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A ground-up approach to estimate the likelihood of business interruption

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ABSTRACT

Critical infrastructure (such as transportation, water, electric power, wastewater, and telecommunications) provides the conveyance of goods, services, and resources to communities, which are vital for economic activities. Buildings, bridges, and other structures and components of the infrastructure might be subject to natural and anthropogenic hazards, which may lead to a reduction or loss of functionality of infrastructure. Businesses may experience disruptions because of (i) direct damage to the business properties and facilities, (ii) reduction or loss of functionality of the supporting critical infrastructure, or (iii) impact on social systems affecting the availability of the workforce at a specific business and supporting businesses as well as customers. Current approaches do not model the functionality of business properties and supporting infrastructure, generally obtaining only empirical estimates of business interruption losses. Such predictions might not be applicable to different businesses, supporting infrastructure, social systems, locations, and hazards; and they would in general not reflect possible changes in the built environment due to mitigation strategies and interventions. This paper proposes a mathematical formulation that overcomes the limitations in the current approaches. The proposed formulation models and quantifies the likelihood of business interruption with a ground-up approach by incorporating the dependency of business operations on physical structures, infrastructure and social systems. The paper illustrates the proposed formulation by investigating the business interruption of an example food retail store in Seaside, Oregon subject to a seismic hazard.

1. Introduction

Critical infrastructure such as transportation, water, electric power, and wastewater provides a continuous flow of goods and services, serving as a foundation of the well-being and economic prosperity of communities [1–5]. Past catastrophic events highlight the vulnerability of critical infrastructure to natural and anthropogenic hazards, as well as emphasize the need for the development of mitigation strategies, urban planning and public policies that can help reduce the impact of hazards [4,6,7]. Natural hazards (e.g., floods, earthquakes, and hurricanes) and anthropogenic hazards (e.g., accidents and terrorist attacks) might damage critical infrastructure and lead to their reduction or loss of functionality [8–10].

Business interruption may occur because of (i) direct damage to the business properties and facilities, (ii) reduction or loss of functionality of the supporting critical infrastructure, or (iii) impact on social systems affecting the availability of the workforce at a specific business and supporting businesses as well as customers.

In addition to the obvious dependency of businesses on the business properties and facilities, businesses usually depend on several supporting infrastructure (e.g., transportation, water, electric power, and wastewater). As a result, reduction (or loss) of functionality of the supporting infrastructure typically affects business operations. For example, businesses need to be physically accessible by employees and customers, as well as by their suppliers to receive goods and sell products, thereby disruption in the transportation infrastructure may result in business interruption due to impaired access of employees and customers, and of the delivery of supplies. Businesses' ability to run their activities and sell their products also depends on the availability of a sufficient number of employees and customers. The impact on social systems (e.g., in terms of casualties and/or population dislocation) may lead to a reduction in the number of employees and customers, resulting in business interruption.

The effect of the damage to business properties and facilities, loss of functionality of the supporting critical infrastructure, and impact on the social systems on business interruption depends on the type of business [11]. Business interruption may range from the reduction (or loss) of production and sales to temporary or permanent closure [12,13]. For

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instance, damage to business facilities may range from minor inconvenience to shutdown of business operations whether business activities can be relocated (e.g., professional services such as law and consulting firms) or not (e.g., businesses in the manufacturing and industry sector.) Similarly, the effect of reduction or loss of functionality of the transportation, water, electric power, and wastewater infrastructure may vary from partial inoperability up to immediate closure of the business activities. Finally, the impact on social systems affecting the available workforce may affect business interruption differently in case business activities require specialized personnel (e.g., engineering firms, banks, and hospitals) or not (e.g., retail stores and restaurants.)

Modeling and predicting the consequences of natural hazards on critical infrastructure has been the subject of much research. Past studies have focused on predicting the immediate impact on physical components of critical infrastructure, as well as on modeling the effects in terms of provision of goods and services (e.g., Refs. [8,14–18]). In addition, interdisciplinary research has focused on estimating the societal impact, coupling the effects of natural hazards on critical infrastructure with socio-economic characteristics of communities (e.g. Refs. [5,6,19,20,21–27]).

There is a need for a mathematical formulation for estimating the losses due to business interruption in order to understand businesses' vulnerabilities, inform mitigation and recovery strategies, as well as insurers of their liability (e.g., Refs. [28–30]). Nevertheless, past studies (e.g. Refs. [12,31]), have not modeled the functionality of business properties and supporting infrastructure, generally obtaining only empirical estimates of business interruption losses. Past studies generally focused on modeling businesses at an aggregated level to measure the economic impacts at a regional scale (e.g., Refs. [32–36].) However, studies at the regional scale cannot provide specific recommendations on factors (e.g., infrastructure characteristics) that influence individual businesses, do not capture the specific business' vulnerabilities, and cannot inform mitigation and recovery strategies at the individual business level. Only a few studies focused on assessing the relevant factors that determine the survival of a business after a catastrophic event at the individual business level. However, in general, researchers only developed empirical models calibrated by regression analysis using data from specific past events (e.g. Refs. [37–45]). For example, Kajitani and Tatano [46] focused on the effects of natural hazards on business interruption using survey data after the occurrence of the Tokai Earthquake. Xiao and Peacock [47] focused on identifying the value of business disasters plans on business continuity, providing some evidence on the status of business disaster planning, mitigation, and preparedness, as well as the effectiveness of these activities on loss reduction. However, their work is again based on a single event (i.e., Hurricane Ike) and a specific area with its specific infrastructure. As a result, while some research on business interruption focused on identifying the factors influencing business interruption, the developed empirical models only serve to better understand the past event (by identifying the important factors that help explain what happened), but cannot be used to predict the likelihood of business interruption for future events. Empirical models available in the literature are generally not applicable to different businesses, supporting infrastructure, social systems, locations, and hazards; and, in general, they do not reflect possible changes in the built environment due to mitigation strategies and interventions.

This paper proposes a mathematical formulation that overcomes the limitations in the current approaches. In particular, the proposed formulation models and quantifies the likelihood of business interruption with a ground-up approach by incorporating the dependency of business operations on physical structures, infrastructure, and social systems. The proposed formulation starts by estimating the direct physical damage to the business properties, the impact on the functionality of the supporting infrastructure, and the changes in the social systems. Then, we integrate the effects of the individual causes that may lead to business interruption in a matrix-based system reliability method to estimate the likelihood of business interruption. As an example, the proposed mathematical formulation is applied to a hypothetical food retail store located in Seaside, Oregon subject to a seismic event. The example is an illustration of the proposed formulation. This paper differs from the available literature because the proposed modeling of the dependency of business operations on physical structures, infrastructure, and social systems by using a ground-up approach allows (i) to predict the likelihood of business interruption for future events, (ii) inform mitigation and recovery strategies, and (iii) advise insurers of their liability. The formulation is general and applicable to different businesses, supporting infrastructure, social systems, locations, and hazards; and can reflect the changes in the built environment due to mitigation strategies and possible interventions.

Following this introduction, Section 2 introduces the fundamental aspects of business interruption. Section 3 presents the proposed formulation to estimate the likelihood of business interruption, integrating direct damage to business properties, reduction or loss of functionality of the supporting critical infrastructure, and impact on social systems. Section 4 illustrates the proposed formulation estimating the likelihood of the business interruption of an example food retail store following a seismic event.

2. Business interruption

Business interruption is typically defined as a disruption of the normal operations of a firm [48] and is usually divided into ordinary and contingent business interruption [49]. Ordinary business interruption refers to disruptions due to on-site damage. Contingent business interruption, instead, refers to disruptions to off-site sources, such as disruption in the supply chain or supporting critical infrastructure on which the business depends. A rigorous modeling of business interruption requires incorporating both ordinary and contingent business interruption. Distinguishing between the two types helps estimate the likelihood of survival of a business after the occurrence of a catastrophic event [50].

Ordinary and contingent business interruption may be described by the following causes that are crucial to estimate the likelihood of business interruption [49,51]:

(i) **Physical damage to plants/facilities and/or equipment.** Although it is intuitive that the facilities and the equipment of a business need to be undamaged or at least still functional to carry business activities regularly, there is a need to specify their role in the business operations. The consequences on business interruption are different based on the business sector. Activities that are usually carried in office headquarters may be relocated more easily than activities carried in factories (e.g., production), which may not be possible or significantly harder to relocate. As an example, most of the businesses in the World Trade Center area were able to mitigate their business interruption losses relocating relatively quickly [52]. Conversely, factories that involve production processes might mitigate business interruption losses adopting some retrofit strategies (e.g., Ref. [53]).

(ii) **Supply-chain disruptions.** Business interruption may also occur when there is a loss (or reduction) of inputs or outputs due to damage to the supply-chain. A business may be forced to close either if it is not able to restock (due to disruption to suppliers) or it is not able to sell its products (due to disruption to its customers or access to them.) Also, the consequences of a catastrophic event may be felt across different regions and even countries. Therefore, even businesses that are physically undamaged may have to close due to disruption throughout the supply-chain. As an example, automobile manufacturers located in the United States and in Europe experienced an interruption in their business due to damage to their part producers in Japan after the 2011 Great East Japan Earthquake [54,55].

(iii) **Redundancy in the business network.** Businesses may mitigate overall business interruption by shifting their activities to different locations that have suffered less damage, or for which inputs/outputs through the supply-chain are less affected.

There are also other more indirect, and less intuitive, causes that may affect the likelihood of business interruption [31,52]. In the remaining of this section, we provide more details about these two indirect factors that have been less studied in the literature.

(iv) **Damage to supporting infrastructure.** Critical infrastructure such as transportation, water, and electrical power infrastructure, provide the conveyance of goods, services, and resources to businesses to be functional [2–4,10]. Critical infrastructure usually depend on each other to jointly provide the production and distribution of goods and services [1,5]. As a result, even if an infrastructure has no physical damage, there might be a loss (or reduction) of functionality of such infrastructure due to physical damage of an infrastructure on which it depends (e.g., Refs. [9,56–64]).

Businesses need to have access to infrastructure services (e.g., water, electric power, and wastewater) to run their activities. The loss (or reduction) of functionality of supporting infrastructure may, therefore, cause business interruption. As an example, considering the water and the electric power infrastructure, they are designed to provide vital resources from a source to the business facilities. Previous studies (e.g. Refs. [31,60]), observed that businesses usually consider electric power crucial for their ability to do business, such that they would shut down immediately due to lack of electric power. Also, an industrial building may be forced to close (for safety reasons) if there is low water pressure to guarantee the sprinkler system functionality, yet both business facilities themselves and/or equipment might not experience physical damage. In addition to the resources provided at the business facilities, there is the need of physical connection between businesses and employees, as well as customers to conduct regular business activities, insofar as supplies and products need to be able to go in and out of the business facilities. The transportation infrastructure ensure the mobility of goods and people across space [65]. Disruptions in the transportation infrastructure may result in business interruptions due to obstructions in the employees' ability to go to work, delivery and transport of products or supplies, and customers' access.

(v) **Social systems and employees/customers' profile.** Business interruption also depends on the availability of a sufficient number of existing employees to carry business activities and customers. The modeling of the likelihood of business interruption, therefore, needs to take into account of the societal impact of a hazard to properly estimate the available employees [49] and customers. Businesses may experience interruption (in terms of being open or functional as usual) if there are no or fewer employees attending work.

After the occurrence of a catastrophic event, employees and customers' priority is their safety, as well as the one of their family. In addition, having access to primary needs might induce people to dislocate, affecting the number of available employees and customers at a specific site. As a result, businesses may experience a reduction in the available workforce and customers also because of either possible casualties or population dislocation.

Building damage, including nonstructural and structural damage, coupled with other socio-economic factors that characterize the individuals' vulnerability [66], usually influence the number and the severity of casualties [67,68]. As an example, in a seismic scenario, casualty estimates depend on nonstructural damage in smaller earthquakes, whereas in severe earthquakes, structural damage and induced collapses control the number of casualties. In general, the severity of casualties is classified into four different levels [67,69,70]. Severity 1 is defined as injuries requiring basic medical aid that could be administered by paraprofessionals (e.g., sprain, a severe cut requiring stitches, a minor burn (first degree or second degree on a small part of the body), or a bump on the head without loss of consciousness) that, while not life threatening, may impede employees ability to attend work; Severity 2 is defined as injuries that require a greater degree of medical care and use of medical technology such as x-rays or surgery, yet do not expect to progress to a life-threatening condition; Severity 3 is defined as injuries that pose an immediate life-threatening condition; Severity 4 is defined as instantaneous mortality. Each severity of casualties influences differently the employees and customers' pool in terms of their ability to go to work or purchase products and services. Therefore, in estimating the likelihood of business interruption, it is important to predict the number and severity of casualties.

In addition, after the occurrence of a catastrophic event, the human response depends on the ability of the interconnected system of critical infrastructure to provide services. The available workforce and customers may be reduced due to population dislocation resulting from the lack of functionality of critical infrastructure (e.g., potable water and power) [27].

3. Mathematical formulation to estimate the likelihood of business interruption

This paper proposes a mathematical formulation to estimate the likelihood of business interruption incorporating the damage to business properties, along with the dependence of businesses on the supporting critical infrastructure and social systems in terms of available employees and customers. The proposed mathematical formulation includes a reliability analysis to model the direct physical damage to the business properties, and the impact to the functionality of the supporting infrastructure and social systems. The relative effects of the highlighted causes are then integrated in a matrix-based system reliability method to estimate the likelihood of business interruption.

3.1. Glossary for the modeling of infrastructure to estimate the likelihood of business interruption

This section formalizes the definitions of some of the fundamental terms related to the infrastructure modeling to estimate the likelihood of business interruption.

Capacity: Capacity of a component (or of an infrastructure) is defined as a measure of its ability to generate the specific goods and services belonging to the scope of the individual infrastructure. For example, capacity of the potable water infrastructure refers to its ability to deliver a certain amount of water of minimum quality and at a minimum pressure; capacity of the transportation infrastructure refers to its ability to allow the movement of people and goods at a desired rate. The capacity of an infrastructure is typically spatially distributed over its different components (e.g., water distribution nodes) and varies with time. Furthermore, for an infrastructure, there may exist multiple capacity measures to capture different goods and services generation (e.g., water quality and water pressure.)

Demand: Demand for a component (or for an infrastructure) is defined as a measure of the request from its users in terms of the goods and services provided by the individual infrastructure (e.g., Ref. [71]). For example, the demand on the potable water infrastructure refers to the request of water from the different users. Similar to its capacity, the demand of an infrastructure is typically spatially distributed over its different components and varies with time. Furthermore, for an infrastructure, there may exist multiple demand measures corresponding to the different capacity measures. A “failure” of a component (or of an infrastructure) may occur when the demand exceeds its capacity [18, 72]. When the demand on an infrastructure is less than its capacity, then, only a portion of its capacity is used to satisfy the imposed demand (s). Such portion is also called the supply [73] (see later for a complete definition.)

Infrastructure: The infrastructure is herein defined as the physical object representing the collection of elements (components) that function together to produce the capability needed to meet a specific demand (s).

Network: A network is defined as the mathematical representation of an infrastructure [14,27,64]. A network is characterized by a set of connected objects with attributes other than the topology of their relations [74].

State variables: State variables are the variables that describe the state of the components of an infrastructure. State variables represent physical quantities that are specific to the individual components of a specific infrastructure, such as material properties and geometry for structural components. Capacity and demand measures of an infrastructure are typically defined as a function of state variables. Life-cycle processes such as deterioration and recovery activities may affect the state variables over time and can be modeled in terms of changes in the state variables (e.g., Ref. [75]).

Supply: Supply is defined as the portion of the capacity that is utilized by an infrastructure to meet an imposed demand [73]. The supply of an infrastructure (along with its capacity and demand) is needed to measure the infrastructure performance in terms of derived measures such as reliability and functionality.

3.2. Modeling the physical infrastructure: graph theory-based models

Critical infrastructure are usually modeled based on the established concepts of graph theory (e.g., Refs. [14,64,76]). Defining the generic k^{th} network mathematically as $G^{(k)} = (V^{(k)}, E^{(k)})$ means representing the set $V^{(k)}$ of $N^{(k)}$ vertices, as well as the set $E^{(k)}$ of $M^{(k)}$ edges. Two generic nodes $v_i^{(k)}$ and $v_j^{(k)}$ are connected if it is possible to define a finite sequence of nodes and edges (i.e., a path or a walk in $G^{(k)}$) from $v_i^{(k)}$ to $v_j^{(k)}$. Adjacency tables, which provide information on the connectivity of the network, have been used to fully describe mathematically the network topology [9,62,63,77]. Moreover, adjacency tables can also capture the topology of directed networks, which is, for instance, a required feature in flow-based approaches. The adjacency table of the generic k^{th} network $G^{(k)}$ is the square matrix $\mathbf{A}^{(k)}$ of order $N^{(k)}$, with elements $a_{ij}^{(k)}$ equal to either 1 if there exists a link $e_{ij}^{(k)} = (v_i^{(k)}, v_j^{(k)}) \in E^{(k)}$, or 0 otherwise, and $a_{ii}^{(k)} = 0$. As an example, for a transportation network, nodes denote the collection of different locations that constitute the set of origins and destinations (e.g., distribution centers, employees' residential building, and business plants/facilities.) Furthermore, the set of nodes can also contain the set of bridges (i.e., bridges over waterways, highways), as commonly done in bridge engineering (e.g., Ref. [96]). Similarly, edges symbolize the line segments (e.g., roads, bus lines) that connect the nodes. In addition, since the transportation network can be described as a directed network, the directed adjacency table will result being generally asymmetric.

The mathematical modeling of any supporting critical infrastructure network requires a precise characterization of the taxonomy, the footprint, and the granularity of the considered network [27,78]. Based on the function that critical infrastructure components fulfill, we identify: 1) origin nodes $v_{O,i}^{(k)}$ that generate the goods and services (e.g., distribution centers and warehouses, and employees' residencies); 2) destination nodes $v_{D,i}^{(k)}$ that receive and use the goods (e.g., retail stores, factories, and customers); and 3) transmission nodes $v_{T,i}^{(k)}$ that ensure the transmission of goods and services from generation to distribution components. The network footprint is defined in order to fully include the nodes of the supply-chain that can be affected by the occurrence of a damaging event (i.e., distribution centers and warehouses.) In addition, the purpose of the network modeling may shape the definition of the network granularity. As an example, the modeling of the movement of goods (e.g., from distribution centers to retail stores) on a road transportation network might narrow the model only to major roads, such as highways and main routes with no specific need to include minor or local roads, or other transportation modes like railway and overwater. In general, spatially hybrid modeling granularity is also possible when we model business interruption, since critical infrastructure encompass over large areas [79]. Further details on the models of network topology for different granularities can be found in Guidotti et al. [27].

3.2.1. Modeling the structural performance and functionality of infrastructure

With a flow-based approach it is possible to associate to the set of nodes and edges of the generic network $G^{(k)}$ a mapping (i.e., weights) that describes the performance and functionality of the network [9,14]. As an example, the mapping for a node of the transportation network may correspond to physical quantities able to capture its characteristics like the traffic capacity for a bridge. Similarly, for the set of edges the mapping may represent the flow of goods between two nodes. Mathematically, we can represent the flow over a network as follows:

$$\left\{ \psi^{(k)} \;:\; E^{(k)} \to \mathbb{R} \,\middle|\, \forall e_m^{(k)} \in E^{(k)} \;\exists\; \psi^{(k)}_{e_m^{(k)}} \Rightarrow \psi^{(k)}_{e_m^{(k)}} = \psi^{(k)}\left(e_m^{(k)}\right) \right\} \tag{1}$$

We now introduce the model $\mathbf{x}^{(k)}(v_i^{(k)}, t)$ of the state variables of the i^{th} node belonging to the generic k^{th} network $v_i^{(k)}$ at time t as [75,80]

$$\mathbf{x}^{(k)}\left(v_i^{(k)}, t\right) = \mathbf{x}^{(k)}\left[\mathbf{x}_0^{(k)}\left(v_i^{(k)}, t = 0\right), t, \mathbf{Z}^{(k)}(t); \boldsymbol{\Theta}_{\mathbf{x}^{(k)}}\right] \tag{2}$$

where $\mathbf{x}_0^{(k)}(v_i^{(k)}, t = 0)$ represents the vector of the state variables of the i^{th} node belonging to the k^{th} network at a reference time t; $\mathbf{Z}^{(k)}(t)$ is the vector of external conditions/variables at time t that includes environmental conditions/variables (e.g., temperature, and relative humidity) and shock intensity measures at the site of the node; and $\boldsymbol{\Theta}_{\mathbf{x}^{(k)}}$ is a vector that includes the parameters of the state model $\mathbf{x}^{(k)}(v_i^{(k)}, t)$. To mathematically describe the response of a generic component of the considered network, we can use the predicted value of $\mathbf{x}^{(k)}(v_i^{(k)}, t)$ in existing capacity and demand models. The general expression for the capacity and demand of a component is as follows:

$$\begin{cases} c^{(k)}(t) = c^{(k)}\left[\mathbf{x}^{(k)}\left(v_i^{(k)}, t\right); \boldsymbol{\Theta}_{c^{(k)}}\right] \\ d^{(k)}(t) = d^{(k)}\left[\mathbf{x}^{(k)}\left(v_i^{(k)}, t\right), \mathbf{Z}^{(k)}(t); \boldsymbol{\Theta}_{d^{(k)}}\right] \end{cases} \tag{3}$$

where $c^{(k)}[\mathbf{x}^{(k)}(v_i^{(k)}, t); \boldsymbol{\Theta}_{c^{(k)}}]$ and $d^{(k)}[\mathbf{x}^{(k)}(v_i^{(k)}, t), \mathbf{Z}^{(k)}(t); \boldsymbol{\Theta}_{d^{(k)}}]$ are the predicted capacity and demand of the component at time t, $\boldsymbol{\Theta}_{c^{(k)}}$ and $\boldsymbol{\Theta}_{d^{(k)}}$ are, respectively, the set of parameters of the capacity and demand model. As an example, in a transportation network, one can use the capacity models in Gardoni et al. [72] and the demand models in Gardoni et al. [71] for reinforced concrete bridges.

The capacity and demand models in Eq. (3) can be used to write the limit-state function as $g^{(k)}(t) = c^{(k)}(t) - d^{(k)}(t)$. Fragility functions are commonly defined as the conditional probability of attaining or exceeding a specified performance level given a (set of) hazard intensity measure(s) [18,71,72,93]. The conditional probability of being in a particular damage state is computed as the difference between the fragility curves [94]. Thus, we can obtain the fragility functions of any network component at any time t given the occurrence of a shock with an intensity $\mathbf{Z}^{(k)}(t)$ as $F^{(k)}[\mathbf{Z}^{(k)}(t); \boldsymbol{\Theta}^{(k)}] := P[g^{(k)}(t) \leq 0 | \mathbf{Z}^{(k)}(t)]$, where $\boldsymbol{\Theta}^{(k)} = (\boldsymbol{\Theta}_{\mathbf{x}^{(k)}}, \boldsymbol{\Theta}_{c^{(k)}}, \boldsymbol{\Theta}_{d^{(k)}})$. Following Gardoni et al. [72], different estimates of the fragility functions $F^{(k)}[\mathbf{Z}^{(k)}(t); \boldsymbol{\Theta}^{(k)}]$ can be obtained depending on how we treat the model parameters. A point estimate of the fragility function is obtained as $\widehat{F}^{(k)}[\mathbf{Z}^{(k)}(t)] = F^{(k)}[\mathbf{Z}^{(k)}(t); \widehat{\mathbf{\Theta}}^{(k)}]$, using a point estimate of $\widehat{\mathbf{\Theta}}^{(k)}$, in place of $\mathbf{\Theta}^{(k)}$. In general, the mean value of $\mathbf{\Theta}^{(k)}$ or the maximum likelihood estimate (MLE) $\mathbf{\Theta}^{(k)}_{\text{MLE}}$ can be used. However, point estimates do not incorporate the epistemic (statistical) uncertainties in the model parameters $\mathbf{\Theta}^{(k)}$. Alternatively, we can account for the uncertainty in $\mathbf{\Theta}^{(k)}$ and obtain a predictive estimate of the fragility as $\tilde{F}^{(k)}[\mathbf{Z}^{(k)}(t)] = \int F^{(k)}[\mathbf{Z}^{(k)}(t); \mathbf{\Theta}^{(k)}] f(\mathbf{\Theta}^{(k)}) d\mathbf{\Theta}^{(k)}$, where $f(\mathbf{\Theta}^{(k)})$ is the probability density function of $\mathbf{\Theta}^{(k)}$. Furthermore, we can obtain confidence bounds on the estimate of the fragility functions to identify the effect of the statistical uncertainties in the model parameters [72]. First, we can define the reliability index as

$$\beta\left(\mathbf{Z}^{(k)}(t); \mathbf{\Theta}^{(k)}\right) = \Phi^{-1}\left\{1 - F^{(k)}\left[\mathbf{Z}^{(k)}(t); \mathbf{\Theta}^{(k)}\right]\right\} \tag{4}$$

where $\Phi^{-1}(\cdot)$ indicates the inverse of the standard normal CDF. The variance of $\beta(\mathbf{Z}^{(k)}(t); \mathbf{\Theta}^{(k)})$ can then be estimated by a first-order Taylor expansion as

$$\sigma^2_\beta\left[\mathbf{Z}^{(k)}(t)\right] \approx \nabla_{\mathbf{\Theta}^{(k)}}\beta\left[\mathbf{Z}^{(k)}(t)\right]\mathbf{\Sigma}_{\mathbf{\Theta}^{(k)}\mathbf{\Theta}^{(k)}}\nabla_{\mathbf{\Theta}^{(k)}}\beta\left[\mathbf{Z}^{(k)}(t)\right]^{\mathrm{T}} \tag{5}$$

where $\nabla_{\mathbf{\Theta}^{(k)}}\beta[\mathbf{Z}^{(k)}(t)]$ is the gradient of $\beta[\mathbf{Z}^{(k)}(t)]$ evaluated at the mean value and $\mathbf{\Sigma}_{\mathbf{\Theta}^{(k)}\mathbf{\Theta}^{(k)}}$ is the posterior covariance matrix. Then, we can obtain, for example, bounds on the reliability index considering one standard deviation away from the mean that represent approximately 15% and 85% probability levels as

$$\left\{\Phi\left\{-\tilde{\beta}\left[\mathbf{Z}^{(k)}(t)\right] - \sigma_\beta\left[\mathbf{Z}^{(k)}(t)\right]\right\}, \Phi\left\{-\tilde{\beta}\left[\mathbf{Z}^{(k)}(t)\right] + \sigma_\beta\left[\mathbf{Z}^{(k)}(t)\right]\right\}\right\} \tag{6}$$

The fragility functions at the network component level provide only an estimate of the impact of a catastrophic event at the individual component level [27]. Infrastructure are composed of multiple components, and they are usually interconnected and (inter)dependent, leading to cascading impacts as disruptions of one component may spread to other dependent nodes within the network. At the network-level, we can mathematically describe the capacity and demand models as follows [27]:

$$\begin{cases} C^{(k)}(t) = C^{(k)}\left[\mathbf{X}^{(k)}(t); \mathbf{\Theta}_{C^{(k)}}\right] \\ D^{(k)}(t) = D^{(k)}\left[\mathbf{X}^{(k)}(t), \mathbf{Z}^{(k)}(t); \mathbf{\Theta}_{D^{(k)}}\right] \end{cases} \tag{7}$$

where $\mathbf{X}^{(k)}(t) = [\mathbf{x}^{(k)}(v_1^{(k)}, t), \ldots, \mathbf{x}^{(k)}(v_N^{(k)}, t)]$ represents the vector of the state variables of all the $N^{(k)}$ nodes in $G^{(k)}$; $\mathbf{\Theta}_{C^{(k)}}$ and $\mathbf{\Theta}_{D^{(k)}}$ are the set of parameters of the capacity and demand model. Once we have defined the capacity and demand models at the network-level, the flow introduced in Eq. (1) can be solved as an optimization problem that tends to minimize the total cost of the flows over the edges of the network subject to the traditional flow conservation and capacity constraints [81]. In general, we can express the optimization problem as

$$\begin{aligned}
&\text{minimize} && \sum_{e_m^{(k)}:\left\{e_m^{(k)} \in E^{(k)}\right\}} \varpi_{e_m^{(k)}} \psi^{(k)}_{e_m^{(k)}} \\
& && \forall v_{T,i}^{(k)} \in V^{(k)}: \sum_{y=1}^{Y^{(k)}} \psi_y^{(k)}\left[\left(\cdot, v_{T,i}^{(k)}\right)\right] = \sum_{z=1}^{Z^{(k)}} \psi_z^{(k)}\left[\left(v_{T,i}^{(k)}, \cdot\right)\right] \\
&\text{subject to} && \forall v_{D,i}^{(k)} \in V^{(k)}: \sum_{i=1}^{N_D^{(k)}}\left\{\sum \psi^{(k)}\left[\left(\cdot, v_{D,i}^{(k)}\right)\right]\right\} \geq D^{(k)}(t) \\
& && \forall e_m^{(k)} \in E^{(k)}: 0 \leq \psi^{(k)}\left(e_m^{(k)}\right) \leq C^{(k)}(t) \\
& && \sum_{i=1}^{N_G^{(k)}}\left\{\sum \psi^{(k)}\left[\left(v_{G,i}^{(k)}, \cdot\right)\right]\right\} - \sum_{i=1}^{N_D^{(k)}}\left\{\sum \psi^{(k)}\left[\left(\cdot, v_{D,i}^{(k)}\right)\right]\right\} = \psi_G^{(k)} - \psi_D^{(k)} = 0
\end{aligned} \tag{8.1)-(8.5}$$

where Eq. (8.1) represents the objective function to minimize; Eq. (8.2) represents the Kirchhoff's law (i.e., the conservation law), which states that the sum of the flows afferent to a (transmission) node equals the sum of the flows efferent from that node [76]; Eq. (8.3) and (8.4) ensure that capacity and demand constraints are satisfied; and Eq. (8.5) is the equilibrium condition that guarantees that the sum of the generated flows equals the sum of the distributed flows. The result obtained from the optimization problem described by the set of Eqs. (8.1)-(8.5) is hereafter called the supply $S^{(k)}(t)$, which measures the amount of goods and services passing through the edges and delivered at the nodes for a selected performance measure [73]. As an example, in a potable water network, we define the amount of water in terms of actual flow on the edges, whereas we can estimate its pressure at the nodes.

Under normal conditions (i.e., in the pre-hazard scenario), critical infrastructure typically guarantee a sufficient supply of goods and services at the destination node. However, after the occurrence of a damaging event, nodes may fail (and might need to be removed from the network), implying changes in the infrastructure ability to meet the imposed demand at time t_{0^+} (the time immediately after the occurrence of the damaging event.)

We introduce the functional threshold, defined as the minimum amount of service at the node to be functional, that can be mathematically expressed as

$$\left\{\gamma^{(k)} \;:\; V^{(k)} \rightarrow \mathbb{R} \,\middle|\, \forall v_n^{(k)} \in V^{(k)} \;\exists\; \gamma^{(k)}_{v_n^{(k)}, t_{0^+}} \Rightarrow \gamma^{(k)}_{v_n^{(k)}, t_{0^+}} = \gamma^{(k)}\left(v_n^{(k)}, t_{0^+}\right)\right\} \tag{9}$$

Thus, the fragility of a generic destination node $v_{D,i^*}^{(k)}$ (e.g., the i^{th} node that provides the k^{th} good and service to a business facility) can be defined as follows:

$$\begin{aligned} F_*^{(k)}\left[\mathbf{Z}^{(k)}(t); \mathbf{\Theta}^{(k)}\right] = P\Big\{ & S^{(k)}\left[\left(v_{D,i^*}^{(k)}, t = t_{0^+}\right)\right] - \gamma_D^{(k)}\left[\left(v_{D,i^*}^{(k)}, t = t_{0^+}\right)\right] \\ & \leq 0 \,\Big|\, \mathbf{Z}^{(k)}(t)\Big\} \end{aligned} \tag{10}$$

In particular, numerical simulations could be used to capture the uncertainty in the network reliability analysis in terms of the reduction or loss of functionality of a network described in Eq. (10).

In addition, critical infrastructure identify a system of interdependent networks that collaboratively operate to produce and distribute a continuous flow of goods and services [1]. We can represent the system of interdependent networks using an augmented adjacency table $\mathbf{A}$ [9]. In the case of K interdependent networks, the $\mathbf{A}^{(k)}$'s (for $k = 1, \ldots, \mathrm{K}$) adjacency matrices of the individual networks (introduced in Section 3.2) are arranged along the main diagonal, whereas the pairwise connections between nodes of different networks are captured in the out-of-diagonal tables. As an example, let us considering two generic networks s and l; the connections between nodes of the two networks can be captured by the table $\mathbf{A}^{(s,l)} = [a_{ij}^{(s,l)}]$, where $a_{ij}^{(s,l)}$ is either 1, if $e_m^{(s,l)} \in E^{(s,l)}$, or 0 otherwise. Also, following Sharma and Gardoni [73], we can incorporate the interdependency between networks through interface functions, obtaining the modified capacity and demand estimates of the k^{th} network $C'^{(k)}(t)$ and $D'^{(k)}(t)$. The modified estimates of the supply, $S'^{(k)}(t)$, can then be obtained using $C'^{(k)}(t)$ and $D'^{(k)}(t)$ in the set of Eqs. (8.1)-(8.5). The modified estimates of the supply, $S'^{(k)}(t)$, can then be used in Eq. (10) to estimate the fragility of a generic destination node $v_{D,i^*}^{(k)}$ (e.g., the i^{th} node that provides the k^{th} good and service to a business facility) considering the networks' dependencies and interdependencies. Moreover, the modified estimates of the supply, $S'^{(k)}(t)$, can be used to obtain the joint probability mass function of the system of K interdependent networks, in terms of provision (or not) of the needed goods and services to the business facility as follows:

$$F_*^{(1,\dots,\mathrm{K})}\left[\mathbf{Z}^{(1,\dots,\mathrm{K})}(t);\boldsymbol{\Theta}^{(1,\dots,\mathrm{K})}\right]=P\Big\{S'^{(1)}\left[\left(v_{D,i^*}^{(1)},t=t_{0^+}\right)\right]-\gamma_D{}^{(1)}\left[\left(v_{D,i^*}^{(1)},t=t_{0^+}\right)\right]\le 0,\dots, S'^{(\mathrm{K})}\left[\left(v_{D,i^*}^{(\mathrm{K})},t=t_{0^+}\right)\right]-\gamma_D{}^{(\mathrm{K})}\left[\left(v_{D,i^*}^{(\mathrm{K})},t=t_{0^+}\right)\right]\le 0\Big|\mathbf{Z}^{(1,\dots,\mathrm{K})}(t)\Big\} \tag{11}$$

In conclusion, Eq. (11) identifies the state of the K critical infrastructure that support the regular business operations.

3.3. Estimating the likelihood of business interruption: a matrix-based system reliability

In addition to capturing the loss or reduction in functionality of the physical infrastructure, modeling the likelihood of business interruption also requires incorporating the damage to business properties, as well as the impact on social systems (discussed later in Section 3.4.) As for the damage to the business properties, building-specific fragility functions (e.g., based on the building type and year of construction) can be used to estimate the conditional probability of attaining or exceeding a specified performance level given a (set of) hazard intensity measure(s). As an example, building-specific fragility functions for tilt-up buildings (widely used for low-rise structures that require a large open space such as distribution centers, retail stores, and other commercial and industrial facilities) could be found in Bai et al. [82].

We propose to estimate the likelihood of business interruption adopting a matrix-based system reliability analysis [16]. Let us consider the occurrence of business interruption as a system event whose k^{th} cause ($k=1,\dots,\mathrm{K},\mathrm{K}+1$), corresponding to the k^{th} critical infrastructure, or the business properties, may have δ_r distinct states, $r=1,\dots,\omega$. The sample space of the component events can then be divided into $\Omega=\prod_{r=1}^{\omega}\delta_r$ mutually exclusive and collectively exhaustive (MECE) events, denoted by ξ_q, $q=1,\dots,\Omega$. Therefore, a general system event (i.e., the occurrence of business interruption) denoted as $\Xi^{(sys)}$ can be represented by the vector $\boldsymbol{\alpha}_{\{\xi_q\in\ \Xi^{(sys)}\}}$, where $\boldsymbol{\alpha}_{\{\xi_q\in\ \Xi^{(sys)}\}}$ is an incident vector such that the p^{th} component $\alpha_{\{\xi_q\in\ \Xi^{(sys)}\},p}=1$, when $\xi_q\in\ \Xi^{(sys)}$ and $\alpha_{\{\xi_q\in\ \Xi^{(sys)}\},p}=0$, otherwise. Also, let $p_q:=P(\xi_q)$, $q=1,\dots,\Omega$, being the probability of the event ξ_q. The conditional probability of business interruption, $P\left[\Xi^{(sys)}|\mathbf{IM}\right]$, where $\mathbf{IM}$ represents the vector of the hazard characteristics (e.g., in the case of an earthquake it may be the magnitude, depth, and distance of the seismic event), can be easily calculated as the sum of the probabilities of the events ξ_q's due to the mutual exclusiveness of the events ξ_q. In particular, following the discussion in Section 2, a business interruption occurs when at least one of the factors causing a business interruption occurs. Therefore, the conditional probability of business interruption corresponds to occurrence probability of the union of the $\mathrm{K}+1$ causing events. Following Kang et al. [16], we can compute the vector $\boldsymbol{\alpha}_{\{\xi_q\in\ \Xi^{(sys)}\}}:=\boldsymbol{\alpha}^{\Xi^{(1)}\cup\Xi^{(2)}\cup\dots\cup\Xi^{(\mathrm{K})}\cup\Xi^{(\mathrm{K}+1)}}\equiv\boldsymbol{\alpha}^{\Xi^{(sys)}}$, where $\Xi^{(k)}$, for $k=1,\dots,\mathrm{K},\mathrm{K}+1$, denotes the failure event of the k^{th} cause, and $\Xi^{(sys)}$ is the failure system event, as

$$\boldsymbol{\alpha}^{\Xi^{(1)}\cup\Xi^{(2)}\cup\dots\cup\Xi^{(\mathrm{K})}\cup\Xi^{(\mathrm{K}+1)}}=\mathbf{1}-\left(\mathbf{1}-\boldsymbol{\alpha}^{\Xi^{(1)}}\right)\odot\left(\mathbf{1}-\boldsymbol{\alpha}^{\Xi^{(2)}}\right)\odot\dots\odot\left(\mathbf{1}-\boldsymbol{\alpha}^{\Xi^{(\mathrm{K})}}\right)\odot\left(\mathbf{1}-\boldsymbol{\alpha}^{\Xi^{(\mathrm{K}+1)}}\right) \tag{12}$$

where $\mathbf{1}$ is the all-ones vector of the same size as the event vector, and "$\odot$" corresponds to the Hadamard product (i.e., the element-by-element multiplication.) However, when estimating the conditional probability of business interruption, it is easier to compute the complementary probability (i.e., when the system does not fail). Therefore, the vector $\boldsymbol{\alpha}_{\{\xi_q\in\ \Xi^{(sys)}\}^{\mathrm{C}}}:=\boldsymbol{\alpha}^{\overline{\Xi^{(1)}\cup\Xi^{(2)}\cup\dots\cup\Xi^{(\mathrm{K})}\cup\Xi^{(\mathrm{K}+1)}}}\equiv\boldsymbol{\alpha}^{\overline{\Xi}^{(sys)}}$, where $\{\xi_q\in\ \Xi^{(sys)}\}^{\mathrm{C}}$ denotes the complementary set of survival events, can be expressed as $\boldsymbol{\alpha}:=\boldsymbol{\alpha}^{\overline{\Xi^{(1)}\cup\Xi^{(2)}\cup\dots\cup\Xi^{(\mathrm{K})}\cup\Xi^{(\mathrm{K}+1)}}}=\mathbf{1}-\boldsymbol{\alpha}^{\Xi^{(1)}\cup\Xi^{(2)}\cup\dots\cup\Xi^{(\mathrm{K})}\cup\Xi^{(\mathrm{K}+1)}}$. As for the probability vector $\mathbf{p}$, it can be constructed, based on Eq. (11), by an iterative matrix procedure such that

$$\begin{aligned}&\mathbf{p}_{[1]}=\left\{F_*^{(1)}\left[\mathbf{Z}^{(1)}(t);\boldsymbol{\Theta}^{(1)}\right]\quad \overline{F}_*^{(1)}\left[\mathbf{Z}^{(1)}(t);\boldsymbol{\Theta}^{(1)}\right]\right\}^{\mathrm{T}}\\ &\mathbf{p}_{[k]}=\left\{\begin{matrix}\mathbf{p}_{[k-1]}\cdot F_*^{(k)}\left[\mathbf{Z}^{(k)}(t);\boldsymbol{\Theta}^{(k)}\right]\\ \mathbf{p}_{[k-1]}\cdot \overline{F}_*^{(k)}\left[\mathbf{Z}^{(k)}(t);\boldsymbol{\Theta}^{(k)}\right]\end{matrix}\right\}\\ &\text{for } k=2,3,\dots,\mathrm{K},\mathrm{K}+1\end{aligned} \tag{13}$$

Thus, we can compute the conditional probability of business interruption as

$$P\left[\Xi^{(sys)}|\mathbf{IM}\right]=1-\sum\nolimits_{\{\xi_q\in\ \Xi^{(sys)}\}^{\mathrm{C}}}P\left(\xi_q\right)=1-\boldsymbol{\alpha}^{\mathrm{T}}\mathbf{p} \tag{14}$$

When estimating $P\left[\Xi^{(sys)}|\mathbf{IM}\right]$, numerical simulations could be used to propagate the uncertainty in the vector $\mathbf{p}$.

3.3.1. Estimating importance measures using conditional probabilities

Computing the conditional probability of business interruption adopting a matrix-based system reliability analysis also allows us to easily compute byproducts such as importance measures, which are able to capture the relative contributions of subsystems to the likelihood of business interruption. In particular, importance measures are relevant to develop specifically tailored mitigation strategies to reduce, for instance, the likelihood of business interruption. Following a similar approach proposed in Kang et al. [16], we propose to use the conditional probability of a component event given the system failure as an importance measure of the component ($CIM^{(k)}$). Based on the definition of conditional probability, $CIM^{(k)}$ can be computed as

$$CIM^{(k)}=P\left(\Xi^{(k)}\middle|\Xi^{(sys)}\right)=\frac{P\left(\Xi^{(k)}\cap\Xi^{(sys)}\right)}{P\left(\Xi^{(sys)}\right)} \tag{15}$$

Therefore, in a matrix-based system reliability method, $CIM^{(k)}$, computed as the ratio between probability of a new system event $\breve{\Xi}^{(sys)}$ and the system event of interest $\Xi^{(sys)}$, can be expressed as

$$CIM^{(k)}=\frac{P\left(\breve{\Xi}^{(sys)}\right)}{P\left(\Xi^{(sys)}\right)}=\frac{\breve{\boldsymbol{\alpha}}^{\mathrm{T}}\mathbf{p}}{1-\boldsymbol{\alpha}^{\mathrm{T}}\mathbf{p}} \tag{16}$$

where $\breve{\boldsymbol{\alpha}}$ represents the vector event corresponding to $\breve{\Xi}^{(sys)}$, which can be obtained using the matrix-based system reliability method [16] by matrix manipulation, whereas there is no need to re-compute the probability vector. In addition, Eq. (16) can be used to estimate the probability mass function of the number of causes, and the importance measures of the component in each scenario, simply identifying the Boolean description of the vector $\breve{\boldsymbol{\alpha}}$.

3.4. Integrating physical infrastructure and social systems in the modeling of business interruption

The proposed formulation also considers the societal impact of a hazard to properly estimate the available employees to carry business activities. The set of available employees can be modeled as weights, introduced in Section 3.2.1, associated to a critical infrastructure network (e.g., the road transportation network.) A social weight, υ, representing the social systems characteristics at a node $v_i^{(k)}$ can be mathematically expressed as $\upsilon\left(v_i^{(k)}\right) = \upsilon\left[\mathbf{X}^{(1,\ldots,K)}, \mathbf{Z}^{(1,\ldots,K)}(t), \boldsymbol{\lambda}; \boldsymbol{\Theta}^{(1,\ldots,K)}, \boldsymbol{\Theta}_\lambda\right]$, where $\boldsymbol{\lambda}$ and $\boldsymbol{\Theta}_\lambda$ represent, respectively, the vector of the employees' socio-economic characteristics (e.g., race, income), and the corresponding set of parameters.

In order to consider the changes in the weights after the occurrence of a catastrophic event, first we estimate the structural and non-structural damage to the residential building inventory using fragility functions (e.g., Refs. [82–86]). Based on the estimate of the physical damage to buildings, we can estimate the corresponding casualties (e.g., Refs. [66–68,87]). Employees estimated to be casualties with Severity 1 or higher, for example, can be removed from the set of available employees, implying a change in the social weight $\upsilon(v_i^{(k)})$. Moreover, the set of potential available employees may be reduced due to population dislocation, which could be due to physical damage to residential buildings (e.g. Ref. [88]), or lack of services to the residential buildings [25,27]. Therefore, also employees estimated to dislocate need to be removed from the set of available employees. Finally, the available workforce is estimated considering employees that are not injured, dead, or dislocated and that have physical access to the business facility.

4. Example: estimating the likelihood of business interruption of a food retailer in Seaside, Oregon

This section presents the proposed mathematical formulation to estimate the conditional probability of business interruption of a hypothetical food retail store located in Seaside, Oregon. Seaside is a coastal city located in Clatsop County with 6440 off-season inhabitants according to the 2010 census data [25]. As a disrupting event, we consider a hypothetical earthquake originated from the Cascadia Subduction Zone with magnitude $M_W = 7.0$, at a depth of 10 km, located 25 km southwest of Seaside (45°48′23.1″N, 124°06′04.3″W). Ground Motion Prediction Equations [89] are used to obtain maps of the Peak Ground Acceleration (PGA) and Spectral Acceleration (S_a) at the natural period of the vulnerable components considered in this paper (i.e., bridges, pumping stations, tanks, and pipelines, generators, substations, and transmission lines, business properties and facilities, and residential buildings.) In the presented example, we consider the dependency of the food retail store activities on the integrity of the business property, the supporting infrastructure (i.e., transportation, water and electric power), and the social systems in terms of available employees.

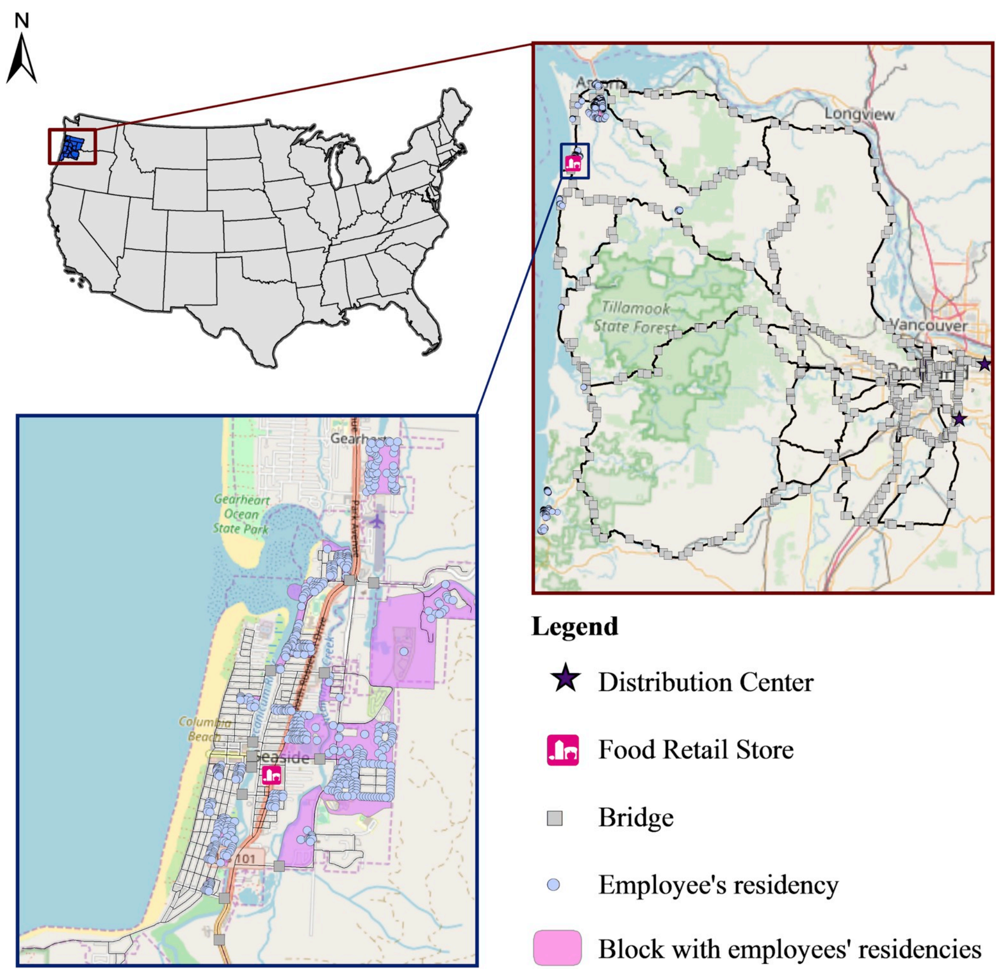

Fig. 1. Developed model of the transportation network.

4.1. Damage to the buildings

As for the business properties, in this example, we consider the food retail store as the only vulnerable building, assuming that the two distribution centers located in Portland are sufficiently far from the seismic source. Furthermore, we assumed that the food retail store is a (low-code) wood W2 structural system following the definition provided in HAZUS-MH [67]. Therefore, we estimate the damage, i.e., extensive or complete structural damage, to the food retail store using the corresponding building-specific fragility function according to the assumed building type (i.e., HAZUS-MH [67].)

As for the residential buildings (later used to estimate the employees' availability), we use the building inventory of Seaside, Oregon described in Park et al. [90], which includes a taxonomy of the building types later used to select the appropriate fragility. We estimate the damage to the residential buildings using fragility functions (i.e., HAZUS-MH [67], and [87]) based on the building type, considering four possible damage states (i.e., insignificant, moderate, heavy, and complete), as proposed in Bai et al. [94].

4.2. Damage to the transportation infrastructure

The data for the model of the road transportation infrastructure are available from the Census Bureau (https://www.census.gov/geo/maps-data/data/tiger.html). The data include the different type of roads (i.e., local roads, routes, highways, and interstates), which are used to define the set of edges of the network. The bridge inventory is available from the U.S Department of Transportation in the Federal Highway Administration section (https://www.fhwa.dot.gov/bridge/nbi/ascii.cfm), which includes the bridge taxonomy used in the paper to select the appropriate fragility functions. Data include bridge locations, as well as geometry characteristics, year built (that characterize the state variables **x** for these network components.)

As for the footprint of the transportation network, we consider an area that extends to Portland, Oregon, where we assumed that there are two distribution centers providing the goods to the food retail store in Seaside. In terms of the granularity of the transportation network, we model all of the roads within the city of Seaside, and only the major roads, such as highways and main routes outside of Seaside (as edges between the food retail store located in Seaside and the distribution centers.) Thus, we developed the model with hybrid granularity for the

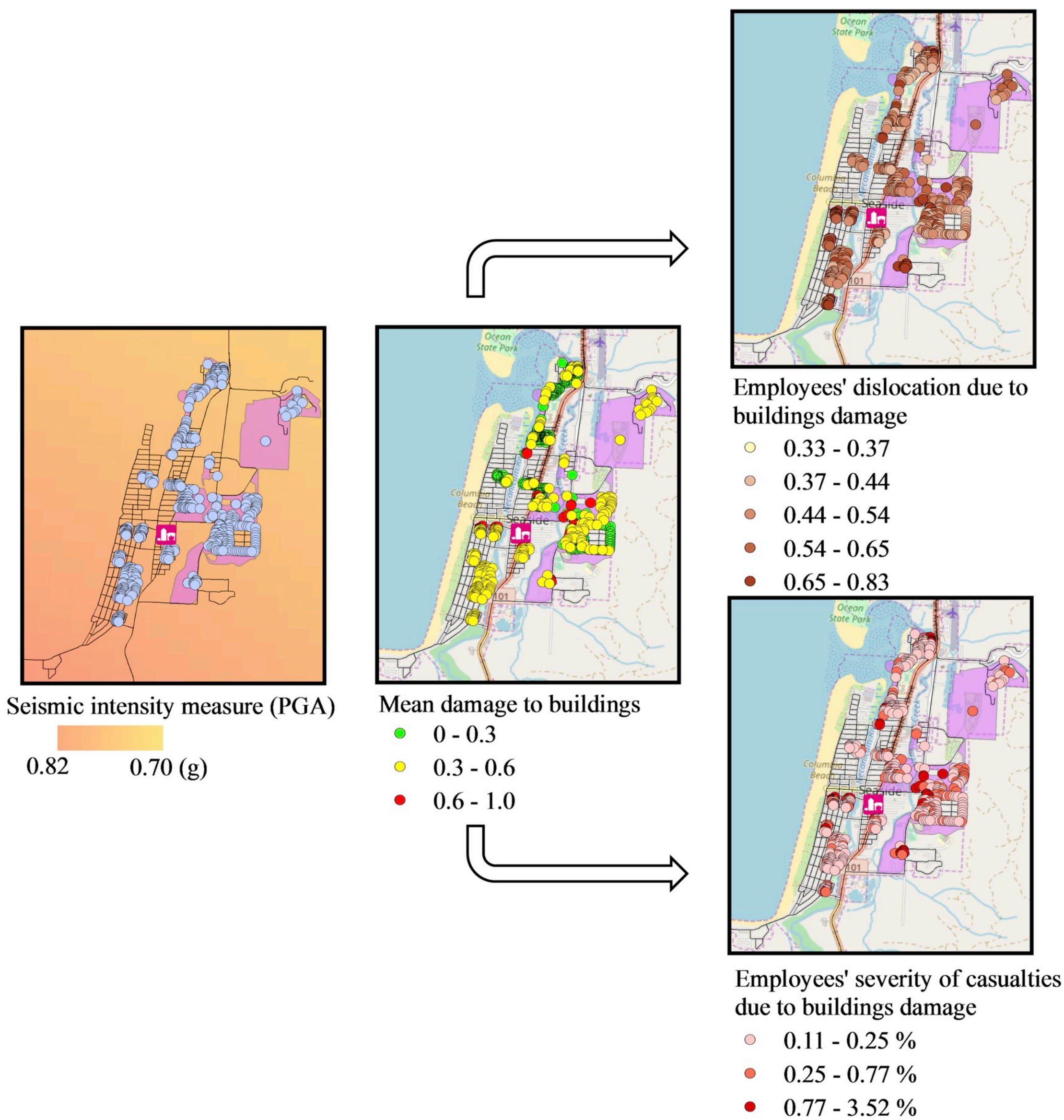

Fig. 2. (Left hand side) Map of the seismic intensity measure (PGA); (Center) mean damage to the Seaside employees' residencies. (Right hand side) Map of employees' dislocation and severity of casualties due to damage to buildings.

transportation network, as shown in Fig. 1. Fig. 1 also includes information on the spatial location of the food retail store, distribution centers, and employees' residencies. Additional details on the transportation network and the bridges can be found in Nocera and Gardoni [78].

In this example, the bridges are considered as the vulnerable nodes of the transportation network (i.e., the only components that may impede the mobility along the transportation network). Capacity and demand models as those in Eq. (3) are used to obtain fragility functions, as well as the conditional probability of being in a particular damage state. We consider five different damage states: none, slight, moderate, heavy, and complete (as described in Section 3), and we consider a bridge to be closed when its damage state is either moderate, or heavy, or complete [91]. In particular, for the reinforced concrete bridges, we used the probabilistic capacity and demand models in Gardoni et al. [71,72] and obtained the estimates of the fragility functions performing a reliability analysis. To compute the conditional probability of being in a particular damage state (i.e., slight, moderate, and heavy), we considered capacity drift values of 1, 2, and 4% based on Simon et al. [95]. For the steel and

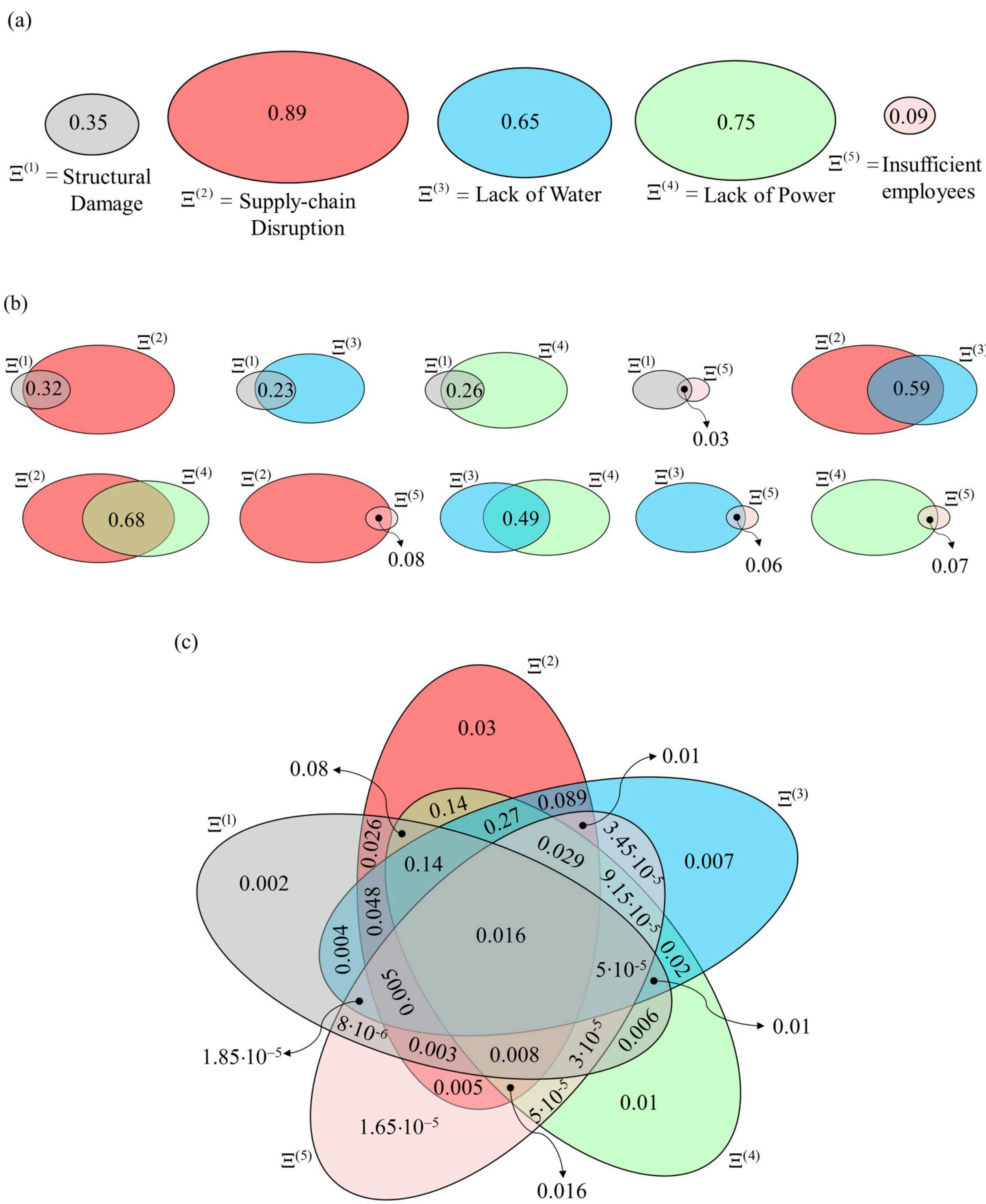

Fig. 3. Venn diagram representation of the causes of business interruption considering (a) the individual events; (b) the intersection of each pair of the events; (c) all five considered events.

wood bridges, we adopt fragility curves from HAZUS-MH [67]. The impact of the disruptions in the transportation network on the business activities is twofold: disruption in the supply-chain in terms of availability of goods to sell, and impaired access of employees (discussed later). As for the disruption to the supply-chain, we assume that the business experiences an interruption when the food retail store is disconnected from both distribution centers.

4.3. Damage to the water and power infrastructure

To create the water network, we use the information and output of the analyses in Guidotti et al. [27]. Specifically, we assume that the food retail store experiences an interruption when the operating pressure at the distribution node is equal or below 0. The impact of the earthquake on the power network is modeled in this example assuming that the probability of lack of power at the food retail store follows a Beta distribution with mean 0.75 and standard deviation 0.12. The assumed values are consistent with the Oregon resilience plan (https://www.oregon.gov/oem/Documents/Oregon_Resilience_Plan_Final.pdf). While for this example these values are assumed, one can do a power flow analysis of the damaged electric power network as in Sharma and Gardoni [10] to estimate them. The probability of lack of power is assumed to be a random variable to capture the uncertainties that would be reflected in a power flow analysis.

4.4. Integration of physical infrastructure and social systems

As for the employees of the food retail store, data on the spatial locations of their residences are available from the Census Bureau in the Longitudinal Employer-Households Dynamics Origin-Destination Employment Statistics (LODES) section (https://lehd.ces.census.gov/data/). The LODES data consist in a table where there is a one-to-one correspondence between city block of the employees' residencies and the block of the workplace. Because the developed model requires information at the individual building level, we randomly allocate employees within a block to each building. We use the information from the LODES data to assign the weights introduced in Section 3.4 to the nodes of the developed model of the transportation network (i.e., in terms of number of employees for a given location.) Because there is uncertainty in the exact location of the employees within each block, we use a Monte Carlo Simulation (MCS) to integrate the uncertainty in the impact of social systems on the likelihood of business interruption. At each run of the MCS, employees are randomly associated to a building within the known block from the LODES data.

We estimate the probability that an employee is unavailable either because he or she is a casualty of Level 1–4, or because he or she has to dislocate. The conditional probability of the severity of a casualty as well as the probability of dislocation are modeled as functions of the structural damage of his/her residence. Specifically, we estimate the conditional probability mass function (PMF) of the casualty severity according to HAZUS-MH [67]. While, a logistic regression model [88] is used to compute the probability of household dislocation for a given structural damage (as described in Section 4.1) as a function of the social characteristics (i.e., race). The estimated available employees are then used as the nodal weights $v(v_i^{(k)})$ in the transportation network as described in Section 3.4.

For each run of the MCS, Eqs. (8.1)-(8.5) are used to estimate the number of employees out of the available ones that are physically connected to the food retail store, i.e. $S^{(k)}[(v_D^{(k)}, t = t_{0^+})]$, and we check whether the limit state function $S^{(k)}[(v_D^{(k)}, t = t_{0^+})] - \gamma_D^{(k)}[(v_D^{(k)}, t = t_{0^+})]$ in Eq. (10) is positive or not, where D is the destination food retail store and k indicates the transportation network. For this example, we assume that the functional threshold $\gamma_D^{(k)}[(v_D^{(k)}, t = t_{0^+})]$ equals to 1/3 of the regular available employees. To compute $F_*^{(k)}$ in Eq. (10), we use the generated MCS with a termination criterion based on the coefficient of variation (COV) equal to 0.05.

Fig. 2 shows a zoom-in on Seaside of the seismic intensity, the mean damage to the buildings, where the mean is computed following Bai et al. [94], as well as the cascading effects in terms of the severity of casualties and employees' dislocation.

4.5. Business interruption and importance measures

Based on the five considered causes, we identify $\Omega = 2^5 = 32$ mutually exclusive collectively exhaustive events as discussed in Section 3.3. We construct the vector $\boldsymbol{\alpha}$ as $\boldsymbol{\alpha}_{\{\xi_q \in\ \Xi^{(sys)}\}^C} = [\,0 \quad 0 \quad \dots \quad 1\,]^T_{32\times 1}$. Next, we construct the probability vector $\mathbf{p}$ as in Eq (13) (i.e., $\mathbf{p} = \left[F_*^{(1)} F_*^{(2)} F_*^{(3)} F_*^{(4)} F_*^{(5)} \quad \overline{F}_*^{(1)} F_*^{(2)} F_*^{(3)} F_*^{(4)} F_*^{(5)} \quad F_*^{(1)} \overline{F}_*^{(2)} F_*^{(3)} F_*^{(4)} F_*^{(5)} \quad \dots \quad \overline{F}_*^{(1)} \overline{F}_*^{(2)} \overline{F}_*^{(3)} \overline{F}_*^{(4)} \overline{F}_*^{(5)}\right]^T_{32\times 1}$.)
To propagate the uncertainty in the probability of business interruption, we model the probabilities in $\mathbf{p}$ as random variables. We estimate the conditional probability of business interruption $P\left[\Xi^{(sys)} | \mathbf{IM}\right]$ as in Eq. (14) using a MCS where for each run, we use a realization of $\mathbf{p}$ and compute $P\left[\Xi^{(sys)} | \mathbf{IM}\right] = 1 - \boldsymbol{\alpha}^T \mathbf{p}$. A termination criterion based on the COV of $P\left[\Xi^{(sys)} | \mathbf{IM}\right]$ equal to 0.05 is adopted in the analysis.

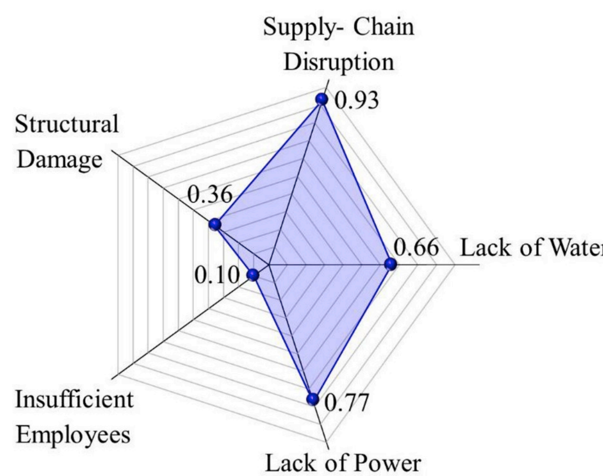

Fig. 4. CIM of the causes of business interruption.

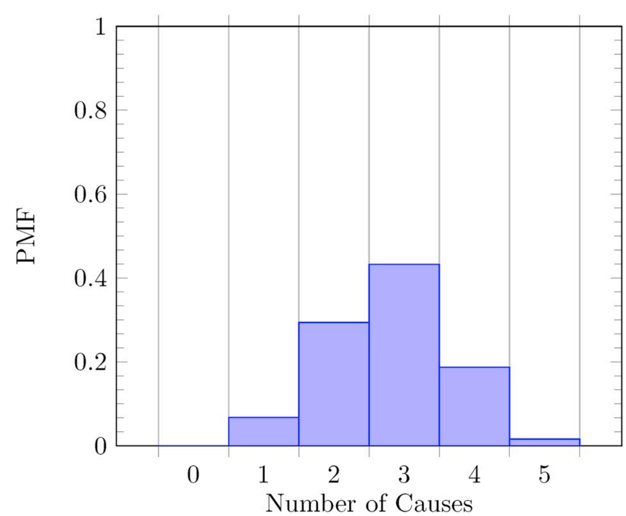

Fig. 5. Probability mass function of the number of causes of business interruption.

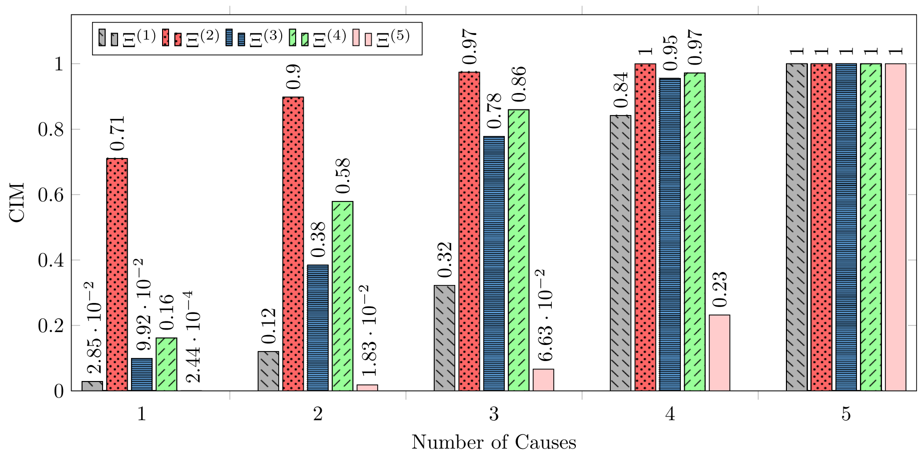

Fig. 6. CIMs of the causes of business interruption in the five cases.

Fig. 3(a) shows the probability of each event $\Xi^{(k)}$ for the considered scenario earthquake in a Venn diagram representation. We can see that the considered earthquake is most likely to create a disruption to the supply-chain, while the least likely event is having insufficient employees. Fig. 3(b) shows the probability of the intersection of each pair of events. We see that the probabilities of the intersections are significant, so the events are not mutually exclusive, and we cannot add the probabilities in Fig. 3(a) to obtain the probability of business interruption (this is why we have to use Eq. (14).) Fig. 3(c) shows the contributions of each sole cause to $P[\Xi^{(sys)}|\mathbf{IM}]$. Adding all contributions gives that for the considered scenario earthquake $P[\Xi^{(sys)}|\mathbf{IM}] = 0.98$. If $P[\Xi^{(sys)}|\mathbf{IM}]$ was computed only considering structural damage (as most commonly done in practice), $P[\Xi^{(sys)}|\mathbf{IM}]$ would be estimated as 0.35 (capturing only about one third of the actual probability). The results show that considering only the structural damage as a possible cause of business interruption would lead to significantly underestimating its probability.

Because the events $\Xi^{(k)}$ are not mutually exclusive, mitigation strategies targeted at one of the causes of business interruption would not reduce the probability of business interruption by the probability of that specific event. To understand which intervention is most beneficial, we need to look at the measures of importance defined in Section 3.3.1. Following Section 3.3.1, we estimate $CIM^{(k)}$ using Eq. (16), i.e. $P(\Xi^{(k)})/(1-\boldsymbol{\alpha}^{\mathrm{T}}\mathbf{p})$ for $k = 1,\dots,5$. For this example, $\breve{\boldsymbol{\alpha}}$ is defined as $\boldsymbol{\alpha}^{\Xi^{(k)}}$, for $k = 1,\dots,5$. Fig. 4 shows the computed $CIM^{(k)}$ of the considered causes of business interruption. The plot shows that the probability of structural damage being one of the causes of business interruption is 0.36. Similarly, the probability of having supply-chain disruption, lack of water, lack of power, or insufficient employees when experiencing business interruption are equal to 0.93, 0.66, 0.77, and 0.10. As noted for Fig. 3, considering structural damage as the possible sole cause of business interruption would significantly underestimate $P[\Xi^{(sys)}|\mathbf{IM}]$.

Following Section 3.3.1, we also estimate the probability of having only one cause of business interruption, exactly two causes, and up to exactly five causes. As for Eq. (16), to estimate the PMF of the number of causes of $\Xi^{(sys)}$, we construct a new vector $\breve{\boldsymbol{\alpha}}$ for the five cases as follows:

(i) $\breve{\boldsymbol{\alpha}} := \boldsymbol{\alpha}^{\Xi^{(1)}\overline{\Xi}^{(2)}\overline{\Xi}^{(3)}\overline{\Xi}^{(4)}\overline{\Xi}^{(5)} \cup \overline{\Xi}^{(1)}\Xi^{(2)}\overline{\Xi}^{(3)}\overline{\Xi}^{(4)}\overline{\Xi}^{(5)} \cup \dots \cup \overline{\Xi}^{(1)}\overline{\Xi}^{(2)}\overline{\Xi}^{(3)}\overline{\Xi}^{(4)}\Xi^{(5)}}$;

(ii) $\breve{\boldsymbol{\alpha}} := \boldsymbol{\alpha}^{\Xi^{(1)}\Xi^{(2)}\overline{\Xi}^{(3)}\overline{\Xi}^{(4)}\overline{\Xi}^{(5)} \cup \Xi^{(1)}\overline{\Xi}^{(2)}\Xi^{(3)}\overline{\Xi}^{(4)}\overline{\Xi}^{(5)} \cup \dots \cup \overline{\Xi}^{(1)}\overline{\Xi}^{(2)}\overline{\Xi}^{(3)}\Xi^{(4)}\Xi^{(5)}}$;

(iii) $\breve{\boldsymbol{\alpha}} := \boldsymbol{\alpha}^{\Xi^{(1)}\Xi^{(2)}\Xi^{(3)}\overline{\Xi}^{(4)}\overline{\Xi}^{(5)} \cup \Xi^{(1)}\Xi^{(2)}\overline{\Xi}^{(3)}\Xi^{(4)}\overline{\Xi}^{(5)} \cup \dots \cup \overline{\Xi}^{(1)}\overline{\Xi}^{(2)}\Xi^{(3)}\Xi^{(4)}\Xi^{(5)}}$;

(iv) $\breve{\boldsymbol{\alpha}} := \boldsymbol{\alpha}^{\Xi^{(1)}\Xi^{(2)}\Xi^{(3)}\Xi^{(4)}\overline{\Xi}^{(5)} \cup \Xi^{(1)}\Xi^{(2)}\Xi^{(3)}\overline{\Xi}^{(4)}\Xi^{(5)} \cup \dots \cup \overline{\Xi}^{(1)}\Xi^{(2)}\Xi^{(3)}\Xi^{(4)}\Xi^{(5)}}$; and

(v) $\breve{\boldsymbol{\alpha}} := \boldsymbol{\alpha}^{\Xi^{(1)}\Xi^{(2)}\Xi^{(3)}\Xi^{(4)}\Xi^{(5)}}$.

Fig. 5 shows the PMF of the number of causes of $\Xi^{(sys)}$. The results show that the occurrence of business interruption due to three out of the five causes is the most likely scenario. This means that the most effective mitigation strategy would likely have to address three causes of business interruption.

In addition, we estimate the $CIM^{(k)}$ of the causes of $\Xi^{(sys)}$ in the five different scenarios. Fig. 6 shows the results. The numbers represent the probabilities that the specific event is one the causes of the occurrence of $\Xi^{(sys)}$ in the five cases. Fig. 6 shows that supply-chain disruption, lack of water, and lack of power are the most likely causes of the occurrence of $\Xi^{(sys)}$. Instead, structural damage to the business properties generally plays a minor role to $P[\Xi^{(sys)}|\mathbf{IM}]$. The results from the PMF in Fig. 5, along with the $CIM^{(k)}$ in Fig. 6 can be used as a guidance in selecting the most effective mitigation strategies to reduce $P[\Xi^{(sys)}|\mathbf{IM}]$.

5. Conclusions

Current formulations to estimate the likelihood of business interruption due to the occurrence of a natural hazard only consider the probability of damage to the business properties. However, there are other possible causes of business interruption like damage to the critical supporting infrastructure like water and power, and lack of available employees. This paper proposed a mathematical formulation to estimate the likelihood of business interruption incorporating the dependency of business operations on business properties, critical infrastructure, and social systems. The proposed formulation starts by modeling and quantifying the direct physical damage to the business properties, the impact to the functionality of the supporting infrastructure, and the changes in the social systems due to the occurrence of a natural hazard. Then, the proposed formulation integrated the effects of the individual causes that may lead to business interruption in a matrix-based system reliability method to estimate the likelihood of business interruption.

The paper illustrated the proposed formulation modeling the likelihood of business interruption of a hypothetical food retail store located in Seaside, Oregon subject to a scenario earthquake originated from the Cascadia subduction zone. The example considers as the supporting infrastructure the transportation, water, and electrical power. As for the dependency on social systems, the example considers the reduction of available employees due to possible casualties and dislocation. For the considered example, the probability of business interruption is computed considering different possible combinations of causes. Finally, importance measures are estimated to rank the causes of business interruption. The importance measures can help in selecting most efficient mitigation strategies. For this example, we see that the supporting critical infrastructure make significant contributions to the probability of business interruption. So, considering only the damage to the business properties would significantly underestimate such probability. While the example focused on a food retail store subject to an earthquake, the proposed formulation is general and applicable to different category of businesses, critical infrastructure, and hazards.

Notes

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