Proposal for testing the electric Aharonov-Bohm effect with superconductors

Thomas C. Bachlechner[1] and Matthew Kleban[2]
[1]*Department of Physics, University of California San Diego, La Jolla, California 92093, USA*
[2]*Department of Physics, New York University, New York 10003, USA*

(Received 7 October 2019; revised manuscript received 9 April 2020; accepted 14 April 2020; published 4 May 2020)

The phase of the wave function of charged matter is sensitive to the value of the electric potential, even when the matter never enters any region with nonvanishing electromagnetic fields. Despite its fundamental character, this archetypal electric Aharonov-Bohm effect has evidently never been observed. We propose an experiment to detect the electric potential through its coupling to the superconducting order parameter. A potential difference between two superconductors will induce a relative phase shift that is observable via the DC Josephson effect even when no electromagnetic fields ever act on the superconductors, and even if the potential difference is later reduced to zero. This is a type of electromagnetic memory effect, and would directly demonstrate the physical significance of the electric potential.

DOI: 10.1103/PhysRevB.101.174504

I. INTRODUCTION

Electrodynamics is conventionally described using scalar and vector potentials, even though in classical physics only the electric and magnetic field strengths are observable. It was noticed long ago that the potentials themselves have direct physical significance in that they can affect the phase of the quantum mechanical wave function of charged matter even in regions where the field strengths vanish [1–3]. Such phase differences can then be observed with interference experiments.

In their seminal paper [2], Aharonov and Bohm describe two archetypal versions of their effect. The best known version today is magnetostatic, with vanishing electric field and a magnetic field **B** that is nonzero only within a solenoidal tube. In this situation the vector potential is (necessarily) nonvanishing [4] outside the tube, and charged particles that propagate around it will experience a phase shift proportional to the magnetic flux in the tube, despite never entering the region of nonzero **B**. This magnetic Aharonov-Bohm (AB) effect was observed long ago [5,6].

The obvious electric counterpart is a setup where charged particles propagate only in regions of vanishing electric field **E**, but different potential due to the presence of nonzero electric fields somewhere else. The experiment proposed in Ref. [2] was to pass two electron beams through Faraday cages, with a different time-varying voltage applied to each. To our knowledge this experiment has yet to be carried out, nor has the electric version of the AB effect been experimentally verified.

In this paper we propose a simple and feasible experimental setup employing superconductors. This could verify the physical significance of the electric potential in a region of vanishing fields. For our purposes it is essential that the charged particles are never exposed to nonzero electric fields. Such an effect—where the interaction is between charges and potentials in a region of vanishing field strengths—is sometimes referred to as Aharonov-Bohm type I, while effects that can be explained by interactions in regions of nonvanishing field strengths are referred to as type II [7]. There have been some experimental studies of the electric AB effect that failed to verify the effect [8], while others made positive observations [9,10]. However, these two experiments constitute measurements of the type-II effect in that the charged particles traversed regions of nonzero electric field. In this paper we are interested exclusively in the type-I effect that (to our knowledge) has yet to be tested [11].

Consider two superconductors that are initially connected so that their phase difference is zero, $\Delta\theta = 0$. Subsequently, the superconductors are placed on either side of a large planar capacitor. In the temporal gauge (where $\boldsymbol{E} = -\dot{\boldsymbol{A}}$) Gauss' law reads

$$\rho = -\boldsymbol{\nabla} \cdot \dot{\boldsymbol{A}}. \tag{1}$$

Charging the capacitor and maintaining a fixed voltage across it changes the initially vanishing vector potential, $\boldsymbol{A}(t = 0) = \boldsymbol{0}$, to a pure gradient, $\boldsymbol{A}(t) = \boldsymbol{\nabla}\lambda(x, t)$, that is nonvanishing between the capacitor plates. Assuming the capacitor plates are very large or that the superconductors are enclosed in Faraday cages, the electric field at the superconductors remains zero at all times. The function $\lambda(x, t)$ depends linearly on time while a constant voltage on the capacitor is maintained, and is independent of position outside but sufficiently near the capacitor plates and inside any Faraday cages [12].

We can eliminate the nontrivial vector potential by the gauge transformation $\boldsymbol{A} \to \boldsymbol{A} - \boldsymbol{\nabla}\lambda(x, t)$, where again $\lambda(x, t) = \int dx A_x$ is constant in space but takes different values on either side of the capacitor. The gauge transformation acts on the phase of the superconductors as $\theta \to \theta - q\lambda/\hbar$, showing that a time-dependent gauge configuration $\lambda \propto t$ is equivalent to $\Delta\theta \propto t$—a phase difference that increases in proportion to time and to the voltage on the capacitor. This is illustrated in Fig. 1.

Once the capacitor is discharged the time dependence disappears and the phase difference becomes constant. As

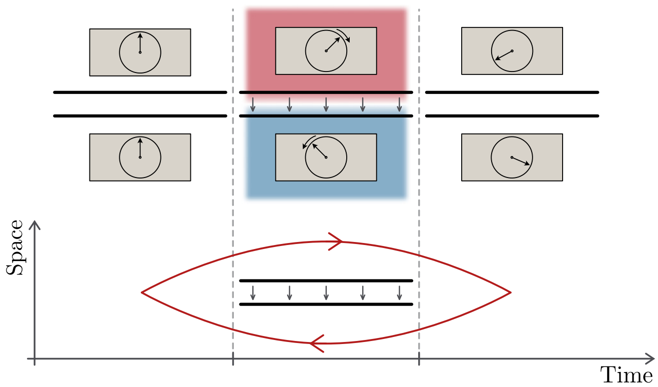

FIG. 1. Basic setup (top) and contour of the Wilson loop (bottom). An electric potential difference V maintained for a time T across two superconductors induces a relative superconducting phase difference $\Delta\theta = qVT/\hbar$. The pointers illustrate the phases.

long as the superconductors remain isolated from any further influences, this phase difference remains eternally imprinted, and can (in principle) be observed at any later time by reconnecting the superconductors and using the DC Josephson effect. This is an example of what is termed "electromagnetic memory" in Refs. [13–15]. Together with the more general question of the physical significance and infrared dynamics of gauge potentials, it is relevant for many questions of interest to ongoing research, including soft theorems (see Ref. [16] for a recent pedagogical review) and is related to gravitational memory and black hole information loss [17].

Taken together, the trajectories of the superconductors trace out a closed loop in space-time that we illustrate in Fig. 1—initially connected, then separated to either side of the capacitor, and then again connected. The Wilson loop integral $\oint A_\mu dx^\mu$ (with A_μ the four-potential) is gauge invariant and nonzero when taken along this loop.

II. PREVIOUS WORK

The experiment originally proposed by Aharonov and Bohm to measure the electric version of their eponymous effect was an electron interference experiment where a time-dependent voltage V is applied to the exterior of two Faraday cages while the electron beams pass through them [2]. If the voltage difference is nonzero only during the time T when the electrons are well contained inside the cages, the electrons never enter a region with nonzero electric field. Nevertheless, applying a different potential to each cage would induce a relative phase shift of

$$\Delta\theta = \frac{q\Delta VT}{\hbar} \tag{2}$$

in the electron wave functions, where V coincides with A_0 in the Coulomb gauge. The beams can then be interfered in order to measure the phase shift. However, in order to prevent the electrons from entering a region of nonzero fields, the typically high velocity of electron beams would require the voltage to be switched on and off extremely rapidly, over times of order 10^{-9} s. This is challenging and would induce strong nonadiabatic fields [18,19]. An interesting

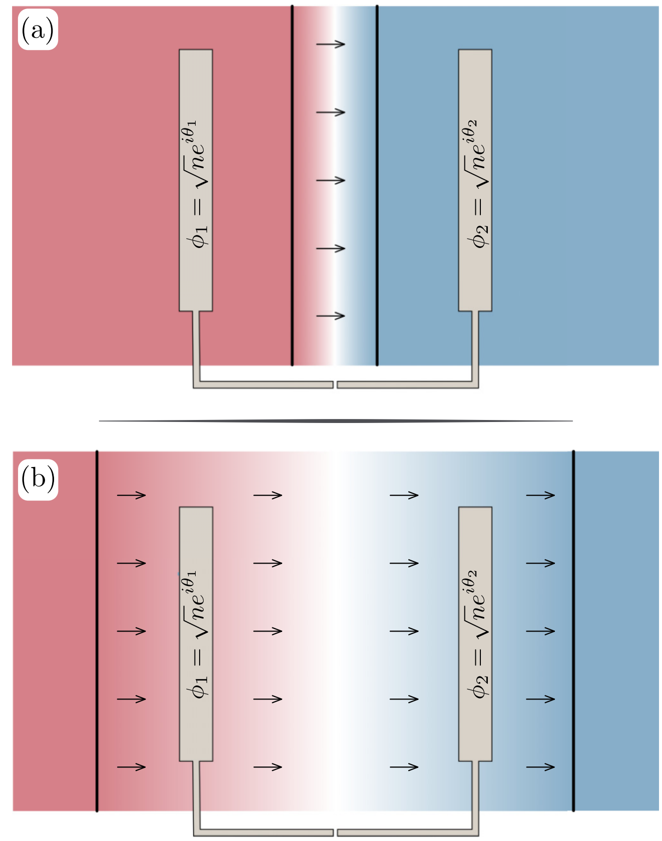

FIG. 2. Two superconductors subject to an electric potential difference V and connected by a thin superconducting wire with a junction. Positive/negative electric potential is illustrated as red/blue shading, and the arrows indicate the magnitude and direction of the field. (a) A charged capacitor is in between the two superconductors, creating a potential difference but zero electric field on the superconductors. (b) The capacitor encloses the two superconductors, creating a field that acts directly on the superconductors.

application of the electric Aharonov-Bohm effect was proposed in Ref. [20], but so far no positive observations have been made.

Instead of using charged particles directly, in this work we propose to employ the superconducting Cooper pair condensate in a quantum interference experiment that is sensitive to the electric potential in a region of vanishing fields. Using superconductors allows a more experimentally feasible test of the significance of the scalar potential. A related gedanken experiment was proposed in the context of "electromagnetic memory" [14].

Reference [21] proposed an experiment measure the relative phase shift between two superconductors induced a gravitational potential difference, analogous to the electric Aharonov-Bohm effect we discuss here.

III. EXPERIMENTAL SETUP

The experiment consists of two superconductors that are each embedded between large planar capacitors as shown in Fig. 2. The superconductors are described by the Ginzburg-Landau order parameters $\phi_{1,2} = \sqrt{n}e^{i\theta_{1,2}}$, where n denotes the constant Cooper pair density and $\theta_{1,2}$ are the time-dependent phases. Thin superconducting wires originate from each of the superconductors and terminate at an insulating junction located far from the capacitors. The junction contains a phase-dependent gradient of the order parameter that allows Cooper pairs to tunnel. This leads to an observable supercurrent described by the first Josephson relation

$$I = I_c \sin(\Delta\theta),$$

where I_c is the critical current that depends on the detailed configuration of the junction [22–24].

We present two distinct experimental setups corresponding to different configurations of the capacitor plates, shown in Figs. 2(a) and 2(b). In configuration (a) [Fig. 2(a)] a voltage difference is induced between the two superconductors, which lie in regions of vanishing field. In configuration (b) [Fig. 2(b)] there is both a nonzero voltage difference and a nonvanishing electric field acting on the superconductors.

Both setups result in an electric potential difference of $V(t)$ between the superconductors. The Cooper pairs carry a charge $q = -2e$, so the gauge-coupling term in the action, $S \supset \frac{2e}{\hbar} \oint A_\mu dx^\mu$ (with A_μ the four-potential), yields an associated phase shift given by the second Josephson relation,

$$\Delta\theta = \frac{2e}{\hbar} \int V(t) dt.$$

The path $V(t)$ is imprinted as a memory in the relative phase. The electric field at the superconductors vanishes in setup (a) [Fig. 2(a)], but acts locally in setup (b) [Fig. 2(b)]. Therefore, experimental verification of the second Josephson relation in setup (a) [Fig. 2(a)] acts as an indication that the scalar potential is physical (type-I electric Aharonov-Bohm), while setup (b) [Fig. 2(b)] represents a more conventional Josephson setup junction and could serve as a control (type-II electric Aharonov-Bohm). In both setups the relation between the relative phase and the voltage difference is identical, but the electric field configurations differ.

A constant voltage of 1 μV will shift the frequency of an AC Josephson current by about 5 GHz. An application of a short voltage pulse of 10 nV over a period of 10 ns would shift the relative phase by π. A precise measurement this phase shift may be difficult due to the small timescales and voltages involved, but any phase shift induced by the configuration in Fig. 2(a) would demonstrate the existence of the type-I electric Aharonov-Bohm effect.

There are several systematic effects that can impact the observations or their interpretations. First, in setup (a) [Fig. 2(a)] we attempted to place the Cooper pair condensate in a region that is entirely separated spatially from the region of nonvanishing electric fields. However, since we also need to observe a current and moving the superconductors is difficult, we rely on superconducting wires leading to the junction. These wires will experience the nonvanishing fringe fields of the capacitor. Phase coherence is maintained within each superconductor, and the number density of Cooper pairs is spatially constant, so we can separate the Ginzburg-Landau action for the condensate into two additive contributions: one from the wires and one from the bulk of the superconductors. The phase couples linearly to the potentials, so if we denote by $\epsilon = \mathcal{V}_{\rm wire}/\mathcal{V}_{\rm bulk}$ the relative volume between the wire and the bulk of the superconductor, we expect the systematic error from fringe fields in the second Josephson relation to be no larger than ϵ. Assuming superconductors of volume 1 cm^3, the volume ratio can be as small as $\epsilon \gtrsim 10^{-11}$ with μm-scale wires. Conversely, if electric fields instead of potentials were fundamental, the second Josephson relation would lead to a phase velocity that is multiplied by the small factor of $\mathcal{O}(\epsilon)$. If realized in nature, a positive direct and unambiguous observation of this significantly weaker effect would actually be easier.

Second, thermal noise will induce nonvanishing electric fields within the Faraday cages. Assuming the conductivity of copper and a frequency bandwidth of 10^6 Hz, a 1-cm Faraday cage contains thermal voltage fluctuations of around 10^{-11} V. These fluctuations induce a negligible relative voltage difference between the superconductors.

IV. CONCLUSIONS

A positive observation would provide experimental evidence for the electric Aharonov-Bohm effect. Conversely, a negative observation ruling out this effect would be of profound importance for our understanding of quantum gauge theories and consistent with a holonomic theory of quantum electrodynamics [25]. Either observation would be of elementary importance.

ACKNOWLEDGMENTS

We thank M. Arvanitaki, R. Dynes, D. Green, R. Flauger, P. Kraus, L. McAllister, J. McGreevy, J. Shabani, and K. Van Tilburg for useful discussions. The work of T.B. was supported in part by DOE under Grant No. DE-SC0009919 and by the Simons Foundation SFARI 560536. The work of M.K. is supported by the NSF through Grants No. PHY-1214302 and No. PHY1820814. This work was performed in part at the Aspen Center for Physics, which is supported by National Science Foundation Grant No. PHY-1607611.

[1] W. Ehrenberg and R. E. Siday, Proc. Phys. Soc., London, Sect. B **62**, 8 (1949).
[2] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).
[3] Y. Aharonov and D. Bohm, Phys. Rev. **123**, 1511 (1961).
[4] More precisely, there must be a component with nonzero curl.
[5] R. G. Chambers, Phys. Rev. Lett. **5**, 3 (1960).
[6] G. Möllenstedt and W. Bayh, Naturwissenschaften **49**, 81 (1962).
[7] Y. Aharonov, in *Foundations of Quantum Mechanics in the Light of New Technology: Selected Papers from the Proceedings of the First Through Fourth International Symposia on Foundations of Quantum Mechanics*, edited by S. Nakajima *et al.* (World Scientific, Singapore, 1996), pp. 8–17.

[8] P. G. N. de Vegvar, G. Timp, P. M. Mankiewich, R. Behringer, and J. Cunningham, Phys. Rev. B **40**, 3491 (1989).

[9] A. van Oudenaarden, M. H. Devoret, Y. V. Nazarov, and J. E. Mooij, Nature (London) **391**, 768 (1998).

[10] W. G. van der Wiel, Y. V. Nazarov, S. De Franceschi, T. Fujisawa, J. M. Elzerman, E. W. G. M. Huizeling, S. Tarucha, and L. P. Kouwenhoven, Phys. Rev. B **67**, 033307 (2003).

[11] Several references [9,26] refer to an article in a conference proceedings [27]. Unfortunately, we have not been able to retrieve this article, and no details are given in any of the papers that cite it.

[12] In the approximation that the capacitor plates are infinitely large and located at $x = 0$ and $x = L$, one simply has $\lambda(t, x) = 0$ for $x < 0$, $\lambda(t, x) = (x/L) \int V(t)dt$ for $0 < x < L$, and $\lambda(t, x) = \int V(t)dt$ for $x > L$.

[13] L. Bieri and D. Garfinkle, Class. Quantum Grav. **30**, 195009 (2013).

[14] L. Susskind, arXiv:1507.02584.

[15] S. Pasterski, J. High Energy Phys. 09 (2017) 154.

[16] A. Strominger, arXiv:1703.05448.

[17] S. W. Hawking, M. J. Perry, and A. Strominger, Phys. Rev. Lett. **116**, 231301 (2016).

[18] G. Badurek, H. Weinfurter, R. Gähler, A. Kollmar, S. Wehinger, and A. Zeilinger, Phys. Rev. Lett. **71**, 307 (1993).

[19] G. Schtz, A. Rembold, A. Pooch, P. Schneeweiss, A. Rauschenbeutel, I.-S. Hwang, and A. Stibor, Ultramicroscopy **158**, 65 (2015).

[20] A. Arvanitaki, S. Dimopoulos, A. A. Geraci, J. Hogan, and M. Kasevich, Phys. Rev. Lett. **100**, 120407 (2008).

[21] M. A. Hohensee, B. Estey, P. Hamilton, A. Zeilinger, and H. Müller, Phys. Rev. Lett. **108**, 230404 (2012).

[22] B. D. Josephson, Phys. Lett. **1**, 251 (1962).

[23] B. D. Josephson, Rev. Mod. Phys. **36**, 216 (1964).

[24] B. D. Josephson, Rev. Mod. Phys. **46**, 251 (1974).

[25] T. C. Bachlechner, arXiv:1910.08080.

[26] S. Washburn, H. Schmid, D. Kern, and R. A. Webb, Phys. Rev. Lett. **59**, 1791 (1987).

[27] H. Schmid, in *Proceedings of the 8th European Congress on Electron Microscopy*, edited by A. Csanady, P. Rolich, and D. Szabo (Programme Committee of the 8th European Congress on Electron Microscopy, Budapest, 1984), p. 285–286.