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Editorial

Timely and Useful Data to Improve Classroom Instruction

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Over the past several decades, educators have become increasingly intent on using data to inform decision-making at all levels of the educational system (Cho & Wayman, 2014; Mandinach, 2012; Means et al., 2010). The underlying reasoning is sound: Better decisions can be made with relevant data. Policymakers have reasoned that instructional decisions made by teachers that are based on data relevant to the classroom will help students to learn and achieve more. Indeed, from the introduction of No Child Left Behind (NCLB) to the current policies of the Every Student Succeeds Act (ESSA), the emphasis on using data to improve instructional decision-making has increased. However, despite increasing pressure from multiple levels for more data-informed instructional decisions, teachers often lack the kinds of timely student data that would help them make better decisions (Bill & Melinda Gates Foundation, 2015; Tsai & Tosh, 2019). There is still a great deal for educators and policymakers to learn about the nature of the data that can be used by teachers to inform their instructional planning and implementation.

In this editorial, we address the fourth of five overarching problems that we believe the field must address to make future progress on the teaching and learning of mathematics: Researchers and educators need to ensure that teachers have access to data that help them make better decisions when planning, implementing, evaluating, and improving instruction. In our previous editorials (Cai et al., 2020, 2020a, 2020b), we suggested that the daily classroom lesson might be a useful grain size for tackling the first three problems that we have discussed. Similarly, in this editorial, we consider what kinds of data teachers need to plan, implement, and improve the mathematics lessons that they teach. We also refer to the teacher–researcher partnerships that we have described in earlier editorials (e.g., Cai et al., 2018b) to define the setting in which teachers and researchers work together to access and test the usefulness of data. The major questions that guide our discussion in this editorial are as follows: What kinds of data do teachers need to make more informed instructional decisions and improve student learning? How should these data be collected and processed? In what form should these data be stored, packaged, and presented to be most useful?

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What Kinds of Data Do Teachers Need?

What Data Have Teachers Already Been Using?

Before considering the data that teachers need, we must consider the data that they have already been using. Teachers have always used classroom data to inform their instructional decisions (Rothkopf, 2009). Student work and grades on assignments and tests, students' solution strategies that are visible during the classroom lesson, students' contributions to classroom discourse, and students' expressions of affect are all part of the daily stream of data that teachers can draw from as they decide what to do next in their classrooms. These data possess many characteristics that teachers find useful (Daly, 2012; Hiebert et al., 2002; Leinhardt, 1990). For example, these data are tightly connected with practice. They arise from learning activities and interactions that constitute the implementation of a daily lesson. The data are also concrete and specific. They reveal how students respond to particular instructional activities and how they think about particular mathematical problems. However, these data are often gathered informally and unsystematically, and they are idiosyncratic to each individual teacher. These data are typically no longer considered once teachers move to the next lesson or unit, and, even when not lost, they are not stored in forms that allow them to be used in the future or shared with other teachers.

Mathematics education researchers have gathered data for teacher use in a more formal and systematic way to provide insights into students' conceptions and misconceptions and to describe learning trajectories for some mathematical topics (Lobato & Walters, 2017). A noteworthy example is Cognitively Guided Instruction (CGI; Carpenter et al., 1996, 1998), an extensive, long-term effort to gather, organize, and make accessible to teachers a comprehensive map of young children's strategies for solving early arithmetic problems. These data are used by teachers to understand the development of their students' thinking about, and strategies for, arithmetic problem solving. This knowledge can also act as a framework for teachers to make sense of the data they themselves collect from observing their own students and can be used as a rationale for making further instructional decisions.

Another example of researchers gathering and analyzing data to enable better instructional decisions by teachers is the long-term development of Realistic Mathematics Education (RME; Gravemeijer, 2004; Streefland, 1991). RME is based on several core principles, including that instruction should begin with problems that are experientially real to students and that students' learning of mathematics involves progressive mathematization in which they build up levels of understanding by formalizing their informal strategies and representations (van den Heuvel-Panhuizen, 2003). In this case, researchers have followed RME design principles to transform data on students' thinking about several specific mathematics topics into local instructional theories and curriculum materials that aimed to put data into the hands of the teacher in a form that is codified for all teachers (van den Heuvel-Panhuizen, 2020).

Research programs of the type and scale represented by CGI and RME are not common because they require significant resources and they depend on gathering data that are useful for teachers and that generalize across large populations of students. Finding developmental trajectories of students' solution strategies, like those identified for early arithmetic problems, is much more challenging as the students get older and the mathematics becomes more complex. Creating a curriculum based on students' thinking faces similar challenges, and its effectiveness depends on constant evaluation and revision (Gravemeijer & van Eerde, 2009), a process that requires a steady stream of significant resources.

Because there are few research programs like those described above, the expectation is for teachers to use data that are readily available. In particular, ubiquitous standardized achievement data are often the focus of discussions on data-driven decision-making (Bill & Melinda Gates Foundation, 2015). Moreover, the accountability demands of NCLB required standardized test data to become a key part of data-driven decision-making (Hamilton et al., 2009). However, much of the standardized achievement data that are currently available cannot be easily used by teachers to make instructional decisions. For example, it is impossible for teachers to review end-of-the-year summary achievement data and devise ways of improving their instruction. These data are too late to help teachers plan, enact, and evaluate daily lessons, and they are often aggregated, which precludes identifying specific types and levels of student understanding. Another kind of data that teachers are often asked to use come from observations of teachers' teaching, often conducted for evaluation purposes (Cohen & Goldhaber, 2016; Whitehurst et al., 2014). These data can be difficult for many teachers to use because they do not provide the concrete, detailed information that teachers need about their students' thinking to plan and enact more effective lessons. Under current conditions in the United States, it is simply unfair and unproductive to ask all teachers to make data-driven instructional decisions.

What Kinds of Data Do Teachers Need?

What types of data are not readily available to most teachers but could play an important role in their instructional decision-making? Mandinach and Jackson (2012) noted that teachers could use many kinds of data for instructional decision-making. In addition to assessment data, these include students' responses to classroom tasks, activities, and projects, as well as data not specific to the classroom, such as data related to demographics, behavior, medical status, language, and special education status. In fact, by relaxing the requirement to base decisions on formal randomized controlled studies, ESSA seems to have opened a door for basing decisions on more varied types of data. A significant problem for mathematics education researchers is to understand which of these types of data are relevant and useful for teachers to make informed instructional decisions.

In this editorial, we constrain our search by focusing on data that can help teachers plan, implement, evaluate, and improve instructional lessons. In addition, we value data that possess the characteristics that teachers naturally find useful—data tied to practice and data that are concrete and specific (Bill & Melinda Gates Foundation, 2015; Clandinin & Connelly, 1991; Hiebert et al., 2002; Leinhardt, 1990). A final parameter that we apply is that data on student thinking have proven especially valuable when these data enable teachers to anticipate how their students might respond to instructional tasks, which, in turn, helps teachers plan and implement lessons (Stein et al., 2008; Supovitz, 2012). These parameters help to focus the questions that we propose for future research.

One question is this: What kinds of data do teachers need to plan lessons versus implement lessons versus evaluate and improve lessons? We suspect that somewhat different data might be more useful for some purposes than others. For example, data about past performance of the students currently in a teacher's class might be more useful for planning a lesson because they help the teacher know what the students' understandings going into the lesson are like. However, notes from other teachers who have taught a similar lesson might be more useful for implementing the lesson because they provide information on the breadth of possible student responses to the tasks in the lesson. Data gathered during a lesson on how different students respond to each instructional activity and on how different students solve each mathematical problem could give teachers rich and timely information on the effectiveness of each activity enacted during the lesson. This knowledge could be used to plan the next day's lesson and to improve the lesson for the next time it is taught.

A second question probes further into the kinds of data that best help teachers plan lessons. In what ways do longitudinal data on individual students' mathematical development or on groups of students with similar profiles (perhaps with respect to a particular mathematical topic or learning trajectory) help teachers plan lessons for specific topics? If teachers had information about their students' recent history with a specific topic—such as how their students thought about a topic, what misconceptions they had entering the lesson, and what aspects of the topic they understood well—could teachers use this information to plan a more effective lesson? Suppose teachers had data from their students' histories on the solution strategies they used for particular mathematical topics and problems. These data could be useful because solution strategies reveal students' thinking, and creating a record of students' strategies could help teachers predict how their students might think about problems presented during a new lesson. At some level of aggregation across classes or time, this kind of data overlaps with professional knowledge for teaching that we have discussed in previous editorials as the foundation for long-term improvement in mathematics teaching (Cai et al., 2020a). This includes knowledge about how students in a given class are likely to respond to particular aspects of instructional tasks plus knowledge of patterns that are observed across classes.

A third question is this: What data do teachers (or researchers) need on a broader scale to systematically improve instruction beyond one or two classrooms? That is, what data do teacher–researcher partnerships need to benefit from what other partnerships have learned about planning, implementing, and improving their own lessons? That is, what data could one partnership collect that would be most useful to other partnerships? Because teachers are fully engaged in teaching their own classes, researchers would likely need to take the lead on collecting these data. In addition, individual teachers often have little time for planning, reflecting, or revising lessons, so they are not inclined to worry about what is happening in other classrooms. What data could be gathered across teachers and classrooms to help more teachers benefit from what others have learned to plan and implement better lessons for their students?

In our July 2018 editorial (Cai et al., 2018c), we discussed three categories of data that teacher–researcher partnerships could gather to improve mathematics teaching and learning at the lesson level: in-the-moment data, short-term data, and long-term data. This categorization provides another way of asking many of the same questions that we posed above. For example, in-the-moment data are those data likely to be most useful for implementing the lesson, and short-term data are useful for planning the next lessons on the basis of students' responses in the current lesson. We have included the table from the July 2018 editorial here for convenience. Although these data seem to us to be likely candidates for the data that teachers need, they provide only a possible beginning point. The questions posed above must be answered to understand whether and how these or other data can be used most effectively to improve instructional decisions.

How Can the Data That Teachers Need Be Collected and Processed?

The questions that we posed in the previous section about the data that teachers need to plan, implement, and revise lessons are worth answering only if we can imagine and develop ways of gathering these data. Without knowing the final answers to the earlier questions, it is still quite clear that gathering data like these will require new ways of working. Teachers are too busy dealing with the demands of teaching to launch new data collection efforts. Researchers are often too disconnected from the classroom to see the kinds of context-rich and concrete data that teachers are likely to need. In previous editorials, we have discussed a potential shift in the dominant research paradigm, one that would partner teachers and researchers to address instructional problems. These partnerships could provide the structure that enables the joint activity likely required to gather the relevant data like those described above. However, an important question is What methods of collaboration would allow researchers and teachers to gather and process data to improve instruction?

It is possible that data across teacher–researcher partnerships could be pooled to create a rich source of information that could be accessed by a network of partnerships. Perhaps the collection of these data could be considered a form of crowdsourcing. Teachers and researchers could harvest the best ideas for their setting by looking at the data across many classrooms. This might be especially useful for building a repertoire of possible student responses to particular instructional tasks. This, in turn, would allow teachers to anticipate how their students might respond and then plan for instruction accordingly. Although it is possible to imagine the benefits of these large sets of data, it is not easy to imagine exactly how they could be collected. One possibility is using technologies that currently exist to gather and mine large datasets (Bill & Melinda Gates Foundation, 2015; Chen et al., 2005; Hamilton et al., 2009; Wayman, 2005b). Technological tools, appropriately deployed, have the potential to alleviate some of the burden of gathering and processing data. Multiple technological tools are already available, including software that monitors student progress, data dashboards, web-based tutoring tools, classroom response systems (such as clickers), student data portals, social-network tools, and software for analyzing and visually displaying data.

What technologies could help partnerships collect data at a scale that teachers would find useful? How could available technologies be adapted to collect data at a level of detail that teachers need and at a large enough scale to detect patterns currently masked by the limitations of small samples (and the human mind)? We will explore aspects of this question in the next editorial when we take up the fifth overarching problem that we propose—the nature and purpose of technologies that could enhance mathematics education experiences for all students.

We conclude this section by returning to a process that many teachers often use naturally and informally to collect data—formative assessment. A great deal has been written about formative assessment, largely as a method for gathering data that teachers can use to adjust their instruction while they are teaching (Wiliam, 2007/2008). More recently, rapidly developing technologies for analyzing human actions have provided tools that could potentially be used to help teachers quickly assess student thinking and understanding in real time (Silver & Mills, 2018). These tools could be deployed during lessons to monitor small-group discussion, analyze student work, and even gauge students' affect.

In the terms that we have used, formative assessment data are usually used to implement lessons. Formative assessments could also be used to gather data to test hypotheses that are built into the plan for the lesson and that could be preserved to evaluate and improve the lesson after instruction. Teacher–researcher partnerships could plan in advance the data that they want to collect during lessons so that teachers are less burdened with creating spontaneous assessments while they are teaching (see, e.g., Herbel-Eisenmann & Cirillo, 2009, as an example of team decision-making around data collection). Although formative assessments often yield data that are tied to practice and specific to a teacher's needs, they are often used idiosyncratically and unsystematically. An important question is How can formative assessments be designed and administered to collect reliable data that can be used by individual teachers and contribute to the data base from which many teachers can draw?

How Should Data Be Stored, Packaged, and Presented to Maximize Its Usefulness?

Assuming that the field identifies the kinds of data that are particularly useful for informing teachers' instructional decision-making and is prepared to gather these data, the next question is evident: How can these data be stored, packaged, and presented so they are most accessible and useful for teachers? Data-driven decision-making "is about making actionable the data by transforming them into usable knowledge (Mandinach et al., 2008; Williams & Hummelbrunner, 2011)" (Mandinach, 2012, p. 73). Data that can only be accessed and reviewed months or years after being collected and that are warehoused in difficult-to-access databases have little power to drive instructional decisions. Similarly, data that are presented in inscrutable or disjointed displays or aggregated in ways that are not immediately relevant to teachers' questions are of little practical value. The following sections consider the ways in which data can be stored, packaged, and presented to make them as useful as possible.

Timeliness and Access: Critical Concerns

As we indicated, one persistent issue with attempts to incorporate student achievement data into instructional decision-making, especially data from standardized tests, has been the time lag between the administration of the assessment and the availability of the student data (Coburn et al., 2009; Marsh et al., 2006). Depending on the type of data being collected, some delay is inevitable to allow for processing. And, depending on the purpose of the data, some delay is fine. However, as shown in Table 1, in-the-moment data must be available immediately to make instructional decisions in the moment, and short-term data must be available after the lesson to help plan and revise tomorrow's lesson. Focusing on the lesson as the unit of instruction helps identify what kinds of data are needed at what point in the instructional process, but questions remain about how to process classroom data so the data that teachers most need during the lesson are quickly accessible.

Not all data are immediately useful or relevant; some data, for example data that address learning trajectories for individual students, become useful over longer spans of time. We can imagine a scenario in which teacher–researcher partnerships have access to their students' learning trajectories based on past performance and can use these trajectories to plan lessons. Learning trajectories relevant for a particular mathematical topic would allow partnerships to identify patterns in students' past performance that could help them prepare for upcoming lessons, such as planning for the different kinds of experiences their students might need. Although the time lag that is likely needed to process these data is not a problem, there remain major questions about how these data can be processed to capture the key understandings and misconceptions for individual students and how they can be indexed to allow access when needed.

Transforming Data Into Usable Knowledge

How should data be stored, packaged, and presented so that teachers can easily use them? Answering this question is packed with challenges. Even if the most useful data are collected, and the issues of timing are resolved, there remain many questions about how to represent data so that they make sense to teachers, how to store and index data so that they can be retrieved when needed, and how to build a growing fund of data that can be shared and accessed by teachers facing similar instructional decisions.

We propose that the space within which research is conducted to answer the question about storing, packaging, and presenting data could again be bounded by focusing on classroom lessons. Within the domain of daily lessons, it is possible to imagine research programs that could address questions about the best ways to store, package, and present data so that they can inform the planning, implementing, and revising of instruction. In addition, we believe that there are advantages to using instructional artifacts such as written lesson plans, classroom materials, and student assessments as the storage receptacles for lesson-level data (Cai et al., 2018a). If written lesson plans are the storage container, the data gathered that could be relevant for teaching a particular lesson are stored in the lesson plan where they are likely to be noticed and used. The plans also provide an

Table 1

Framework for Collecting, Analyzing, and Using Data on Students' Mathematical Learning Experiences

Timeframe	Cognitive	Noncognitive
In the moment		
Data	• Students' conceptions and misconceptions	• Students' engagement with tasks
	• Students' unexpected responses	• Students' affect or frustration level
		• Students' participation in discourse
Goals	• Address in-the-moment, particular misconceptions among subgroups of students and provide immediate support	• Enact support for students who are disengaged or discouraged
		• Identify how students are being positioned within the classroom and shape classroom discourse to provide them with a voice
Short term		
Data	• Students' conceptions, misconceptions, and unexpected responses	• Factors that affect students' engagement with a task
	• Students' solution strategies	• Students' confidence both before and after solving a problem
	• Students' ways of thinking	• Classroom norms of participation
	• Students' insights	
Goals	• Identify groups of students with similar conceptions, misconceptions, or ways of thinking to inform the next lesson plan	• Identify groups of students who are experiencing different levels of motivation or engagement with the lesson to inform the next lesson plan
Long term		
Data	• Data across classrooms and research sites	• Connections between affect and achievement
Goals	• Longitudinally examine changes in students' cognitive learning outcomes and teachers can track the progress of individual students	• Longitudinally examine changes in students' affect related to their learning
	• Develop explanatory theories that connect teaching and learning for particular groups of students	

indexing system for relevant data that is intuitive for teachers. For example, teachers would find data that would help them plan the first lesson on adding fractions with unlike denominators in the written plan for that lesson. Conceptualizing the lesson plan as the container for the data also encourages researchers and teachers to annotate lesson plans with these data in ways that help teachers plan, implement, and then revise this lesson. Within this space, the question of how to store, package, and present data so that they are useful for teachers becomes more manageable.

We can identify related questions that come into view when instructional artifacts are used to make relevant data accessible to teachers. Because concrete artifacts make it possible to share data across teachers and classrooms, one question is How can teacher–researcher partnerships share artifacts plus their embedded data so that what is learned at one site can be used at other sites? How can networks be structured so that data-sharing is timely and useful? Also, because artifacts can be revised and updated as new data are collected, another question is How can teacher–researcher partnerships, and networks of such partnerships, use instructional artifacts to support the continuous improvement of instruction? How can different partnerships communicate to ensure that the current version of an artifact is the most up-to-date, data-based version that is possible? Given the important role of networks in these issues and questions, researchers may find the tools of social network theory and analysis useful (Daly, 2012; Farley-Ripple & Buttram, 2015).

In many discussions of data use by teachers, educators and policymakers ask about the skills that teachers need to use the data (Datnow & Hubbard, 2015; Means et al., 2011; Reeves et al., 2016). In fact, it has been suggested for some time that teachers do not use research to inform their teaching because, in part, they lack the skills or motivation to do so (Anderson & Biddle, 1991). Our view, given the questions that we have posed and our perspective on the importance of teacher–researcher partnerships, is that it is premature to describe the specific skills that teachers might need because the field does not yet know what forms the data could or should take nor does it know how researchers and teachers can best partner around data use. In particular, data that are useful for teachers are, by definition, data that teachers can use without a completely new set of skills. An important consideration for the research questions that we have posed is that the data should be available in a form that teachers can use given their current skills and the intense demands on their time that make learning a new set of skills burdensome.

However, it is possible to talk in general terms about the skills and dispositions that would facilitate teachers' use of data of any kind. Mandinach (2012) has described the need for "*pedagogical data literacy* (Mandinach, 2009a, 2009b, 2010b)" (p. 73), a term used to signify the fact that teachers will need to use data in ways that translate the information into creating better learning opportunities for students. To describe the landscape of activities through which teachers engage with data to improve instruction, Means et al. (2011) identified five skill areas: data location (finding relevant pieces of data in a data system or display), data comprehension (understanding what the data signify), data interpretation (figuring out what the data mean), instructional decision-making (making instructional choices that address the issue identified through the data), and question posing (generating relevant questions and hypotheses that can be addressed through the available data). The authors described some of the struggles that teachers had with these skills. Although it is likely that teachers would benefit from acquiring new skills to become regular users of data, and research has shown that teachers' use of data increases with appropriate support (Tsai & Tosh, 2019), we envision that the need for many of the skills from Means et al.'s (2011) list could be mitigated by the ways that researchers find for data to be gathered, processed, packaged, presented, accessed, and stored to be optimally useful for teachers. In addition, we suspect that teachers will develop the skills that they need to use these data by collaborating with researchers and other teachers in recurring cycles of planning, testing, and improving lessons as part of teacher–researcher partnerships (Cai et al., 2018b; Wayman, 2005a). A final question that we pose in this editorial addresses the issue of teachers' capacity to use data: How can teacher–researcher partnerships be structured and practiced so that teachers develop the abilities to use the new forms of data to inform their instruction?

Conclusions

Given the increasing rhetoric around data-driven decision-making, we suspect that teachers are likely to feel pressure to use data to improve their instruction. However, the data often available have few, if any, of the characteristics that would make the data useful for informing instruction. We believe this setting is ripe for impactful research, and we centered our fourth major problem for the field on determining the kinds of data that could be most useful for teachers. Questions that seem especially important and productive address the various aspects of the data gathering, processing, representing, and accessing process. One suggestion that we offered, to be continued in the final editorial in our series, is to explore the ways in which technology could extend human abilities to collect, process, and present large sets of detailed classroom data. We know that data on student thinking is important, but gathering and processing the ways in which each student thinks about each mathematical topic can quickly become overwhelming. One of the issues that we take up in our final editorial is how technological tools could amplify what is currently possible with traditional methods.

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