Quantum Computing for Enhancing Grid Security

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***Abstract*—This paper introduces quantum computing as a necessary and viable tool in addressing the needs of a modernized power grid. The application of quantum computing in enhancing physical security of the grid – an increasingly difficult problem to solve– is investigated. A comparative study based on mathematically proven computing performance measures shows the merits of the proposed method and further unveils the potential benefits of quantum computing in improving grid performance.**

***Index Terms*—Power grid, Quantum Computing, Security.**

I. Introduction

ELECTRIC utilities have been largely successful in providing reliable and affordable power through the traditional structure of the power grid. This structure, however, is changing due to the emergence of new customer-deployed technologies, the shift towards more renewable and carbon-free generation, and the increasing intensity and frequency of natural disasters as a result of climate change. As the grid is being modernized, i.e., more distributed, digitalized, and decarbonized, the role of data is becoming more important in grid observability and controllability [1]. More data is accordingly equivalent to an increased need for data analytics to extract information. What is being largely ignored is that current computational technology may not be adequate to address the needs of a modernized grid. An example is the grid security analysis that is limited by the capabilities of current computational technologies. This paper contributes to this emerging field of research by investigating the importance of adopting quantum computing in ensuring a secure, resilient, and reliable grid. A gate-based quantum algorithm is proposed to solve the grid security problem; an important problem that needs to be consistently addressed by system operators.

II. Quantum Computing

Classical computers work by converting information to a series of binary digits, or bits, and operating on these bits using integrated circuits (ICs) containing billions of transistors. There are only two feasible values for each bit, 0 or 1. Computers process the data by manipulating these bits. A quantum computer also reflects data as a series of bits, called qubits. Like a normal bit, a qubit could be either 0 or 1, but unlike a regular bit, a qubit may also be simultaneously in both states. This ability to simultaneously occupy all possible binary states results in a potentially exponential scaling advantage in the number of qubits in a system [2], [3].

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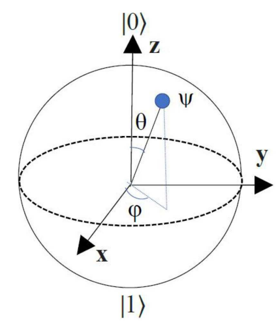

Fig. 1. Bloch sphere represents the set of all possible states for a single qubit.

An imaginary sphere can be thought of as a qubit, as shown in Fig. 1. The north and the south poles correspond to the states $|0\rangle$ and $|1\rangle$, respectively. A classical bit can be a qubit at either of the two poles of the sphere. The qubit state, however, can be mapped onto any point on the surface of this unit sphere. The state of a single qubit can be represented by:

$$|\psi\rangle = k_0|0\rangle + k_1|1\rangle \tag{1}$$

where

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \text{ and } \quad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix} \tag{2}$$

Values of k_0 and k_1 are restricted by the condition below,

$$|k_0|^2 + |k_1|^2 = 1 \tag{3}$$

Qubits behave on the principles of quantum physics and encompass two characteristics of superposition and entanglement. Superposition is the ability of a quantum system to be in multiple states at the same time. Entanglement is the correlation that exists between quantum particles. Relying on these two characteristics, a quantum computer can process a significantly large number of calculations simultaneously, using quantum interference to measure outcomes of interest.

III. Quantum Computing in Power Grids

Based on the characteristics of a qubit, as discussed, with N qubits a total of 2^N numbers can be operated simultaneously. This property (called superposition) is similar to the concept of parallel computing in classical computers. This implies that a computer that uses this property can process data much faster while using less energy than a classical computer.

There are many power system problems that require significant processing capabilities to address size and scalability issues. Some of these issues are addressed by reformulation, approximations, or parallel processing. However, the growing size and needs require a solution from the exact system (without any approximations) in a timely manner. Many examples of this can be found, such as AC optimal power flow, security-constrained unit commitment, contingency analysis, and transient stability, to name a few.

A. Grid Security Applications

The power system security represents the system ability to successfully handle unexpected events (such as the effects of component failures). Security is commonly evaluated by considering probable component failures and calculating resultant overloads on other parts of the grid. Security considerations enable grid operators to better prepare for and react to outages through carefully planned preventive and corrective actions [4]. Security analysis may require solving potentially thousands of power flow scenarios, each considering a different contingency, and accordingly assessing the state of the system.

The existing security analysis methods merely focus on the outage of a limited number of grid components, typically one (the common $N-1$ reliability criterion). Ongoing research focuses on developing new uncertainty-based and statistical methods to provide probabilistic solutions accounting for uncertainty. One major reason to look into probabilistic methods is the lack of adequate computational capabilities to perform higher order deterministic studies, i.e., $N-m$. This need is heightened due to the growing number of natural disasters that result in simultaneous outage of several grid components and thus call for much more complex studies.

Equations (4)-(5) present power flow problem formulation, where m is an index for buses, mn is used as an index for lines, and s is an index for scenarios. These two equations represent real and reactive nodal load balances, where p and q are net real and reactive nodal injections, respectively. V is voltage magnitude and θ is voltage angle. G and B reprsent grid characteristics and are obtained from admittance matrix.

$$p_m^s = \sum_n V_m^s V_n^s (G_{mn} \cos \theta_{mn}^s + B_{mn} \sin \theta_{mn}^s) \; \forall m, \forall s \quad (4)$$

$$q_m^s = \sum_n V_m^s V_n^s (G_{mn} \sin \theta_{mn}^s - B_{mn} \cos \theta_{mn}^s) \; \forall m, \forall s \quad (5)$$

In contingency analysis the state of the system in response to any potential contingency, i.e., the outage of units and/or lines, is evaluated. The index of interest in this formulation is s, representing contingency scenarios. All equations should be solved for all contingency scenarios. For $N-1$ studies, the number of scenarios would be N and $s \in \{1, , N\}$, for $N-2$ studies we would have $s \in \{1, , N(N-1)/2\}$ scenarios which shows a considerable increase in the problem size.

To solve the power flow problem a linear system of equations should be solved. In a DC power flow, this linear system of equations should be solved only once as the problem is already linear. In an AC power flow, however, equations are linearized around the operating point in each iteration, and accordingly a linear system of equations is solved. Therefore, independent of the power flow method used for contingency analysis, i.e., AC or DC, a linear system of equations has to be solved.

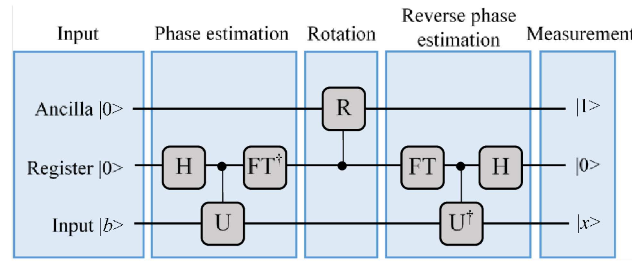

Fig. 2. Quantum algorithm for solving the security analysis problem.

IV. Solution Algorithm

Assume that each power flow scenario has M equations and M unknowns, and can be shown as a linear system of equations, $Cy = h$. In the power flow problem, y is the vector of unknowns, C is the matrix of coefficients (obtained from physics of the system as well as contingency scenarios), and h is the vector of solutions (obtained from system parameters). The solution can be achieved as $y = C^{-1}h$, assuming that C is an invertible matrix.

Matrix C should be Hermitian (i.e., a matrix that its complex matrix is equal to its own conjugate transpose) to apply the quantum algorithm that is proposed next. This is the case for a DC power flow, but not for an AC power flow. To resolve this issue we reformulate the problem as follows and solve for the new vectors.

$$\begin{pmatrix} 0 & C \\ C^\dagger & 0 \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} h \\ 0 \end{pmatrix} \quad (6)$$

The coefficient matrix in this case is Hermitian so the proposed quantum algorithm can be applied. We rename the new matrices to reach the form of $Ax = b$ and solve for this problem. By rescaling the x and b vectors to $||x|| = ||b|| = 1$, these vectors can be represented as quantum states $|x\rangle$ and $|b\rangle$. The solution is therefore to find state $|x\rangle$ such that

$$|x\rangle = \alpha A^{-1}|b\rangle \quad (7)$$

where

$$\alpha^{-1} = ||A^{-1}|b\rangle|| \quad (8)$$

For the coefficient matrix A, consider the eigenbasis $\{|u_j\rangle\}$ and rescaled eigenvalues $\{\lambda_j\}$, i.e., $A = \sum_j \lambda_j |u_j\rangle\langle u_j|$. The state $|b\rangle$ can be expanded in the eigenbasis as

$$|b\rangle = \sum_j \beta_j |u_j\rangle \quad (9)$$

and accordingly $|x\rangle$ can be calculated as

$$|x\rangle = \sum_j \beta_j \frac{1}{\lambda_j} |u_j\rangle \quad (10)$$

This equation can be solved using the HHL algorithm proposed in [5]. This algorithm proceeds in four steps: phase estimation, controlled rotation, uncomputation, and measurement, as discussed in the following and shown in Fig. 2.

Three inputs are considered in this quantum algorithm, including a single ancilla qubit initialized in $|0\rangle$, a register of n qubits of working memory initialized in $|0\rangle^{\otimes n}$, and an input state initialized in $|b\rangle$ as defined in (9). The ancilla qubit is defined to enable a reverse computing task (required in quantum computing). Ancilla qubits are not usually uncomputed and their values are known a priori.

Phase estimation: Phase estimation finds the eigenvalues (or phases) of an eigenvector of a unitary operator, i.e., a controlled unitary with a change of basis that maps the eigenvalues onto the working memory [6]. Phase estimation employs a Hadamard gate (H), followed by a unitary operator (U) and an inverse quantum Fourier transform (FT) to determine $\{\lambda_j\}$ the eigenvalues of A. The Hadamard gate creates superposition by mapping the basis state $|0\rangle$ to $(|0\rangle + |1\rangle)/\sqrt{2}$ and ensures that the measurements will have equal probabilities to become 0 or 1. The unitary operator is defined as $U = \exp(2\pi i 2^{n-1} A)$, where n depends on the number of eigenvalues, or in other words, on what binary digit of the eigenvalue needs to be read out. The quantum Fourier transform estimates the eigenvalues of the resultant qubits. Using phase estimation, the following transformation is achieved where $|\lambda_j\rangle$ represents the binary representation of λ_j.

$$|0\rangle^{\otimes n}|b\rangle \Rightarrow \sum_j \beta_j |u_j\rangle|\lambda_j\rangle \tag{11}$$

Controlled rotation: The ancilla qubit is initialized in state $|0\rangle$ and further used to extract the eigenvalues of A^{-1} from $|\lambda_j\rangle$. A controlled rotation $R(\lambda_j^{-1})$ is applied to this qubit to transform the system to

$$\sum_j \beta_j |u_j\rangle|\lambda_j\rangle \left(\sqrt{1-\gamma^2/\lambda_j^2}|0\rangle + \gamma/\lambda_j|1\rangle\right) \tag{12}$$

Uncomputation: A reverse phase estimation is applied to uncompute the results of the previous step. The memory register is accordingly disentangled and reset to $|0\rangle^{\otimes n}$ which will further result in

$$\sum_j \beta_j |u_j\rangle \left(\sqrt{1-\gamma^2/\lambda_j^2}|0\rangle + \gamma/\lambda_j|1\rangle\right) \tag{13}$$

Measurement: The ancilla qubit will be measured. Conditioned on seeing $|1\rangle$, the output state will be $\sum_j \beta_j \gamma/\lambda_j |u_j\rangle$, which corresponds to our expected result state $|x\rangle$.

The HHL algorithm creates the $|x\rangle$ solution state that we desire. However, when $|x\rangle$ is measured, we don't learn each coefficient in the solution state and instead, the $|x\rangle$ state vector collapses to one bit string, with a probability based on the amplitudes. In this case, we are able to measure scalar-output functions of the $|x\rangle$ state vector. In other words, instead of looking for the solution $|x\rangle$, we look for the expectation value of some operator associated with $|x\rangle$, e.g., $\langle x|M|x\rangle$ for a matrix M. Fortunately this is the case in the contingency analysis problem, in which we are looking for line overloads or voltage violations and can find those as a function of calculated variables.

V. Comparative Analysis

Consider the standard IEEE 300-bus test system [7]. This system has 300 buses, 69 generators, 304 transmission lines and 195 loads. For an $N-1$ study, the set of power flow equations should be solved 373 times (i.e., 69+304). For an N-2 study, this number goes up to 69,000, and for N-3 it will be close to 8.5 million. If we assume each run takes an average of 100 ms, it would take close to 10 days to run simulations for all contingency cases under a $N-3$ scenario. For higher contingency scenarios or larger systems, even the strongest classical computers may fail to perform this simulation in a timely manner. Considering Λ as the number of unknowns, classical computers solve linear system of equations in a timescale of order Λ while the proposed quantum computing algorithm is proved to find the solution in a timescale of order $\log(\Lambda)$. Based on this speed up, the N-3 study can be performed in around 0.7 s instead of 10 days as in a classical computer. Therefore, employing the HHL algorithm in a quantum computer can confer an exponential speedup over the best classical algorithm. It is worth noting that the line outage distribution factor (LODF) method can also be used instead of solving power flow equations. This method, although faster than solving a linear system of equations in a classical computer, will still show scalability issues, i.e., an exponential computation time increase, when used for larger systems and higher number of contingency scenarios.

TABLE I
COMPUTATION TIME

Contingency type	N-1	N-2	N-3
Classical computer	37 sec	2 hrs	10 days
Quantum computer	0.3 sec	0.5 sec	0.7 sec

VI. Discussions and Conclusion

As stakeholders seek to integrate additional advanced technologies to support efforts to provide higher levels of sustainability and resilience, there is a need to develop the enabling capabilities to meet those goals. Quantum computing, as discussed in this paper, represents an enabling technology that can help make power systems more reliable, resilient and sustainable. In this paper, mathematically proven computing performance measures were used to show the speed up over classical computers. There still doesn't exist a quantum computer with a large number of qubits that can solve the discussed security problem. This problem will be resolved in near future as the quantum computing technology progresses. Moreover, the originally-envisioned HHL algorithm is also best suited to fault-tolerant quantum hardware with millions of qubits. However, recent proposals [8], [9] have adapted HHL for near-term quantum computers, which have dozens of noisy qubits. These proposals feature a hybrid scheme that leverages the separate strengths of quantum and classical computation; an appealing technique for near-term application.

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