Generalized Automatic Surface Reconstruction for CEM Simulations

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***Abstract*—We present a robust method for automatic high-quality reconstruction and refinement of surface meshes for application in both traditional and ultra-high-order computational electromagnetics (CEM) tools. The generation of optimal surface meshes, especially for higher-order geometries, demonstrates significant promise to improve the accuracy and efficiency in CEM.**

***Keywords*—adaptive refinement, automatic surface meshing, computational electromagnetics, higher-order methods, Ricci-Flow, optimization, surface integral equations, surface reconstruction**

I. Introduction

Geometrically higher-order meshes, while more efficient than traditional, first-order modeling schemes, present significant challenges to general applicability due to the complexity in generating quality meshes. Instead, applications of new surface integral equation (SIE) techniques rely on semi-manual meshing strategies, a time-consuming and impractical process.

Though often neglected in computational electromagnetics (CEM) research, the geometric discretization problem holds incredible potential to increase accuracy and reduce computational expenses. Unnecessarily fine geometric discretization, for instance, when applied to the method of moments in the surface integral equation formulation generates an equivalently inefficient and large linear system. Different element types as well, such as quadrilateral, provide unique benefits yet incur new challenges related to geometric constraints. The corner angles in quadrilateral meshing, for example, commonly neglected by existing techniques, heavily influences the accuracy of integration procedures [1]-[2].

The success of higher-order methods particularly relies on proper element construction to maximize orthogonality of the local basis functions [3]-[4]. By optimizing and enforcing high-quality surfaces – even from low quality parent surfaces – the accuracy and efficiency of SIE solvers can see dramatic improvements [5].

II. Discrete Surface Ricci-Flow

As opposed to traditional surface discretization methods, parametrization-based approaches offer significant advantages in full reconstruction and refining of low-quality meshes for optimal performance in CEM application, and for conversion to a variety of element types and orders extremely inexpensively.

While higher-order elements and different element types – like quadrilaterals – have (and continue) to gain in popularity in numerical methods, it is also imperative for a general discretization tool to produce optimal first-order (flat) and triangular elements. By leveraging the discrete surface Ricci-Flow (DSRF), an initial (potentially low-quality) triangle parent mesh can be used to generate these new high-quality meshes of arbitrary construction extremely efficiently.

DSRF assists in the generation of conformal map from a parent surface to its parametric representation implied by its Euler characteristic and which satisfies the Gauss-Bonnet theorem [6]-[7]. By approximating the conformal deformation in the Ricci-Flow procedure with a specified circle-packing metric, this parametric surface can be computed for extremely inexpensive full reconstruction and mesh refinement.

Figs. 1 and 2 show examples of DSRF generated triangular and quadrilateral SIE meshes of a fighter jet and a NASA almond, of extremely high quality.

The conformality of the mapping preserves corner angles constructed on the parametric surface, enhancing and

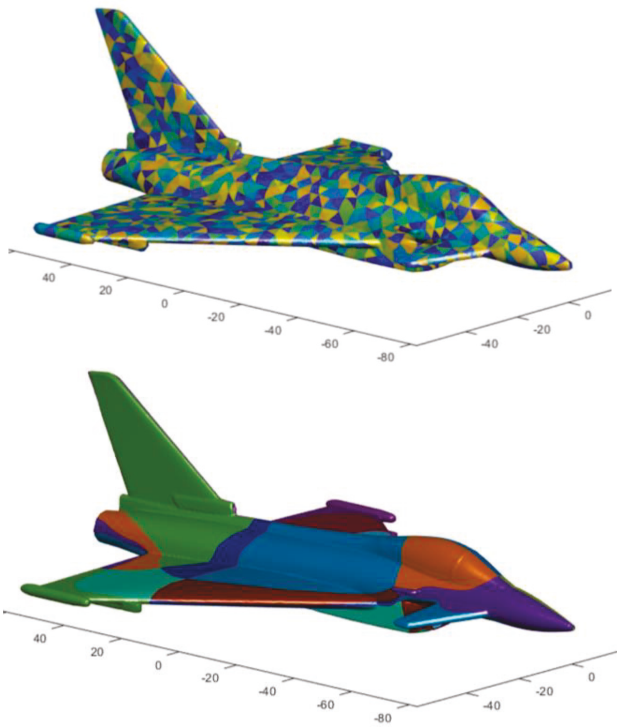

Fig. 1. DSRF generated jet meshes. (Top) 1801 10th order triangles, (Bottom) 16 64th order quadrilaterals.

guaranteeing many advantageous qualities in the reconstruction, such as local orthogonality of the basis functions, while leaving the macroscopic structure of the surface undistorted.

The same parametric realization of an arbitrary surface can produce high-quality discretizations, as seen in Figs. 1 and 2, including for geometrically extremely high-order elements, and continuous and discontinuous topologies.

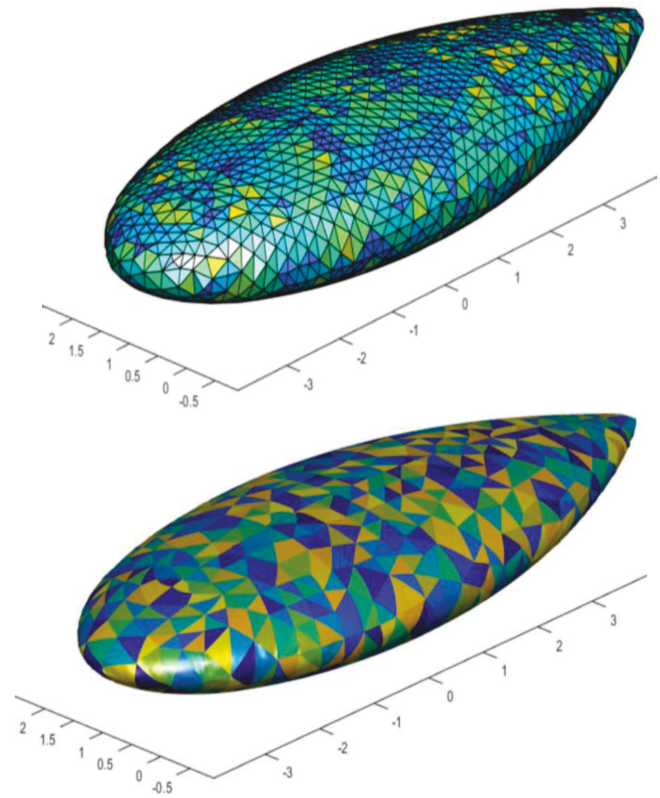

Fig. 2. DSRF generated NASA almond meshes. (Top) 1879 1^{st} order triangles, (Bottom) 549 5^{th} order triangles.

Both the fine and sharp features of both geometries are accurately preserved while producing high-quality individual elements. As seen in the two jet models in Fig. 1, the complicated characteristics of the surface, such as the rear fin and wings are perfectly preserved in the higher-order triangle and higher-order quadrilateral discretizations. Likewise for the NASA almond in Fig. 2, the sharp end is well preserved automatically for sufficient sampling density and with minimal computational effort. Mathematical guarantees provided by the DSRF process ensure this element quality, corner angle uniformity, and local orthogonality of basis functions.

The low computational cost associated with reconstructing surfaces from a precomputed map also permits efficient and effective automated refinement and optimization based on characteristics of the surface discretization, such as element sizing or density, and characteristics of the solution. Instead of a uniform grid sampling in the parametric domain – which produces inhomogeneous element sizing for surfaces with non-uniform curvature – adaptively refining the sampling density helps to optimize element size for a user-defined number of elements with respect to constraints on the topology, element type, and element orders. Tailoring a given discretization for optimal scattered field computation in a specific direction, for example, is also extremely simple and efficient.

Moreover, the ease of adaptive refinement with DSRF eliminates semi-manual mesh adjustment, or expert user intervention in the generation of optimal surface discretization, extending therefore the practicality of geometrically higher-order methods especially.

By coupling the geometric discretization and CEM problems with DSRF iterative adaptive refinement, error reduction and efficiency improvement strategies can be conducted automatically and without significant computational expense.

III. Conclusions

DSRF-based reconstruction strategies present a robust and fully automated method to generalized surface reconstruction for optimal CEM simulations. Whether desiring flat triangles, or extremely high-order quadrilateral elements, DSRF provides key mathematical guarantees in the construction of high-quality elements. Further, the low computational cost associated with mapping between a surface and its parametric representation provides the ability to conduct efficient iterative adaptive refinement. Such refinement can effectively target issues or concerns with qualities of surface, and in the accuracy of the CEM solution which allows a direct coupling of the discretization and CEM problem.

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