may also lie inside the zero to one range, with clustering at zero or one.

Several studies have investigated the determinants of bank loan recovery rates ((Altman et al., 2005; Acharya et al., 2007; Khieu et al., 2012) and (Altman and Kalotay, 2014)). However, the systemically time-varying RR response to different credit and economic cycles does not appear to have been captured. Therefore, assuming linear recovery relations through cycles could potentially lead to an inaccurate assessment, particularly over periods of economic downturn (Resti, 2002; Altman et al., 2005).

Furthermore, the current literature on corporate debt recovery determinants does not focus specifically on bank loans. Altman and Kalotay (2014) combine bank loans with corporate bonds, whereas Mora (2015) investigates corporate bonds. A bank loan is fundamentally different to other securities, as they are typically senior to corporate bonds, making bank loan recoveries more likely due to a different repayment hierarchy. Furthermore, banks generally have direct access to their customers' financial information with the ability to demand special purpose financial reports and can also force covenant compliance; e.g. if any financial ratio or loan condition is breached. Therefore, given their privileged access to non-public information, banks may be quicker to impose bankruptcy procedures than other key stakeholders, potentially achieving better recoveries than other investors. Additionally, banks are able to take control of the underlying assets as their fixed/floating charges allows them to liquidate assets to get paid ahead of other creditors.

While Khieu et al. (2012) studied the RR of defaulted bank loans, their performance through to recoveries during the 2008/09 financial crisis and beyond has not been analysed. They also impose parametric assumptions/constraints embedded within the models they employ – which imposes assumptions about the recovery distributions that, may be quite different from the observed recoveries. Furthermore, their QMLE method, where recoveries are modelled using a Bernoulli likelihood, does not naturally accommodate observed recoveries that fit inside the unit interval due to clustering at the unit level. In addition, Khieu et al. (2012) consider a linear model for RRs, where the errors are effectively assumed to be normal – an assumption that is contrary to the observed RR distribution. There is also an assumed constant and negative relation between the RR and probability of default, see also the discussion in Altman et al. (2005). In contrast, we argue that the relation between probability of default and RR, while potentially negative may in fact be different under good times and bad times. To our knowledge, no prior study has considered the extent to which such interactions change with economic conditions/cycles, and it is this gap in the literature that we attempt to address here.

The unsatisfactory features of the existing parametric approaches in the recovery context have led to the development of more flexible models. Nonparametric methods have been shown to sometimes outperform their parametric counterparts in terms of accommodating non-linear relations between observed recoveries and certain conditioning variables (Qi and Zhao, 2011). However, Bastos (2010) and Qi and Zhao (2011) find such flexible predictive models are more likely to over fit the data and do not tend to work well in predicting future defaulted loan recoveries. Yao et al. (2015) use a support vector machine methodology to improve out-of-sample prediction, however they do not report parameter estimates or confidence intervals to help gauge the individual importance of the potential RR determinants. Similarly, regression trees can become overly complex and appear to produce results sensitive to assumed distribution and the dataset used. For example Bastos (2010) and Qi and Zhao (2011) report very distinctive trees based on different datasets. See Höcht et al. (2011) for a very similar analysis for European defaulted loans.

Modeling the recovery determinants has shown them to be a function of the individual loan characteristics, firm characteristics or fundamentals, industry variables, recovery process variables and macroeconomic factors. However, Altman et al. (2005) demonstrates a negative association between an aggregate measure of the underlying default rate over a given period and the average recovery, suggesting that changes in the underlying credit environment can also impact on recoveries. Hu and Perraudin (2002) observe a similar negative relation from data covering the period 1971–2000. In response to such relation, Bruche and Gonzalez-Aguado (2010) define a two state latent credit cycle variable and suggests that a 99% credit VaR (the Value at Risk for a portfolio consisting of bank loans and corporate bonds) is underestimated by more than 1.5% of the total outstanding amount if the credit cycle is omitted. The focus of this research is to enhance understanding of processes for the recovery of corporate loans, addressing the limitations of the abovementioned literature. This is done by investigating recoveries in relation to appropriate firm and loan variables, in a time variant framework allowing for different economic circumstances including major shocks such as the financial crisis of 2008/09.

3. Data

We investigate RRs from Moody's Ultimate Recovery Database over the period 1987 through to 2015 resulting in a set of 1,611 defaulted bank loans of US firms originated by an array of syndicated lending. Typically, the defaulted loans have debt values, at the time of default, of greater than \$50 million (\$50M) – namely this study investigates a unique loan segment, i.e. the larger corporate loans. This approach is consistent with other empirical studies within the recovery literature (Khieu et al. (2012) and Altman and Kalotay (2014)).

The same recovery variable is used in the current study. It is defined as the nominal settlement recovery amount discounted back from each settlement instruments trading date to the last date cash was paid on the individual defaulted instrument, using its own effective interest rate. This key recovery measure takes account of the time value of money for the effective settlement period. Our sample is split between term loans (48%) and revolvers (52%) i.e. loans that can be repaid and redrawn any number of times within a term. About 7.6% of the recoveries have some type of reorganization plan that shareholders have approved prior to, or at the time of, the bankruptcy filing. Frequency plots (histograms) of the RR for the sample period are shown in Fig. 1. Note the data concentration on the extreme right boundary of both graphs. While extreme modes associated with RR values at zero and unity are present, the mode at zero is almost negligible compared to that associated with full recovery. This feature is in line with the tendency towards left skewness typically associated

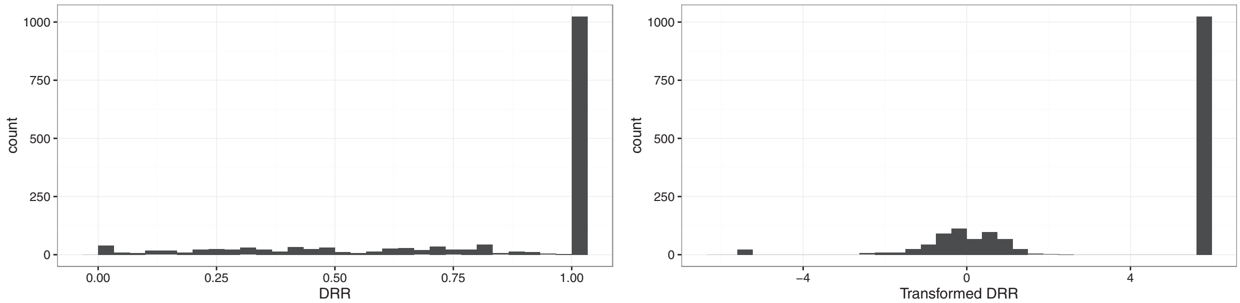

Fig. 1. Histograms of the discounted recovery rates (left panel) and of the transformed discounted recovery rates (**y**) (right panel), 1987–2015.

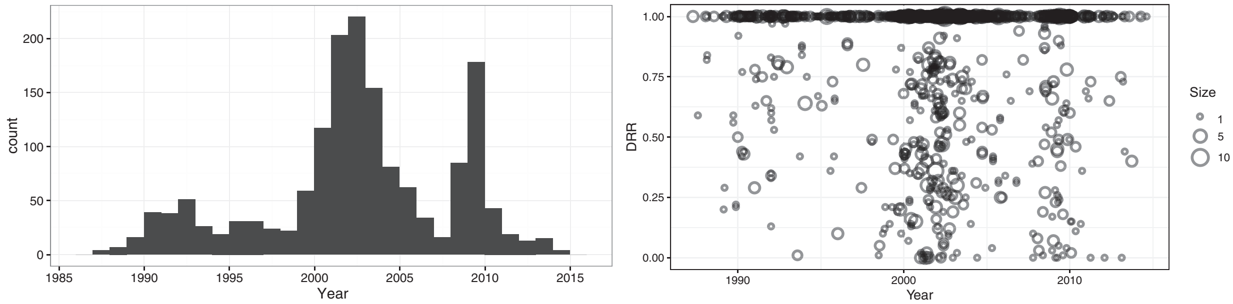

Fig. 2. Number of defaults (left panel) and (discounted) recovery rate outcomes (right panel) over the period 1987–2015. The 'Size' of each circle in the plot on the right is determined by the number of loans with that DRR value.

with RRs for bank loans Altman and Kalotay (2014). Given that the observed distribution of the RR is neither symmetric nor unimodal, the use of the average or median recovery as a single summary measure for the entire distribution is potentially very misleading. In particular, for this sample the average RR is 80.8%, while the median value is 100%, indicating that the distribution is indeed skewed to the left. We also plot the frequency of observed defaults and corresponding RRs by calendar year in Fig. 2. The diameter ('Size') of the plotting symbol, an open circle, reflects the number of loans for the given RR value. It can be seen that the RR varies over time, with many more observed during the early 2000s and over the period 2007–2009. For the same time period, however, more low RRs are observed in our sample. This suggests the potential for a time-varying model for RRs, since RRs vary throughout different periods in our sample.

Further to the RR data, we also compile a complete set of loan recovery determinants associated with each of the observed RRs, as used by Khieu et al. (2012) for consistency and comparison. Broadly speaking, the available determinants address: loan characteristics, recovery process, borrower characteristics, as well as other conditions associated with the loan or default period. The observed determinants relating to borrower characteristics are obtained through manual matching of these firms with Standard & Poorâs Compustat firms based on both CUSIP numbers and also firm names. Definitions for each of these determinants are provided in Table 1.

In addition to the definitions shown in Table 1, we also report in Table 2 the descriptive statistics for the RR and the selected determinants. Overall, the summary statistics of the RR determinants for the sample period here are similar to those found in Emery (2007) and Khieu et al. (2012), suggesting that in aggregate the data are by and large in line with those of earlier studies. Furthermore, as per Khieu et al. (2012), the firm characteristics are measured one year before default. In terms of loan characteristics, however, our dataset reports larger average loan sizes than Khieu et al. (2012) ($224M compared to $142M), with similar increases in the average term loan size, the average revolver value and the values of loans secured by all assets. With respect to recovery process characteristics, the average length of time a sample firm stays in default is 13 months with a maximum of 13 years. Most of the firms in the sample defaulted in a non-prepackaged bankruptcy, whereas 10% went through prepackaged bankruptcy and 13% had private workouts. Similar to McConnell et al. (1996) and Khieu et al. (2012) the mean RR for loans with a reorganization plan lie between those for loans resolved through traditional bankruptcy and those for loans utilising other forms of default resolution. With respect to borrower characteristics, the mean and median cash flows, relative to total assets, are 16% and 9%, respectively.

4. A hierarchical econometric model for bank loan recovery rates

The starting point to our investigation is determining the role of a complete set of recovery determinants, as explored by Khieu et al. (2012). However in this research we implement a more enhanced and flexible modelling framework, extending the sample beyond the GFC capturing different economic conditions. The proposed methodology addresses two key challenges previously identified in the RR modelling literature, namely that (i) the observed RR distribution has a distinctive

Table 1
Definitions of the RR determinants.

Name	Definition
Loan characteristics	
(1) LOANSIZE($M)	The dollar amount (in millions of dollars) of the facility at the time of issuance.
(2) LOANTYPE	A dummy variable equal to one if the loan is a term loan (fixed tenure and not recallable on demand), and equal to zero if it is a revolver (short-term revolving and recallable on demand).
(3) LOANTYPE × FIRMSIZE	The product of LOANTYPE and FIRMSIZE.
(4) ALLASSETCOLL	A dummy variable equal to one if the loan is secured by all firm assets, and zero otherwise.
(5) INVENTRECIVECOLL	A dummy variable equal to one if the loan is secured by inventory, accounts receivable, or both, and zero otherwise.
(6) OTHERCOLL	A dummy variable equal to one if the loan is secured differently from the other types, and zero otherwise.
Recovery process characteristics	
(7) PREPACK	A dummy variable equal to one if the bankruptcy is through a pre-packaged bankruptcy, and zero otherwise.
(8) RESTRUCTURE	A dummy variable equal to one if default is resolved by out-of-court restructuring, including distressed exchange offers, and zero otherwise.
(9) OTHERDEFAULT	A dummy variable, equal to one if default is resolved by other methods than an out-of-court restructuring, pre-packaged formal bankruptcy, and zero otherwise.
(10) TIMETOEMERGE	The length of time (in months) between bankruptcy or restructuring and emergence, often known as resolution time.
(11) TIMETOEMERGE^2	TIMETOEMERGE squared.
(12) PREPACK × TIMETOEMERGE	The product of PREPACK and TIMETOEMERGE.
Borrower characteristics	
(13) FIRMSIZE	The market value of firm-level assets one year before default. The market value is calculated as the book value of long-term and short-term debt plus the number of common shares outstanding.
(14) FIRMPPE	Firm asset tangibility, measured as net property, plant, and equipment over total book assets one year before default.
(15) FIRMCF	Firm cash flows, measured EBITDA (earning before interest, tax and depreciation and amortization) over total book assets one year before default.
(16) FIRMLEV	Firm leverage, measured as total long-term debt plus debt in current liabilities over total book assets one year before default.
(17) EVERDEFAULTED	A dummy variable equal to one if the firm has defaulted before, and zero otherwise.
Loan conditions	
(18) GDP	The annual GDP growth rate measured one year before default.
(19) INDDISTRESS	A dummy variable equal to one if the industry median stock returns in the year default is less than -30%, and zero otherwise. The stock returns are calculated without the defaulting firms and the industry is defined according to the three-digit SIC codes.
(20) AIS	The credit spread (in percent) at the time of loan origination over LIBOR of the drawn loan that defaulted.

(non-Gaussian) shape, and (ii) when plotted over time the RRs appear to exhibit varying behavior – possibly owing to differences in the underlying probability of default. The first issue is addressed through the use of a finite Gaussian mixture model, first implemented on a combined loan and bond dataset by Altman and Kalotay (2014). This approach enables the RR determinants to be stochastically connected to the observed RRs through a latent predictive regression structure. In addition, to capture cyclical aspects such as the impact of the GFC, we augment the model structure with a Markov switching mechanism within the predictive regression model.

In this framework, the regression coefficients depend on the state of the credit cycle, where the state corresponds to either a credit upturn or a credit downturn – i.e. a good state or a bad one. The coefficient estimates we obtain for each credit state provide insight into the pro-cyclical effects of RR determinants. To combine the Gaussian mixture components, the latent predictive regression and Markov switching mechanism, a hierarchical model is developed and estimated using a fully Bayesian inferential approach. The Bayesian approach enables a flexible hierarchical structure, which is estimated jointly and has the benefit of the consideration of each model component individually (i.e., marginally) while accounting for the uncertainty present in the remaining components.

Before detailing the form of the hierarchical model, also known as a 'state space' model, and describing the associated Bayesian inferential framework, following Altman and Kalotay (2014) we transform the observed RRs from the unit interval to the real line via the inverse of the cumulative distribution function (cdf) associated with the standard normal distribution, denoted by $\Phi^{-1}(\cdot)$. Specifically, if RR_i denotes the observed (appropriately discounted) RR value associated with defaulted loan i, we obtain the transformed RR value, denoted by y_i and given by

$$y_i = \Phi^{-1}(RR_i^*) \tag{1}$$

where

$$RR_i^* = \begin{cases} \epsilon & \text{if } RR_i = 0 \\ RR_i & \text{if } 0 < RR_i < 1 \\ 1 - \epsilon & \text{if } RR_i = 1, \end{cases}$$

Table 2
Descriptive statistics of the discounted RR and of the determinants of bank loan recoveries, by determinant category. By column corresponding to the variable indicated in the far left-hand column, the following statistics are reported: sample size (n), sample mean (Mean), sample median (Median), 25% quantile (1st Qu), 75% quantile (3rd Qu), sample minimum (Min), sample maximum (Max) and whether variable is binary [Y] or not [N] (Binary).

Variable	n	Mean	Median	1st Qu	3rd Qu	Min	Max	Binary
RR	1611	0.81	1.00	0.66	1.00	0.00	1.00	N
Loan characteristics								
(1) LOANSIZE($M)	1611	224.5	96.0	35.0	208.5	1.0	11150.0	N
(2) LOANTYPE	1611	0.48	0	0	1	0	1	Y
(3) LOANTYPE × FIRMSIZE	1611	813.5	0.0	0.0	692.3	0.0	60631.9	N
(4) ALLASSETCOLL	1611	0.62	1	0	1	0	1	Y
(5) INVENTRECIVECOLL	1611	0.10	0	0	0	0	1	Y
(6) OTHERCOLL	1611	0.17	0	0	0	0	1	Y
Recovery process characteristics								
(7) PREPACK	1611	0.08	0	0	0	0	1	Y
(8) RESTRUCTURE	1611	0.13	0	0	0	0	1	Y
(9) OTHERDEFAULT	1611	0.01	0	0	0	0	1	Y
(10) TIMETOEMERGE	1611	13.49	9.67	2.59	18.42	0	156.33	N
(11) TIMETOEMERGE^2	1611	427.17	93.51	6.682	339.11	0.00	24439.07	N
(12) PREPACK × TIMETOEMERGE	1611	0.21	0.00	0.00	0.00	0.00	11.87	N
Borrower characteristics								
(13) FIRMSIZE	1611	1654.8	665.5	227.4	1365.7	0	60631.92	N
(14) FIRMPPE	1611	0.54	0.44	0.13	0.82	0	9.73	N
(15) FIRMCF	1611	0.16	0.09	0.05	0.14	0	23.48	N
(16) FIRMLEV	1611	1.08	0.94	0.77	1.26	0	4.9	N
(17) EVERDEFAULTED	1611	0.15	0	0	0	0	1	Y
Loan conditions								
(18) GDP	1611	2.61	2.80	0.98	4.09	0.06	4.79	N
(19) INDDISTRESS	1611	0.18	0	0	0	0	1	Y
(20) AIS	1611	0.04	0.03	0.02	0.04	0	0.3	N

for $i = 1, 2, \ldots, n$. As is typical, before transformation is undertaken, the values of RR_i at zero are replaced with a small positive value, ϵ, and values at unity are replace with $1 - \epsilon$, so that the y_i values are all finite.[2] It is the distribution of these y_i values that we model, however it is important to note that no information is lost by using this one-to-one transformation, and it may be reversed if needed, for example if a predictive distribution for raw RR values is desired. Note that positive y_i values result whenever the original $RR_i > 0.5$. We now turn to the hierarchical model specification and the Bayesian inferential framework used to estimate it. Subsequently, Section 4.1 first details the Gaussian mixture model where membership to each mixture component is predicted by a latent regression on RR determinants. The regression coefficients here are shown in their static form, without the Markov switching component, which is described later in Section 4.2. Section 4.3 then summarizes a computational strategy suitable for Bayesian inference to be conducted for the full dynamic model. Details regarding the algorithms required and implementation of the computational strategy are given in the Appendix.

4.1. A mixture model from recovery rate determinants

Having transformed each original RR observation, y_i is then treated as arising from one of J distinct Gaussian distributions, with the jth distribution having mean and variance denoted by μ_j and σ_j^2, with $\mu_1 < \mu_2 < \cdots < \mu_J$. From the investor's perspective, recovery outcomes from a mixture component having a larger mean will be preferred (e.g. the J^{th} mixture component is preferred over the $(J-1)$st, etcetera, with the first component being least desired), with the ordering imposed to retain the ability to interpret each of the categories. For the current context, we set $J = 4$.

Next, the connection between the mixture components and the RR determinants occurs through a latent ordered probit regression framework (Albert and Chib, 1993), which permits a range of explanatory variables, including loan, borrower, recovery process and other loan conditions, to characterize the probability of y_i being in component j of the Gaussian mixture. In particular, the determinants associated with loan i, denoted by $x_{1,i}, x_{2,i}, \ldots, x_{K,i}$, are related to a latent variable z_i through the regression equation

$$z_i = \beta_0 + \beta_1 x_{1,i} + \cdots + \beta_K x_{K,i} + \varepsilon_i, \qquad (2)$$

with $\varepsilon_i \sim N(0,\ 1)$. The latent (unobserved) z_i is referred to as a *predictive score* for defaulted loan i, while the vector $\boldsymbol{\beta} = (\beta_0, \beta_1, \ldots, \beta_K)'$ contains the regression coefficients that describe the marginal impact of each of the determinants on this predictive score. The predictive score for loan i in (2) relates to each of the J Gaussian mixture components via a set of so-called 'cut-points', $\mathbf{c} = (c_0, \ldots, c_J)$, with $c_0 = -\infty, c_1 = 0, c_J = +\infty$, so that when in fact loan i belongs to group j, the jth mixture probability may be calculated as $\Pr(c_{j-1} < z_i \le c_j)$. Although the values of c_0, c_1 and c_J are fixed for identification purposes (see again Albert and Chib (1993)) the locations of the remaining cut-points (here c_2 and c_3) are treated

[2] We use $\epsilon = 1 \times 10^{-8}$.

as unknowns to be estimated. The values of μ_j and σ_j^2 are also estimated, essentially being determined by those y_i that are predicted by the regression to fall in category j.

Up to this point, the approach used here largely follows that of Altman and Kalotay (2014), apart from our use of a wider set of determinants as discussed in Section 1. However, as described in the next section, we introduce an additional Markov switching component to the framework, so that the impact of the RR determinants is able to vary with the credit environment, whether good or bad, at the time of default. We also note the introduction of a LASSO prior for $\boldsymbol{\beta}$ in Section 4.3.1, to assist with the large number of determinants when undertaking Bayesian inference for this model.

4.2. The credit cycle

To incorporate the notion of an underlying dynamic credit cycle, a two-state hidden Markov switching component is added to the mixture model with latent predictive regression framework outlined in Section 4.1. The introduction of the latent binary Marko-switching state process has a long history as a viable methodology in the literature as a mechanism for proxying economic conditions (see Goldfeld and Quandt, 1973 and Hamilton, 1989, with key applications in economics and finance referenced in Hamilton (2010)). The model postulates that the underlying conditions, i.e. the latent credit state variable, drives common dependence in the recovery rate regression coefficients. Thus, regime-specific coefficients for the recovery determinants are obtained.

In the dynamic model, the binary credit cycle state variable for time (year) t is denoted by S_t, and takes on either the value of zero or one, depending on the underlying credit environment prevalent at the time of default.[3] The credit cycle states are normalized so that $S_t = 0$ corresponds to a low recovery period (a downturn, or bad credit state), while $S_t = 1$ corresponds to a high recovery period (an upturn, or good credit state). Transition to each credit cycle state at time t from a relevant state one period earlier, time $t-1$, is governed by the probabilities

$$\begin{aligned}
&\text{Prob}(S_t = 0|S_{t-1} = 0) = p\\
&\text{Prob}(S_t = 1|S_{t-1} = 0) = 1 - p\\
&\text{Prob}(S_t = 1|S_{t-1} = 1) = q\\
&\text{Prob}(S_t = 0|S_{t-1} = 1) = 1 - q.
\end{aligned} \tag{3}$$

According to the transition probabilities in (3), if the credit cycle at time $t-1$ is in a low recovery state (i.e $S_{t-1} = 0$), then the chance of remaining in this bad state at time t equal to p, with $0 < p < 1$, while the chance of moving to the good recovery state (i.e.with $S_t = 1$) is equal to $1-p$. On the other hand, if $S_{t-1} = 1$, then the chance of remaining in the good recovery state at time t is equal to q, with $0 < q < 1$, and the chance of moving to the bad recovery state at time t is given by $1-q$.

Using the Markov switching device, two sets of regression coefficients are obtained for the recovery determinants: $\boldsymbol{\beta_0} = (\beta_{0,0}, \beta_{1,0}, \ldots, \beta_{K,0})'$ relating to the predictive scores in credit cycle downturns, and $\boldsymbol{\beta_1} = (\beta_{0,1}, \beta_{1,1} \ldots, \beta_{K,1})'$ applicable during credit cycle upturns. To link the latent credit cycle states to the available data, let t_i denote the time associated with the default of loan i, so that S_{t_i} indicates the relevant state of the credit cycle at the time of default of loan i. The predictive regression coefficient vector $\boldsymbol{\beta_0}$ will apply for predicting z_i if $S_{t_i} = 0$, whereas the vector $\boldsymbol{\beta_1}$ will apply for predicting z_i if $S_{t_i} = 1$. Hence, by adding the Markov switching component, the regression coefficients in (2) become state dependent, and the predictive regression for loan i becomes

$$z_i = \beta_{0,S_{t_i}} + \beta_{1,S_{t_i}} x_{1,i} + \cdots + \beta_{K,S_{t_i}} x_{K,i} + \varepsilon_i, \tag{4}$$

where again $\varepsilon_i \sim \mathcal{N}(0, 1)$, for $i = 1, 2, \ldots, n$. Now that the regression coefficients in the predictive regression are state dependent, the estimated values of the vectors $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_0}$ will provide insight into the differentiated impact of RR determinants in good times and bad.

4.3. Bayesian inference

Like Altman and Kalotay (2014), we take a Bayesian approach when estimating the proposed model, an approach that offers several advantages over the perhaps more familiar frequentist strategy. The outcome of any Bayesian inferential procedure is a full joint probability distribution for all unknowns, including both parameters and latent variables. This outcome distribution, referred to the joint posterior distribution, characterizes all that is known about the parameters, and the credit states, prediction scores and Gaussian mixture allocations for each loan. From this joint posterior, the corresponding marginal distribution for any individual parameter or state variable (or indeed any subset of these) will automatically and coherently account for uncertainty in the remaining unknown variables. This is of particular importance when working with a hierarchical model, such as the one we advocate here.

An added advantage of using a hierarchical model within a Bayesian framework is that computation to produce the posterior can be undertaken efficiently using Markov chain Monte Carlo (MCMC) techniques. As a further advantage, Bayesian inference yields a finite sample analysis, conditioning only on the available data, whereas a corresponding frequentist inferential method would typically require assumptions about the behavior of estimators as the sample size increases without

[3] In this study, $t = 1$ corresponds to 1987, the first year of the available sample period.

bound. This is important in empirical applications, such as the one undertaken here, where the number of RR observations is limited relative to the number of unknowns being estimated.

4.3.1. Prior distribution incorporating the Bayesian LASSO

We conservatively adopt a relatively non-informative prior distribution with *a priori* independence assumed between $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_J)'$, $\boldsymbol{\sigma^2} = (\sigma_1^2, \sigma_2^2, \ldots, \sigma_J^2)'$, **c**, $\boldsymbol{\beta_0}$, $\boldsymbol{\beta_1}$, p and q. Apart from the prior specified for the state-dependent predictive regression coefficients, $\boldsymbol{\beta_0}$ and $\boldsymbol{\beta_1}$ discussed below, the prior components are chosen from the appropriate conditionally conjugate family, thereby enabling fast computation of the posterior distribution via MCMC.

For the predictive regression vectors, $\boldsymbol{\beta_0}$ and $\boldsymbol{\beta_1}$, we introduce the use of the Bayesian LASSO prior of Park and Casella (2008). As is now widely recognized the LASSO encourages a sparse regression model by down-weighting certain covariates when a large number of regression terms are used (Nazemi and Fabozzi, 2018), as is the case here. Effectively, the LASSO will tend to reduce the size of the estimated regression coefficients to account for correlation (multi-collinearity) or other dependence between the available recovery determinants, favouring putting weight on regressors whose association with the response variable (here the latent predictive scores $\mathbf{z} = (z_1, z_2, \ldots, z_n)$) can be estimated with relative certainty. In this way, predictive information shared by different determinants is not 'double counted' when fitting the model, and therefore we reduce the potential for overstating the significance of individual determinants. The Bayesian LASSO achieves this reduction, or *shrinkage*, through the choice of the prior distribution for $\boldsymbol{\beta_0}$ and $\boldsymbol{\beta_1}$. This prior distribution for each regression vector in the dynamic credit cycle context relies critically on certain additional so-called shrinkage parameters, denoted by λ_0^2 and λ_1^2, respectively, with a single shrinkage parameter, denoted by λ^2 used for the static latent regression model. These shrinkage parameters are included as unknowns, and are also estimated here within the Bayesian framework.

We note that many existing studies have considered the predictive performance for RRs. For instance, Altman and Kalotay (2014) investigate the predictive performance of different models using a set of variables for debt seniority, collateralization and industry classification. In the Bayesian paradigm, Barbieri and Berger (2004) point out that a model with highest posterior probability is not necessarily optimal for prediction, instead, optimal predictive models are 'median probability models'. In any case, all such prediction strategies are available under our framework as the full joint posterior distributions is obtained.

5. Results

The results reported in this section are based on two model implementations described in Section 4: (i) the *static* version, corresponding to Section 4.1 where the predictive regression coefficient variables are assumed to be constant over the entire sample period, and; (ii) the *dynamic* version described in Section 4.2, where the latent time-varying credit cycle is included by using a Markov-switching state process to incorporate economic conditions. The LASSO priors are used in both cases, with a single shrinkage parameter used in the static version and two shrinkage parameters used in the dynamic version. Note that while we have information regarding the entire joint posterior distribution, where the term estimate is used it will generally refer to the mean of the posterior distribution for the relevant quantity. Uncertainty in such an estimate will be indicated by a 95% so-called highest posterior density (HPD) interval taken as the shortest single interval associated with 95% marginal posterior probability. These Bayesian point and interval estimates are used as a convenient way to summarize the marginal posterior distributions, and are obtained from the MCMC output based on 100,000 MCMC draws retained following a 5000 burn-in period.

5.1. Recovery mixture components

As alluded to in Section 3, given RR observations are clustered at zero (zero recovery) and one (full recovery), following Altman and Kalotay (2014), we apply a $J = 4$ Gaussian mixture model to transformed RRs. Table 3 provides details of the features of the estimated Gaussian mixture components that result from each of the two models fitted to the dataset considered. For each case, the estimated mean and standard deviation parameters for each Gaussian component are provided, along with its corresponding mixture weight and median RR. The estimated components labeled 1 and 4 effectively concentrate, with the same relative proportions for both the dynamic and static specifications, on point masses corresponding to RR values at zero and one, respectively. However, the two interior mixture components (labeled 2 and 3) show differences across these attributes, notably in the third mixture component. As the two models correspond to different latent predictive regression structures – one static (i.e. without imposing the Markov switching credit states) and the other dynamic – two separate estimation results are shown. The mean parameter for the jth component, μ_j, and the corresponding standard deviation, σ_j^2, is determined by observed RRs with predictive regressions corresponding to outcomes that fall in mixture component j.

5.2. The latent credit cycle

The latent Markov switching states are introduced into the dynamic model to condition the RR determinants on the underlying economic state and thereby account for the time-series variation in the observed RRs in the analysis. Estimates of q, the probability of remaining in a good credit state, from one year to the next, and p, the probability of the economy

Table 3
Estimated Gaussian mixture components. Posterior mean and 95% HPD interval (in parentheses) for each mixture component as indicated in the first row, and for (a) the component mean parameter (Mean (μ_j)) in row two; (b) the component standard deviation parameter (Std (σ_j)); (c) the Implied weight, as given by the proportion of observations allocated to the mixture component; and (d) Mean RR, corresponding to the inversion of (1); for each of the four mixture components as labeled by $j = 1, 2, 3,$ and 4, for the static model (columns 2–5) and the dynamic model (columns 6–9).

	Static model				Dynamic model			
Component (j)	*1*	*2*	*3*	*4*	*1*	*2*	*3*	*4*
(a) Mean (μ_j)	-5.61	−0.85	0.21	5.61	−5.61	−1.31	0.09	5.61
	(−5.72,−5.50)	(−1.77,−0.32)	(0.03,0.46)	(5.61,5.61)	(−5.72,−5.50)	(−2.00,−0.52)	(−0.02,0.24)	(5.61,5.61)
(b) Std (σ_j)	0.08	0.54	0.41	0.00	0.08	0.44	0.48	0.00
	(0.05,0.13)	(0.21,0.83)	(0.29,0.53)	(0.00,0.00)	(0.05,0.13)	(0.14,0.90)	(0.38,0.58)	(0.00,0.00)
(c) Implied weight	0.01	0.10	0.25	0.64	0.01	0.04	0.31	0.64
	(0.01,0.01)	(0.02,0.20)	(0.15,0.33)	(0.64,0.64)	(0.01,0.01)	(0.01,0.11)	(0.24,0.34)	(0.64,0.64)
(d) Mean RR	0.00	0.25	0.63	1.00	0.00	0.10	0.54	1.00
	(0.00,0.00)	(0.04,0.37)	(0.51,0.68)	(1.00,1.00)	(0.00,0.00)	(0.02,0.30)	(0.49,0.59)	(1.00,1.00)

Table 4
Dynamic model transition probabilities. Estimated posterior mean and 95% HDP (in parenthesis) for each possible transition probability associated with a one period transition from credit state S_{t-1}, shown by row in the first column, to a new credit state S_t, shown by corresponding entry in columns 2 and 3. The final row provides the estimated overall long-run probability of being in the good credit state $\Pr(S_t = 1)$ resulting from the dynamic model.

	$\Pr(S_t \mid S_{t-1})$	
	$S_t = 0$	$S_t = 1$
$S_{t-1} = 0$	0.53	0.47
	(0.24,0.83)	(0.17,0.76)
$S_{t-1} = 1$	0.67	0.33
	(0.40,0.88)	(0.12,0.60)
Steady state: $\Pr(S_t = 1)$	0.61	
	(0.38,0.83)	

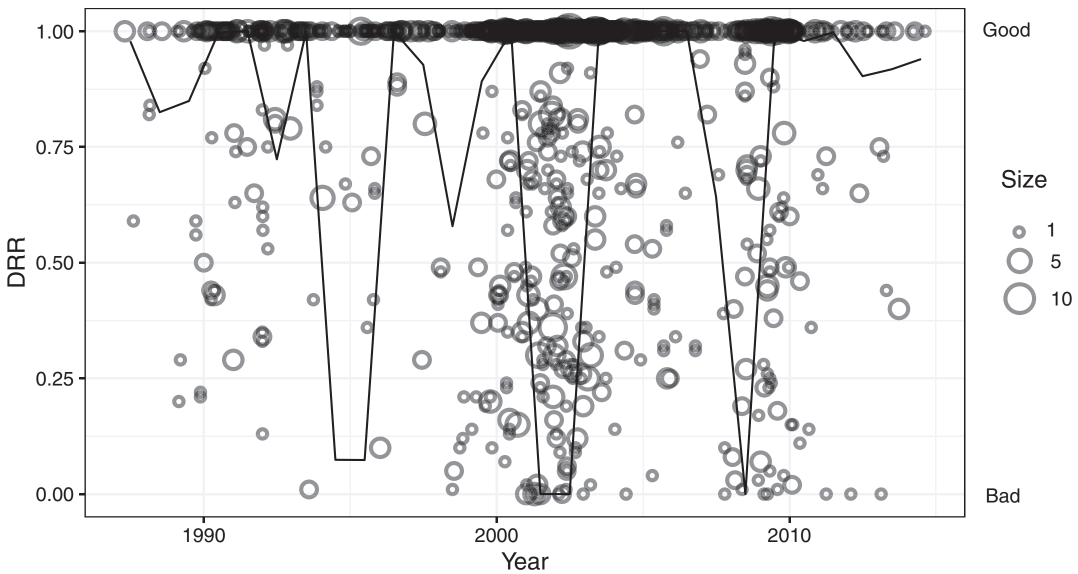

Fig. 3. DRRs plotted over the 1987–2015 period and aligned by calender year, with the 'Size' of each circle is determined by the number of loans. The superimposed line graph represents the estimated probability of being in a good credit state (when the state variable is equal to one) implied by the dynamic model..

remaining in a bad credit state, are given in Table 4, with the corresponding estimated steady state (or long-run) probabilities being 61% and 39% for the good and bad states, respectively, as indicated by the final row of Table 4. A line graph of the estimated probabilities for being in the good credit state during each specific year during the given sample period is overlaid on a plot of RR outcomes is shown in Fig. 3. This figure appears to show that the latent credit cycle estimated by the dynamic model captures the main periods when many RRs are low, notably in 2002/03 and in 2008/09.

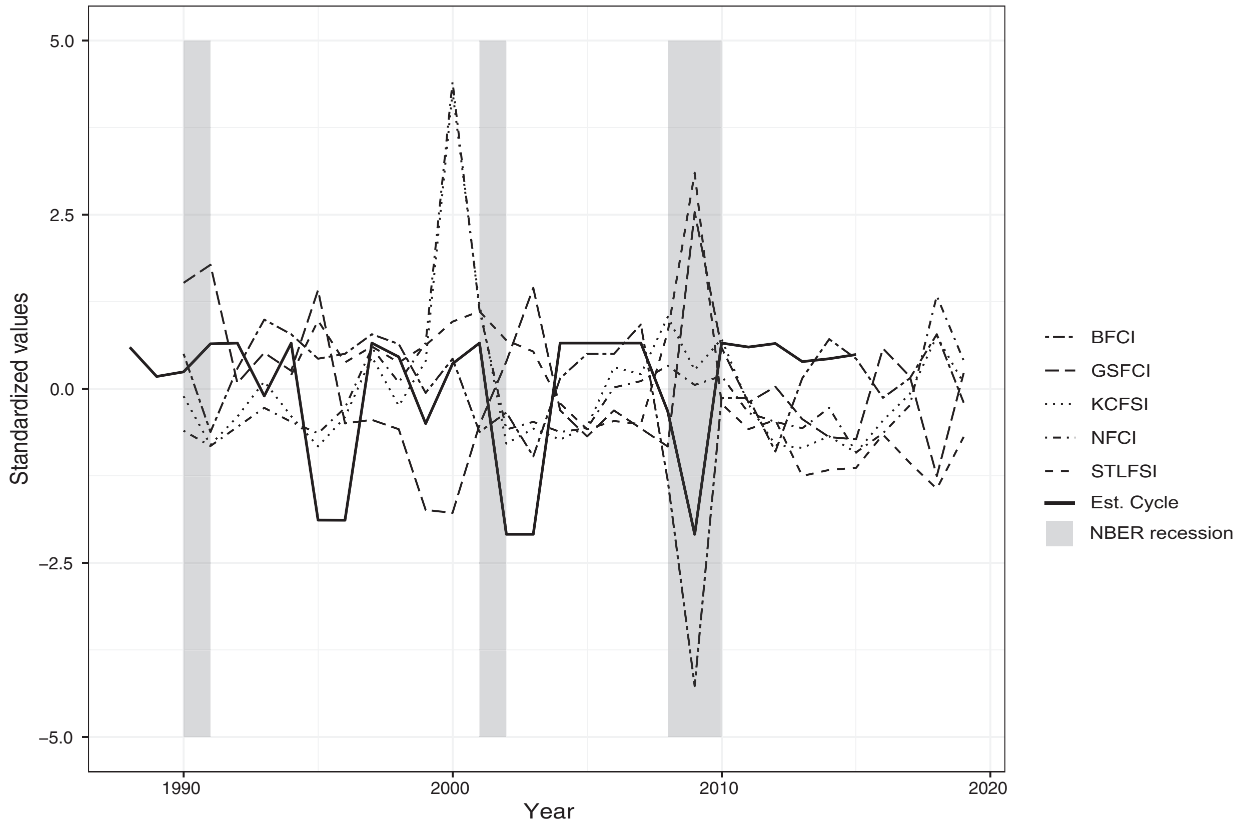

Fig. 4. Market condition indicators, shown for January of each year 1994–2015. All indicator series are standardized to have zero mean and unit variance. The gray shaded rectangles represent NBER recession periods.

Table 5
Pairwise correlations between the estimated latent credit cycle (Dynamic model) and each of five available financial stress indicators, namely the Goldman Sachs Financial Conditions Index (GSFCI), Bloomberg Financial Conditions Index (BFCI), Chicago Fed National Financial Conditions Index (CNFCI), the Kansas City Financial Stress Index (KCFSI), and the St. Louis Fed Stress Index (STLFSI), along with a binary series containing the NBER indicator of recessions. *Significant correlations in bold font, with corresponding *p*-value below. Number of observations (*n* pairs) used for each calculation shown in final row.

Indicator	BFCI	GSFCI	KCFSI	NFCI	STLFSI	NBER
Correlation	**0.452***	**-0.423***	0.136	0.110	**-0.595***	−0.157
p-value	0.020	0.031	0.5060	0.594	0.004	0.425
n pairs	26	26	26	26	22	28

To gauge whether the collection of estimated probabilities for being in the good credit state shown in Fig. 3 can reasonably be interpreted as an estimated credit cycle, we calculate pairwise correlations between our estimated credit cycle and several available market stress indicators.[4] We consider each of the following stress indicators: the Bloomberg Financial Conditions Index (BFCI, see bloomberg.com), the Goldman Sachs Financial Conditions Index (GSFCI, see https://www.goldmansachs.com/), the Kansas City Financial Stress Index (KCFSI, see www.kansascityfed.org/research/indicatorsdata/kcfsi), the National Financial Conditions Index (NFCI, see fred.stlouisfed.org/series/NFCI), and the St. Louis Fed Stress Index (STLFSI, see https://fred.stlouisfed.org/series/STLFSI) series. We also include the correlation between the estimated credit cycle and a binary series indicating periods of recession and expansion, according to the National Bureau of Economic Research (NBER, see www.nber.org/cycles.html). All indicators are displayed together in Fig. 4, with each series standardized for display purposes. The benchmark comparison is made in January each year, with recession attributed to a given year if the NBER identifies six or more months of that year in recession. The correlations are provided in the second row of Table 5, followed by the corresponding *p*-value and the number of pairs (*n* pairs) available for each in the subsequent two rows. While both GSFCI and STLFSI are negatively oriented, meaning they tend to increase when the credit market is declining, the other

[4] We thank an anonymous referee for this useful suggestion.

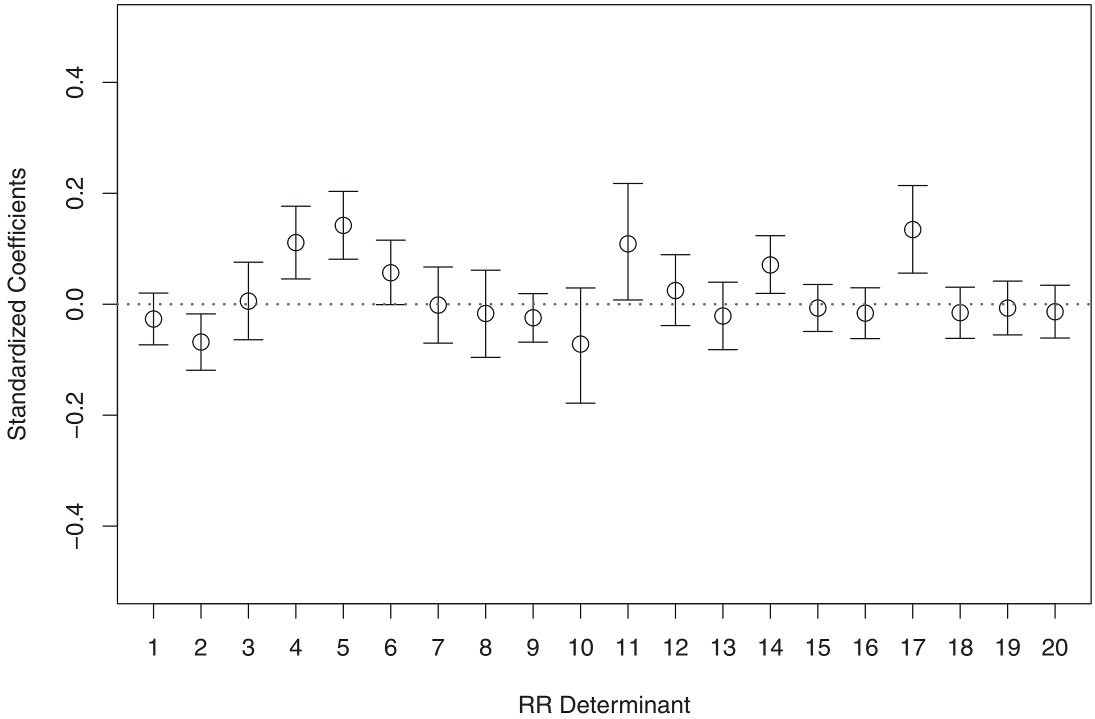

Fig. 5. Static model: Posterior mean estimates (○) of individual β_k coefficients, for variable $k = 1, 2, \ldots, K = 20$, with corresponding 95% HPDs indicated by the vertical bars. The variables are: (1) LOANSIZE, (2) LOANTYPE, (3) LOANTYPE × FIRMSIZE, (4) ALLASSETCOLL, (5) INVENTRECIVECOLL, (6) OTHERCOLL, (7) PREPACK, (8) RESTRUCTURE, (9) OTHERDEFAULT, (10) TIMETOEMERGE, (11) TIMETOEMERGE^2, (12) PREPACK × TIMETOEMERGE, (13) FIRMSIZE, (14) FIRMPPE, (15) FIRMCF, (16) FIRMLEV, (17) EVERDEFAULTED, (18) INDUSTRESS, (19) GDP, (20) AIS.

indices and our estimated credit cycle are positioned to move in the same direction as the credit market. Regardless of the sign, however, we note that significant correlations are found (at the 95% percent level) between our estimated credit cycle and each of BFCI, GSFCI and STLFSI. No significant correlation is found with the others, including the NBER recession periods. We take this lack of significance with the NBER recession periods, together with the fact that our estimated credit cycle *does* correlate with three alternative market condition indictors, as justification that our interpretation of estimated credit cycle.[5]

5.3. The predictive regressions

To demonstrate the importance of allowing for temporal variation in economic conditions, we contrast the inferential results from the dynamic and static Bayesian models. As discussed in Section 4, the models are developed using a Bayesian LASSO to control for multi-collinearity arising from competing and highly correlated RR determinants. Figs. 5 and 6 illustrate the significance of each of the variables after applying the Bayesian LASSO, with Fig. 5 showing the signficance of parameters in the static model, and Fig. 6 showing those for the dynamic case, with the top panel of the latter figure corresponding to significance for the bad credit state and the lower panel corresponding to the significance of determinants under the good credit state. In all cases, interval estimates for variables that cross the vertical axis at zero indicate a lack of (marginal) significance for that variable in the relevant model.

In Table 6 we report an alternative summary of these predictive regression estimation results, again for each of the static and dynamic models, in this case showing the sign only of the significant coefficients along with those obtained previously in Khieu et al. (2012). In column two, we report the signs of the significant RR determinants identified in column one under our static model resulting from the Bayesian approach and corresponding to data from 1987–2015. Columns three and four of Table 6 report the sign of significant RR determinants under the Bayesian dynamic model, with β_0 corresponding to the bad credit state (i.e. when $S_t = 0$) and with β_1 corresponding to the good credit state (i.e. when $S_t = 1$). For comparative purposes, the sign of the significant coefficients of these determinants corresponding to frequentist inference using OLS and QMLE methodologies, and relating to data from 1997–2007 (as reported in Khieu et al. (2012)), are provided in columns five and six. This is done to illustrate the contribution made by static vs. dynamic versions, and the need to allow for variation in

[5] We also analysed correlations between the NBER series and our estimated credit cycle at one and two lead and lag periods, and also did not find any of significance.

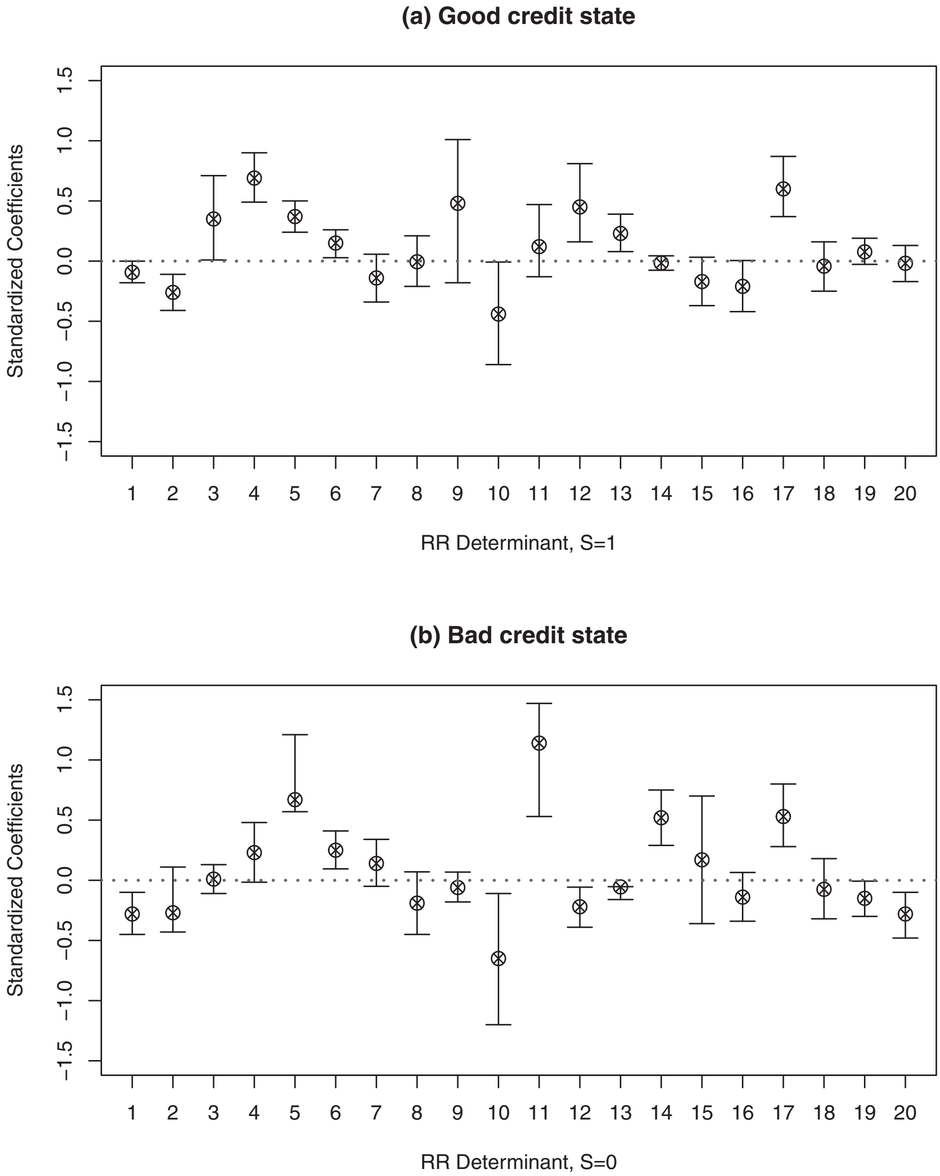

Fig. 6. Dynamic model: Posterior mean estimates (○) of $\beta_{0,k}$ for the good credit state (top panel) and $\beta_{1,k}$ for the bad credit state (bottom panel), for variable $k = 1, 2, \ldots, K = 20$, along with corresponding 95% credible intervals indicated by the vertical bars. The variables are: (1) LOANSIZE, (2) LOANTYPE, (3) LOANTYPE × FIRMSIZE, (4) ALLASSETCOLL, (5) INVENTRECIVECOLL, (6) OTHERCOLL, (7) PREPACK, (8) RESTRUCTURE, (9) OTHERDEFAULT, (10) TIMETOEMERGE, (11) TIMETOEMERGE^2, (12) PREPACK × TIMETOEMERGE, (13) FIRMSIZE, (14) FIRMPPE, (15) FIRMCF, (16) FIRMLEV, (17) EVERDEFAULTED, (18) INDUSTRESS, (19) GDP, (20) AIS.

the impact of RR determinants under different credit conditions. The reported models are more parsimonious relative to the existing literature due to our use of a LASSO, though both are consistent overall regarding the relevance of RR determinants. While the numerical values of the estimated frequentist and Bayesian coefficients themselves are not directly comparable, we can compare their statistical significance and their sign.

We note that only three loan characteristic variables appear to be important for explaining RRs in bad times, whereas there is evidence that six variables are relevant during good times. As for the OLS and QMLE results of Khieu et al. (2012), the loansize (LOANSIZE) determinant does not appear significant in the Bayesian static model. However, once the credit cycle is incorporated this determinant does appear to be important.[6] In the case of recovery process and borrower characteristics, we find that a fewer number of variables are important in bad times and a greater number in good times. Also, and im-

[6] Although significant in both good and bad states, the magnitude of the estimated marginal impact under the good state is relatively small.

Table 6
The sign of significant RR determinants under each of the Bayesian models and of those from Khieu et al. (2012).

Recovery Determinant	Static model	Dynamic model		Khieu et al. (2012)	Khieu et al. (2012)
	β	β_0 (Bad)	β_1 (Good)	β	β
	Bayes	Bayes	Bayes	OLS	QMLE
Loan characteristics					
(1) LOANSIZE		–	–		
(2) LOANTYPE	–		–	–	–
(3) LOANTYPE × FIRMSIZE			+	+	+
(4) ALLASSETCOLL	+		+	+	+
(5) INVENTRECIVECOLL	+	+	+	+	+
(6) OTHERCOLL		+	+	+	+
Recovery process characteristics					
(7) PREPACK					
(8) RESTRUCTURE					
(9) OTHERDEFAULT					
(10) TIMETOEMERGE		–	–	–	
(11) TIMETOEMERGE^2		+		+	
(12) PREPACK × TIMETOEMERGE		–	+	+	
Borrower characteristics					
(13) FIRMSIZE		–	+		
(14) FIRMPPE	+	+			
(15) FIRMCF					
(16) FIRMLEV				–	
(17) EVERDEFAULTED	+	+	+	+	+
Loan conditions					
(18) GDP				+	+
(19) INDDISTRESS		–		–	
(20) AIS		–			

portantly, the relations for some variables change from negative to positive. Finally, and consistent with Khieu et al. (2012), the Bayesian posterior distribution for the static model shows no relation between RR and the all-in-spread (AIS). However, when we allow for different economic conditions, a negative relation is indeed found between AIS and RR, but only during bad times. This finding has clear implications for planning counter-cyclical capital allocations in operational risk modelling.

We now examine these variables more closely using our dynamic model. Table 7 reports fully detailed numerical summaries of the static and dynamic model Bayesian posterior distributions. The results for each type of RR determinant grouping are discussed in detail over the next several subsections.

5.3.1. Loan characteristics

In line with Dermine and De Carvalho (2006), we find loan size to be negatively associated with RRs. Irrespective of being in a good or bad cycle, from a bank's perspective, the larger the loan amount, the less likely the bank will be able to recover subsequent to default. Larger loans are generally organized around a syndicate banking arrangement; hence, as more providers are involved, lower RRs are realized once they enter foreclosure. This finding is contrary to those of Acharya et al. (2007), as banks granting larger loans are meant to have less asymmetric information and more bargaining power during the bankruptcy process. However, it seems that as loan sizes increase and default occurs during a downturn, banks are less likely to recover their outstanding debts.

Loan type is not useful to explain RR levels in bad times, irrespective of whether the credit granted is a term loan or a revolver. During an upturn, however they can contribute to explaining RRs with respect to revolver loans. Khieu et al. (2012) find a similar significant relation and argue that since revolvers typically have a shorter duration and are therefore reviewed more often, so banks are able to reassess their clientsâ credit profiles and seek further collateral if necessary.

The literature emphasizes the importance of collateral with respect to higher RRs emanating from secured loans where more secured loans imply higher RRs (Altman and Kishore (1996), Araten et al. (2004) and Van de Castle and Keisman (1999)). Our study contributes to the literature by showing that during good times, we report similar results to those of Khieu et al. (2012), i.e., a significant positive relation between the RR and total assets used as collateral. While during bad times total assets is not significant; the dynamic model also shows that assets such as inventory, receivables and other more liquid assets do appear to be important for recovering a higher RR across both latent credit cycle states, particularly during bad times.

5.3.2. Recovery process characteristics

The existence of pre-arranged recovery processes for bankruptcy and out-of-court restructuring in the event of default-triggered failure is examined. Marginally, our dynamic model suggests that pre-packaged processes do not have a significant linear relation with RRs subsequent to default in either good or bad times. This finding, evident from Tables 6 and 7, also

Table 7
Bayesian estimates of the regression coefficients based on 100,000 retained MCMC draws (with 5000 burn-in) from each marginal posterior as indicated by the column heading. Results in the dynamic case shown in column three and four correspond to estimates conditioned on the latent credit cycle state, with $\boldsymbol{\beta_0}$ corresponding to the bad state, and $\boldsymbol{\beta_1}$ corresponding to the good state. MPM denotes the marginal posterior mean and 95% HPD (in parentheses) denotes the 95% higher posterior density interval. For the static model, the MPM of the squared shrinkage parameter is $\lambda^2 = 3.33$ the static case, whereas for the dynamic case, the (conditional) MPMs are $\lambda_0^2 = 2.98$ and $\lambda_1^2 = 2.71$, corresponding to the bad and good states, respectively.

	Static model	Dynamic model	
Standardized Parameter	β MPM (95% HPD)	β_0 (Bad) MPM (95% HPD)	β_1 (Good) MPM (95% HPD)
Loan characteristics			
(1) LOANSIZE($M)	−0.026 (−0.081, 0.030)	**-0.28** (−0.45, −0.10)	**-0.093** (−0.18, −0.00054)
(2) LOANTYPE	**-0.068** (−0.13, −0.0079)	−0.27 (−0.43, 0.11)	**-0.26** (−0.41, −0.11)
(3) LOANTYPE × FIRMSIZE	0.025 (−0.050, 0.10)	0.0098 (−0.11, 0.13)	**0.35** (0.0094,0.71)
(4) ALLASSETCOLL	**0.11** (0.032,0.19)	0.23 (−0.016, 0.48)	**0.69** (0.49,0.90)
(5) INVENTRECIVECOLL	**0.14** (0.069,0.22)	**0.67** (0.57,1.21)	**0.37** (0.24,0.50)
(6) OTHERCOLL	0.057 (−0.011, 0.13)	**0.25** (0.095,0.41)	**0.15** (0.028,0.26)
Recovery process characteristics			
(7) PREPACK	−0.0024 (−0.084, 0.080)	0.14 (−0.050, 0.34)	−0.14 (−0.34, 0.057)
(8) RESTRUCTURE	−0.016 (−0.11, 0.076)	−0.19 (−0.45, 0.07)	−0.0056 (−0.21, 0.21)
(9) OTHERDEFAULT	−0.024 (−0.077, 0.028)	−0.060 (−0.18, 0.068)	0.48 (−0.18, 1.01)
(10) TIMETOEMERGE	−0.072 (−0.20, 0.048)	**-0.65** (−1.20, −0.11)	**-0.44** (−0.86, −0.0078)
(11) TIMETOEMERGE^2	0.11 (−0.0093, 0.24)	**1.14** (0.53,1.47)	0.12 (−0.13, 0.47)
(12) PREPACK × TIMETOEMERGE	0.0064 (−0.076, 0.090)	**-0.22** (−0.39, −0.057)	**0.45** (0.16,0.81)
Borrower characteristics			
(13) FIRMSIZE	−0.022 (−0.095, 0.050)	**-0.055** (−0.16, −0.053)	**0.23** (0.079,0.39)
(14) FIRMPPE	**0.071** (0.011,0.13)	**0.52** (0.29,0.75)	−0.016 (−0.076, 0.044)
(15) FIRMCF	−0.0065 (−0.057, 0.045)	0.17 (−0.36, 0.70)	−0.17 (−0.37, 0.032)
(16) FIRMLEV *	−0.016 (−0.072, 0.038)	−0.14 (−0.34, 0.065)	−0.21 (−0.42, 0.0050)
(17) EVERDEFAULTED	**0.13** (0.041,0.22)	**0.53** (0.28,0.80)	**0.60** (0.37,0.87)
Loan conditions			
(18) GDP	−0.0072 (−0.065, 0.050)	−0.075 (−0.32, 0.18)	−0.042 (−0.25, 0.16)
(19) INDDISTRESS	−0.015 (−0.070, 0.039)	**-0.15** (−0.30, −0.0071)	0.077 (−0.027, 0.19)
(20) AIS	−0.013 (−0.070, 0.043)	**-0.28** (−0.48, −0.080)	−0.018 (−0.17, 0.13)

holds for the static model and is in line with the results of Khieu et al. (2012) who employed both OLS and QMLE methodologies in a static setting. We also note, however, that the literature also suggests companies that pre-package appear to be more financially sound (Ryan, 2008). In addition, with respect to distressed exchanges, it transpires that firms undertaking pre-packaging are more solvent at the time of re-organization than are bankrupt firms (Franks and Torous, 1994).

To reconcile this apparent contradiction of whether prepackaging is beneficial, we focus first on the the good credit state regime of the dynamic model. When underlying economic conditions are favourable, the marginal effect due to an increase in the time to emergence is negative. However, this effect is softened under prepackaged arrangements. That is, we see an overall positive effect on recovery rates due to prepackaging – but only in good times. During bad times, any impact due to prepackaging is completely outweighted by the nonlinear effect from the TIMETOEMERGE^2 variable, whose effect increases dramatically the longer resolution is delayed.

Whereas Khieu et al. (2012) report (in their Table 3) strong significance for these variables under OLS, and find at best only weak significance (at most at the 90% level) for these variables under the QMLE. We also do not find any significance under our static model. Yet, we find the nonlinear effects from delayed resolution only hold true for bad times, while in good times prepackaging has a positive effect. These results contrast to our findings for the dynamic model, and highlight the importance of accounting for the temporal variation present in bank loan RRs.

5.3.3. Borrower characteristics

The literature is not definitive on whether firm size impacts RRs. Large firms may signal higher bankruptcy costs, resulting in lower RRs. Conversely, larger firms are expected to present less information asymmetry problems to creditors, hence facilitating any restructuring process and improving recoveries from lenders. Large firms also have more negotiating power than small firms. As per Khieu et al. (2012), we do not find a significant relation between firm size and RRs with the static model. However, our dynamic model reveals a significant negative (positive) relation with RRs and firm size during bad (good) times. During bad times, the larger the firm, the greater the negative impact on RRs. In good times, this is reversed; larger firms are associated with greater RRs. This could be a sign of loan mispricing: recoverable assets are being over-valued prior to bankruptcy in bad times. Conversely, during good times, these asset values are more likely realizable and consistent with higher RRs.

The level of a firm's tangible assets, namely property, plant and equipment (FIRMPPE), is thought to be positively related to the RR (Acharya et al., 2007), that is, banks are more likely to recover outstanding loans when firms report tangible assets on their balance sheet. We find likewise, but only during bad times. (We note that Acharya et al. (2007) include bonds that are generally unsecured, in their sample.) We also find that firm cash flow and leverage are not significantly related to RRs. Like Khieu et al. (2012), as our dataset excludes bonds and focuses only on loans (which are likely to be secured by tangible assets), this contrast is not surprising. Finally, consistent with the literature, we find that prior defaults (as indicated by the variable EVERDEFAULTED) are significant and positively related to RRs in both good and bad times.

5.3.4. Loan conditions

We find no signicant relation between the prevailing Gross Domestic Product growth rate (GDP) and RRs, however this is likely due to the fact that we control for the underlying economic conditions state using a Markov switching approach. While we note that this finding differs from Khieu et al. (2012), even in the static context, we also note that our mixture model with LASSO approach is aimed at addressing the clustering of RR values along with possible redundancy in the determinants. Hence, the two approaches are, in a sense, not directly comparable. We do, however, obtain a significant negative relation between industry distress (measured by stock returns, via the INDDISTRESS variable) and RRs in bad times.

The literature suggests a negative relation between the AIS and the RR. Both Hu and Perraudin (2002) and Altman et al. (2005) report a negative association, although the former uses bond default data. Khieu et al. (2012) report no relation between the AIS and the RR. We find that AIS is significantly negatively related to RR, but only in bad times, we suggest is consistent with banks being less likely to recover under a larger AIS during bad times. This finding is partially consistent with Altman et al. (2005), although, as previously noted, that study does not distinguish between good and bad times.

5.4. Model evaluation

We evaluate the relative performance of the proposed dynamic model against the corresponding static version proposed by Altman and Kalotay (2014) by undertaking a Bayesian hypothesis test using a so-called *Bayes factor*. When two models are considered *a priori* equally likely, then the Bayes factor may be interpreted as the posterior odds, or weight of evidence, in favour of one model – here the dynamic model – against another. This standard method may be used to compare Bayesian models, including non-nested ones. The Bayes factor itself is calculated as the ratio of average likelihood values that result from each of the dynamic amd static models, respectively, where the averaging occurs with respect to the corresponding prior distribution. To ensure a well-defined Bayes factor, the prior distributions must be *proper*[7] meaning that the prior distributions each integrate to one. Using the data over the entire sample period, we calculate a Bayes factor of $3.87e+14$, corresponding to posterior log-odds[8] of 33.59 in favour of the dynamic model. As a log-Bayes factor greater than about 5 is commonly viewed as *very strong* evidence, our value of 33.59 represents very strong evidence in favour of the dynamic model, based on the available sample. For a full discussion of Bayes Factors, see Kass and Raftery (1995).

Noting that the logarithm of a product of factors is equal to the sum of logarithms of the individual factors themselves, the log-Bayes factor can be decomposed into a sum of differences in the log-predictive likelihoods that result from each of the models, corresponding to a breakdown of predictions made over the sample period. Following Geweke and Amisano (2010), the terms in this sum may be accumulated to illustrate the periods when one model out-performs the other. Fig. 7 displays the cumulative annual differences from 1987 to 2015, corresponding to the observed sample period. As we take model the two models as *a priori* equally likely, the initial difference at the start of the sample period is equal to zero. At the end of the sample period, the calculated cumulative predictive likelihoods are equal to -3376.27 and -3409.86 for the dynamic model and the static model, respectively, resulting in the positive difference of 33.59, corresponding to the value noted above. Over the sample period, and particularly after 2002, the proposed dynamic model progressively accumulates evidence against the static one. While the Bayes factor is calculated within-sample, it nevertheless corresponds

[7] In cases where the same parameters appear in both models, this requirement can often be relaxed.

[8] We refer throughout to the natural logarithm.

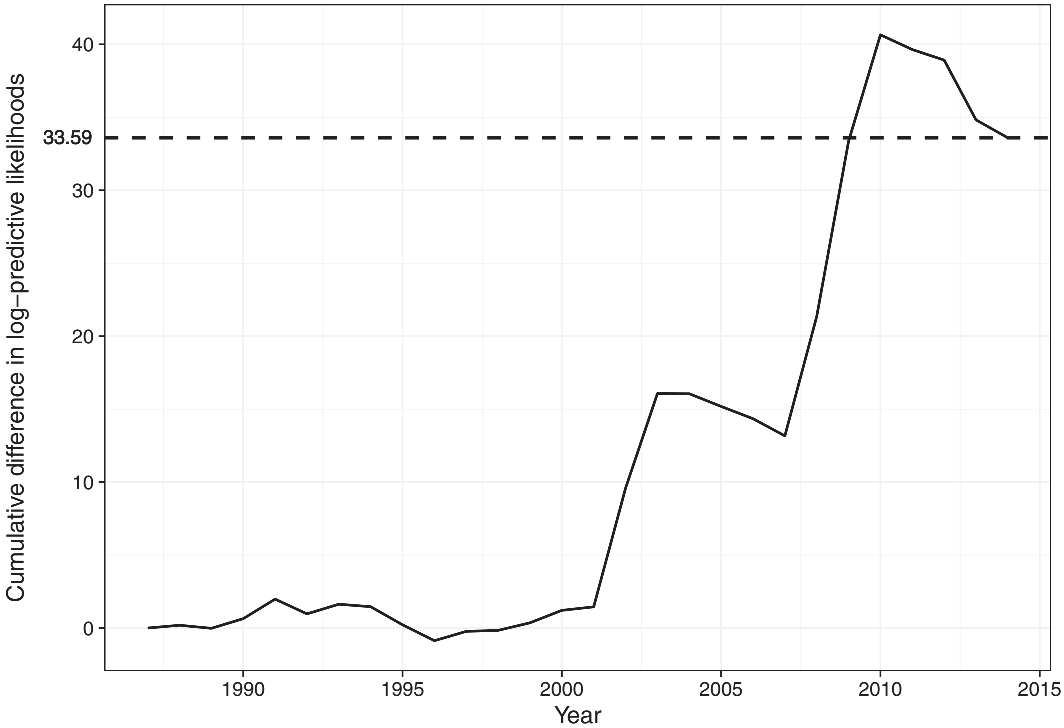

Fig. 7. Difference in cumulative log predictive likelihoods for the dynamic model against the static model. The large positive value after 2002 indicates when the dynamic model strongly outperforms the static model. The final value associated with 2015 and corresponding to the log Bayes factor is quite large, indicating strong evidence in favour of the dynamic model.

to a cumulative out-of-sample assessment. This approach is commonly used in a variety of econometric settings – see for example Maneesoonthorn et al. (2012), though there are others.

6. Discussion and conclusion

Using US bank default loan data from Moody's Ultimate Recovery Database and covering the pre- and post-GFC period, this paper develops a dynamic predictive model for RRs emanating from large bank loans. We model the distinctive features of the RR distribution, incorporating a large number of variables as possible determinants. Some of these variables features appear insignificant under a static model approach, but become important in a dynamic setting when a latent credit cycle is considered. The estimated credit cycle obtained from the dynamic model tallies with several existing indicators of financial stress and is shown, using a Bayes factor, to provide a better description of the observed RRs. Primarily, the Bayesian approach is able to handle the hierarchical specification that is built to explain the complex relation between various potential RR determinants and the empirical distribution of RRs. It is the first paper to incorporate time-series variation into the probabilistic modelling of bank loan RRs, proposing a Bayesian hierarchical framework that enables inference of a latent credit cycle. We also introduce the use of a LASSO prior to encourage the most relevant RR determinants to be found, despite potentially confounding evidence of correlation between observed RR determinants. Our temporal conditioning allows us to discriminate between good and bad states of the credit cycle and hence we report new findings. Thus, this paper contributes to the literature in different and additional ways.

The approach taken enables us to find that some loan characteristics such as those using specific types of collateral hold different explanatory power in good times and bad. We report that certain recovery process variables, such as the length of time between default and resolution for loans with pre-packaged recoveries, differ in their importance in relation to RRs, depending on the state of the credit cycle (in this case being negatively related to RR in bad times while being positively related in good times). Only a few borrower characteristics and loan conditions appear to be relevant across the cycle. The defaulting firm size and asset liquidity and tangibility can imply different relations with RRs depending on conditions. Finally, by allowing for variation in the level of the all-in-spread at the initialisation of a loan, on top of the latent dynamic cycle states, we confirm a negative relation with RR but one that is only significant during bad times.

Our results illustrate the importance of utilizing dynamic models that allow for time-varying conditions, as there is significant variation in the explanatory power of the variables analyzed depending on conditions, yielding new insights previously unavailable from the established literature. This variation in significance in variables across good and bad times occurs in several of our variables, supporting the need for a dynamic approach. Hence, our results yield vital implications for the banking sector, notably providing empirical support for the latest addition to the Basel framework concerning the importance of planning and activating counter-cyclical capital buffers during economic downturns. Applying such a buffer would not only enable banks to absorb increased losses but would also assist in achieving the broad macro-prudential goals of protecting the banking sector in periods of excess aggregate credit growth, and from the build-up of system-wide risk.

The notion that RR is driven by a systemic risk component that becomes more pronounced during bad times is evident from the results reported in our dynamic model. Loan size and type are also critical features, especially during bad times. Such features need to be priced within the cost of financing, as some banks are less likely to recover loans when the economy is entering a downturn. These important differential impacts, during bad and good times, suggests that RRs have a large element of systemic risk that needs to be factored in during the pricing of loan finance contracts. As RRs are an integral part of credit risk, this aspect should attract an additional risk premium allowing for a differentiated credit risk exposure.

Under the new regulatory regime, banks are required to provide more timely and forward-looking information. It is no longer necessary for a credit event to have occurred before a potential credit loss is recognized. This paradigm shift is in being cognizant of the credit cycle and to update the banks' loan loss provision in line with their recovery rate expectations. In summary, we find several variables are important for explaining RRs, depending on the state of the credit cycle. This has major implications for the counter-cyclicality of regulatory capital and operational risk management. The potential risk of not allowing for economic conditions in addressing such factors will result in either underestimating the relevant credit risk, or overestimating it. Both of these eventualities could potentially result in negative consequences, such as more expensive loans and under or over provision of equity capital. This in turn would result in desirable customers leaving to access financing at cheaper rates from alternative institutions more effective in correctly pricing loans through the pro-cyclical process.

Declaration of Competing Interest

The authors declare that they do not have any financial or nonfinancial conflict of interests

Appendix A. Implementation details for Bayesian analysis

The model detailed in Section 4 provides a characterization of the distribution of the observed RRs via a predictive regression of the RR mixture component on a large collection of RR determinants, with the regression coefficients in turn dependent upon the current state of an underlying credit cycle state variable. In the static regression setting, calculation of the posterior distribution via MCMC follows the approach of Altman and Kalotay (2014), who in turn rely upon the methodology details provided in Albert and Chib (1993). Our implementation here is similar, including in the dynamic case where we include the additional hierarchical layer containing the Markov switching variables, except for the use of the alternative LASSO prior specification on the latent RR regression parameter coefficient vector(s).

A.1. Likelihood function

The relevant likelihood for Bayesian analysis is the joint probability density function (pdf) of the complete set of measurements, denoted by $\mathbf{y} = (y_1, y_2, \ldots, y_n)$, together with the latent Markov switching state variables, $\mathbf{S} = (S_1, S_2, \ldots, S_T)$, all conditional upon the collection of parameters, $(\mu, \boldsymbol{\sigma^2}, \boldsymbol{\beta_0}, \boldsymbol{\beta_1})$. Unfortunately, even if the sequence of latent credit state variables, $\mathbf{S}$, were known, calculation of the likelihood function is not available in closed form, and consequently the Bayesian posterior is also not available. However, owing to the relation between the Gaussian mixture model for each y_i, the cut-points $\mathbf{c}$ and the latent predictive regression in (2), we can express the joint pdf of $\mathbf{y}$ and $\mathbf{z}$ conditional on $\mathbf{S}$, given by the product of

$$p(\mathbf{y}, \mathbf{z}, \mathbf{z}^* | \mathbf{S}, \boldsymbol{\psi}, \mathbf{x}) \propto \prod_{i=1}^{n} \prod_{j=1}^{J} \frac{1}{\sigma_j} \phi\left(\frac{y_i - \mu_j}{\sigma_j}\right) \times \mathbb{I}(c_{j-1} < z_i \le c_j) \times \mathbb{I}(z_i^* = j) \times \phi(z_i - \mathbf{x}_i'((1 - S_{t_i})\boldsymbol{\beta_0} + S_{t_i}\boldsymbol{\beta_1})), \tag{5}$$

where $\phi(\cdots)$ denotes the pdf of the standard normal distribution, $\mathbb{I}(\cdot)$ is the indicator function so that $\mathbb{I}(A) = 1$ if event A is true and is equal to zero otherwise, $\boldsymbol{\psi} = (\boldsymbol{\mu}, \boldsymbol{\sigma^2}, \boldsymbol{\beta_0}, \boldsymbol{\beta_1}, \mathbf{c})$, and the joint pdf of the Markov switching states $\mathbf{S}$, given by

$$p(\mathbf{S}|\boldsymbol{\psi}) = p(S_1|p, q) \prod_{t=2}^{T} p(S_t|S_{1:t-1}, p, q), \tag{6}$$

where $p(S_t|S_{1:t-1}, p, q)$ is given in (3), and $p(S_1|p, q)$ arising from the long-run marginal probability given by

$$S_1|p, q \sim \text{Bernoulli}((1-p)/(2-p-q)). \tag{7}$$

The likelihood function is then the product of (5) and (6). Note that the factors in these equations are expressed conditionally given the parameters $(\boldsymbol{\psi}, p, q)$ and also given the regression covariates $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K]$, where $\mathbf{x}_k = (x_{k,1}, x_{k,2}, \ldots, x_{k,n})'$.

A.2. Priors

To complete a fully Bayesian analysis, we must put a (joint) prior distribution over the unknown parameters. We take these priors to be relatively diffuse, so that the data will dominate the analysis. Specifically, the prior mean μ_j and variance σ_j^2 for the jth mixture component of the RR distribution are taken as independent normal (N) and inverse gamma (IG) distributions, with

- $\mu_j \overset{ind}{\sim} N(\bar{\mu}_j \;\bar{V}_{\mu,j})$, where $\bar{\mu}_j = 0$ and $\bar{V}_{\mu,j} = 100$ for $j = 1, \ldots, J$, and
- $\sigma_j^2 \overset{ind}{\sim} IG(\bar{a}_j \;\bar{b}_j)$, where $\bar{a}_j = 3$ and $\bar{b}_j = 1$ denote the scale and shape parameters, respectively, for $j = 1, \ldots, J$.

To avoid the well known label switching problems in the finite mixture model, we impose the same identification restrictions, $\mu_1 < \cdots < \mu_J$, as in Koop et al. (2007).[9] The joint prior distribution for the cut-point vector $\mathbf{c}$ is completely diffuse, while the prior distributions for $\boldsymbol{\beta_0}$ and $\boldsymbol{\beta_1}$, corresponding to the Bayesian LASSO for the coefficients of the RR determinants under the bad and good credit cycle states, respectively, are specified hierarchically using independent scale mixture of normals for each. These are given by

- $\boldsymbol{\beta_0} \mid \sigma_\varepsilon^2, \boldsymbol{\tau_0} \sim N(\mathbf{0}_K, \sigma_\varepsilon^2 \mathbf{I}_K D_{\boldsymbol{\tau_0}})$, with $D_{\boldsymbol{\tau_0}} = diag(\tau_{0,1}, \tau_{0,2}, \ldots, \tau_{0,K})$, and
- $\boldsymbol{\beta_1} \mid \sigma_\varepsilon^2, \boldsymbol{\tau_1} \sim N(\mathbf{0}_K, \sigma_\varepsilon^2 \mathbf{I}_K D_{\boldsymbol{\tau_1}})$, with $D_{\boldsymbol{\tau_1}} = diag(\tau_{1,1}, \tau_{1,2}, \ldots, \tau_{1,K})$,

with I_K denoting the $K-$ dimensional identity matrix and the mixing variables (also known as local shrinkage parameters) given by $\boldsymbol{\tau_0} = (\tau_{0,1}, \tau_{0,2}, \ldots, \tau_{0,K})$ and $\boldsymbol{\tau_1} = (\tau_{1,1}, \tau_{1,2}, \ldots, \tau_{1,K})$ and with the variance $\sigma_\varepsilon^2 = 1$ held fixed as used in the latent ordered probit regression. In addition, following Park and Casella (2008), we use the following independent (hyper) priors

- $\tau_{0,1}, \tau_{0,2}, \ldots, \tau_{0,K} \mid \lambda_0^2 \overset{ind}{\sim} Exp(\lambda_0^2/2)$, and
- $\tau_{1,1}, \tau_{1,2}, \ldots, \tau_{1,K} \mid \lambda_1 \overset{ind}{\sim} Exp(\lambda_1^2/2)$,

where $Exp(s)$ denotes the exponential distribution with mean value $1/s$. Then, the (hyper) prior for the two global LASSO parameters λ_0^2 and λ_1^2 is given by independent distributions

- $\lambda_0^2 \sim \text{Gamma}(\bar{r}, \bar{\delta})$, and
- $\lambda_1^2 \sim \text{Gamma}(\bar{r}, \bar{\delta})$,

where $\bar{r} = 3$ and $\bar{\delta} = 1$. Finally, we have priors for the Markov switching probabilities, corresponding to the parameters in (3), given by

- $p \sim \text{Beta}(\bar{u}_{0,0}, \bar{u}_{0,1})$ and $q \sim \text{Beta}(\bar{u}_{1,0}, \bar{u}_{1,1})$,

with $\bar{u}_{0,0}$, $\bar{u}_{0,1}$, $\bar{u}_{1,0}$ and $\bar{u}_{1,1}$ all set equal to 0.5, as per the algorithm of Kim and Nelson (2001). Collectively, the joint prior distribution is specified over the entire collection of unknown parameters $\boldsymbol{\theta} = (\boldsymbol{\psi}, p, q, \boldsymbol{\tau_0}, \boldsymbol{\tau_1}, \lambda_0, \lambda_1^2)$ is given by

$$\begin{aligned} p(\boldsymbol{\theta}) = {} & p(\boldsymbol{\mu}) \; p(\boldsymbol{\sigma^2}) \; p(\mathbf{c}) \\ & \times p(\boldsymbol{\beta_0} \mid \boldsymbol{\tau_0}) \; p(\boldsymbol{\beta_1} \mid \boldsymbol{\tau_1}) \\ & \times p(\boldsymbol{\tau_0} \mid \lambda_0^2) \; p(\boldsymbol{\tau_1} \mid \lambda_1^2) \; p(p) \; p(q). \end{aligned}$$

A.3. MCMC algorithm

The marginal posterior distribution is obtained using a basic Gibbs sampling approach, where the parameters and latent variables are each drawn recursively from the relevant (full) conditional posteriors. Given the prior distribution, the gth iteration of the Gibbs sampler proceeds as follows:

- Step 1: draw the mixture indicators and the predictive scores $\mathbf{z}^{*(\mathbf{g})}, \mathbf{z}^{(\mathbf{g})} | \mathbf{y}, \boldsymbol{\mu}^{(g-1)}, \boldsymbol{\sigma}^{2(g-1)}, \mathbf{c}^{(g-1)}, \boldsymbol{\beta_0}^{(g-1)}, \boldsymbol{\beta_1}^{(g-1)}, \mathbf{c}^{(g-1)}, \mathbf{S}^{(g-1)}$
- Step 2: draw the regression parameters $\boldsymbol{\beta}_0^{(g)} | \mathbf{y}, \mathbf{z}^{(g)}, \mathbf{S}^{(g-1)}, \boldsymbol{\tau}^{(g-1)}$ and $\boldsymbol{\beta}_1^{(g)} | \mathbf{y}, \mathbf{z}^{(g)}, \mathbf{S}^{(g-1)}, \boldsymbol{\tau}^{(g-1)}$
- Step 3: draw the shrinkage parameters (via augmentation) $\lambda_0^{2(g)}, \tau_0^{(g)} \mid \boldsymbol{\beta}_0^{(g)}$ and $\lambda_1^{2(g)}, \tau_1^{(g)} \mid \boldsymbol{\beta}_1^{(g)}$

[9] For a detailed discussion of alternative solutions to the label switching problem, see Frühwirth-Schnatter (2006).

- Step 4: draw each of the J cut-points
$c_j^{(g)}|\mathbf{z}^{(g)}, \boldsymbol{c}_{/j}$
- Step 5: draw the latent Markov states
$\mathbf{S}^{(g)}|y, \mathbf{z}^{(g)}, \boldsymbol{\beta_0}^{(g)}, \boldsymbol{\beta_1}^{(g)}, p^{(g-1)}, q^{(g-1)}$
- Step 6: draw the Markov transition probabilities
$p^{(g)}, q^{(g)}|\mathbf{S}^{(g)}$
- Step 7: draw the vector of mean parameters for the Gaussian mixture distribution
$\boldsymbol{\mu}^{(g)}|y, \mathbf{z}^{(g)}, \boldsymbol{c}^{(g)}, \boldsymbol{\sigma}^{2(g-1)}$ and $\boldsymbol{\sigma}^{2(g)}|y, \mathbf{z}^{(g)}, \boldsymbol{c}^{(g)}, \mu^{(g)}$.
- Step 8: draw the vector of variance parameters for the Gaussian mixture distribution
$\boldsymbol{\sigma}^{2(g)}|y, \mathbf{z}^{(g)}, \boldsymbol{c}^{(g)}, \mu^{(g)}$.

We note that in Step 3 each shrinkage parameter, either λ_0^2 and λ_1^2 (or λ^2 in the static case), is generated by first sampling an augmentation vector, $\tau_0^{(g)}$ and $\tau_0^{(g)}$, respectively, from the corresponding distribution that conditions on the relevant previous draw of the shrinkage parameter. This approach follows as per Park and Casella (2008). The new draws of the shrinkage parameters are then sampled from the full conditional distributions that utilize the augmentation vectors, i.e. $\lambda_0^{2(g)} \sim p(\lambda_0^2 \mid \tau_0^{(g)}, \boldsymbol{\beta}_0^{(g)})$ and $\lambda_1^{2(g)} \sim p(\lambda_1^2 \mid \tau_1^{(g)}, \boldsymbol{\beta}_1^{(g)})$, respectively. The values of the $\tau_0^{(g)}$ and $\tau_1^{(g)}$ are not required for any additional part of the MCMC algorithm and may be discarded at the end of each iteration. We also note for Step 4 that $\mathbf{c}_{/j} = \{c_0, c_1, \ldots, c_{j-1}, c_{j+1}, \ldots, c_J\}$, denoting the vector $\mathbf{c}$ but with the jth element excluded. Since the priors are conditionally conjugate for all unknowns, the relevant conditional posterior distributions are all derived analytically, ensuring a fast algorithm for sampling from the full joint posterior distribution.

Mixture indicator vector z^* and the latent predictive score vector z

We jointly sample the $\boldsymbol{z}^*$ and $\boldsymbol{z}$ vectors, conditionally on all other variables and the data, by first sampling the vector of mixture indicator variables, $\boldsymbol{z}^*$, *marginal* of the predictive score vector $\boldsymbol{z}$, before subsequently sampling $\boldsymbol{z}$ itself. The mixture indicator variable components $z_1^*, \ldots, z_N^*$ are conditionally independent and hence are sampled independently from multinomial distributions. In particular, the probability that z_i^* is assigned to mixture j is given by

$$w_{i,j} = \frac{\left[\Phi(\mathbf{x}_i'\beta_{S_{t_i}} - c_{j-1}) - \Phi(\mathbf{x}_i'\beta_{S_{t_i}} - c_j)\right] \phi(y_i; \mu_j, \sigma_j^2)}{\sum_{j=1}^J \left[\Phi(\mathbf{x}_i'\beta_{S_{t_i}} - c_{j-1}) - \Phi(\mathbf{x}_i'\beta_{S_{t_i}} - c_j)\right] \phi(y_i; \mu_j, \sigma_j^2)},$$

for each $j = 1, \ldots, J$, where $\Phi(\ \cdot\ )$ denotes the cdf of a standard normal random variable. These probabilities are derived from integrating (5) over $\boldsymbol{z}$ and then normalising, with $\sum_{j=1}^J w_{i,j} = 1$ for every $i = 1, \ldots, n$.

Proceeding next to sampling $\boldsymbol{z}$, we note that for every i (and conditional on the relevant latent credit state coefficient $\boldsymbol{\beta}_{\mathbf{S}_{\mathbf{t_i}}}$ corresponding to the latent credit state S_{t_i} at the time of default for observation i, the full data vector $\mathbf{y}$ and other parameters), the predictive scores, z_i for $i = 1, 2, \ldots, N$ are independent, and z_i has a truncated normal distribution with

$$p(z_i|\mathbf{c}, \beta_0, \beta_1, z_i^*) \sim \mathcal{TN}_{(c_{z_i^*-1}, c_{z_i^*})}(\mathbf{x}_i'\beta_{S_{t_i}}, 1),$$

where $c_{z_{i-1}^*}$ and $c_{z_i^*}$ are the lower and upper bound parameters, respectively. This result again follows directly from (5).

State-dependent regression coefficients β_0 and β_1

The latent data $\mathbf{z}$ and recovery determinants $\boldsymbol{x}$ are divided into $\mathbf{Z}_0$, $\mathbf{Z}_1$ and $\boldsymbol{X}_0 = [\boldsymbol{x}_{1,0}, \boldsymbol{x}_{2,0}, \ldots, \boldsymbol{x}_{k,0}]$, $\boldsymbol{X}_1 = [\boldsymbol{x}_{1,1}, \boldsymbol{x}_{2,1}, \ldots, \boldsymbol{x}_{k,1}]$ respectively, according the latent Markov state S_{t_i}. Given the data, $\mathbf{y}$ and $\mathbf{z}$, Markov states, S_{t_i}, and the local shrinkage parameter, $\tau_0^2 = (\tau_{1,0}, \ldots, \tau_{k,0})$ and $\tau_1^2 = (\tau_{1,1}, \ldots, \tau_{k,1})$, the conditional posterior distribution for $\beta_{S_{t_i}} = (\beta_0, \beta_1)'$ is given by

$$p(\beta_{S_{t_i}}|\mathbf{y}, \mathbf{z}, \mathbf{S}, \tau) \sim \mathcal{N}(D_{S_{t_i}} d_{S_{t_i}}, D_{S_{t_i}}),$$

where

$$D_{S_{t_i}} = (\mathbf{x}_{S_{t_i}}' \mathbf{x}_{S_{t_i}} + \mathrm{diag}(\tau^2)^{-1})^{-1} \text{ and } d_{S_{t_i}} = \mathbf{x}_{S_{t_i}}' y_{S_{t_i}}.$$

Shrinkage parameters τ^2 and λ^2

For each credit state, the local shrinkage parameters $\tau_1^2, \ldots, \tau_k^2$ are conditionally independent, with

$$p(1/\tau_j^2|\beta_j, \lambda^2) \sim InvGaussian(\bar{\bar{\mu}}_j, \lambda^2),$$

where *InvGaussian* denotes an Inverse Gaussian distribution and

$$\bar{\bar{\mu}}_j = \sqrt{\frac{\lambda^2}{\beta_j^2}},$$

for $j = 1, \ldots, k$. With a conjugate prior, the full conditional distribution of λ^2 is given by

$$p(\lambda^2|\tau^2) \sim \Gamma(\bar{\bar{r}}, \bar{\bar{\delta}}),$$

where Γ denotes a gamma distribution with shape parameter $K + \bar{r}$ and rate parameter $\sum_{j=1}^{K} \tau_j^2/2 + \bar{\delta}$.

Cut-points c

We follow Albert and Chib (1993) and use diffuse priors for all cut-points $c_2, \ldots, c_{J-1}$. For identification purposes, we set $c_0 = -\infty$, $c_1 = 0$ and $c_J = \infty$ as it is common in any other studies using an ordered probit model. The joint conditional posterior for the cut-points $j = 2, \ldots, J-1$ (recall that $c_0 = -\infty, c_1 = 0$, and $c_J = \infty$) is given by,

$$p(c_j|c_{/j}, \mathbf{z}, \mathbf{z}^*) \sim \mathcal{U}(l_j, u_j),$$

where $\mathcal{U}(l_j, u_j)$ denotes a uniform distribution with

$$l_j = \max\{c_{j-1}, \max\{z_i : z_i^* = j\}\},$$
$$u_j = \min\{c_{j+1}, \min\{z_i : z_i^* = j+1\}\}.$$

The latent Markov states S

We use the efficient block sampling algorithm of Carter and Kohn (1994) and Frühwirth-Schnatter (1994) to generate **S**, which is known as *forward filtering, backward sampling* (FFBS). Hamilton (1989) provides the following filtering algorithm to calculate the filtered probabilities for **S**. Let z_t be vector contains all z_i observed in year t, the Hamilton filter consists of, for $t = 1, \ldots, T$,

- predict

$$\begin{bmatrix} \Pr(S_t = 0|z_t) \\ \Pr(S_t = 1|z_t) \end{bmatrix} = \begin{bmatrix} p & 1-q \\ 1-p & q \end{bmatrix} \begin{bmatrix} \Pr(S_t = 0|z_{t-1}) \\ \Pr(S_t = 1|z_{t-1}) \end{bmatrix},$$

- update

$$\Pr(S_t = 0|z_t) \propto p(z_t|S_t = 0) \Pr(S_t = 0|z_t) \text{ and}$$
$$\Pr(S_t = 1|z_t) \propto p(z_t|S_t = 1) \Pr(S_t = 1|z_t),$$

where

$$p(z_t|S_t = 0) = \prod_{i=1}^{n_t} p(z_{i_t}|S_t = 0) \text{ and}$$
$$p(z_t|S_t = 1) = \prod_{i=1}^{n_t} p(z_{i_t}|S_t = 1).$$

The latent Markov states are then simulated sequentially, for $t = T, T-1, \ldots, 1$. Given S_{t+1}, the parameter in the Bernoulli distribution for each t is calculated by

$$\Pr(S_t = 1|z_{1:T})/(\Pr(S_t = 0|z_{1:T}) + \Pr(S_t = 1|z_{1:T})),$$

where

$$\Pr(S_t = 0|z_{1:T}) \propto \Pr(S_t = 0|z_t) \Pr(S_t = 0|S_{t+1}) \text{ and}$$
$$\Pr(S_t = 1|z_{1:T}) \propto \Pr(S_t = 1|z_t) \Pr(S_t = 1|S_{t+1}).$$

The Markov transition probabilities p and q

Conditional on **S**, the transition probabilities, p and q are independent of the data. Since we have assigned Beta prior distributions to the transition probabilities, the conditional posterior distributions are given by

$$p(p, q|\mathbf{S}) \propto p(p, q) p(\mathbf{S} \mid p, q),$$

where $p(\mathbf{S}|p, q)$ describes the joint probabilities associated with the latent Markov-switching states. Prior independence (of p and q) implies posterior independence here, and hence p and q may be jointly sampled according to

$$p|\mathbf{S} \sim \mathcal{B}(\bar{u}_{0,0} + n_{0,0}, \bar{u}_{0,1} + n_{0,1}) \text{ and}$$
$$q|\mathbf{S} \sim \mathcal{B}(\bar{u}_{1,1} + n_{1,1}, \bar{u}_{1,0} + n_{1,0}),$$

where $\mathcal{B}(a, b)$ denotes the Beta distribution on (0,1), having mean $\frac{a}{a+b}$ and variance $\frac{ab}{(a+b)^2(a+b+1)}$, here with

$$n_{1,0} = \sum_{t=1}^{T}\sum_{i=1}^{n_t} S_{t_i}|S_{t-1} = 0,$$

$$n_{1,1} = \sum_{t=1}^{T}\sum_{i=1}^{n_t} S_{t_i}|S_{t-1} = 1,$$

and $n_{0,0} = n_{S_t=0} - n_{1,0}$ and $n_{0,1} = n_{S_t=1} - n_{1,1}$.

The Gaussian mixture means μ

Given the independent conjugate priors, the individual μ_j, for $j = 1, \ldots, J$, may be sampled independently from

$$\mu_j|\mathbf{y}, \mathbf{z}, \sigma_j^2 \sim \mathcal{TN}_{(\mu_{j-1},\mu_{j+1})}(D_{\mu_j} d_{\mu_j}, D_{\mu_j}),$$

where

$$D_{\mu_j} = \left(\sum_{i=1}^{n} \mathbb{I}(z_i^* = j)/\sigma_j^2 + \bar{V}_{\mu_j}^{-1}\right)^{-1},$$

and

$$d_{\mu_j} = \sum_{i=1}^{n} \mathbb{I}(z_i^* = j)y_i/\sigma_j^2 + \bar{V}_{\mu_j}\bar{\mu}_j.$$

The Gaussian mixture variances σ^2

The individual variance parameters σ_j^2 for mixture components $j = 1, \ldots, J$, are sampled independently conditional on μ_j and $\mathbf{z^*}$ given by

$$p(\sigma_j^2|\mathbf{y}, \mathbf{z}^*, \mu_j) \sim IG(\bar{\bar{a}}_j, \bar{\bar{b}}_j),$$

with

$$\bar{\bar{a}}_j = \frac{\sum_{i=1}^{n} \mathbb{I}(z_i^* = j)}{2} + \bar{a}_j,$$

and

$$\bar{\bar{b}}_j = \bar{b}_j^{-1} + \frac{1}{2}\sum_{i=1}^{n} \mathbb{I}(z_i^* = j)(y_i - \mu_j)^2.$$

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