Momentum-Based Policy Gradient Methods

Feihu Huang [1] **Shangqian Gao** [1] **Jian Pei** [2] **Heng Huang** [1 3]

Abstract

Policy gradient methods are a class of powerful algorithms in reinforcement learning (RL). More recently, some variance reduced policy gradient methods have been developed to improve sample efficiency and obtain a near-optimal sample complexity $O)\epsilon^{-3}+$for finding an ϵ-stationary point of non-concave performance function in model-free RL. However, the practical performances of these variance reduced policy gradient methods are not consistent with their near-optimal sample complexity, because these methods require large batches and strict learning rates to achieve this optimal complexity. In the paper, thus, we propose a class of efficient momentum-based policy gradient methods, which use adaptive learning rates and do not require large batches. Specifically, we propose a fast important-sampling momentum-based policy gradient (IS-MBPG) method by using the importance sampling technique. We also propose a fast Hessian-aided momentum-based policy gradient (HA-MBPG) method via using the semi-Hessian information. Moreover, we prove that the IS-MBPG and HA-MBPG methods reach the best known sample complexity $O)\epsilon^{-3}+$without large batches. In particular, we present a non-adaptive version of IS-MBPG method (i.e., IS-MBPG*), which has a simple monotonically decreasing learning rate. We prove that the IS-MBPG* also reaches the best known sample complexity of $O)\epsilon^{-3}+$without large batches. In the experiments, we use some benchmark tasks to demonstrate the effectiveness of our algorithms.

[1]Department of Electrical and Computer Engineering, University of Pittsburgh, Pittsburgh, USA [2]School of Computing Science, Simon Fraser University, Vancouver, Canada [3]JD Finance America Corporation, Mountain View, CA, USA. Correspondence to: Heng Huang <heng.huang@pitt.edu>.

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1. Introduction

Reinforcement Learning (RL) has achieved great success in solving many sequential decision-making problems such as autonomous driving (Shalev-Shwartz et al., 2016), robot manipulation (Deisenroth et al., 2013), the game of Go (Silver et al., 2017) and natural language processing (Wang et al., 2018). In general, RL involves a Markov decision process (MDP), where an agent takes actions dictated by a policy in a stochastic environment over a sequence of time steps, and then maximizes the long-term cumulative rewards to obtain an optimal policy. Due to easy implementation and avoiding policy degradation, policy gradient method (Williams, 1992; Sutton et al., 2000) is widely used for finding the optimal policy in MDPs, especially for the high dimensional continuous state and action spaces. To obtain the optimal policy, policy gradient methods directly maximize the expected total reward (also called as performance function $J)\theta+$) via using the stochastic first-order gradient of cumulative rewards. Recently, policy gradient methods have achieved significant empirical successes in many challenging deep reinforcement learning applications (Li, 2017) such as playing Go game and robot manipulation.

Thus, policy gradient methods have regained much interest in reinforcement learning, and some corresponding algorithms and theory of policy gradient (Fellows et al., 2018; Fujimoto et al., 2018; Papini et al., 2018; Haarnoja et al., 2018; Xu et al., 2019a; Shen et al., 2019; Cheng et al., 2019b;a; Wang et al., 2019a) have been proposed and studied. Since the classic policy gradient methods (e.g., REINFORCE (Williams, 1992), PGT (Sutton et al., 2000), GPOMDP (Baxter & Bartlett, 2001) and TRPO (Schulman et al., 2015a)) approximate the gradient of the expected total reward based on a batch of sampled trajectories, they generally suffer from large variance in the estimated gradients, which results in a poor convergence. Following the standard stochastic gradient method (Robbins & Monro, 1951), these gradient-based algorithms require $O)\epsilon^{-4}+$samples for finding an ϵ-approximated stationary point, i.e., $\mathbb{E}\sqrt{\ }\sqrt{\ }J)\theta+\geq\epsilon$. Thus, recently many works have begun to study to reduce variance in the policy gradient methods. For example, the early variance reduced policy methods (Greensmith et al., 2004; Peters & Schaal, 2008) mainly focused on using unbiased baseline functions

Table 1. Convergence properties of the representative variance-reduced policy algorithms on the *non-oblivious* model-free RL problem for finding an ϵ-approximated stationary point, i.e., $\mathbb{E}\|\nabla J(\theta)\| \leq \epsilon$. Our algorithms (IS-MBPG, IS-MBPG* and HA-MBPG) and REINFORCE are **single-loop** algorithms, while the other algorithms are **double-loops**, which need the outer-loop and inner-loop mini-batch sizes. Note that Papini et al. (2018) only remarked that apply the ADAM algorithm (Kingma & Ba, 2014) to the SVRPG algorithm to obtain an adaptive learning rate, but did not provide any theoretical analysis about this learning rate.

Algorithm	Reference	Sample Complexity	Batch-Size	Adaptive Learning Rate
REINFORCE	Williams (1992)	$O)\epsilon^{-4}+$	$O)\epsilon^{-2}+$	
SVRPG	Papini et al. (2018)	$O)\epsilon^{-4}+$	$O)\epsilon^{-2}+$& $O)\epsilon^{-2}+$	
SVRPG	Xu et al. (2019a)	$O)\epsilon^{-10/3}+$	$O)\epsilon^{-4/3}+$& $O)\epsilon^{-2}+$	
HAPG	Shen et al. (2019)	$O)\epsilon^{-3}+$	$O)\epsilon^{-1}+$& $O)\epsilon^{-2}+$	
SRVR-PG	Xu et al. (2019b)	$O)\epsilon^{-3}+$	$O)\epsilon^{-1}+$& $O)\epsilon^{-2}+$	
IS-MBPG	Ours	$O)\epsilon^{-3}+$	$O)2+$	✓
HA-MBPG	Ours	$O)\epsilon^{-3}+$	$O)2+$	✓
IS-MBPG*	Ours	$O)\epsilon^{-3}+$	$O)2+$	

to reduce the variance. Schulman et al. (2015b) presented the generalized advantage estimation (GAE) to discover the balance between bias and variance of policy gradient. Then Gu et al. (2016) applied both the GAE and linear baseline function to reduce variance. Recently, Mao et al. (2018); Wu et al. (2018) proposed the input-dependent and action-dependent baselines to reduce the variance, respectively. More recently, Cheng et al. (2019b) leveraged the predictive models to reduce the variance to accelerate policy learning.

Recently, the variance reduced gradient estimators such as SVRG (Johnson & Zhang, 2013; Allen-Zhu & Hazan, 2016; Reddi et al., 2016), SAGA (Defazio et al., 2014), SARAH (Nguyen et al., 2017), SPIDER (Fang et al., 2018) and SpiderBoost (Wang et al., 2019b) have been successful in the oblivious supervised learning. However, the RL optimization problems are *non-oblivious*, i.e., the distribution of the samples is non-stationarity and changes over time. Thus, Du et al. (2017); Xu et al. (2017); Wai et al. (2019) first transform the original non-oblivious policy evaluation problem into some oblivious subproblems, and then use the existing variance reduced gradient estimators (such as SVRG and SAGA) to solve these subproblems to reach the goal of reducing the large variance in the original RL problem. For example, Du et al. (2017) first transforms the empirical policy evaluation problem into a quadratic convex-concave saddle-point problem via linear function approximation, and then applies the variants of SVRG and SAGA (Palaniappan & Bach, 2016) to solve this oblivious saddle-point problem.

More recently, Papini et al. (2018); Xu et al. (2019a;b); Shen et al. (2019) further have developed some variance reduced policy gradient estimators directly used in the non-oblivious model-free RL, based on the existing variance reduced techniques such as SVRG and SPIDER used in the oblivious supervised learning. Moreover, Xu et al. (2019a;b); Shen et al. (2019) have effectively improved the sample complexity by using these variance reduced policy gradients. For example, two efficient variance reduced policy gradient methods, i.e, SRVR-PG (Xu et al., 2019b) and HAPG (Shen et al., 2019) have been proposed based on the SPIDER, and reach a sharp sample complexity $O)\epsilon^{-3}+$ for finding an ϵ-stationary point of non-concave performance function ($\mathbb{E}$ $J)\theta+ \geq \epsilon$), which improves the standard complexity $O)\epsilon^{-4}+$(Williams, 1992) by a factor of $O)\epsilon^{-1}+$. Since a lower bound of complexity $O)\epsilon^{-3}+$for recently proposed variance reduction techniques is established in (Arjevani et al., 2019), both the SRVR-PG and HAPG obtain a near-optimal sample complexity $O)\epsilon^{-3}+$. However, the practical performances of these variance reduced policy gradient methods are not consistent with their near-optimal sample complexity, because these methods require large batch size and strict learning rates to achieve this optimal complexity.

In the paper, thus, we propose a class of efficient momentum based policy gradient methods, which use adaptive learning rates and do not require large batches. Specifically, our algorithms only need one trajectory at each iteration and use adaptive learning rates based on the current and historical stochastic gradients. Note that Pirotta et al. (2013) has studied the adaptive learning rates for policy gradient methods, which only focuses on Gaussian policy. Moreover, Pirotta et al. (2013) did not consider sample complexity and can not improve it. While our algorithms not only provide the adaptive learning rates that are suitable for any policies, but also improve sample complexity.

Contributions

Our main contributions are summarized as follows:

1) We propose a fast important-sampling momentum-based policy gradient (IS-MBPG) method with adaptive learning rate, which builds on momentum-based variance reduction technique (Cutkosky & Orabona, 2019; Tran-Dinh et al., 2019) and importance sampling.

2) We propose a fast Hessian-aided momentum-based policy gradient (HA-MBPG) method with adaptive learning rate, which builds on momentum-based variance reduction technique and semi-Hessian information.
3) We study the sample complexity of our methods, and prove that both the IS-MBPG and HA-MBPG methods reach the best known sample complexity of $O(\epsilon^{-3})$ without large batches (see Table 1).
4) We propose a non-adaptive version of IS-MBPG method, i.e., IS-MBPG*, which has a simple monotonically decreasing learning rate. We prove that it also reaches the best known sample complexity of $O(\epsilon^{-3})$ without large batches.

After our paper is accepted, we find that three related papers (Xiong et al., 2020; Pham et al., 2020; Yuan et al., 2020) more recently are released on arXiv. Xiong et al. (2020) has studied the adaptive Adam-type gradient (PG-AMSGrad) method, and has proved that the PG-AMSGrad still have a high sample complexity of $O(\epsilon^{-4})$. Subsequently, Pham et al. (2020); Yuan et al. (2020) have proposed the policy gradient methods, i.e., ProxHSPGA and STORM-P, respectively, which also build on the momentum-based variance reduced technique. Although both the ProxHSPGA and STORM-P also reach the best known sample complexity of $O(\epsilon^{-3})$, these methods still rely on the large batches to obtain this sample complexity and does not provide an efficient adaptive learning rate as our methods.

Notations

Let $\|\cdot\|$ denote the vector ℓ_2 norm and the matrix spectral norm, respectively. We denote $a_n = O(b_n)$ if $a_n \geq cb_n$ for some constant $c > 1$. $\mathbb{E}[X]$ and $\mathbb{V}[X]$ denote the expectation and variance of a random variable X, respectively. $\mathbb{E}_{\tau_t}[\cdot] = \mathbb{E}_{\tau_t}[\cdot|\tau_1, \ldots, \tau_{t-1}]$ for any $t \geq 3$.

2. Background

In the section, we will review some preliminaries of standard reinforcement learning and policy gradient.

2.1. Reinforcement Learning

Reinforcement learning is generally modeled as a discrete time Markov Decision Process (MDP): $\mathcal{P} = \{\mathcal{S}, \mathcal{B}, \mathcal{R}, \mathcal{S}, \gamma, \rho_0\}$. Here $\mathcal{S}$ is the state space, $\mathcal{B}$ is the action space, and ρ_0 denotes the initial state distribution. $\mathcal{R}(s'|s, a)$ denotes the probability that the agent transits from the state s to s' under taking the action $a \in \mathcal{B}$. $\mathcal{S}(s, a) = \{\cup \mathcal{B} \subseteq \mathbb{R}\}$ $R, R)R > 1$ is the bounded reward function, *i.e.,* the agent obtain the reward $\mathcal{S}(s, a)$ after it takes the action a at the state s, and $\gamma \in (0, 1)$ is the discount factor. The policy $\pi(a|s)$ at the state s is represented by a conditional probability distribution $\pi_\theta(a|s)$ associated to the parameter $\theta \in \mathbb{R}^d$.

Given a time horizon H, the agent can collect a trajectory $\tau = \{s_0, a_0, \ldots, s_{H-1}, a_{H-1}\}$ under any stationary policy. Following the trajectory τ, a cumulative discounted reward can be given as follows:

$$\mathcal{S}(\tau) = \prod_{h=0}^{H-1} \gamma^h \mathcal{S}(s_h, a_h), \tag{1}$$

where γ is the discount factor. Assume that the policy π_θ is parameterized by an unknown parameter $\theta \in \mathbb{R}^d$. Given the initial distribution $\rho_0(s_0)$, the probability distribution over trajectory τ can be obtain

$$p(\tau|\theta) = \rho(s_0) \int_{h=0}^{H-1} \mathcal{R}(s_{h+1}|s_h, a_h) \pi_\theta(a_h|s_h). \tag{2}$$

2.2. Policy Gradient

The goal of RL is to find an optimal policy π_θ that is equivalent to maximize the expected discounted trajectory reward:

$$\max_{\theta \in \mathbb{R}^d} J(\theta) = \mathbb{E}_{\tau \sim p(\tau|\theta)}[\mathcal{S}(\tau)] = \int \sqrt{\mathcal{S}(\tau)} p(\tau|\theta) d\tau. \tag{3}$$

Since the underlying distribution p depends on the variable θ and varies through the whole optimization procedure, the problem (3) is a *non-oblivious* learning problem, which is unlike the traditional supervised learning problems that the underlying distribution p is stationary. To deal with this problem, the policy gradient method (Williams, 1992; Sutton et al., 2000) is a good choice. Specifically, we first compute the gradient of $J(\theta)$ with respect to θ, and obtain

$$\begin{aligned} \nabla J(\theta) &= \int \sqrt{\mathcal{S}(\tau)} \nabla p(\tau|\theta) d\tau = \int \sqrt{\mathcal{S}(\tau)} \frac{\nabla p(\tau|\theta)}{p(\tau|\theta)} p(\tau|\theta) d\tau \\ &= \mathbb{E}_{\tau \sim p(\tau|\theta)}\left[\sqrt{\mathcal{S}(\tau)} \nabla \log p(\tau|\theta) \mathcal{S}(\tau)\right]. \end{aligned} \tag{4}$$

Since the distribution $p(\tau|\theta)$ is unknown, we can not compute the exact full gradient of (4). Similar for stochastic gradient descent (SGD), the policy gradient method samples a batch of trajectories $\mathcal{M} = \{\tau_i\}_{i=1}^{|\mathcal{B}|}$ from the distribution $p(\tau|\theta)$ to obtain the stochastic gradient as follows:

$$\nabla J(\theta) = \frac{2}{|\mathcal{M}|} \sum_{i \in \mathcal{B}} \nabla \log p(\tau_i|\theta) \mathcal{S}(\tau_i).$$

At the t-th iteration, the parameter θ can be updated:

$$\theta_{t+1} = \theta_t + \eta_t \nabla_\theta J(\theta), \tag{5}$$

where $\eta_t > 0$ is a learning rate. In addition, since the term $\nabla \log p(\tau_i|\theta)$ is independent of the transition probability $\mathcal{R}$,

we rewrite the stochastic gradient $\nabla J(\theta)$ as follows:

$$\nabla J(\theta) = \frac{1}{|\mathcal{B}|}\sum_{i\in\mathcal{B}} g(\tau_i, \theta) \quad (6)$$
$$= \frac{1}{|\mathcal{B}|}\sum_{i\in\mathcal{B}} \sum_{h=0}^{H-1} \nabla_\theta \log \pi_\theta(a_h^i, s_h^i) \Big(\prod_{h=0}^{H-1} \gamma^h \mathcal{S}(s_h^i, a_h^i)\Big),$$

where $g(\tau_i, \theta)$ is an unbiased stochastic gradient based on the trajectory τ_i, *i.e.,* $\mathbb{E}[g(\tau_i, \theta)] = \nabla J(\theta)$. Based on the above gradient estimator in (6), we can obtain the existing well-known gradient estimators of policy gradient such as the REINFORCE, the PGT and the GPOMDP. Due to $\mathbb{E}[\nabla_\theta \log \pi_\theta(a, s)] = 0$, the REINFORCE adds a constant baseline b and obtains a gradient estimator as follows:

$$g(\tau_i, \theta) = \sum_{h=0}^{H-1} \nabla_\theta \log \pi_\theta(a_h^i, s_h^i) \Big(\sum_{h=0}^{H-1} \gamma^h \mathcal{S}(s_h^i, a_h^i) - b\Big).$$

Further, considering the fact that the current actions do not rely on the previous rewards, the PGT refines the REINFORCE and obtains the following gradient estimator:

$$g(\tau_i, \theta) = \sum_{h=0}^{H-1}\sum_{j=h}^{H-1} \gamma^j \mathcal{S}(s_j^i, a_j^i) - b_j \nabla_\theta \log \pi_\theta(a_h^i, s_h^i).$$

Meanwhile, the PGT estimator is equivalent to the popular GPOMDP estimator defined as follows:

$$g(\tau_i, \theta) = \sum_{h=0}^{H-1}\sum_{j=0}^{h} \nabla_\theta \log \pi_\theta(a_j^i, s_j^i) \gamma^h \mathcal{S}(s_h^i, a_h^i) - b_h.$$

3. Momentum-Based Policy Gradients

In the section, we propose a class of fast momentum-based policy gradient methods based on a new momentum-based variance reduction method, i.e., STORM (Cutkosky & Orabona, 2019). Although the STORM shows its effectiveness in the *oblivious* learning problems, it is not well suitable for the *non-oblivious* learning problem , where the underlying distribution $p(\tau|\theta)$ depends on the variable θ and varies through the whole optimization procedure. To deal with this challenge, we will apply two effective techniques, i.e., *importance sampling* (Metelli et al., 2018; Papini et al., 2018) and *Hessian-aided* (Shen et al., 2019), and propose the corresponded policy gradient methods, respectively.

3.1. Important-Sampling Momentum-Based Policy Gradient

In the subsection, we propose a fast important-sampling momentum-based policy gradient (IS-MBPG) method by using the importance sampling technique. Algorithm 1 describes the algorithmic framework of IS-MBPG method.

Since the problem (3) is *non-oblivious* or *non-stationarity* that the underlying distribution $p(\tau|\theta)$ depends on the variable θ and varies through the whole optimization procedure, we have $\mathbb{E}_{\tau\sim p(\tau|\theta)}[g(\tau|\theta)] \neq \nabla J(\theta)$. Given τ sampled from $p(\tau|\theta)$, we define an importance sampling weight

$$w(\tau|\theta', \theta) = \frac{p(\tau|\theta')}{p(\tau|\theta)} = \prod_{h=0}^{H-1} \frac{\pi_{\theta'}(a_h|s_h)}{\pi_\theta(a_h|s_h)} \quad (7)$$

to obtain $\mathbb{E}_{\tau\sim p(\tau|\theta)}[w(\tau|\theta', \theta) g(\tau|\theta')] = \nabla J(\theta)$. In Algorithm 1, we use the following momentum-based variance reduced stochastic gradient

$$u_t = (1-\beta_t)\underbrace{\big(u_{t-1} - g(\tau_t|\theta_t) + w(\tau_t|\theta_{t-1}, \theta_t) g(\tau_t|\theta_{t-1})\big)}_{\text{SARAH}} + \underbrace{\beta_t g(\tau_t|\theta_t)}_{\text{SGD}},$$

where $\beta_t \in [0, 1]$. When $\beta_t = 1$, the IS-MBPG will reduce to the REINFORCE. When $\beta_t = 0$, it will reduce to the SRVR-PG.

Let $e_t = u_t - \nabla J(\theta_t)$. It is easily verified that

$$\mathbb{E}\|e_t\|^2 = \mathbb{E}\big\|(1-\beta_t)e_{t-1} + (1-\beta_t)\underbrace{\big(g(\tau_t|\theta_t) - \nabla J(\theta_t)\big)}_{=T_1} + \beta_t \nabla J(\theta_t)\big\|^2$$
$$\times \underbrace{\big(g(\tau_t|\theta_t) - w(\tau_t|\theta_{t-1}, \theta_t) g(\tau_t|\theta_{t-1})\big)}_{=T_2} + \nabla J(\theta_t) - \nabla J(\theta_{t-1})\big\|^2$$
$$= (1-\beta_t)^2 \mathbb{E}\|e_{t-1}\|^2 + \beta_t^2 \mathbb{E}\|T_1\|^2 + (1-\beta_t)^2 \mathbb{E}\|T_2\|^2. \quad (8)$$

where the last equality holds by $\mathbb{E}_{\tau_t\sim p(\tau|\theta_t)}[T_1] = 0$ and $\mathbb{E}_{\tau_t\sim p(\tau|\theta_t)}[T_2] = 0$. By Cauchy-Schwarz inequality, we can obtain

$$\mathbb{E}\|e_t\|^2 \le (1-\beta_t)^2 \mathbb{E}\|e_{t-1}\|^2 + 3\beta_t^2 \mathbb{E}\|T_1\|^2 + 3(1-\beta_t)^2 \mathbb{E}\|T_2\|^2. \quad (9)$$

Since $O(\|T_2\|^2) = O(\|\theta_t - \theta_{t-1}\|^2) = O(\eta_t^2 \|u_t\|^2)$, we can choose appropriate η_t and β_t to reduce the variance of stochastic gradient u_t. From the following theoretical results, our IS-MBPG algorithm can generate the adaptive and monotonically decreasing learning rate $\eta_t \in (0, \frac{1}{2L}]$, and the monotonically decreasing parameter $\beta_t \in (0, 1]$.

3.2. Hessian-Aided Momentum-Based Policy Gradient

In the subsection, we propose a fast Hessian-aided momentum-based policy gradient (HA-MBPG) method via using the semi-Hessian information. Algorithm 2 describes the algorithmic framework of HA-MBPG method.

In Algorithm 2, at the 8-th step, we use an unbiased term Λ^t *i.e.,* $\mathbb{E}_{\tau_t\sim p(\tau|\theta_t(\alpha))}[\Lambda^t] = \nabla J(\theta_t) - \nabla J(\theta_{t-1})$ instead of the biased term $g(\tau|\theta_t) - g(\tau|\theta_{t-1})$. To construct the term

Algorithm 1 Important-Sampling Momentum-Based Policy Gradient (**IS-MBPG**) Algorithm

Input: Total iteration T, parameters $\}k, m, c|$ and initial input θ_1;
for t [$2, 3, \ldots, T$ **do**
  if t [2 **then**
    Sample a trajectory τ_1 from $p)\tau|\theta_1+$, and compute u_1 [$g)\tau_1|\theta_1+$;
  else
    Sample a trajectory τ_t from $p)\tau|\theta_t+$, and compute u_t [$\beta_t g)\tau_t|\theta_t+0$)2 $\beta_t+]u_{t-1}$ 0 $g)\tau_t|\theta_t+$ $w)\tau_t|\theta_{t-1}, \theta_t+g)\tau_t|\theta_{t-1}+($, where the importance sampling weight $w)\tau_t|\theta_t($1$, \theta_t+$can be computed by using (7);
  end if
  Compute G_t [$g)\tau|\theta_t+;$
  Compute η_t [$\frac{k}{(m+\sum_{i=1}^{t} G_i^2)^{1/3}}$;
  Update θ_{t+1} [θ_t 0 $\eta_t u_t$;
  Update β_{t+1} [$c\eta_t^2$;
end for
Output: θ_ζ chosen uniformly random from $\}\theta_t|_{t=1}^{T}$.

Algorithm 2 Hessian-Aided Momentum-Based Policy Gradient (**HA-MBPG**) Algorithm

Input: Total iteration T, parameters $\}k, m, c|$ and initial input θ_1;
for t [$2, 3, \ldots, T$ **do**
  if t [2 **then**
    Sample a trajectory τ_1 from $p)\tau|\theta_1+$, and compute u_1 [$g)\tau_1|\theta_1+$;
  else
    Choose α uniformly at random from $]1, 2\hat{}$, and compute $\theta_t)\alpha+[$ $\alpha\theta_t$ 0)2 $\alpha-\theta_{t-1}$;
    Sample a trajectory τ_t from $p)\tau|\theta_t)\alpha+$, and compute u_t [$\beta_t w)\tau_t|\theta_t, \theta_t)\alpha+g)\tau_t|\theta_t+0$)2 $\beta_t+u_{t-1}0$ $\Lambda_t[$, where $w)\tau|\theta_t, \theta_t)\alpha+$and Λ_t can be computed by using (7) and (11), respectively;
  end if
  Compute G_t [$g)\tau|\theta_t+;$
  Compute η_t [$\frac{k}{(m+\sum_{i=1}^{t} G_i^2)^{1/3}}$;
  Update θ_{t+1} [θ_t 0 $\eta_t u_t$;
  Update β_{t+1} [$c\eta_t^2$;
end for
Output: θ_ζ chosen uniformly random from $\}\theta_t|_{t=1}^{T}$.

Λ^t, we first assume that the function $J)\theta+$is twice differentiable as in (Furmston et al., 2016; Shen et al., 2019). By the Taylor's expansion (or Newton-Leibniz formula), the gradient difference $J)\theta_t+$ $J)\theta_{t-1}+$can be written as

$$J)\theta_t+ \quad J)\theta_{t-1}+[\ ] \sqrt{\int_0^1} {}^2J)\theta_t)\alpha+d\alpha($$ v_t, (10)

where v_t [θ_t θ_{t-1} and $\theta_t)\alpha+[$ $\alpha\theta_t$ 0)2 $\alpha-\theta_{t-1}$ for some α / $]1, 2\hat{}$. Following (Furmston et al., 2016; Shen et al., 2019), we obtain the policy Hessian ${}^2J)\theta+$as follows:

$${}^2J)\theta+[\ \mathbb{E}_{\tau\sim p(\tau|\theta)}] \quad \text{mpl}\ p)\tau|\theta+ \ \text{mpl}\ p)\tau|\theta+^T$$
$$0 \quad {}^2\,\text{mpl}\ p)\tau|\theta+[\mathcal{S})\tau+($$
$$[\ \mathbb{E}_{\tau\sim p(\tau|\theta)}] \quad \Psi)\tau|\theta+ \ \text{mpl}\ p)\tau|\theta+^T 0 \quad {}^2\Psi)\tau|\theta+($$

where $\Psi)\tau|\theta+[\ \int_{h=0}^{H-1} \int_{j=h}^{H-1} \gamma^j r)s_j, a_j+\text{mpl}\ \pi_\theta)a_h, s_h+$. Given the random tuple $)\alpha, \tau+$, where α samples uniformly from $]1, 2\hat{}$ and τ samples from the distribution $p)\tau|\theta_t)\alpha+$, we can construct Λ_t as follows:

$$\Lambda_t \ \dot{+} \ {}^{\text{a}\,2})\theta_t)\alpha+, \tau+v_t, \quad (11)$$

where $\mathbb{E}_{\tau\sim p(\tau|\theta_t(\alpha))}]\,{}^{\text{a}\,2})\theta_t)\alpha+, \tau+\hat{}[$ ${}^2J)\theta_t)\alpha+$and

$${}^{\text{a}\,2})\theta_t, \tau+[\quad \Psi)\tau|\theta_t)\alpha+ \ \text{mpl}\ p)\tau|\theta_t)\alpha+^T$$
$$0 \quad {}^2\Psi)\tau|\theta_t)\alpha+.$$

Note that $\mathbb{E}_{\alpha\sim U[0,1]}$ ${}^2J)\theta_t)\alpha+\hat{}$ [$\int_0^1$ ${}^2J)\theta_t)\alpha+d\alpha$ implies the unbiased estimator ${}^2J)\theta)\&\%+$ with $\&\%$ uniformly sampled from $]1, 2\hat{}$. Given $\&\%$, we have $\mathbb{E}_{\tau\sim p(\tau|\theta_t(\bar{\alpha}))}]\,{}^{\text{a}\,2})\theta_t)\&\%, \tau+\hat{}$ [${}^2J)\theta_t)\&\%+$. According to the equation (10), thus we have $\mathbb{E}_{\alpha\sim U[0,1],\ \tau\sim p(\tau|\theta_t(\alpha))}]\Lambda_t\hat{}$ [$J)\theta_t+$ $J)\theta_{t-1}+$, where $U]1, 2\hat{}$denotes the uniform distribution over $]1, 2\hat{}$.

Next, we rewrite (11) as follows:

$$\Lambda_t [\quad \text{mpl}\ p)\tau|\theta_t)\alpha+^T v_t[\ \Psi)\tau|\theta_t)\alpha+$$
$$0 \quad {}^2\Psi)\tau|\theta_t)\alpha+v_t. \quad (12)$$

Considering the second term in (12) is a time-consuming Hessian-vector product, in practice, we use can the finite difference method to estimate ${}^2\Psi)\tau|\theta_t)\alpha+v_t$ as follows:

$${}^2\Psi)\tau|\theta_t)\alpha+v_t$$
$$\to \frac{\Psi)\tau|\theta_t)\alpha+0\ \delta v_t+ \quad \Psi)\tau|\theta_t)\alpha+ \quad \delta v_t+}{3\delta} v_t$$
$$[\quad {}^2\Psi)\tau|\theta_t)\alpha+v_t, \quad (13)$$

where $\delta > 1$ is very small and $\theta_t)\alpha+$ / $]\theta_t)\alpha+$ $\delta v_t, \theta_t)\alpha+0$ $\delta v_t\hat{}$ is obtained by the mean-value theorem. Suppose $\Psi)\tau|\theta+$is L_2-second-order smooth, we can upper bound the approximated error:

$$\sqrt{} \ {}^2\Psi)\tau|\theta_t)\alpha+v_t \quad {}^2\Psi)\tau|\theta_t)\alpha+v_t \sqrt{} \geq L_2 \sqrt{} v_t \sqrt{} \delta. \quad (14)$$

Thus, we take a sufficiency small δ to obtain arbitrarily small approximated error.

In Algorithm 2, we use the following momentum-based variance reduced stochastic gradient

$$u_t = \beta_t w(\tau|\theta_t,\theta_t)\alpha+g(\tau|\theta_t+0)2 \quad \beta_t+u_{t-1}\,0\,\Lambda_t[,$$

where $\beta_t \in]1,2\hat{}$. When $\beta_t = 2$, the HA-MBPG will reduce to the REINFORCE. When $\beta_t = 1$, it will reduce to the HAPG.

Let $e_t = u_t - \nabla J(\theta_t+$ It is also easily verified that

$$\mathbb{E}]e_t\hat{}[= \mathbb{E}])2 \quad \beta_t+e_{t-1}\,0\,\beta_t)\underbrace{w(\tau|\theta_t,\theta_t)\alpha+g(\tau|\theta_t+}_{=T_3} \quad \underbrace{\nabla J(\theta_t+}+$$

$$0)2 \quad \beta_t+\underbrace{\Lambda_t \quad \nabla J(\theta_t+0\ \nabla J(\theta_{t-1}+}_{=T_4}[($$

$$[)2 \quad \beta_t+\mathbb{E}]e_{t-1}\hat{}, \tag{15}$$

where the last equality holds by $\mathbb{E}_{\tau\sim p(\tau|\theta_t(\alpha))}]T_3\hat{}[= 1$ and $\mathbb{E}_{\tau\sim p(\tau|\theta_t(\alpha))}]T_4\hat{}[= 1$. Similarly, by Cauchy-Schwarz inequality, we can obtain

$$\mathbb{E}\sqrt{\|e_t\|^2} \geq)2 \quad \beta_t+^2\mathbb{E}\sqrt{\|e_{t-1}\|^2}\,0\,3\beta_t^2\mathbb{E}\sqrt{\|T_3\|^2}$$
$$0\ 3)2 \quad \beta_t+^2\mathbb{E}\sqrt{\|T_4\|^2}. \tag{16}$$

Since $O)\sqrt{\|T_4\|^2}+[\quad O)\sqrt{\|\theta_t \quad \theta_{t-1}\|^2}+[\quad O)\eta_t^2\sqrt{\|u_t\|^2}+$ we can choose appropriate η_t and β_t to reduce the variance of stochastic gradient u_t. From the following theoretical results, our HA-MBPG algorithm can also generate the adaptive and monotonically decreasing learning rate $\eta_t \in)1, \frac{1}{2L}\hat{}$, and the monotonically decreasing parameter $\beta_t \in)1,2\hat{}$.

3.3. Non-Adaptive IS-MBPG*

In this subsection, we propose a non-adaptive version of IS-MBPG algorithm, i.e., IS-MBPG*. The IS-MBPG* algorithm is given in Algorithm 3. Specifically, Algorithm 3 applies a simple monotonically decreasing learning rate η_t, which only depends on the number of iteration t.

4. Convergence Analysis

In this section, we will study the convergence properties of our algorithms, i.e., IS-MBPG, HA-MBPG and IS-MBPG*. *All related proofs are provided in supplementary document.* We first give some assumptions as follows:

Assumption 1. *Gradient and Hessian matrix of function* $\log \pi_\theta)a|s+$*are bounded,* i.e., *there exist constants* $M_g, M_h > 1$ *such that*

$$\sqrt{\|\nabla_\theta \log \pi_\theta)a|s+\|^2} \geq M_g, \quad \sqrt{\|\nabla^2_\theta \log \pi_\theta)a|s+\|^2} \geq M_h. \tag{17}$$

Assumption 2. *Variance of stochastic gradient* $g)\tau|\theta+$*is bounded,* i.e., *there exists a constant* $\sigma > 1$*, for all* π_θ *such that* $\mathbb{V})g)\tau|\theta+[\quad \mathbb{E}\sqrt{\|g)\tau|\theta+ \quad J)\theta+\|^2} \geq \sigma^2$.

Assumption 3. *Variance of importance sampling weight* $w)\tau|\theta_1,\theta_2+[\quad p)\tau|\theta_1+/p)\tau|\theta_2+$*is bounded,* i.e., *there exists a constant* $W > 1$*, it follows* $\mathbb{V})w)\tau|\theta_1,\theta_2+\geq W$ *for any* $\theta_1,\theta_2 \in \mathbb{R}^d$ *and* $\tau \approx p)\tau|\theta_2+$

Algorithm 3 IS-MBPG* Algorithm

Input: Total iteration T, parameters $\}k, m, c|$ and initial input θ_1;
for $t = 2, 3, \ldots, T$ **do**
  if $t = 2$ **then**
    Sample a trajectory τ_1 from $p)\tau|\theta_1+$ and compute $u_1 = g)\tau_1|\theta_1+$
  else
    Sample a trajectory τ_t from $p)\tau|\theta_t+$ and compute $u_t = \beta_t g)\tau_t|\theta_t+0)2 \quad \beta_t+]u_{t-1}\,0\ g)\tau_t|\theta_t+ w)\tau_t|\theta_{t-1},\theta_t+g)\tau_t|\theta_{t-1}+($;
  end if
  Compute $\eta_t = \frac{k}{(m+t)^{1/3}}$;
  Update $\theta_{t+1} = \theta_t\,0\ \eta_t u_t$;
  Update $\beta_{t+1} = c\eta_t^2$;
end for
Output: θ_ζ chosen uniformly random from $\}\theta_t|_{t=1}^T$.

Assumptions 1 and 2 have been commonly used in the convergence analysis of policy gradient algorithms (Papini et al., 2018; Xu et al., 2019a;b; Shen et al., 2019). Assumption 3 has been used in the study of variance reduced policy gradient algorithms (Papini et al., 2018; Xu et al., 2019a;b). Note that the bounded importance sampling weight in Assumption 3 might be violated in practice. For example, because when using neural networks (NNs) as the policy, small perturbations in θ might raise a large gap in the point probability due to some activation functions in NNs. Thus, we usually clip the importance sampling weights to make our algorithms (i.e., IS-MBPG and HA-MBPG) more effective. Based on Assumption 1, we give some useful properties of stochastic gradient $g)\tau|\theta+$and $\nabla^2)\theta_t,\tau+$ respectively.

Proposition 1. *(Proposition 4.2 in (Xu et al., 2019b)) Suppose* $g)\tau|\theta+$*is the PGT estimator. By Assumption 1, we have*

1) $g)\tau|\theta+$ *is* $\hat{L}$*-Lipschitz differential, i.e.,* $\sqrt{\|g)\tau|\theta+ - g)\tau|\theta'+\|^2} \geq L\sqrt{\|\theta \quad \theta'\|^2}$ *with* $\hat{L} = M_h R/)2 \quad \gamma+^2$;
2) $\nabla J)\theta+$*is* $\hat{L}$*-smooth, i.e.,* $\sqrt{\|\nabla^2 J)\theta+\|^2} \geq \hat{L}$;
3) $g)\tau|\theta+$*is bounded, i.e.,* $\sqrt{\|g)\tau|\theta+\|^2} \geq G$ *for all* $\theta \in \mathbb{R}^d$ *with* $G = M_g R/)2 \quad \gamma+^2$.

Since $\sqrt{\|\nabla J)\theta+ - \mathbb{E}]g)\tau|\theta+\|^2} \geq \mathbb{E}\sqrt{\|g)\tau|\theta+\|^2} \geq G$, Proposition 1 implies that $\nabla J)\theta+$is G-Lipschitz. Without loss of generality, we use the PGT estimator to generate the gradient $g)\tau|\theta+$in our algorithms, so $G_t = \sqrt{\|g)\tau|\theta+\|^2} \geq G$.

Proposition 2. *(Lemma 4.1 in (Shen et al., 2019)) Under Assumption 1, we have for all* θ

$$\sqrt{\|\nabla^2)\theta,\tau+\|^2} \geq \frac{H^2 M_g^4 R^2\,0\ M_h^2 R^2}{)2 \quad \gamma+^4} - L^2. \tag{18}$$

Since $\nabla^2 J(\theta) + [\mathbb{E}][\nabla \log p(a|\theta, \tau) + \nabla \log p(a|\theta, \tau) \geq \mathbb{E}\, \nabla \log p(a|\theta, \tau) + \geq L$, Proposition 2 implies that $J(\theta) +$ is L-smooth. Let L [$\max(\tilde{L}, L +$, so $J(\theta) +$ is L-smooth.

4.1. Convergence Analysis of IS-MBPG Algorithm

In the subsection, we analyze the convergence properties of the IS-MBPG algorithm. For notational simplicity, let B^2 [L^2 0 $3G^2 C_w^2$ with C_w [$\sqrt{H(3HM_g^2 0\ M_h +)W 0\ 2+}$.

Theorem 1. *Assume that the sequence* $\{\theta_t\}_{t=1}^T$ *be generated from Algorithm 1. Set* k [$O)\frac{G^{2/3}}{L} + c$ [$\frac{G^2}{3k^3 L} 0\ 215B^2$, m [$\max\{3G^2,)3Lk +^3,)\frac{ck}{2L} +^3|$ *and* η_0 [$\frac{k}{m^{1/3}}$, *we have*

$$\mathbb{E}\sqrt{\nabla J(\theta_\zeta) +} \geq \frac{\sqrt{3'} m^{1/6} 0\ 3'^{3/4}}{\sqrt{T}} 0\ \frac{3\sqrt{\sigma^{1/3}}}{T^{1/3}}, \quad (19)$$

where $' = \frac{1}{k}\ 27) J^* \quad J(\theta_1 + 0\ \frac{m^{1/3}}{8B^2 k}\sigma^2 0\ \frac{c^2 k^3}{4B^2}\ \max)T 0\ 3+$ *with* $J^* = \min_\theta J(\theta) + < 0 \in$.

Remark 1. *Since* $' = O)\max)T +$, *Theorem 1 shows that the IS-MBPG algorithm has* $O)\sqrt{\max)T} + / T^{\frac{1}{3}} +$*convergence rate. The IS-MBPG algorithm needs* 2 *trajectory to estimate the stochastic policy gradient* u_t *at each iteration, and needs* T *iterations. Without loss of generality, we omit a relative small term* $\sqrt{\max)T} +$. *By* $T^{-\frac{1}{3}} \geq \epsilon$, *we choose* T [ϵ^{-3}. *Thus, the IS-MBPG has the sample complexity of* $2 \times T$ [$O)\epsilon^{-3} +$*for finding an* ϵ*-stationary point.*

4.2. Convergence Analysis of HA-MBPG Algorithm

In the subsection, we analyze the convergence properties of the HA-MBPG algorithm.

Theorem 2. *Assume that the sequence* $\{\theta_t\}_{t=1}^T$ *be generated from Algorithm 2, and let* k [$O)\frac{G^{2/3}}{L} + c$ [$\frac{G^2}{3k^3 L}$ 0 $63L^2$, m [$\max\{3G^2,)3Lk +^3,)\frac{ck}{2L} +^3|$ *and* η_0 [$\frac{k}{m^{1/3}}$, *we have*

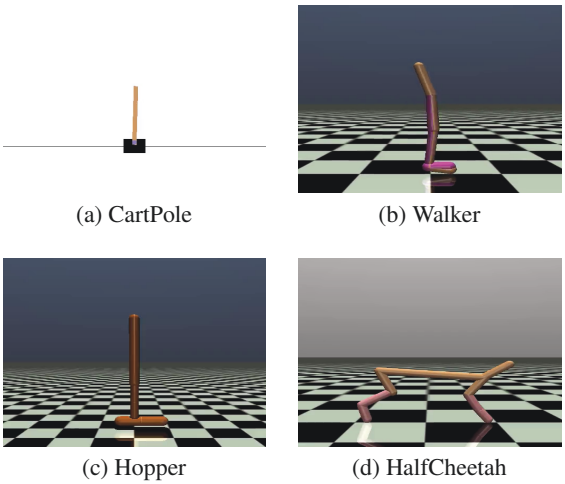
$$\mathbb{E}\sqrt{\nabla J(\theta_\zeta) +} \geq \frac{\sqrt{3\Phi} m^{1/6} 0\ 3\Phi^{3/4}}{\sqrt{T}} 0\ \frac{3\sqrt{\Phi\sigma^{1/3}}}{T^{1/3}},$$

where Φ [$\frac{1}{k}\ 27) J^* \quad J(\theta_1 + 0\ \frac{m^{1/3}}{4L^2 k}\sigma^2 \quad 0\ \frac{(W+1)c^2 k^3}{2L^2}\ \max)T 0\ 3+$ *with* $J^* = \min_\theta J(\theta) + < 0 \in$.

Remark 2. *Since* Φ [$O)\max)T +$, *Theorem 2 shows that the HA-MBPG algorithm has* $O)\sqrt{\max)T} + / T^{\frac{1}{3}} +$*convergence rate. The HA-MBPG algorithm needs* 2 *trajectory to estimate the stochastic policy gradient* u_t *at each iteration, and needs* T *iterations. Without loss of generality, we omit a relative small term* $\sqrt{\max)T} +$. *By* $T^{-\frac{1}{3}} \geq \epsilon$, *we choose* T [ϵ^{-3}. *Thus, the HA-MBPG has the sample complexity of* $2 \times T$ [$O)\epsilon^{-3} +$*for finding an* ϵ*-stationary point.*

4.3. Convergence Analysis of IS-MBPG* Algorithm

In the subsection, we give the convergence properties of the IS-MBPG* algorithm.



(a) CartPole (b) Walker (c) Hopper (d) HalfCheetah

Figure 1. Four environments we used. (a) Cartpole: balance a pole on a cart; (b) Walker: make a 2D robot walk; (c) Hopper: make a 2D robot hop; (d) HalfCheetah: make a 2D cheetah robot run.

Theorem 3. *Assume that the sequence* $\{\theta_t\}_{t=1}^T$ *be generated from Algorithm 3, and let* B^2 [L^2 0 $3G^2 C_w^2$, $k > 1$ c [$\frac{1}{3k^3 L}$ 0 $215B^2$, m [$\max\{3,)3Lk +^3,)\frac{ck}{2L} +^3|$ *and* η_0 [$\frac{k}{m^{1/3}}$, *we have*

$$\mathbb{E}\sqrt{\nabla J(\theta_\zeta) +}\Big[\ \frac{2}{T}\prod_{t=1}^{T}\mathbb{E}\sqrt{\nabla J(\theta_t) +} \geq \frac{\sqrt{\nabla -}\, m^{1/6}}{\sqrt{T}} 0\ \frac{\sqrt{\nabla -}}{T^{1/3}},$$

where [$\frac{1}{k}\ 27) J^* \quad J(\theta_1 + 0\ \frac{m^{1/3}}{8B^2 k}\sigma^2 0\ \frac{c^2 k^3 \sigma^2}{4B^2}\ \max)T 0\ 3+$ *with* $J^* = \min_\theta J(\theta) + < 0 \in$.

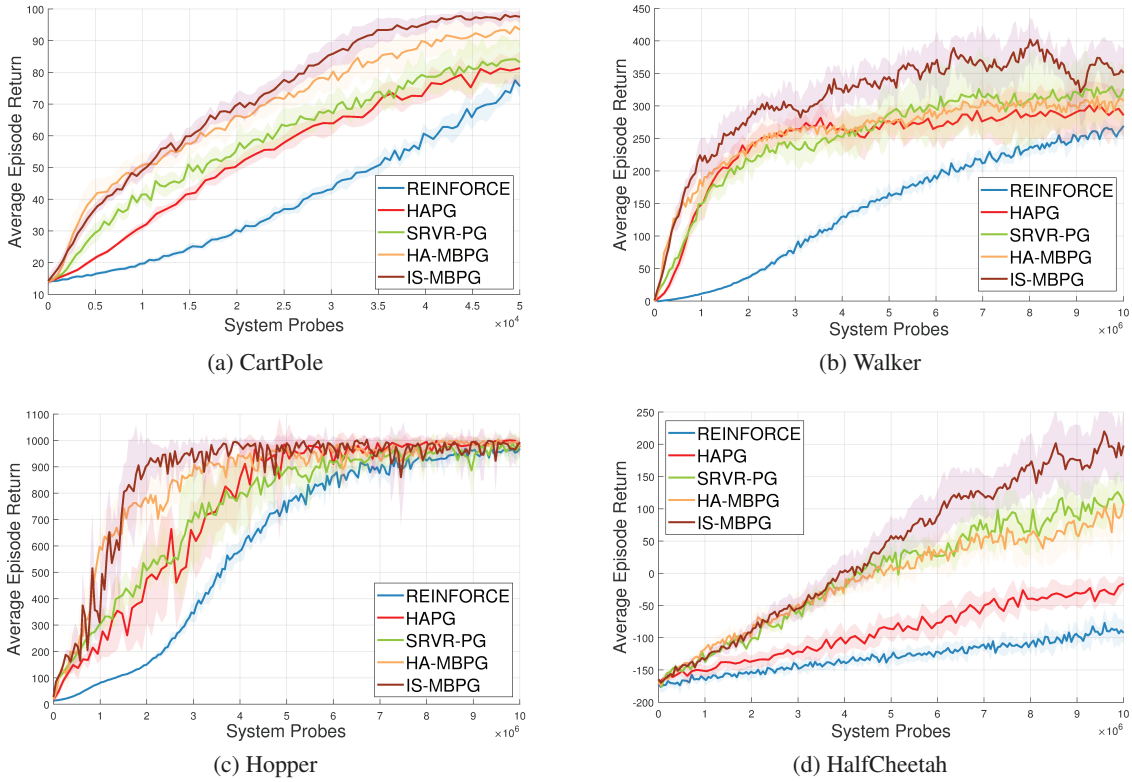
Remark 3. *Since* [$O)\max)T +$, *Theorem 3 shows that the IS-MBPG* algorithm has* $O)\sqrt{\max)T} + / T^{\frac{1}{3}} +$*convergence rate. The IS-MBPG* algorithm needs* 2 *trajectory to estimate the stochastic policy gradient* u_t *at each iteration, and needs* T *iterations. Without loss of generality, we omit a relative small term* $\sqrt{\max)T} +$. *By* $T^{-\frac{1}{3}} \geq \epsilon$, *we choose* T [ϵ^{-3}. *Thus, the IS-MBPG* also has the sample complexity of* $2 \times T$ [$O)\epsilon^{-3} +$*for finding an* ϵ*-stationary point.*

5. Experiments

In this section, we demonstrate the performance of our algorithms on four standard reinforcement learning tasks, which are CartPole, Walker, HalfCheetah and Hopper. The first one is a discrete task from classic control, and the later three tasks are continuous RL task, which are popular MuJoCo environments (Todorov et al., 2012). Detailed description of these environments is shown in Fig. 1.

5.1. Experimental Setup

In the experiment, we use Categorical Policy for CartPole, and Gaussian Policy for all the other environments. All Policies are parameterized by the fully connected neu-



(a) CartPole (b) Walker

(c) Hopper (d) HalfCheetah

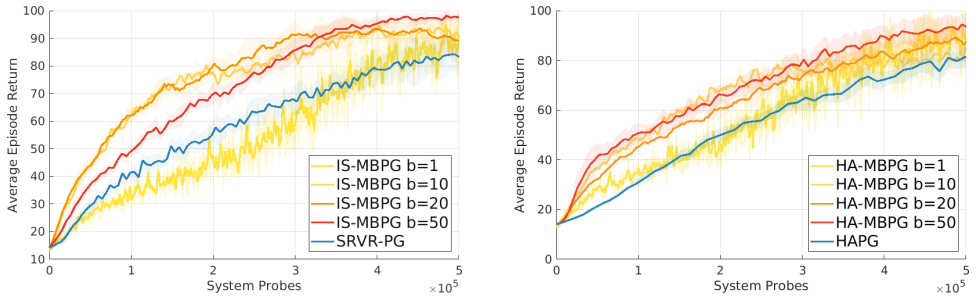
Figure 2. Experimental results of our algorithms (IS-MBPG and HA-MBPG) and baseline algorithms at four environments.

ral network. The detail of network architecture and activation function used are shown in the Appendix A. The network settings are similar to the HAPG (Shen et al., 2019) algorithm. We implement our algorithms by using garage (garage contributors, 2019) and pytorch (Paszke et al., 2019). Note that Previous works mostly use environments implemented by old versions of garage, while latest version of garage directly use environments from gym (Brockman et al., 2016). As a result, there might be an inconsistency of the reward calculation between this paper and previous works due to the difference of environment implementation.

In the experiments, we compare our algorithm with the existing two best algorithms: Hessian Aided Policy Gradient (HAPG) (Shen et al., 2019), Stochastic Recursive Variance Reduced Policy Gradient (SRVR-PG) (Xu et al., 2019b) and a baseline algorithm: REINFORCE (Sutton et al., 2000). For a fair comparison, the policies of all methods use the same initialization, which ensures that they have similar start point. Moreover, to ease the impact of randomness, we run each method 10 times, and plot mean as well as variance interval for each of them.

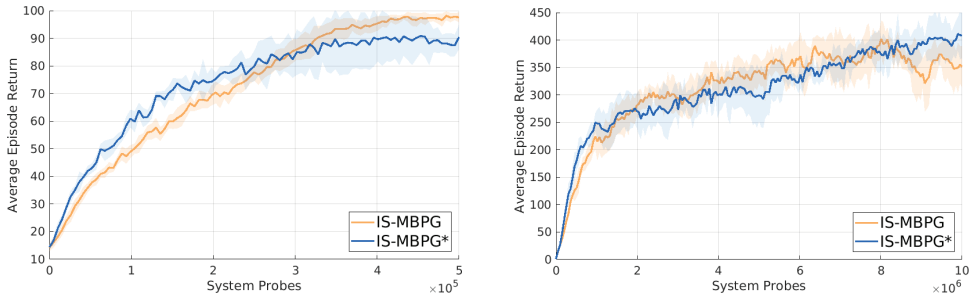
In addition, for the purpose of fair comparison, we use the same batch size $|\mathcal{M}|$ for all algorithms, though our algorithms do not have a requirement on it. HAPG and SRVR-PG have sub-iterations (or inner loop), and requires additional hyper-parameters. The inner batch size for HAPG and SRVR-PG is also set to be the same value. For all the other hyper-parameters, we try to make them be analogous to the settings in their original paper. One may argue that our algorithms need three hyper-parameters k, m and c to control the evolution of learning rate while for other algorithms one hyper parameter is enough to control the learning rate. However, it should be noticed that our algorithms do not involve any sub-iterations unlike HAPG and SRVR-PG. Introducing sub-iterations itself naturally bring more hyper-parameters such as the number of sub-iteration and the inner batch size. From this perspective, the hyper-parameter complexity of our algorithms resembles HAPG and SRVR-PG. The more details of hyper-parameter selection are shown in the Appendix A.

Similar to the HAPG algorithm, we use the **system probes** (i.e., the number of state transitions) as the measurement of sample complexity instead of number of trajectories. The reason of doing so is because each trajectory may have different length of states due to a failure flag returned from the environment (often happens at the beginning of training). Besides this reason, if using the number of trajectories as complexity measurement and the environment can return a failure flag, a faster algorithm may have a lot more system probes given the same number of trajectories. We also



(a) IS-MBPG (b) HA-MBPG

Figure 3. Different batch sizes for our algorithms (IS-MBPG and HA-MBPG) at CartPole environment.



(a) CartPole (b) Walker

Figure 4. Results of IS-MBPG and IS-MBPG* algorithms at CartPole and Walker environment.

use average episode return as used in HAPG (Shen et al., 2019).

5.2. Experimental Results

The results of experiments are presented in Fig. 2. In the CartPole environment, our IS-MBPG and HA-MBPG algorithms have better performances than the other methods. In the Walker environment, our algorithms start to have more advantages. Specifically, the average return of IS-MBPG and HA-MBPG grows rapidly at the beginning of training. Moreover, our IS-MBPG algorithm achieves the best final performance with a obvious margin. HA-MBPG performs similar compared to SRVR-PG and HAPG, though it has an advantage at the beginning. In Hopper environment, our IS-MBPG and HA-MBPG algorithms are significantly faster compared to all other methods, while the final average reward are similar for different algorithms. In HalfCheetah environment, IS-MBPG, HA-MBPG and SRVR-PG performs similarly at the beginning. In the end of training, IS-MBPG can achieve the best performance. We note that HAPG performs poorly on this task, which is probably because of the normalized gradient and fixed learning rate in their algorithm. For all tasks, HA-MBPG are always inferior to the IS-MBPG. One possible reason for this observation is that we use the estimated Hessian vector product instead of the exact Hessian vector product in HA-MBPG algorithm, which brings additional estimation error to the algorithm.

In Fig. 3, we plot the average reward when changing batch size in CartPole environment. From Fig. 3, we find that when $31($ of the original batch size, our HA-MBPG and IS-MBPG algorithms still outperform the HAPG and SRVR-PG algorithms, respectively. When the batch size is 1, our HA-MBPG and IS-MBPG algorithms still reach a good performance. These results demonstrate that our HA-MBPG and IS-MBPG algorithms are not sensitive to the selection of batch size. Fig. 4 shows that the non-adaptive IS-MBPG* algorithm also has similar performances as the adaptive IS-MBPG algorithm.

6. Conclusion

In the paper, we proposed a class of efficient momentum-based policy gradient methods, which do not require large batches and use adaptive learning rates. Specifically, we proposed a fast IS-MBPG method by using the importance sampling technique. At the same time, we also presented a fast HA-MBPG method via using the semi-Hessian information. Moreover, we proved that both IS-MBPG and HA-MBPG methods reach the best known sample complexity $O)\epsilon^{-3}+$without large batches. In particular, we also presented a non-adaptive version of IS-MBPG method (i.e., IS-MBPG*), which has a simple monotonically decreasing learning rate. We proved that the IS-MBPG* also reaches the best known sample complexity of $O)\epsilon^{-3}+$without large batches.

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Environments	CartPole	Walker	Hopper	HalfCheetah
Horizon	100	500	1000	500
Baseline	None	Linear	Linear	Linear
Neural Network sizes	$9 \bigcirc 9$	$75 \bigcirc 75$	$75 \bigcirc 75$	$75 \bigcirc 75$
Number of timesteps	$6 \bigcirc 21^5$	$2 \bigcirc 21^7$	$2 \bigcirc 21^7$	$2 \bigcirc 21^7$
Batch size $\|\mathcal{M}\|$	50	100	50	100
HAPG $\|\mathcal{M}_{\text{sub}}\|$	10	10	10	10
SRVR-PG $\|\mathcal{M}_{\text{sub}}\|$	10	10	10	10
HAPG n_{sub}	5	10	10	10
SRVR-PG n_{sub}	3	2	2	2
IS-MBPG/HA-MBPG k	0.75	0.75	0.75	0.75
IS-MBPG/HA-MBPG c	2	2	1	1
IS-MBPG/HA-MBPG m	2	12	3	3
REINFORCE learning rate η	0.01	0.01	0.01	0.01
HAPG learning rate η	0.01	0.01	0.01	0.01
SRVR-PG learning rate η	0.1	0.1	0.1	0.1

Table 2. Hyper-parameter Details. $|\mathcal{B}_{\text{sub}}|$ represents sub-iteration (inner-loop) batch size. n_{sub} represents number of sub-iterations, which is called p in the original paper of HAPG and m in the oringinal paper of SRVR-PG. Although the learning rate η of HAPG is given a small value $\eta = \epsilon$ in the theoretical analysis, we choose learning rate $\eta = 0.01$ as given in the experiments of (Shen et al., 2019).

A. Supplementary Materials for "Momentum-Based Policy Gradient Methods"

In this section, we first provide the details of hyper-parameter selection for the algorithms in Table 2. Table 2 also shows that the detail of network architecture and activation function used in the experiments. Next, we study the convergence properties of our algorithms. We begin with giving some useful lemmas.

Lemma 1. *(Lemma 1 in (Cortes et al., 2010)) Let $w(x) = P(x)/Q(x)$ be the importance weight for distributions P and Q. The following identities hold for the expectation, second moment, and variance of $w(x)$*

$$\mathbb{E}[w(x)] = 1, \ \mathbb{E}[w^2(x)] = d_2(P\|Q), \ \mathbb{V}[w(x)] = d_2(P\|Q) - 1, \tag{20}$$

where $d_2(P\|Q) = 2^{D(P\|Q)}$, and $D(P\|Q)$ is the Rēnyi divergence between distributions P and Q.

Lemma 2. *Under Assumptions 1 and 3, let $w(\tau|\theta_{t-1}, \theta_t) = g(\tau|\theta_{t-1})/g(\tau|\theta_t)$, we have*

$$\mathbb{V}[w(\tau|\theta_{t-1}, \theta_t)] \leq C_w^2 \sqrt{\|\theta_t - \theta_{t-1}\|^2}, \tag{21}$$

where $C_w = \sqrt{H(3HM_g^2(M_h - 1)W^2 + 2)}$.

Proof. This proof can easy follow the proof of Lemma 6.1 in (Xu et al., 2019a). □

Lemma 3. *Under Assumption 1, let $e_t = \nabla J(\theta_t) - u_t$. Given $1 \geq \eta_t \geq \frac{1}{2L}$ for all $t \geq 2$, we have*

$$\mathbb{E}[\|\nabla J(\theta_{t+1}) - u_{t+1}\|^2] \leq \mathbb{E}[\|\nabla J(\theta_t) - u_t\|^2] - \frac{4\eta_t}{5}\|e_t\|^2 + \frac{\eta_t}{9}\|\nabla J(\theta_t)\|^2. \tag{22}$$

Proof. Let $e_t = \nabla J(\theta_t) - u_t$. By using $J(\theta)$ is L-smooth, we have

$$\begin{aligned}
\mathbb{E}[\|\nabla J(\theta_{t+1}) - u_{t+1}\|^2] &\leq \mathbb{E}[\|\nabla J(\theta_t) - u_t\|^2] + \|\nabla J(\theta_{t+1}) - \nabla J(\theta_t)\|^2 + \frac{L}{3}\|\theta_{t+1} - \theta_t\|^2 - [\mathbb{E}[\|\nabla J(\theta_t) - u_t\|^2] \\
&\quad + \eta_t \|\nabla J(\theta_t) - u_t\|^2 + \frac{L\eta_t^2}{3}\|u_t\|^2] \\
&\leq (1 - \eta_t)\|\nabla J(\theta_t) - u_t\|^2 + \eta_t\|\nabla J(\theta_t) - u_t\|^2 + \frac{L\eta_t^2}{3}\|u_t\|^2 \\
&\leq \frac{\eta_t}{3}\|\nabla J(\theta_t) - u_t\|^2 + (\frac{\eta_t}{3} - 2)\|L\eta_t(u_t - \nabla J(\theta_t))\|^2 \\
&\leq \frac{\eta_t}{3}\|\nabla J(\theta_t) - u_t\|^2 - \frac{\eta_t}{5}\|u_t - \nabla J(\theta_t)\|^2 \\
&\leq \frac{4\eta_t}{5}\|e_t\|^2 - \frac{\eta_t}{9}\|\nabla J(\theta_t)\|^2,
\end{aligned} \tag{23}$$

where the second inequality holds by Young's inequality, and the third inequality holds by $1 < \eta_t \geq \frac{1}{2L}$, and the last inequality follows by $\nabla J(\theta_t)\|^2 \geq 3\|\nabla J(\theta_t) - u_t\|^2 [3\|u_t\|^2 0\ 3\|\nabla J(\theta_t)\|^2$. □

A.1. Convergence Analysis of IS-MBPG Algorithm

In this subsection, we analyze the convergence properties of IS-MBPG algorithm. For notational simplicity, let $J(\theta)$ denote $\nabla_\theta J(\theta)$.

Lemma 4. *Assume that the stochastic policy gradient u_t be generated from Algorithm 1, and let $e_t = u_t - \nabla J(\theta_t)$, we have*

$$\mathbb{E}\|\eta_{t-1}^{-1} e_t\|^2 \leq (1-\eta_t)^2 \|e_{t-1}\|^2 + 3\beta_t^2\eta_{t-1}^{-1}G_t^2 + (1-\eta_t)^2\|\nabla J(\theta_t) - \nabla J(\theta_{t-1})\|^2 + 3\beta_t^2 B^2\eta_{t-1}^2 + B^2\eta_{t-1}\|\nabla J(\theta_{t-1})\|^2,$$

where $B^2 = L^2 + 3G^2C_w^2$ with $C_w = \sqrt{H(3HM_g^2 + M_h^2)W^{-2}}$.

Proof. By the definition of u_t in Algorithm 1, we have

$$u_t - u_{t-1} = \beta_t u_{t-1} + (1-\beta_t)\tau_t(\theta_t) + (1-\beta_t)(g(\tau_t|\theta_t) - g(\tau_t|\theta_{t-1}, \theta_t)). \tag{24}$$

Then we have

$$\begin{aligned}
\mathbb{E}\|\eta_{t-1}^{-1} e_t\|^2 &= \mathbb{E}\|\eta_{t-1}^{-1}(\nabla J(\theta_{t-1}) - u_{t-1}) + (\nabla J(\theta_t) - \nabla J(\theta_{t-1})) + (u_t - u_{t-1})\|^2 \\
&= \mathbb{E}\|\eta_{t-1}^{-1}(\nabla J(\theta_{t-1}) - u_{t-1}) + (\nabla J(\theta_t) - \nabla J(\theta_{t-1})) + (1-\beta_t)u_{t-1} - \beta_t g(\tau_t|\theta_t) \\
&\quad + (1-\beta_t)(g(\tau_t|\theta_t) - g(\tau_t|\theta_{t-1}, \theta_t))\|^2 \\
&= \mathbb{E}\|(1-\eta_{t-1})\eta_{t-1}^{-1}(\nabla J(\theta_{t-1}) - u_{t-1}) + (1-\beta_t)(\nabla J(\theta_t) - \nabla J(\theta_{t-1})) \\
&\quad + (1-\beta_t)(g(\tau_t|\theta_t) - g(\tau_t|\theta_{t-1}, \theta_t)) + \beta_t(\nabla J(\theta_t) - g(\tau_t|\theta_t))\|^2 \\
&= \mathbb{E}\|(1-\eta_{t-1})\eta_{t-1}^{-1}(\nabla J(\theta_{t-1}) - u_{t-1}) + (1-\beta_t)(\nabla J(\theta_t) - \nabla J(\theta_{t-1})) \\
&\quad + (1-\beta_t)(g(\tau_t|\theta_t) - g(\tau_t|\theta_{t-1}, \theta_t)) + \beta_t(\nabla J(\theta_t) - g(\tau_t|\theta_t))\|^2 \\
&\leq (1-\eta_{t-1})^2\mathbb{E}\|\nabla J(\theta_{t-1}) - u_{t-1}\|^2 + 3\beta_t^2\eta_{t-1}^{-1}\mathbb{E}\|\nabla J(\theta_t) - g(\tau_t|\theta_t)\|^2 \\
&\quad + 3(1-\beta_t)^2\mathbb{E}\|g(\tau_t|\theta_t) - g(\tau_t|\theta_{t-1}, \theta_t)\|^2 + (1-\beta_t)^2\|\nabla J(\theta_t) - \nabla J(\theta_{t-1})\|^2 \\
&\leq (1-\eta_{t-1})^2\mathbb{E}\|e_{t-1}\|^2 + 3\beta_t^2\eta_{t-1}^{-1}\mathbb{E}\|g(\tau_t|\theta_t)\|^2 + 3(1-\beta_t)^2\eta_{t-1}^{-1}\mathbb{E}\|g(\tau_t|\theta_t) - w(\tau_t|\theta_{t-1}, \theta_t)g(\tau_t|\theta_{t-1})\|^2 \\
&\quad + (1-\beta_t)^2\|\nabla J(\theta_t) - \nabla J(\theta_{t-1})\|^2 \\
&\leq (1-\eta_{t-1})^2\mathbb{E}\|e_{t-1}\|^2 + 3\beta_t^2\eta_{t-1}^{-1}G_t^2 + 3(1-\beta_t)^2\eta_{t-1}\underbrace{\mathbb{E}\|g(\tau_t|\theta_t) - w(\tau_t|\theta_{t-1}, \theta_t)g(\tau_t|\theta_{t-1})\|^2}_{=T_1},
\end{aligned} \tag{25}$$

where the forth equality holds by $\mathbb{E}_{\tau_t\sim p(\tau|\theta_t)}[g(\tau_t|\theta_t)] = \nabla J(\theta_t)$ and $\mathbb{E}_{\tau_t\sim p(\tau|\theta_t)}[g(\tau_t|\theta_t) - w(\tau_t|\theta_{t-1}, \theta_t)g(\tau_t|\theta_{t-1})] = \nabla J(\theta_t) - \nabla J(\theta_{t-1})$, the first inequality follows by Young's inequality; and the last inequality holds by $\mathbb{E}\|\zeta - \mathbb{E}[\zeta]\|^2 = \mathbb{E}\|\zeta\|^2 - \|\mathbb{E}[\zeta]\|^2 \leq \mathbb{E}\|\zeta\|^2$.

Next, we give an upper bound of the term T_1 as follows:

$$\begin{aligned}
T_1 &= \mathbb{E}\|g(\tau_t|\theta_t) - w(\tau_t|\theta_{t-1}, \theta_t)g(\tau_t|\theta_{t-1})\|^2 \\
&= \mathbb{E}\|g(\tau_t|\theta_t) - g(\tau_t|\theta_{t-1}) + g(\tau_t|\theta_{t-1}) - w(\tau_t|\theta_{t-1}, \theta_t)g(\tau_t|\theta_{t-1})\|^2 \\
&\leq 3\mathbb{E}\|g(\tau_t|\theta_t) - g(\tau_t|\theta_{t-1})\|^2 + 3\mathbb{E}\|(1 - w(\tau_t|\theta_{t-1}, \theta_t))g(\tau_t|\theta_{t-1})\|^2 \\
&\leq 3L^2\|\theta_t - \theta_{t-1}\|^2 + 3G^2\mathbb{E}(1 - w(\tau_t|\theta_{t-1}, \theta_t))^2 \\
&= 3L^2\|\theta_t - \theta_{t-1}\|^2 + 3G^2\mathbb{V}\mathrm{ar}(w(\tau_t|\theta_{t-1}, \theta_t)) \\
&\leq 3(L^2 + 3G^2C_w^2)\|\theta_t - \theta_{t-1}\|^2,
\end{aligned} \tag{26}$$

where the second inequality holds by Proposition 1, and the third equality holds by Lemma 1, and the last inequality follows by Lemma 2.

Combining the inequalities (25) with (26), let $B^2 \geq L^2 + 3G^2C_w^2$, we have

$$\begin{aligned}\mathbb{E}\|\eta_{t-1}^{-1}(\theta_t-\theta_{t-1})+(\eta_{t-1}^{-1}-\beta_t)(\theta_t-\omega_t)\|^2 &\leq (\eta_{t-1}^{-1}-\beta_t)^2\|\theta_t-\theta_{t-1}\|^2+\beta_t^2\mathbb{E}\|\nabla J(\theta_t;\tau_t)-\nabla J(\theta_{t-1};\tau_t)\|^2+3\beta_t^2\eta_{t-1}^{-1}G_t^2\\ &\leq (\eta_{t-1}^{-1}-\beta_t)^2\|\theta_t-\theta_{t-1}\|^2+\beta_t^2L^2\|\theta_t-\theta_{t-1}\|^2+3\beta_t^2\eta_{t-1}^{-1}G_t^2\\ &\leq (\eta_{t-1}^{-1}-\beta_t)^2\|\theta_t-\theta_{t-1}\|^2+\beta_t^2B^2\|\omega_t-\omega_{t-1}\|^2+3\beta_t^2\eta_{t-1}^{-1}G_t^2\\ &\leq (\eta_{t-1}^{-1}-\beta_t)^2\|\theta_t-\theta_{t-1}\|^2+\beta_t^2B^2\eta_{t-1}^2\|\nabla J(\theta_{t-1};\tau_t)\|^2+3\beta_t^2\eta_{t-1}^{-1}G_t^2\\ &\leq 3\beta_t^2\eta_{t-1}^{-1}G_t^2+(\eta_{t-1}^{-1}-\beta_t)^2\eta_{t-1}^2\|\theta_t-\theta_{t-1}\|^2+\beta_t^2B^2\eta_{t-1}^2\|\nabla J(\theta_{t-1};\tau_t)\|^2. \end{aligned} \tag{27}$$

□

Theorem 4. *Assume that the sequence $\{\theta_t\}_{t=1}^T$ be generated from Algorithm 1, and let $B^2 \geq L^2 + 3G^2C_w^2$, $k \geq \frac{G^{2/3}}{L}+c \geq \frac{G^2}{3k^3L} + 215B^2$, $m \geq \max\{3G^2, 3Lk+1, \frac{ck}{2L}+1\}$ and $\eta_0 \geq \frac{k}{m^{1/3}}$, we have*

$$\mathbb{E}\|\nabla J(\theta_\zeta)\| \leq \frac{2}{T}\prod_{t=1}^{T}\mathbb{E}\|\nabla J(\theta_t)\| \leq \frac{\sqrt{3}m^{1/6}+\sqrt{3}\omega^{3/4}}{\sqrt{T}} + \frac{\sqrt{3}\sigma^{1/3}}{T^{1/3}},$$

where $\omega = \frac{1}{k}(J^* - J(\theta_1)) + \frac{m^{1/3}}{8B^2k}\sigma^2 + \frac{c^2k^3}{4B^2}$ and $J^* = \max_\theta J(\theta) < \infty$.

Proof. Due to $m \sim 3Lk+1$, we have $\eta_t \geq \frac{k}{m^{1/3}} \geq \frac{1}{2L}$. Since $\eta_t \geq \frac{1}{2L}$ and $m \sim \frac{ck}{2L}+1$, we have $\beta_{t+1} = c\eta_t^2 \leq \frac{c\eta_t}{2L} \leq \frac{ck}{2Lm^{1/3}} \leq 2$. By Lemma 4, we have

$$\begin{aligned}\mathbb{E}\|\eta_t^{-1}(\theta_{t+1}-\theta_t)-(\eta_t^{-1}-\eta_{t-1}^{-1})(\theta_t-\omega_t)\|^2 &\leq \mathbb{E}\|3\beta_{t+1}^2\eta_t^{-1}G_{t+1}^2+(\eta_t^{-1}-\beta_{t+1})^2\|\theta_{t+1}-\theta_t\|^2\\ &\quad+\beta_{t+1}^2B^2\eta_t^2\|\nabla J(\theta_t)\|^2+(\eta_{t-1}^{-1}-\eta_t^{-1})^2\|\theta_t-\omega_t\|^2\\ &\leq \mathbb{E}\|3c^2\eta_t^3G_{t+1}^2+\underbrace{(\eta_t^{-1}-\beta_{t+1})^2\|\theta_{t+1}-\theta_t\|^2+\eta_{t-1}^{-1}\|\theta_t-\omega_t\|^2}_{=T_2}\\ &\quad+3B^2\eta_t\|\nabla J(\theta_t)\|^2+\eta_{t-1}^{-1}\|\theta_t-\omega_t\|^2\|^2, \end{aligned} \tag{28}$$

where the last inequality holds by $1 < \beta_{t+1} \leq 2$. Since the function $x^{1/3}$ is cancave, we have $(x+y)^{1/3} \leq x^{1/3} + yx^{-2/3}/4$. Then we have

$$\begin{aligned}\eta_t^{-1} - \eta_{t-1}^{-1} + \frac{2}{k} &\leq \frac{m+\sum_{i=1}^{t}G_i^2}{(m+\sum_{i=1}^{t-1}G_i^2)^{1/3}} - (m+\sum_{i=1}^{t-1}G_i^2)^{1/3} \leq \frac{G_t^2}{4(m+\sum_{i=1}^{t-1}G_i^2)^{2/3}}\\ &\leq \frac{G_t^2}{4(m+G^2+\sum_{i=1}^{t}G_i^2)^{2/3}} \leq \frac{G_t^2}{4(m/3+\sum_{i=1}^{t}G_i^2)^{2/3}} \leq \frac{3^{2/3}G_t^2}{4(m+\sum_{i=1}^{t}G_i^2)^{2/3}}\\ &\leq \frac{3^{2/3}G^2}{4k^3}\eta_t^2 \leq \frac{3^{2/3}G^2}{7k^3L}\eta_t \leq \frac{G^2}{4k^3L}\eta_t, \end{aligned} \tag{29}$$

where the third inequality holds by $m \sim 3G^2$, and the sixth inequality holds by $1 < \eta \leq \frac{1}{2L}$.

Next, considering the upper bound of the term T_2, we have

$$\begin{aligned}T_2 &= (\eta_t^{-1}-\beta_{t+1})^2\|\theta_{t+1}-\theta_t\|^2+\eta_{t-1}^{-1}\|\theta_t-\omega_t\|^2\\ &= (\eta_t^{-1}-\eta_{t-1}^{-1}+\eta_{t-1}^{-1}-\beta_{t+1})^2\|\theta_{t+1}-\theta_t\|^2+\eta_{t-1}^{-1}\|\theta_t-\omega_t\|^2\\ &\leq (\eta_t^{-1}-\eta_{t-1}^{-1})^2\|\theta_{t+1}-\theta_t\|^2+2(\eta_t^{-1}-\eta_{t-1}^{-1})\|\theta_{t+1}-\theta_t\|\|\theta_t-\omega_t\|+\eta_{t-1}^{-1}\|\theta_t-\omega_t\|^2\\ &\leq (\eta_t^{-1}-\eta_{t-1}^{-1})^2\eta_t^2\|\nabla J(\theta_t;\tau_t)\|^2+2(\eta_t^{-1}-\eta_{t-1}^{-1})\eta_t\|\nabla J(\theta_t;\tau_t)\|\|\theta_t-\omega_t\|+\eta_{t-1}^{-1}\|\theta_t-\omega_t\|^2\\ &\leq (\eta_t^{-1}-\eta_{t-1}^{-1})\eta_t\|\nabla J(\theta_t;\tau_t)\|+\eta_{t-1}^{-1}\|\theta_t-\omega_t\|^2\\ &\leq \frac{G^2}{4k^3L}\eta_t+3B^2\eta_t\|\nabla J(\theta_t;\tau_t)\|^2+c\eta_t\|\nabla J(\theta_t;\tau_t)\|^2+\|\theta_t-\omega_t\|^2, \end{aligned} \tag{30}$$

where the last equality holds by $c \geq \frac{G^2}{3k^3L} + 215B^2$. Combining the inequalities (28) with (30), we have

$$\mathbb{E}\|\eta_t^{-1}(\theta_{t+1}-\theta_t)-(\eta_t^{-1}-\eta_{t-1}^{-1})(\theta_t-\omega_t)\|^2 \leq \mathbb{E}\|3c^2\eta_t^3G_{t+1}^2+\|\theta_t-\omega_t\|^2+3B^2\eta_t\|\nabla J(\theta_t;\tau_t)\|^2+3B^2\eta_t\|\nabla J(\theta_t)\|^2, \tag{31}$$

We define a *Lyapunov* function $\Psi_t = J(\theta_t) + \frac{1}{128B^2\eta_{t-1}}\|e_t\|^2$ for any $t \geq 2$. Then we have

$$\begin{aligned}
\mathbb{E}[\Psi_{t+1} - \Psi_t] &= \mathbb{E}\big[J(\theta_{t+1}) + J(\theta_t) + \frac{2}{239B^2\eta_t}\|e_{t+1}\|^2 - \frac{2}{239B^2\eta_{t-1}}\|e_t\|^2\big] \\
&\leq \mathbb{E}\big[\frac{4\eta_t}{5}\|e_t\|^2 - \frac{\eta_t}{9}\|\nabla J(\theta_t)\|^2 + \frac{2}{239B^2}(\eta_t^{-1}\|e_{t+1}\|^2 - \eta_{t-1}^{-1}\|e_t\|^2)\big] \\
&\leq \frac{c^2\eta_t^3 G_{t+1}^2}{75B^2} - \frac{\eta_t}{27}\mathbb{E}\|\nabla J(\theta_t)\|^2, \qquad (32)
\end{aligned}$$

where the first inequality holds by the Lemma 3, and the second inequality follows by the above inequality (31). Summing the above inequality (32) over t from 2 to T, we obtain

$$\begin{aligned}
\sum_{t=1}^{T}\mathbb{E}[\eta_t\|\nabla J(\theta_t)\|^2] &\geq \mathbb{E}[27(J^* - J(\theta_1))] - \Psi_{T+1} - \Psi_1 + \sum_{t=1}^{T}\frac{c^2\eta_t^3 G_{t+1}^2}{5B^2} \\
&\geq \mathbb{E}[27(J^* - J(\theta_1))] - \frac{2}{9B^2\eta_0}\mathbb{E}\|e_1\|^2 + \frac{c^2k^3}{5B^2}\sum_{t=1}^{T}\frac{G_{t+1}^2}{(m+\sum_{i=1}^{t}G_i^2)^{1/3}} \\
&\geq \mathbb{E}[27(J^* - J(\theta_1))] - \frac{2}{9B^2\eta_0}\mathbb{E}\|e_1\|^2 + \frac{c^2k^3}{5B^2}\sum_{t=1}^{T}\frac{G_{t+1}^2}{(G^2+\sum_{i=1}^{t+1}G_i^2)^{1/3}} \\
&\geq \mathbb{E}[27(J^* - J(\theta_1))] - \frac{2}{9B^2\eta_0}\mathbb{E}\|e_1\|^2 + \frac{c^2k^3}{5B^2}\sum_{t=1}^{T}(m+G^2+\sum_{i=1}^{t+1}G_i^2)^{1/3} - (m+G^2+\sum_{i=1}^{t}G_i^2)^{1/3}] \\
&\geq \mathbb{E}[27(J^* - J(\theta_1))] - \frac{2}{9B^2\eta_0}\mathbb{E}\|e_1\|^2 + \frac{c^2k^3}{5B^2})(m+G^2+\sum_{i=1}^{T+1}G_i^2)^{1/3} - (m+G^2+\cdots \\
&\geq \mathbb{E}[27(J^* - J(\theta_1))] - \frac{m^{1/3}}{9B^2k}\sigma^2 - \frac{c^2k^3}{5B^2}(m+G^2)^{1/3}\sum_{i=1}^{T+1}\frac{G_i^2}{G^2} + \\
&\geq \mathbb{E}[27(J^* - J(\theta_1))] - \frac{m^{1/3}}{9B^2k}\sigma^2 - \frac{c^2k^3}{5B^2}(m+G^2)^{1/3}T^{2/3}, \qquad (33)
\end{aligned}$$

where $J^* = \max_\theta J(\theta) < \infty$, and the fourth inequality holds by the concavity of the function $(\cdot)^{1/3}$, and the sixth inequality holds by the definition of e_1 and η_0.

By Cauchy-Schwarz inequality, we have $\mathbb{E}[XY]^2 \leq \mathbb{E}[X^2]\mathbb{E}[Y^2]$. Let $X = \sqrt{\eta_T\sum_{t=1}^{T-1}\|\nabla J(\theta_t)\|^2}$ and $Y = \sqrt{2/\eta_T}$, we have

$$\mathbb{E}\Big[\sum_{t=1}^{T}\|\nabla J(\theta_t)\|^2\Big] \leq \mathbb{E}[2/\eta_T]\,\mathbb{E}\Big[\eta_T\sum_{t=1}^{T}\|\nabla J(\theta_t)\|^2\Big]. \qquad (34)$$

Since η_t is decreasing, we have

$$\mathbb{E}\Big[\sum_{t=1}^{T}\|\nabla J(\theta_t)\|^2\Big] \leq \mathbb{E}[2/\eta_T]\,\mathbb{E}\Big[\sum_{t=1}^{T}\eta_T\|\nabla J(\theta_t)\|^2\Big] \leq \mathbb{E}[2/\eta_T]\,\mathbb{E}\Big[\sum_{t=1}^{T}\eta_t\|\nabla J(\theta_t)\|^2\Big]. \qquad (35)$$

Combining the inequalities (33) and (35), we obtain

$$\begin{aligned}
\mathbb{E}\Big[\sum_{t=1}^{T}\|\nabla J(\theta_t)\|^2\Big] &\leq \mathbb{E}\Big[\frac{27(J^* - J(\theta_1)) + \frac{m^{1/3}}{8B^2k}\sigma^2 + \frac{c^2k^3}{4B^2}(m+G^2)^{1/3}T^{2/3}}{\eta_T}\Big] \\
&\quad + [\mathbb{E}]' \cdot (m+\sum_{t=1}^{T}G_t^2)^{1/3}, \qquad (36)
\end{aligned}$$

where $' = \frac{1}{k}\,27(J^* - J(\theta_1)) + \frac{m^{1/3}}{8B^2k}\sigma^2 + \frac{c^2k^3}{4B^2}(m+G^2)^{1/3}T^{2/3}]$.

By Assumption 2, we have $G_t^2 [\quad g)\tau|\theta_t+ \quad J)\theta_t+0 \quad J)\theta_t+^2 \geq 3\ g)\tau|\theta_t+ \quad J)\theta_t+^2 0\ 3 \quad J)\theta_t+^2 \geq 3\sigma^2 0\ 3 \quad J)\theta_t+^2$. Then using the inequality $)a 0\ b+^{1/3} \geq a^{1/3} 0\ b^{1/3}$ for all $a, b > 1$ to the inequality (36), we obtain

$$\begin{aligned}
\Big)\mathbb{E}\Big[\overline{\prod_{t=1}^{T} J)\theta_t+^2}\Big]^2 &\geq \mathbb{E}\Big]\Big[\overline{\prod_{t=1}^{T} J)\theta_t+^2}\Big]^{\sum 2} [\ \mathbb{E}] \prod_{t=1}^{T} J)\theta_t+^2\Big(\\
&\geq \mathbb{E}\Big]\ ')m 0\ 3T\sigma^2+^{1/3} 0\ 3^{1/3\,\prime}\ \prod_{t=1}^{T} J)\theta_t+^2 [^{1/3\sum} \\
&[\ \ ')m 0\ 3T\sigma^2+^{1/3} 0\ 3^{1/3\,\prime}\mathbb{E}\Big]\Big[\overline{\prod_{t=1}^{T} J)\theta_t+^2}\Big]^{\sum 2/3} \\
&\geq\ ')m 0\ 3T\sigma^2+^{1/3} 0\ 3^{1/3\,\prime}\Big]\mathbb{E}\Big[\overline{\prod_{t=1}^{T} J)\theta_t+^2}\Big]^{\sum 2/3},
\end{aligned} \tag{37}$$

where the first inequality holds by the convexity of the function x^2, and the last inequality holds by the concavity of the function $x^{2/3}$. For simplicity, let $Z [\ \sqrt{\int_{t=1}^{T} J)\theta_t+^2}$, we have

$$\mathbb{E}]Z\hat{}\big[^2 \geq\ ')m 0\ 3T\sigma^2+^{1/3} 0\ 3^{1/3\,\prime}\ \ \mathbb{E}]Z\hat{}\big[^{2/3}. \tag{38}$$

The inequality (38) implies that $\mathbb{E}]Z\hat{}\big[^2 \geq 3')m 0\ 3T\sigma^2+^{1/3}$ or $\mathbb{E}]Z\hat{}\big[^2 \geq 3 \times 3^{1/3\,\prime}\ \mathbb{E}]Z\hat{}\big[^{2/3}$. Thus, we have

$$\mathbb{E}]Z\hat{} \geq \frac{\nabla}{3'})m 0\ 3T\sigma^2+^{1/6} 0\ 3'^{3/4}. \tag{39}$$

By Cauchy-Schwarz inequality, then we have

$$\begin{aligned}
\frac{2}{T}\prod_{t=1}^{T}\mathbb{E}\ J)\theta_t+[\ \ \mathbb{E}]\frac{2}{T}\prod_{t=1}^{T} J)\theta_t+\Big(&\geq \mathbb{E}\Big]\sqrt{\frac{2}{T}}\Big[\overline{\prod_{t=1}^{T} J)\theta_t+^2}\Big]^{\sum} \\
&\geq \frac{\nabla}{3'})m 0\ 3T\sigma^2+^{1/6} 0\ 3'^{3/4}}{\nabla T} \\
&\geq \frac{\nabla}{3'}m^{1/6} 0\ 3'^{3/4}}{\nabla T} 0\ \frac{3\nabla'\sigma^{1/3}}{T^{1/3}},
\end{aligned} \tag{40}$$

where the last inequality follows by the inequality $)a 0\ b+^{1/6} \geq a^{1/6} 0\ b^{1/6}$ for all $a, b > 1$. □

A.2. Convergence Analysis of HA-MBPG Algorithm

In this subsection, we analyze the convergence properties of HA-MBPG algorithm.

Lemma 5. *Assume that the stochastic policy gradient u_t be generated from Algorithm 2. Let $e_t [\ u_t \quad J)\theta_t+$ we have*

$$\mathbb{E}]\eta_{t-1}^{-1}\ e_t\ ^2\Big(\geq 5)W 0\ 2+\beta_t^2\eta_{t-1}^{-1}G_t^2 0\ \eta_{t-1}^{-1})2 \quad \beta_t+^2\ 2 0\ 5\eta_{t-1}^2L^2[\mathbb{E}\ e_{t-1}\ ^2 0\ 5)2 \quad \beta_t+^2L^2\eta_{t-1} J)\theta_{t-1}+^2.$$

Proof. By the definition of u_t in Algorithm 2, we have

$$u_t \quad u_{t-1} [\quad \beta_t u_{t-1} 0\ \beta_t w)\tau_t|\theta_t, \theta_t)\alpha+g)\tau_t|\theta_t+0\ )2 \quad \beta_t+\Lambda_t. \tag{41}$$

Then we have

$$
\begin{aligned}
\mathbb{E}\big\|\eta_{t-1}^{-1}\sqrt{e_t}\sqrt{}^2\Big(&\big[\ \mathbb{E}\big\|\eta_{t-1}^{-1}\sqrt{}\ J)\theta_{t-1}+\ u_{t-1}\,0\ J)\theta_t+\ J)\theta_{t-1}+\ )u_t\ u_{t-1}+^2\sqrt{}\Big(\\
&\big[\ \mathbb{E}\big\|\eta_{t-1}^{-1}\sqrt{}\ J)\theta_{t-1}+\ u_{t-1}\,0\ J)\theta_t+\ J)\theta_{t-1}+0\ \beta_t u_{t-1}\ \beta_t w)\tau_t|\theta_t,\theta_t)\alpha+g)\tau_t|\theta_t+\ )2\ \beta_t+\Lambda_t\Big(\\
&\big[\ \mathbb{E}\big\|\eta_{t-1}^{-1}\sqrt{}\ )2\ \beta_t+)\ J)\theta_{t-1}+\ u_{t-1}+0\ \beta_t)\ J)\theta_t+\ w)\tau_t|\theta_t,\theta_t)\alpha+g)\tau_t|\theta_t++ \\
&\quad)2\ \beta_t+\Lambda_t\ )\ J)\theta_t+\ J)\theta_{t-1}++\big[\sqrt{}^2\Big(\\
&\big[\ \eta_{t-1}^{-1})2\ \beta_t+^2\mathbb{E}\sqrt{}\ J)\theta_{t-1}+\ u_{t-1}\sqrt{}^2\,0\ \eta_{t-1}^{-1}\mathbb{E}\big\|\sqrt{}\ \beta_t)\ J)\theta_t+\ w)\tau_t|\theta_t,\theta_t)\alpha+g)\tau_t|\theta_t++ \\
&\quad)2\ \beta_t+\Lambda_t\ )\ J)\theta_t+\ J)\theta_{t-1}++\big[\sqrt{}^2\Big(\\
&\geq \eta_{t-1}^{-1})2\ \beta_t+^2\mathbb{E}\sqrt{}\ J)\theta_{t-1}+\ u_{t-1}\sqrt{}^2\,0\ 3\beta_t^2\eta_{t-1}^{-1}\mathbb{E}\sqrt{}\ J)\theta_t+\ w)\tau_t|\theta_t,\theta_t)\alpha+g)\tau_t|\theta_t+\sqrt{}^2 \\
&\quad 0\ 3)2\ \beta_t+^2\mathbb{E}\sqrt{}\Lambda_t\ )\ J)\theta_t+\ J)\theta_{t-1}++\sqrt{}^2 \\
&\geq \eta_{t-1}^{-1})2\ \beta_t+^2\mathbb{E}\sqrt{e_{t-1}}\sqrt{}^2\,0\ 3\beta_t^2\eta_{t-1}^{-1}\underbrace{\mathbb{E}\sqrt{}\ w)\tau_t|\theta_t,\theta_t)\alpha+g)\tau_t|\theta_t+\sqrt{}^2}_{=T_3}\,0\ 3)2\ \beta_t+^2\eta_{t-1}^{-1}\mathbb{E}\sqrt{\Lambda_t}\sqrt{}^2,
\end{aligned}
\tag{42}
$$

where the forth equality holds by $\mathbb{E}_{\tau_t\sim p(\tau|\theta_t(\alpha))}]w)\tau_t|\theta_t,\theta_t)\alpha+g)\tau_t|\theta_t+\hat{}[\ \ J)\theta_t+$and $\mathbb{E}_{\tau_t\sim p(\tau|\theta_t(\alpha))}]\Lambda_t\hat{}[\ \ J)\theta_t+$ $J)\theta_{t-1}+$; the first inequality follows by Young's inequality; and the last inequality holds by $\mathbb{E}\sqrt{\zeta}\ \ \mathbb{E}]\zeta\hat{}\sqrt{}^2\ [\ \ \mathbb{E}\sqrt{\zeta}\sqrt{}^2$ $)\mathbb{E}]\zeta\hat{}+^2\geq\mathbb{E}\sqrt{\zeta}\sqrt{}^2$.

Next, we give an upper bound of the term T_3 as follows:

$$
\begin{aligned}
T_3\ &[\ \ \mathbb{E}\sqrt{}\ w)\tau_t|\theta_t,\theta_t)\alpha+g)\tau_t|\theta_t+\sqrt{}^2 \\
&\ \ [\ \ \mathbb{E}\sqrt{}\ w)\tau_t|\theta_t,\theta_t)\alpha+g)\tau_t|\theta_t+\sqrt{}\ \ g)\tau_t|\theta_t+0\ \ g)\tau_t|\theta_t+\sqrt{}^2 \\
&\geq 3\mathbb{E}\sqrt{}\ w)\tau_t|\theta_t,\theta_t)\alpha++\ \ 2\big\|g)\tau_t|\theta_t+\sqrt{}^2\,0\ 3\mathbb{E}\sqrt{}\ g)\tau_t|\theta_t+\sqrt{}^2 \\
&\geq 3\mathbb{E}\sqrt{}\ w)\tau_t|\theta_t,\theta_t)\alpha++\ \ 2\sqrt{}^2\mathbb{E}\sqrt{}\ g)\tau_t|\theta_t+\sqrt{}^2\,0\ 3\mathbb{E}\sqrt{}\ g)\tau_t|\theta_t+\sqrt{}^2 \\
&\geq 3)W\,0\ 2+G_t^2,
\end{aligned}
\tag{43}
$$

where the last inequality holds by Proposition 1 and Assumption 3.

Finally, combining the inequalities (42) with (43), we have

$$
\begin{aligned}
\mathbb{E}\big\|\eta_{t-1}^{-1}\sqrt{e_t}\sqrt{}^2\Big(&\geq \eta_{t-1}^{-1})2\ \ \beta_t+^2\mathbb{E}\sqrt{e_{t-1}}\sqrt{}^2\,0\ 5\beta_t^2\eta_{t-1}^{-1})W\,0\ 2+G_t^2\,0\ 3)2\ \ \beta_t+^2\eta_{t-1}^{-1}\mathbb{E}\sqrt{\mathrm{a}}^2)\theta_t,\tau+w\sqrt{}^2 \\
&\geq \eta_{t-1}^{-1})2\ \ \beta_t+^2\mathbb{E}\sqrt{e_{t-1}}\sqrt{}^2\,0\ 5\beta_t^2\eta_{t-1}^{-1})W\,0\ 2+G_t^2\,0\ 3)2\ \ \beta_t+^2\eta_{t-1}^{-1}L^2\mathbb{E}\sqrt{\theta_t\ \ \theta_{t-1}}\sqrt{}^2 \\
&\ \ [\ \ \eta_{t-1}^{-1})2\ \ \beta_t+^2\mathbb{E}\sqrt{e_{t-1}}\sqrt{}^2\,0\ 5\beta_t^2\eta_{t-1}^{-1})W\,0\ 2+G_t^2\,0\ 3)2\ \ \beta_t+^2L^2\eta_{t-1}\ \sqrt{e_{t-1}}\,0\ \ J)\theta_{t-1}+\sqrt{}^2 \\
&\geq 5)W\,0\ 2+\beta_t^2\eta_{t-1}^{-1}G_t^2\,0\ \eta_{t-1}^{-1})2\ \ \beta_t+^2\ 2\,0\ 5\eta_{t-1}^2L^2\big[\mathbb{E}\sqrt{e_{t-1}}\sqrt{}^2\,0\ 5)2\ \ \beta_t+^2L^2\eta_{t-1}\sqrt{}\ J)\theta_{t-1}+\sqrt{}^2,
\end{aligned}
$$

where the second inequality holds by the Proposition 2.

□

Theorem 5. *Assume that the sequence* $\}\theta_t|_{t=1}^T$ *be generated from Algorithm 2, and let* $k\ [\ \ O)\frac{G^{2/3}}{L}+c\ [\ \ \frac{G^2}{3k^3L}\,0\ 63L^2$, $m\ [\ \ \mathrm{n}\,\mathrm{g}\tilde{}\}3G^2,)3Lk+^3,)\frac{ck}{2L}+^3|$ *and* $\eta_0\ [\ \ \frac{k}{m^{1/3}}$, *we have*

$$
\mathbb{E}\sqrt{}\ J)\theta_\zeta+\sqrt{\lfloor}\ \ \frac{2}{T}\prod_{t=1}^{T}\mathbb{E}\sqrt{}\ J)\theta_t+\sqrt{}\geq\frac{\nabla\sqrt{3\overline{\Phi}m^{1/6}}\,0\ 3\Phi^{3/4}}{\nabla\sqrt{\overline{T}}}\,0\ \ \frac{3\nabla\sqrt{\overline{\Phi}\sigma^{1/3}}}{T^{1/3}},
$$

where $\Phi\ [\ \ \frac{1}{k}\ 27)J^*\ \ J)\theta_1++0\ \frac{m^{1/3}}{4L^2k}\sigma^2\,0\ \frac{(W+1)c^2k^3}{2L^2}\,\mathfrak{m})T\,0\ 3+\lfloor$ *with* $J^*\ [\ \ \mathrm{uxs}_\theta\,J)\theta+<0\in.$

Proof. This proof mainly follows the proof of the above Theorem 4. Due to $m\sim)3Lk+^3$, we have $\eta_t\geq\frac{k}{m^{1/3}}\geq\frac{1}{2L}$.

Since $\eta_t \geq \frac{1}{2L}$ and $m \sim \max\{\frac{ck}{2L}, 3\}$, we have $\beta_{t+1} = 1 - c\eta_t^2 \geq 1 - \frac{c\eta_t}{2L} \geq 1 - \frac{ck}{2Lm^{1/3}} \geq \frac{1}{2}$. By Lemma 5, we have

$$\begin{aligned}
\mathbb{E}\left[\left\|\eta_t^{-1}(\nabla_{t+1} - v_{t+1}) - \eta_{t-1}^{-1}(\nabla_t - v_t)\right\|^2\right] &\leq \mathbb{E}\Big[(1-\beta_{t+1})^2\left\|\eta_t^{-1}(\nabla_{t+1} - \nabla(\theta_{t+1}))\right\|^2 + \left\|(\eta_t^{-1} - \eta_{t-1}^{-1})(\nabla_t - \nabla(\theta_t))\right\|^2 \\
&\quad + \left\|(\eta_t^{-1} - \eta_{t-1}^{-1})\nabla(\theta_t) - (\nabla(\theta_{t+1}) - \nabla(\theta_t))\right\|^2\Big] \\
&\leq \mathbb{E}\Big[(1-\beta_{t+1})^2\left\|\eta_t^{-1}(\nabla_{t+1} - \nabla(\theta_{t+1}))\right\|^2 + \underbrace{\left\|(\eta_t^{-1} - \eta_{t-1}^{-1})(\nabla_t - \nabla(\theta_t))\right\|^2}_{=T_4} \\
&\quad + 2\left\|(\eta_t^{-1} - \eta_{t-1}^{-1})\nabla(\theta_t)\right\|^2 + 2\left\|\nabla(\theta_{t+1}) - \nabla(\theta_t)\right\|^2\Big] \\
&\leq \mathbb{E}\Big[(1-\beta_{t+1})^2\left\|\eta_t^{-1}(\nabla_{t+1} - \nabla(\theta_{t+1}))\right\|^2 + T_4 + 2\left\|(\eta_t^{-1} - \eta_{t-1}^{-1})\nabla(\theta_t)\right\|^2 + 2L^2\left\|\theta_{t+1} - \theta_t\right\|^2\Big] \\
&\leq \mathbb{E}\Big[(1-\beta_{t+1})^2\left\|\eta_t^{-1}(\nabla_{t+1} - \nabla(\theta_{t+1}))\right\|^2 + T_4 + 2\left\|(\eta_t^{-1} - \eta_{t-1}^{-1})\nabla(\theta_t)\right\|^2 + 2L^2\eta_t^2\left\|v_t\right\|^2\Big],
\end{aligned} \tag{44}$$

where the last inequality holds by $1 < \beta_{t+1} \leq 2$. Since the function $x^{1/3}$ is concave, we have $(x + y)^{1/3} \leq x^{1/3} + yx^{-2/3}/4$. Then we have

$$\begin{aligned}
\eta_t^{-1} - \eta_{t-1}^{-1} &= \left(\frac{2}{k}\right)\left(m + \sum_{i=1}^{t} G_i^2\right)^{1/3} - \left(m + \sum_{i=1}^{t-1} G_i^2\right)^{1/3} \geq \frac{G_t^2}{4k(m + \sum_{i=1}^{t-1} G_i^2)^{2/3}} \\
&\geq \frac{G_t^2}{4k(m + G^2 + \sum_{i=1}^{t} G_i^2)^{2/3}} \geq \frac{G_t^2}{4k(m/3 + \sum_{i=1}^{t} G_i^2)^{2/3}} \geq \frac{3^{2/3}G_t^2}{4k(m + \sum_{i=1}^{t} G_i^2)^{2/3}} \\
&\geq \frac{3^{2/3}G^2}{4k^3}\eta_t^2 \geq \frac{3^{2/3}G^2}{7k^3L}\eta_t \geq \frac{G^2}{4k^3L}\eta_t,
\end{aligned} \tag{45}$$

where the third inequality holds by $m \sim 3G^2$, and the sixth inequality holds by $1 < \eta \leq \frac{1}{2L}$.

Next, considering the upper bound of the term T_4, we have

$$\begin{aligned}
T_4 &= \left\|(\eta_t^{-1} - \eta_{t-1}^{-1})(1-\beta_{t+1})(\nabla_t - \nabla(\theta_t))\right\|^2 + \left\|(\eta_t^{-1} - \eta_{t-1}^{-1})(\nabla_t - \nabla(\theta_t))\right\|^2 \\
&\leq \left\|\eta_t^{-1} - \eta_{t-1}^{-1}\right\|^2\left\|\nabla_t - \nabla(\theta_t)\right\|^2 \\
&\leq \left\|\eta_t^{-1} - \eta_{t-1}^{-1}\right\|^2\left\|\nabla_t - v_t\right\|^2 \\
&\leq \frac{G^2}{4k^3L}\eta_t \cdot 5L^2\eta_t^2 \cdot c\eta_t\left\|\nabla_t - v_t\right\|^2 = 59L^2\eta_t^2\left\|\nabla_t - v_t\right\|^2,
\end{aligned} \tag{46}$$

where the last equality holds by $c = \frac{G^2}{3k^3L} + 63L^2$. Combining the inequalities (44) with (46), we have

$$\mathbb{E}\left[\left\|\eta_t^{-1}(\nabla_{t+1} - v_{t+1}) - \eta_{t-1}^{-1}(\nabla_t - v_t)\right\|^2\right] \leq \mathbb{E}\Big[(1-\beta_{t+1})^2\left\|\eta_t^{-1}(\nabla_{t+1} - \nabla(\theta_{t+1}))\right\|^2 + 59L^2\eta_t^2\left\|\nabla_t - v_t\right\|^2 + 2L^2\eta_t^2\left\|v_t\right\|^2\Big]. \tag{47}$$

We define a *Lyapunov* function $\Omega_t = f(\theta_t) + \frac{1}{64L^2\eta_{t-1}}\left\|\nabla_t - v_t\right\|^2$ for any $t \geq 2$. Then we have

$$\begin{aligned}
\mathbb{E}[\Omega_{t+1} - \Omega_t] &= \mathbb{E}\Big[f(\theta_{t+1}) + f(\theta_t) + \frac{2}{75L^2\eta_t}\left\|\nabla_{t+1} - v_{t+1}\right\|^2 + \frac{2}{75L^2\eta_{t-1}}\left\|\nabla_t - v_t\right\|^2\Big] \\
&\sim \mathbb{E}\Big[\frac{4\eta_t}{5}\left\|\nabla_t - v_t\right\|^2 + \frac{\eta_t}{9}\left\|\nabla(\theta_t)\right\|^2 + (\frac{2}{75L^2})\left\|\eta_t^{-1}(\nabla_{t+1} - v_{t+1})\right\|^2 + \left\|\eta_{t-1}^{-1}(\nabla_t - v_t)\right\|^2\Big] \\
&\sim \frac{(1-\beta_{t+1})^2}{43L^2}\eta_t^2\left\|\nabla_{t+1} - \nabla(\theta_{t+1})\right\|^2 + \frac{\eta_t}{27}\mathbb{E}\left\|\nabla(\theta_t)\right\|^2,
\end{aligned} \tag{48}$$

where the first inequality holds by the Lemma 3, and the second inequality follows by the above inequality (47). Summing

the above inequality (48) over t from 2 to T, we obtain

$$\begin{aligned}
\prod_{t=1}^{T} \mathbb{E}]\eta_t \sqrt{} J)\theta_t+\sqrt{}^{2}\hat{} &\geq \mathbb{E}]27)\Omega_{T+1} \quad \Omega_1+\hat{}0 \prod_{t=1}^{T} \frac{)W\ 0\ \ 2+e^2\eta_t^3 G_{t+1}^2}{3L^2} \\
&\geq \mathbb{E}]27)J^* \quad J)\theta_1+\hat{}0\ \frac{2}{5L^2\eta_0}\mathbb{E}\ e_1\sqrt{}^{2}0\ \frac{)W\ 0\ \ 2+e^2k^3}{3L^2}\prod_{t=1}^{T}\frac{G_{t+1}^2}{m\,0\ \int\ _{i=1}^{t} G_i^2} \\
&\geq \mathbb{E}]27)J^* \quad J)\theta_1+\hat{}0\ \frac{2}{5L^2\eta_0}\mathbb{E}\ e_1\sqrt{}^{2}0\ \frac{)W\ 0\ \ 2+e^2k^3}{3L^2}\prod_{t=1}^{T}\frac{G_{t+1}^2}{G^2\ 0\ \int\ _{i=1}^{t+1} G_i^2} \\
&\geq \mathbb{E}]27)J^* \quad J)\theta_1+\hat{}0\ \frac{2}{5L^2\eta_0}\mathbb{E}\ e_1\sqrt{}^{2}0\ \frac{)W\ 0\ \ 2+e^2k^3}{3L^2}\prod_{t=1}^{T}\ \ \mathfrak{m})G^2\ 0\ \prod_{i=1}^{t+1} G_i^2+\ \ \mathfrak{m})G^2\ 0\ \prod_{i=1}^{t} G_i^2+[\\
&\geq \mathbb{E}]27)J^* \quad J)\theta_1+\hat{}0\ \frac{2}{5L^2\eta_0}\mathbb{E}\ e_1\sqrt{}^{2}0\ \frac{)W\ 0\ \ 2+e^2k^3}{3L^2})\mathfrak{m})G^2\ 0\ \prod_{i=1}^{T+1} G_i^2+\ \ \mathfrak{m})G^2++ \\
&\geq \mathbb{E}]27)J^* \quad J)\theta_1+\hat{}0\ \frac{m^{1/3}}{5L^2k}\sigma^2\ 0\ \frac{)W\ 0\ \ 2+e^2k^3}{3L^2}\ \mathfrak{m})2\ 0\ \prod_{i=1}^{T+1}\frac{G_i^2}{G^2}+ \\
&\geq \mathbb{E}]27)J^* \quad J)\theta_1+\hat{}0\ \frac{m^{1/3}}{5L^2k}\sigma^2\ 0\ \frac{)W\ 0\ \ 2+e^2k^3}{3L^2}\ \mathfrak{m})T\ 0\ \ 3+, \qquad (49)
\end{aligned}$$

where $J^* \ [\ \ \mathrm{uxs}_\theta\ J)\theta+<\ 0 \in$, and the fourth inequality holds by the concavity of the function $\mathfrak{m})x+$, and the sixth inequality holds by the definition of e_1 and η_0.

By Cauchy-Schwarz inequality, we have $\mathbb{E}]XY^{\prime 2} \geq \mathbb{E}]X^2\,\mathbb{E}]Y^2\hat{}$. Let $X \ [\ \ \sqrt{\eta_T \int\ _{t=1}^{T-1} \sqrt{}\ J)\theta_t+\sqrt{}^{2}}$ and $Y \ [\ \ \sqrt{2/\eta_T}$, we have

$$\mathbb{E}]\prod_{t=1}^{T}\sqrt{}\ J)\theta_t+\sqrt{}^{2}\Big(\geq \mathbb{E}]2/\eta_T\,\mathbb{E}]\eta_T\prod_{t=1}^{T}\sqrt{}\ J)\theta_t+\sqrt{}^{2}\Big(. \qquad (50)$$

Since η_t is decreasing, we have

$$\mathbb{E}]\prod_{t=1}^{T}\sqrt{}\ J)\theta_t+\sqrt{}^{2}\Big(\geq \mathbb{E}]2/\eta_T\,\mathbb{E}]\prod_{t=1}^{T}\eta_T\sqrt{}\ J)\theta_t+\sqrt{}^{2}\Big(\geq \mathbb{E}]2/\eta_T\,\mathbb{E}]\prod_{t=1}^{T}\eta_t\sqrt{}\ J)\theta_t+\sqrt{}^{2}\Big(. \qquad (51)$$

Combining the inequalities (49) and (51), we obtain

$$\begin{aligned}
\mathbb{E}]\prod_{t=1}^{T}\sqrt{}\ J)\theta_t+\sqrt{}^{2}\Big(&\geq \mathbb{E}]\frac{27)J^* \quad J)\theta_1+\hat{}0\ \ \frac{m^{1/3}}{4L^2k}\sigma^2\ 0\ \ \frac{(W+1)c^2k^3}{2L^2}\ \mathfrak{m})T\ 0\ \ 3+}{\eta_T}\Big(\\
&[\ \ \mathbb{E}]\Phi\ m\,0\ \prod_{t=1}^{T} G_t^2[^{1/3}\Big(\qquad (52)
\end{aligned}$$

where $\Phi \ [\ \ \frac{1}{k}\ \ 27)J^* \quad J)\theta_1+\hat{}0\ \ \frac{m^{1/3}}{4L^2k}\sigma^2\ 0\ \ \frac{(W+1)c^2k^3}{2L^2}\ \mathfrak{m})T\ 0\ \ 3+[$.

By Assumption 2, we have $G_t^2 \ [\ \ \sqrt{}\ g)\tau|\theta_t+ \quad J)\theta_t+0 \quad J)\theta_t+\sqrt{}^{2} \geq 3\ \sqrt{}\ g)\tau|\theta_t+ \quad J)\theta_t+\sqrt{}^{2}\ 0\ \ 3\ \sqrt{}\ J)\theta_t+\sqrt{}^{2} \geq 3\sigma^2\ 0$

$3\sqrt{\ }J)\theta_t+\sqrt{}^2$. Then using the inequality $)a\ 0\ b+^{1/3} \geq a^{1/3}\ 0\ b^{1/3}$ for all $a, b > 1$ to the inequality (52), we obtain

$$\begin{aligned}
\Big)\mathbb{E}\ \Big[\overline{\prod_{t=1}^{T}\sqrt{\ }J)\theta_t+\sqrt{}^2}\Big[^2 &\geq \mathbb{E}\Big]\ \Big[\overline{\prod_{t=1}^{T}\sqrt{\ }J)\theta_t+\sqrt{}^2}\Big]^{\sum 2}[\ \ \mathbb{E}]\prod_{t=1}^{T}\sqrt{\ }J)\theta_t+\sqrt{}^2\Big(\\
&\geq \mathbb{E}\Big]\Phi)m\ 0\ \ 3T\sigma^2+^{1/3}\ 0\ \ 3^{1/3}\Phi\ \prod_{t=1}^{T}\sqrt{\ }J)\theta_t+\sqrt{}^2\Big[^{1/3}\sum\\
&[\ \ \Phi)m\ 0\ \ 3T\sigma^2+^{1/3}\ 0\ \ 3^{1/3}\Phi\mathbb{E}\Big]\ \Big[\overline{\prod_{t=1}^{T}\sqrt{\ }J)\theta_t+\sqrt{}^2}\Big]^{\sum 2/3}\\
&\geq \Phi)m\ 0\ \ 3T\sigma^2+^{1/3}\ 0\ \ 3^{1/3}\Phi\Big]\mathbb{E}\ \Big[\overline{\prod_{t=1}^{T}\sqrt{\ }J)\theta_t+\sqrt{}^2}\Big]^{\sum 2/3},
\end{aligned} \tag{53}$$

where the first inequality holds by the convexity of the function x^2, and the last inequality holds by the concavity of the function $x^{2/3}$. For simplicity, let $Z\ [\ \ \sqrt{\int\ \sum_{t=1}^{T}\sqrt{\ }J)\theta_t+\sqrt{}^2}$, we have

$$\mathbb{E}]Z\hat{}\Big[^2 \geq \Phi)m\ 0\ \ 3T\sigma^2+^{1/3}\ 0\ \ 3^{1/3}\Phi\ \ \mathbb{E}]Z\hat{}\Big[^{2/3}. \tag{54}$$

The inequality (54) implies that $\mathbb{E}]Z\hat{}\Big[^2 \geq 3\Phi)m\ 0\ \ 3T\sigma^2+^{1/3}$ or $\mathbb{E}]Z\hat{}\Big[^2 \geq 3\times 3^{1/3}\Phi\ \ \mathbb{E}]Z\hat{}\Big[^{2/3}$. Thus, we have

$$\mathbb{E}]Z\hat{} \geq \sqrt{3\overline{\Phi})}m\ 0\ \ 3T\sigma^2+^{1/6}\ 0\ \ 3\Phi^{3/4}. \tag{55}$$

By Cauchy-Schwarz inequality, then we have

$$\begin{aligned}
\frac{2}{T}\prod_{t=1}^{T}\mathbb{E}\sqrt{\ }J)\theta_t+\sqrt{}[\ \ \mathbb{E}]\frac{2}{T}\prod_{t=1}^{T}\sqrt{\ }J)\theta_t+\sqrt{}\Big(&\geq \mathbb{E}\Big]\ \sqrt{\Big|\frac{2}{T}}\ \Big[\overline{\prod_{t=1}^{T}\sqrt{\ }J)\theta_t+\sqrt{}^2}\Big]^{\sum}\\
&\geq \frac{\sqrt{3\overline{\Phi})}m\ 0\ \ 3T\sigma^2+^{1/6}\ 0\ \ 3\Phi^{3/4}}{\sqrt{\overline{T}}}\\
&\geq \frac{\sqrt{3\overline{\Phi}}m^{1/6}\ 0\ \ 3\Phi^{3/4}}{\sqrt{\overline{T}}}\ 0\ \ \frac{3\sqrt{\overline{\Phi}}\sigma^{1/3}}{T^{1/3}},
\end{aligned} \tag{56}$$

where the last inequality follows by the inequality $)a\ 0\ \ b+^{1/6} \geq a^{1/6}\ 0\ \ b^{1/6}$ for all $a, b > 1$. □

A.3. Convergence Analysis of IS-MBPG* Algorithm

In this subsection, we detailedly provide the convergence properties of our IS-MBPG* algorithm.

Lemma 6. *Assume that the stochastic policy gradient u_t be generated from Algorithm 3, and let $e_t\ [\ \ u_t\ \ \ \ J)\theta_t+$, we have*

$$\mathbb{E}]\eta_{t-1}^{-1}\sqrt{e_t}\sqrt{}^2\Big(\geq 3\beta_t^2\eta_{t-1}^{-1}\sigma^2\ 0\ \ \eta_{t-1}^{-1})2\quad \beta_t+^2\ 2\ 0\ \ 9\eta_{t-1}^2B^2\Big[\mathbb{E}\sqrt{e_{t-1}}\sqrt{}^2\ 0\ \ 9)2\quad \beta_t+^2B^2\eta_{t-1}\sqrt{\ }J)\theta_{t-1}+\sqrt{}^2,$$

where $B^2\ [\ \ L^2\ 0\ \ 3G^2C_w^2$ with $C_w\ [\ \sqrt{H)3HM_g^2\ 0\ \ M_h+)W\ 0\ \ 2+}$

Proof. The proof is the similar to that of Lemma 4. The only difference is that instead of using $3\beta_t^2\eta_{t-1}^{-1}\mathbb{E}\sqrt{\ }J)\theta_t+\ g)\tau_t|\theta_t+\sqrt{}^2 \geq 3\beta_t^2\eta_{t-1}^{-1}\sigma^2$ instead of $3\beta_t^2\eta_{t-1}^{-1}\mathbb{E}\sqrt{\ }J)\theta_t+\ \ g)\tau_t|\theta_t+\sqrt{}^2 \geq 3\beta_t^2\eta_{t-1}^{-1}\mathbb{E}\sqrt{g)\tau_t|\theta_t+\sqrt{}^2}\ [\ \ 3\beta_t^2\eta_{t-1}^{-1}G_t^2$. □

Theorem 6. *Assume that the sequence $\}\theta_t|_{t=1}^{T}$ be generated from Algorithm 3, and let $B^2\ [\ \ L^2\ 0\ \ 3G^2C_w^2$, $k > 1$ $c\ [\ \ \frac{1}{3k^3L}\ 0\ \ 215B^2$, $m\ [\ \ \mathrm{n}\ \mathrm{g}\tilde{}\}3,)3Lk+^3,)\frac{ck}{2L}+^3|$ and $\eta_0\ [\ \ \frac{k}{m^{1/3}}$, we have*

$$\mathbb{E}\sqrt{\ }J)\theta_\varsigma+\sqrt{}[\ \ \frac{2}{T}\prod_{t=1}^{T}\mathbb{E}\sqrt{\ }J)\theta_t+\sqrt{} \geq \frac{\sqrt{-}m^{1/6}}{\sqrt{\overline{T}}}\ 0\ \ \frac{\sqrt{-}}{T^{1/3}},$$

where $\quad[\ \ \frac{1}{k}\ \ 27)J^*\quad J)\theta_1+\!+0\ \ \frac{m^{1/3}}{8B^2k}\sigma^2\ 0\ \ \frac{c^2k^3\sigma^2}{4B^2}\ \mathfrak{m})T\ 0\ \ 3+\Big[$ with $J^*\ [\ \ \mathrm{uxs}_\theta\ J)\theta+<0 \in .$

Proof. This proof mainly follows the proof of the above Theorem 4. Due to $m \sim)3Lk+^{3}$, we have $\eta_t \geq \frac{k}{m^{1/3}} \geq \frac{1}{2L}$. Since $\eta_t \geq \frac{1}{2L}$ and $m \sim)\frac{ck}{2L}+^{3}$, we have $\beta_{t+1} \left[\; c\eta_t^2 \geq \frac{c\eta_t}{2L} \geq \frac{ck}{2Lm^{1/3}} \geq 2$. By Lemma 6, we have

$$\begin{aligned}
\mathbb{E}]\eta_t^{-1} \sqrt{e_{t+1}}\sqrt{}^{2} \quad \eta_{t-1}^{-1}\sqrt{e_t}\sqrt{}^{2}\hat{} &\geq \mathbb{E}\big]3\beta_{t+1}^2\eta_t^{-1}\sigma^2 \, 0 \; \eta_t^{-1})2 \quad \beta_{t+1}+^{2} \; 2\,0 \; 9\eta_t^2B^2\big[\sqrt{e_t}\sqrt{}^{2} \\
&\quad 0 \; 9)2 \quad \beta_{t+1}+^{2}B^2\eta_t \sqrt{} \; J)\theta_t+\sqrt{}^{2} \quad \eta_{t-1}^{-1}\sqrt{e_t}\sqrt{}^{2}\Big(\\
&\geq \mathbb{E}\big]3c^2\eta_t^3\sigma^2 \, 0 \underbrace{\eta_t^{-1})2 \quad \beta_{t+1}+)2\,0 \; 9\eta_t^2B^2+ \quad \eta_{t-1}^{-1}\big[\sqrt{e_t}\sqrt{}^{2}}_{=T_5} 0 \; 9B^2\eta_t \sqrt{} \; J)\theta_t+\sqrt{}^{2}\Big(\qquad (57)
\end{aligned}$$

where the last inequality holds by $1 < \beta_{t+1} \geq 2$. Since the function $x^{1/3}$ is cancave, we have $)x \, 0 \; y+^{1/3} \geq x^{1/3} \, 0 \; yx^{-2/3}/4$. Then we have

$$\begin{aligned}
\eta_t^{-1} \quad \eta_{t-1}^{-1}\left[\; \frac{2}{k}\right) \; m\,0 \; t\big[^{1/3} \quad m\,0 \; t \quad 2\big[^{1/3}\Big[&\geq \frac{2}{4k)m\,0 \; t \quad 2+^{2/3}} \\
&\geq \frac{2}{4k)m/3\,0 \; t+^{2/3}} \geq \frac{3^{2/3}}{4k)m\,0 \; t+^{2/3}} \\
&\geq \frac{3^{2/3}}{4k^3}\eta_t^2 \geq \frac{3^{2/3}}{7k^3L}\eta_t \geq \frac{2}{4k^3L}\eta_t, \qquad (58)
\end{aligned}$$

where the second inequality holds by $m \sim 3$, and the fifth inequality holds by $1 < \eta \geq \frac{1}{2L}$.

Next, considering the upper bound of the term T_5, we have

$$\begin{aligned}
T_5 \left[\quad \eta_t^{-1})2 \quad \beta_{t+1}+)2\,0 \; 9\eta_t^2B^2+ \quad \eta_{t-1}^{-1}\big[\sqrt{e_t}\sqrt{}^{2} \right. \\
\left[\quad \eta_t^{-1} \quad \eta_{t-1}^{-1} \, 0 \; 9B^2\eta_t \quad \beta_{t+1}\eta_t^{-1} \quad 9\eta_t\beta_{t+1}B^2\big[\sqrt{e_t}\sqrt{}^{2} \right. \\
\geq \quad \eta_t^{-1} \quad \eta_{t-1}^{-1} \, 0 \; 9B^2\eta_t \quad \beta_{t+1}\eta_t^{-1}\big[\sqrt{e_t}\sqrt{}^{2} \\
\geq \quad \frac{2}{4k^3L}\eta_t \, 0 \; 9B^2\eta_t \quad c\eta_t\big[\sqrt{e_t}\sqrt{}^{2}\left[\quad :7B^2\eta_t \sqrt{e_t}\sqrt{}^{2}, \qquad (59)
\end{aligned}$$

where the last equality holds by $c \left[\; \frac{1}{3k^3L} \, 0 \; 215B^2$. Combining the inequalities (57) with (59), we have

$$\mathbb{E}]\eta_t^{-1} \sqrt{e_{t+1}}\sqrt{}^{2} \quad \eta_{t-1}^{-1}\sqrt{e_t}\sqrt{}^{2}\hat{} \geq \mathbb{E}\big]3c^2\eta_t^3\sigma^2 \quad :7B^2\eta_t \sqrt{e_t}\sqrt{}^{2} \, 0 \; 9B^2\eta_t \sqrt{} \; J)\theta_t+\sqrt{}^{2}\Big(\qquad (60)$$

We define a *Lyapunov* function $\Psi_t \left[\; J)\theta_t+ \; \frac{1}{128B^2\eta_{t-1}} \sqrt{e_t}\sqrt{}^{2}$ for any $t \sim 2$. Then we have

$$\begin{aligned}
\mathbb{E}]\Psi_{t+1} \quad \Psi_t\hat{}\left[\; \mathbb{E}\big] J)\theta_{t+1}+ \; J)\theta_t+ \; \frac{2}{239B^2\eta_t}\sqrt{e_{t+1}}\sqrt{}^{2} \, 0 \; \frac{2}{239B^2\eta_{t-1}}\sqrt{e_t}\sqrt{}^{2}\Big(\right. \\
\sim \mathbb{E}\big] \quad \frac{4\eta_t}{5}\sqrt{e_t}\sqrt{}^{2} \, 0 \; \frac{\eta_t}{9}\sqrt{} \; J)\theta_t+\sqrt{}^{2} \quad \frac{2}{239B^2})\eta_t^{-1}\sqrt{e_{t+1}}\sqrt{}^{2} \quad \eta_{t-1}^{-1}\sqrt{e_t}\sqrt{}^{2}+\Big(\right. \\
\sim \quad \frac{c^2\eta_t^3\sigma^2}{75B^2} \, 0 \; \frac{\eta_t}{27}\mathbb{E}\sqrt{} \; J)\theta_t+\sqrt{}^{2}, \qquad (61)
\end{aligned}$$

where the first inequality holds by the Lemma 3, and the second inequality follows by the above inequality (60). Summing

the above inequality (61) over t from 2 to T, we obtain

$$\begin{aligned}
\prod_{t=1}^{T} \mathbb{E}]\eta_t \sqrt{\ } J)\theta_t+\sqrt{}^{2}\hat{} &\geq \mathbb{E}]27)\Psi_{T+1} \quad \Psi_1+\hat{}0 \ \prod_{t=1}^{T} \frac{c^2\eta_t^3\sigma^2}{5B^2} \\
&\geq \mathbb{E}]27)J^* \quad J)\theta_1+\hat{}0 \ \frac{2}{9B^2\eta_0}\mathbb{E}\sqrt{e_1}\sqrt{}^{2}0 \ \frac{c^2k^3\sigma^2}{5B^2}\prod_{t=1}^{T}\frac{2}{m\,0\ t} \\
&\geq \mathbb{E}]27)J^* \quad J)\theta_1+\hat{}0 \ \frac{2}{9B^2\eta_0}\mathbb{E}\sqrt{e_1}\sqrt{}^{2}0 \ \frac{c^2k^3\sigma^2}{5B^2}\prod_{t=1}^{T}\frac{2}{t\,0\ 3} \\
&\geq \mathbb{E}]27)J^* \quad J)\theta_1+\hat{}0 \ \frac{2}{9B^2\eta_0}\sigma^2\,0 \ \frac{c^2k^3\sigma^2}{5B^2}\prod_{t=1}^{T}\frac{2}{t\,0\ 3} \\
&\geq \mathbb{E}]27)J^* \quad J)\theta_1+\hat{}0 \ \frac{m^{1/3}}{9B^2k}\sigma^2\,0 \ \frac{c^2k^3\sigma^2}{5B^2}\,\mathfrak{m})T\,0\ 3+, \qquad (62)
\end{aligned}$$

where $J^* \ [\ \text{ux}\text{s}_\theta \, J)\theta+< 0 \in$, and the third inequality is due to $m \sim 3$, and the last inequality holds by $\int_{t=1}^{T} \frac{1}{t+2} \geq \sum_{1}^{T} \frac{1}{t+2}dt \geq \mathfrak{m})T\,0\ 3+$

Since $\eta_t \ [\ \frac{k}{(m+t)^{1/3}}$ is decreasing, we have

$$\begin{aligned}
\prod_{t=1}^{T} \mathbb{E}\sqrt{\ } J)\theta_t+\sqrt{}^{2} &\geq 2/\eta_T \prod_{t=1}^{T} \mathbb{E}]\eta_t \sqrt{\ } J)\theta_t+\sqrt{}^{2}\Big(\\
&\geq \frac{27)J^* \quad J)\theta_1+\hat{}0 \ \frac{m^{1/3}}{8B^2k}\sigma^2\,0 \ \frac{c^2k^3}{4B^2}\,\mathfrak{m})T\,0\ 3+}{\eta_T} \\
&[\ \acute{} \ m\,0\ T\big[^{1/3} \qquad (63)
\end{aligned}$$

where $[\ \frac{1}{k} \ 27)J^* \quad J)\theta_1+\hat{}0 \ \frac{m^{1/3}}{8B^2k}\sigma^2\,0 \ \frac{c^2k^3\sigma^2}{4B^2}\,\mathfrak{m})T\,0\ 3+\big[$.

According to Jensen's inequality, we have

$$\frac{2}{T}\prod_{t=1}^{T} \mathbb{E}\sqrt{\ } J)\theta_t+\geq \sqrt{} \ \frac{2}{T}\prod_{t=1}^{T} \mathbb{E}\sqrt{\ } J)\theta_t+\sqrt{}^{2}\big[^{1/2} \geq \frac{\nabla_- \ m\,0\ T\big[^{1/6}}{\nabla\overline{T}} \geq \frac{\nabla_- \ m^{1/6}}{\nabla\overline{T}}\,0 \ \frac{\nabla_-}{T^{1/3}}, \qquad (64)$$

where the last inequality follows by the inequality $)a\,0\ b+\hat{}1/6 \geq a^{1/6}\,0\ b^{1/6}$ for all $a, b > 1$. □