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Iterative combinatorial auctions for managing product transitions in semiconductor manufacturing

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ABSTRACT
Successful management of product transitions in the semiconductor industry requires effective coordination of manufacturing and product development activities. Manufacturing units must meet demand for current products while also allocating capacity to product development units for prototype fabrication that will support timely introduction of new products into high-volume manufacturing. Knowledge of detailed operational constraints and capabilities is only available within each unit, precluding the use of a centralized planning model with complete information of all units. However, the decision support tools used by the individual units offer the possibility of a decentralized decision framework that uses these local models as components to rapidly obtain mutually acceptable, implementable solutions. We develop Iterative Combinatorial Auctions (ICAs) that achieve coordinated decisions for all units to maximize the firm's profit while motivating all units to share information truthfully. Computational results show that the ICA that uses column generation to update prices outperforms that using subgradient search, obtaining near-optimal corporate profit in low CPU times.

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Product transitions; Lagrangian relaxation; column generation; iterative combinatorial auctions; semiconductor manufacturing

1. Introduction

Effective introduction of new products (product transitions) is an important competitive advantage in many industries (Levinthal and Purohit, 1989; Padmanabhan *et al.*, 1997). A growing body of research has addressed aspects of product transitions (Billington *et al.*, 1998; Lim and Tang, 2006; Bilginer and Erhun, 2010) including capacity management under technological uncertainty (Rajagopalan *et al.*, 1998; Angelus and Porteus, 2002; Wu *et al.*, 2005; Li *et al.*, 2014), supply constraints (Ho *et al.*, 2002), the impact of initial investment in design and process capacity and capabilities on the time-to-market and ramp-up-time of a new product (Carrillo and Franza, 2006; Wu *et al.*, 2009), industry clockspeed (Souza *et al.*, 2004; Carrillo, 2005; Druehl *et al.*, 2009) and cost structure (Souza, 2004). Liang *et al.* (2014) examine the impact of strategic waiting by customers on the timing of the product transition, whereas Klastorin and Tsai (2004) consider the interaction of pricing, timing and product design in a competitive environment. Koca *et al.* (2010) consider the impact of pre-announcement of the transition on demand, and examine the effects of inventory decisions and dynamic pricing. Li *et al.* (2010) study inventory planning decisions during the transition when the new product may substitute for the old one. This body of work, however, has two shortcomings:

1. It treats the firm as a single centralized decision maker. However, in practice the key resource allocation decisions, as well as the domain knowledge and data required to make them, are distributed among different functional groups that have different, potentially conflicting objectives.
2. It treats product transitions as exceptions to routine operations, and in isolation from products not involved in the transition. However, in industries with frequent product transitions, such as the semiconductor industry, their management assumes a distinctly operational aspect. New product introductions can have significant negative impact on other products with which they share resources, especially those currently in high-volume production that generate the firm's revenue. Hence product transitions cannot be managed effectively without explicitly considering the technological constraints governing the resource allocation decisions of the functional units through which the product transition is realized (Ulrich and Eppinger, 2016).

Although the process by which new products move into volume production differs from firm to firm, units involved in product transitions include several product line organizations that each serve a specific market (e.g., microprocessors, mobile devices). Each product line has its own Sales and Marketing (S-MKT) group and Product Development Groups (PDGs), while multiple product lines can be manufactured by a Manufacturing organization (MFG). S-MKT is

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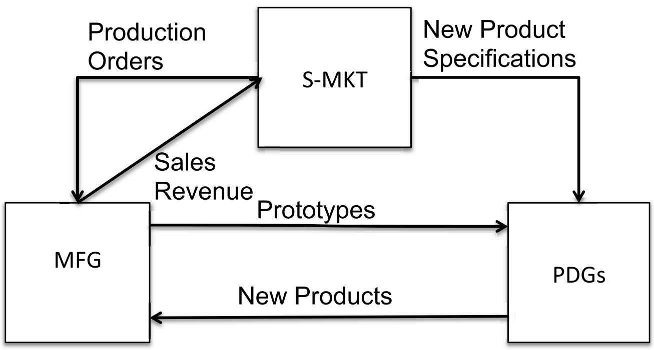

Figure 1. Information flow between S-MKT, MFG, and PDGs.

responsible for generating revenue for the firm through sale of existing products and develops demand forecasts for new products through market research.

Figure 1 shows the information flow between S-MKT, MFG and the PDGs. Based on interaction with their markets and the firm's sales goals, the product line S-MKT units issue production orders to MFG for current products and specifications for new products to their PDGs. The PDGs develop new products based on requests from their S-MKT units, eventually requiring the fabrication of prototypes in MFG facilities. Once MFG makes capacity available and prototypes have been produced, the involved PDG tests them, identifying problems and improving manufacturability and performance. Several cycles of prototype testing and design repair may be required before the product is ready for high-volume production. In addition to these requests from the PDGs, MFG must also address requests from the S-MKT units for products to fill current orders and build inventory positions for the future, generating the revenue needed to operate the company. We stress that this is an ongoing, continuous process. The large number of product transitions in a firm offering multiple product lines with multiple manufacturing facilities precludes a standard response process; each situation requires negotiation between groups and complex resource allocation decisions within each unit.

The problem in its full complexity is clearly extremely challenging, especially given the pervasive uncertainty under which decisions must be taken. Even a completely deterministic environment results in a computationally intractable problem; a relatively simple form of the problem faced by the PDGs reduces to the Resource-Constrained Multiple Project Scheduling Problem, which is strongly NP-hard and notoriously difficult to solve (Kolisch *et al.*, 1995).

The inherently decentralized nature of the decision environment mandates a decentralized solution. Most of the individual units already deploy automated decision support tools to address their (already highly complex) local decision problems. A variety of models to support production planning decisions by MFG are available in the literature (Leachman, 2001; Fordyce *et al.*, 2011; Kacar *et al.*, 2016; Mönch *et al.*, 2018). A variety of models to support the activities of the PDGs have also been proposed (Rash and Kempf, 2012). This suggests the possibility of a decentralized decision support framework that leverages these existing decision support tools with a coordinating mechanism to ensure rapid convergence to a set of decisions acceptable to all units and consistent with the firm's long-term objectives. The rapidly growing literature on combinatorial auctions provides a promising basis for the development of such a framework.

As a first step in this direction, this article studies a simplified version of the problem where the resources available to all units are known with certainty, and demand forecasts for both current and future products have been generated. The Manufacturing (MFG) unit must manage factory capacity to maximize the firm's profit over a finite planning horizon, while several PDGs must manage development activities to have new products ready for high-volume manufacturing (ramp-up) in time to meet their market demand. Factory capacity allocated to the PDGs by MFG reduces short-term revenue, whereas if the PDGs cannot obtain enough factory capacity from MFG for timely prototype fabrication, new products may not be available in the market when needed, resulting in lost revenue and leaving the firm vulnerable to competition.

We propose two Iterative Combinatorial Auction (ICA) frameworks (Parkes and Ungar, 2001) for coordinating decisions between MFG and PDGs. The MFG unit auctions bundles of factory capacity, consisting of specific amounts of capacity in each period in the planning horizon, to the PDGs who, in turn, receive incentive payments from MFG for having new products ready for volume production at specific times. MFG functions as the auctioneer, running a market-clearing algorithm in each iteration to ensure that the decisions of MFG and the PDGs are implementable, i.e., consistent with each other and with all operational constraints for all groups concerned, while seeking to maximize the firm's profit over the planning horizon. We shall refer to such solutions as coordinated solutions, recognizing that our approaches do not guarantee maximization of profit, and hence, are approximate in nature. We assume myopic best response behavior by the PDGs, such that each PDG seeks to obtain the best result it can in the current iteration without considering the possibilities of future iterations. This is a reasonable assumption due to the NP-hard nature of the valuation problem by which the PDGs generate their bids, which can only be increased by incorporating conjectures about what other agents will do in later iterations. In addition, no PDG has any way of knowing when the ICA will end; the possibility that the current iteration may be the last motivates the PDGs to submit their best bids at each iteration.

A potential difficulty in many decentralized resource allocation problems is the possibility that in a given iteration of the ICA, several agents may request the same resources in the same time periods, limiting the number of agents to whom bids can be awarded while leaving others with no allocation. Especially if each PDG submits only the bid that optimizes its local objective, the auctioneer is left with a limited number of bids from which to construct a good quality coordinated solution for the company. Allowing each unit to submit multiple bids in addition to its locally optimal one gives the auctioneer greater flexibility in this regard, since a bid that is suboptimal for one PDG may allow a coordinated solution that yields higher profit for the firm. Hence, an important feature of our ICA approaches is the solicitation of multiple bids from each participating PDG in each iteration of the ICA. This provides the auctioneer with better information on the capabilities of the PDGs, expressed as a range of alternative delivery times for their new products based on alternative resource allocations by MFG. However, this raises two new issues. The first is since the valuation problem solved by the PDGs may be strongly NP-hard (in our case, a Resource-Constrained Project Scheduling Problem), the computational effort involved in providing multiple solutions may be excessive. We address this by assuming that the PDGs develop their solution using a procedure that generates multiple feasible solutions in the course of its computations. A wide range of algorithms, including branch-and-bound methods for solving integer programs (as used in this article), neighborhood search metaheuristics such as Simulated Annealing (Van Laarhoven and Aarts, 1987) and Tabu Search (Glover and Laguna, 1998), or population-based metaheuristics such as Genetic Algorithms (Davis, 1991) meet this condition, although manual procedures may not.

A more subtle issue is that a particular PDG may attempt to improve its local objective at the expense of other PDGs and corporate profit by submitting only its locally optimal solution, effectively claiming this to be its only feasible solution. However, since the auctioneer seeks to select a subset of the submitted bids that maximizes corporate profit over the planning horizon, a PDG whose bids prevent the selection of bids from other PDGs that contribute positively to corporate profit may find itself receiving no factory capacity at all. This essentially represents a decision by the firm not to move that PDG's products into the market, reflecting very negatively on the management of that PDG. Thus, we assume that receiving no factory capacity allocation at all, i.e., having no bid awarded at the end of the ICA procedure, is viewed as catastrophically expensive by each PDG, due to, say, loss of bonuses or profit-sharing income. We show in Appendix B that under the assumption of myopic best response, this very high cost of not receiving any bid incentivizes the PDGs not to withhold information on alternative feasible solutions from the auctioneer, i.e., to disclose truthfully the different "production" possibilities they discover during their search for their preferred bid. We shall refer to this as "truthful" behavior on the part of the PDGs, recognizing that this goes beyond the common definition of truthful behavior in an auction context, which requires only truthful reporting of the valuation of the bids submitted.

The remainder of this article is organized as follows: Section 2 reviews previous related work. In Section 3, we give a centralized formulation of the problem with perfect information, from which we derive the two ICA frameworks presented in Sections 4 and 5. Section 6 presents computational experiments and results, while Section 7 discusses our principal findings and some directions for future work.

2. Previous related work

The work in this article draws on three streams of research literature: that on combinatorial auctions (Section 2.1), distributed resource allocation approaches (Section 2.2) and decentralized resource allocation models, especially in the semiconductor industry (Section 2.3).

2.1. Combinatorial auctions

The field of mechanism design seeks efficient ways to allocate objects among multiple bidders. Auctions are an important subclass of mechanisms under which agents offer bids reflecting their valuation of a set of objects to an auctioneer, who then allocates the objects among the bidders based on their bids to maximize its objective (Osborne and Rubinstein, 1990; Narahari *et al.*, 2009). An auction mechanism thus consists of an *allocation* or *market-clearing* rule through which the auctioneer allocates objects to the bidders based on the bids submitted, and a *payment rule* that determines what the bidders pay for the goods they receive (Abrache *et al.*, 2007; Mishra, 2010).

Combinatorial auctions (Cramton *et al.*, 2007) involve a set S of m discrete objects that are bid for by n bidders. Each bidder i assigns a valuation $v_i(T)$ to each subset $T \subseteq S$ of objects, and submits bids for one or more subsets T. The auctioneer then solves a Winner Determination Problem (WDP) that allocates objects to agents such that one or more bids are accepted from each bidder while maximizing the auctioneer's revenue (De Vries and Vohra, 2003; Abrache *et al.*, 2007; Blumrosen and Nisan, 2007). Thus the solution to the WDP provides the allocation or market-clearing component of the auction mechanism. In this article we implement the XOR bidding language (Parkes and Ungar, 2001) where the objective of the auctioneer is to maximize corporate profit over the planning horizon of interest while accepting at most one bid from each bidder.

The WDP can be formulated as a weighted set packing problem, which is known to be NP-hard (De Vries and Vohra, 2003). Various exact and heuristic approaches for the WDP have been proposed (Andersson *et al.*, 2000; Sandholm, 2002; Jones and Koehler, 2005; Lehmann *et al.*, 2006). If there is no integrality gap between the WDP and its Linear Programming (LP) relaxation (i.e., the WDP has the *integrality property*), the primal and dual solutions to the LP relaxation of the WDP represent a Walrasian equilibrium of the allocation problem (Abrache *et al.*, 2007). Bikhchandani and Ostroy (2002) provide two extended formulations for the WDP that satisfy the integrality property. However, these formulations require exponentially many variables and constraints, rendering them impractical for combinatorial auctions involving divisible goods. O'Neill *et al.* (2005) present an approach for calculating Competitive Equilibrium prices for a general WDP formulation that may not satisfy the integrality property, assuming complete sharing of information between the bidding agents and the auctioneer.

ICAs (Parkes and Ungar, 2001) mitigate these difficulties by requiring each agent to reveal relevant private information incrementally over time. After receiving bids at each iteration, the auctioneer computes a provisional allocation of the goods, and communicates this, together with updated pricing information for each item, to the agents, who then update their bids for the next iteration. The iterations continue until some termination criterion is satisfied, at which point the current provisional allocations become final (De Vries and Vohra, 2003; Abrache *et al.*, 2007).

Based on the information given by the auctioneer to the agents, an ICA can be classified as quantity-setting or price-setting (Villahoz *et al.*, 2010). In a quantity-setting ICA, agents submit price bids for every possible bundle of items, based on which the auctioneer provisionally allocates the objects. Agents adjust their bids for the next iteration based on the provisional allocations in the current iteration. In a price-setting ICA, the auctioneer provides the agents with minimum ask prices for each object or bundle. Agents submit bids, whose value must exceed the minimum ask prices, for one or more bundles of objects. The auctioneer then uses these bids to make provisional allocations and adjust the prices of over- and under-demanded objects or bundles for the next iteration. Under *linear* prices, the price of a bundle is equal to the sum of the prices of the individual objects comprising the bundle; under *nonlinear* prices, this is not the case (Mishra, 2010). Under *anonymous* prices, all agents pay the same price for each object or bundle, while under *nonanonymous* prices different agents can be charged different prices for the same object or bundle. In this article, we model the capacity coordination problem between MFG and PDGs using a price-setting ICA whose linear nonanonymous prices are updated at each iteration using Lagrangian relaxation or column generation. Following the literature, we assume that the PDGs behave as price takers as in a clock auction (Bichler *et al.*, 2013) and submit their desired set of bundles in each period as bids.

Objects traded in an auction can be divisible or indivisible, while the participating agents can be sellers, buyers or both. In this article, new product delivery times are treated as indivisible goods that MFG acquires from the PDGs. Factory capacity in each period is viewed as a divisible good that PDGs acquire from MFG. Thus, MFG acquires a bundle of new product delivery times from PDGs, specifying when each new product will be ready for introduction to manufacturing. Each PDG acquires a bundle of capacity allocations, specifying the capacity allocated to them in each period. This introduces complementarities between the goods for all units. Capacity allocation in a given period is only beneficial to a PDG if it can obtain sufficient capacity in subsequent periods to complete development tasks and deliver the product to MFG for introduction. Since all units act as both buyers and sellers, but bid for goods with significant complementarities, the ICAs discussed in this paper are *multilateral iterative combinatorial* auctions (Abrache *et al.*, 2007).

Private information refers to information known only to the agents which the auctioneer needs to compute an optimal allocation (Mas-Colell *et al.*, 1995). Ideally, the auction's payment rules should ensure that truthful revelation of agents' private information is a dominant strategy, i.e., not revealing private information truthfully results in suboptimal results for the agent (Abrache *et al.*, 2007). For example, in a Generalized Vickery Auction (GVA) (Parkes, 2001; Narahari, 2014), each agent submits a bid for every subset $S \subseteq T$ of goods and the optimal solution to the WDP is used as an allocation rule. Each agent pays the marginal contribution of his (her) participation to the payoffs of all agents except himself (herself), and no agent can do better by giving the auctioneer untruthful information. However, this approach can be difficult to implement, as both the individual agents' valuation problems (the problem each agent must solve to derive their bids) and the WDP may be NP-hard (De Vries and Vohra, 2003; Abrache *et al.*, 2007). Moreover, agents may be unwilling or unable to reveal their private information required by the auctioneer in the allocation rule due to confidentiality restrictions, uncertainty or difficulty in devising valuation functions (Abrache *et al.*, 2007; Arache *et al.*, 2013; Abrache *et al.*, 2014).

Incentive compatibility is a very strong property that motivates agents to truthfully reveal their private information irrespective of other agents' behavior (Narahari, 2014). The GVA is the most famous incentive compatible combinatorial auction mechanism (Parkes, 2001). Gul and Stacchetti (1999), Parkes (2001), Ausubel (2006), and De Vries *et al.* (2007) propose ICA mechanisms that result in the same outcome as a sealed bid GVA under different assumptions on bidder valuations. Mishra and Parkes (2007) develop the price tâtonnement process for general bidder valuations and prove it to be necessary and sufficient for an ascending price ICA to yield GVA outcomes on termination. However, they assume that each agent will truthfully reveal his (her) private information and require several WDPs to be solved optimally in each iteration, which might be computationally expensive. Ausubel and Milgrom (2006) shows that the GVA auction loses its incentive compatible properties if agents have budget constraints, whereas Lavi (2007) argues that implementing incentive compatible mechanisms can be computationally expensive. Considering these difficulties, we do not seek incentive compatibility, but instead prove that our proposed ICA mechanism is Bayesian Incentive Compatible under modest assumptions, such that it is in the best interest of each agent to truthfully reveal his (her) private information provided other agents also truthfully reveal theirs (Mas-Colell *et al.*, 1995).

Dietrich and Forrest (2002) introduce the use of Column Generation (CG) for solving the WDP in Combinatorial Auctions for a given set of bids. Their work is extended by Günlük *et al.* (2005), who propose a branch-and-price framework for solving the WDP for the Federal Communications Commission (FCC) spectrum auction. However, these papers focus on solving the WDP for a known set of bids. In contrast, the current article uses the CG procedure to emulate an ICA, which involves computing prices for items based on the currently available bids and eliciting new bids based on these updated prices from the PDGs at each iteration. Our implementation of the Lagrangian approach follows that of Kutanoglu and Wu (1999) in using the subgradient algorithm, and provides a benchmark for the performance of the CG-based ICA (CG-ICA). Moreover, the ICA proposed in Kutanoglu and Wu (1999) assumes that agents behave truthfully in the proposed

ICA whereas in the current article we show that if not receiving any bid at the end of the auction is prohibitively expensive for each PDG, it is in their best interest to behave truthfully.

2.2. Distributed resource allocation approaches

Bidding mechanisms for distributed resource allocation have been studied in several contexts, notably machine scheduling. Toptal and Sabuncuoglu (2014) provide an extensive review of this body of work, whereas Adhau *et al.* (2012, 2013) apply similar ideas to the problem of distributed project scheduling. Kutanoglu and Wu (1999) were among the first to use the subgradient algorithm to update prices in a price-setting ICA for distributed job-shop scheduling problems. They note that the subgradient algorithm can be viewed as an ICA with a different price tâtonnement from the well-known ascending price tâtonnement (Ausubel and Milgrom, 2002; Ausubel, 2004; Mishra and Parkes, 2007). At each iteration, the subgradient algorithm penalizes the violation of market clearing constraints by updating prices to discourage demand conflicts between agents. De Vries and Vohra (2003) interpret several well-known ICAs such as *iBundle* (Parkes, 2001) and *RAD* (Kwasnica *et al.*, 2005) in the context of the subgradient algorithm. Abrache *et al.* (2013) study an ICA for multilateral procurement using Lagrangian relaxation and the subgradient algorithm. They consider only divisible objects and assume that the production cost functions and valuation functions of sellers and buyers, respectively, are continuous, convex/concave (seller/buyer) and monotonically increasing. These properties of the production cost and valuation functions ensure that the strong duality property holds for the Lagrangian dual. Thus, the allocation and prices devised by the subgradient method are market-clearing and form a Walrasian equilibrium. They do not discuss the incentive compatibility of their mechanism, effectively assuming truthful communication of all necessary information between auctioneer and agents.

Confessore *et al.* (2007) present an ascending price ICA similar to *iBundle* for the Decentralized Resource Constrained Multi-Project Scheduling Problem (DRCMPSP) where each agent faces a Resource Constrained Project Scheduling Problem (RCPSP) involving both local and shared resources. The DRCMPSP is similar to the problem addressed in this article where PDGs solve RCPSPs comprising local resources, such as engineering resources, and shared resources such as factory capacity. An algorithm to solve DRCMPSP would seek to resolve conflicts between different agents over shared resources such that all projects are completed and project makespan is minimized. Araúzo *et al.* (2010) develop a framework for simulating an ICA for the dynamic version of this problem where new agents (with new projects) can join the auction in any time period. They also apply the subgradient method to update prices and iteratively reduce conflicts over shared resources. Song *et al.* (2017) propose a single-shot combinatorial auction for the DRCMPSP which uses different capacity profiles of shared resources for bid elicitation. The auctioneer minimizes the total project completion time after accepting bids from the agents that specify a bundle of indivisible goods (time periods in which shared resources are desired), assuming truthful behaviour by the agents. In the ICAs presented in this article, the auctioneer seeks to maximize corporate profit over the planning horizon, while the PDGs act as both buyers and sellers whose bids involve both divisible (factory capacity in each period) and indivisible goods (new product delivery times). We also show that in the proposed ICAs, PDGs behave truthfully under reasonable assumptions.

2.3. Capacity coordination problems in semiconductor manufacturing

Several authors have applied mechanism design to capacity coordination in the semiconductor industry. Mallik and Harker (2004) and Mallik (2007) consider a semiconductor firm with multiple product line S-MKT units who each request factory capacity for their products from MFG. The authors design a rule to allocate capacity among products and a bonus scheme to ensure that all participants provide truthful information. They show that a bonus is required for truthful reporting from MFG but not for S-MKT. Erkoc and Wu (2005), Karabuk and Wu (2005), and Jin and Wu (2007) address similar problems in the context of capacity reservations, where a S-MKT unit must reserve manufacturing capacity from MFG under uncertain demand. All these mechanisms seek to establish allocation rules and payment policies that will yield the same total profit as a centralized solution with complete information without resorting to auctions. However, the derivation of such mechanisms is extremely difficult in the face of the complex operational constraints encountered in our problem.

Karabuk and Wu (2002, 2003) consider capacity planning under demand and yield uncertainty with two units involved in the capacity planning process: S-MKT seeks to maximize revenue and MFG to minimize costs. They develop multistage stochastic programming models for each unit and integrate these into a single formulation that forces the first-stage decisions of both models to be the same, but point out this would require the two departments to reveal private information, which is unlikely. They then decompose this model into separate S-MKT and MFG models and develop a price-based coordinated solution.

In summary, although ICAs have proven to be a useful and computationally efficient tool for a variety of resource allocation problems, they have not been applied to the problem of coordinating autonomous MFG and PDG units to manage product transitions in the face of complex operational constraints and decentralized information. Direct design of mechanisms through characterization of equilibrium solutions is difficult for this environment in which the valuation problems of the units, particularly the PDGs, are NP-hard and involve significant complementarities. Although the subgradient algorithm has been used as an alternative to the well-known ascending price auction, it provides the auctioneer with limited information on the valuation of goods by the agents, potentially slowing the rate of convergence to a market-clearing solution. The principal contributions of this article over previous work are as follows:

1. We propose two Multilateral ICA frameworks for the problem of managing product transitions in a decentralized environment and compare their properties and performance.
2. The proposed ICAs sell indivisible goods from PDGs to MFG (new product introduction time periods) and multiple divisible goods from MFG to PDGs (factory capacity in each period).
3. Our first approach uses Lagrangian relaxation (Fisher, 1981) of the market-clearing constraints to develop an ICA that uses the subgradient algorithm to update prices.
4. Our second approach develops a CG-based (Dantzig and Wolfe, 1960) ICA and demonstrates its superior computational performance over the Lagrangian Relaxation-based ICA. To the best of our knowledge this is the first time CG has been used to emulate an ICA, as opposed to solving the WDP for a specified set of bids.
5. We show that the proposed ICA mechanisms are Bayesian Incentive Compatible under reasonable assumptions.

3. Centralized formulation under complete information

We begin by formulating a centralized decision model under complete information. Such a centralized formulation is not a viable solution approach since MFG has no knowledge of the constraints faced by the PDGs, which we consider as their private information, but is used to motivate the different price tâtonnements used in our ICAs.

We consider a finite planning horizon of discrete periods $t = 1, \ldots, T$. Let I denote the index set of the PDGs and P_i the set of products under development by $PDG_i, i \in I$. We express factory capacity as the maximum number of wafers that can be manufactured in each period and assume it to be known by both MFG and the PDGs. We can then state the problem as follows, without as yet describing the local constraints of the MFG and PDG units in detail:

$$(\text{C1})\text{Max} \quad \sum_{i=1}^{N}\sum_{t=1}^{T}[\pi_{it}(D_{it} + B_{i,t-1} - B_{it}) - h_{it}U_{it} - r_{it}Q_{it}C_t - b_{it}B_{it}], \tag{1}$$

subject to

$$\textit{MFG Constraints}, \tag{2}$$

$$PDG_i \ \textit{Constraints} \ \forall i \in I, \tag{3}$$

$$Q_{pt} \le y_{pt} \ \forall p \in P_i, \ i \in I, \ t = 1, \ldots, T, \tag{4}$$

$$\hat{a}_{it} \le \bar{a}_{it} \ \forall i \in I, \ t = 1, \ldots, T, \tag{5}$$

Table 1. Parameters and decision variables for Model C1 and associated MFG constraints.

Parameters	*Description*
T	Duration of planning horizon
I	Index set of PDGs
P_i	Set of all products under development with PDG_i
N	Total number of products (old products + new products)
D_{it}	Demand forecast for product i in period t
π_{it}	Per unit revenue of product i in period t
b_{it}	Per unit backordering cost of product i in period t
h_{it}	Per unit inventory holding cost of product i in period t
r_{it}	Release cost of product i in period t.
C_t	Factory capacity in period t
Decision variables	*Description*
U_{it}	Total inventory of product i in period t
B_{it}	Total backorders of product i in period t
Q_{it}	Fraction of factory capacity released for product i in period t
y_{pt}	Binary variable equal to 1 if product p can be manufactured in period t and 0 otherwise
$\bar{a}_{it}$	Fraction of factory capacity allocated to PDG_i in period t
$\hat{a}_{it}$	Fraction of factory capacity used by PDG_i in period t

$$Q_{pt}, \hat{a}_{it}, \bar{a}_{it} \in [0,1], \ y_{pt} \in \{0,1\} \ \forall p \in P_i, \ i \in I, \ t = 1, \ldots, T. \tag{6}$$

In this model Q_{pt} denotes the fraction of available factory capacity allocated by MFG for manufacturing product p in period t. y_{pt} is a binary variable taking the value of 1 if product p is ready to be manufactured (i.e., has completed all its development activities) by period t and 0 otherwise. $\hat{a}_{it}$ denotes the fraction of factory capacity used by PDG_i in period t and $\bar{a}_{it}$ the fraction of factory capacity allocated to PDG_i in period t. The notation used in the models is summarized in Table 1.

The objective (1) maximizes the firm's total contribution (revenue - variable costs) over the planning horizon. Constraints (4) ensure that MFG manufactures a product only when its development is complete, while constraints (5) ensure that the capacity used by each PDG in each period does not exceed its allocation. These constraints balance the supply and demand of factory capacity and new product introduction time periods, and serve as market clearing constraints in our ICAs. We assume that S-MKT has provided demand forecasts for products under development. Manufacturing costs include inventory holding, backordering and silicon wafer release (material) costs; for simplicity we assume that any unsatisfied demand can be backordered. The MFG constraints are:

$$U_{it} = U_{i,t-1} + Q_{it}C_t - (D_{it} + B_{i,t-1} - B_{it}) \quad \forall i = 1\ldots, N \quad t = 1, \ldots, T, \tag{7}$$

$$\sum_{i=1}^{N} Q_{it} + \sum_{i \in I} \bar{a}_{it} \le 1 \quad \forall t = 1, \ldots, T, \tag{8}$$

$$\begin{gathered} U_{it}, B_{it} \ge 0 \quad \forall i = 1, \ldots, N \quad t = 1, \ldots, T, \\ Q_{pt}, \bar{a}_{it} \in [0,1], \ \forall p \in P_i, \ i \in I, \ t = 1, \ldots, T. \end{gathered} \tag{9}$$

Constraints (7) are inventory balance constraints and constraints (8) the factory capacity constraints for each period. These constraints are typical of most production planning problems (Voß and Woodruff, 2006; Missbauer and Uzsoy, 2011). Clearly much more complex formulations

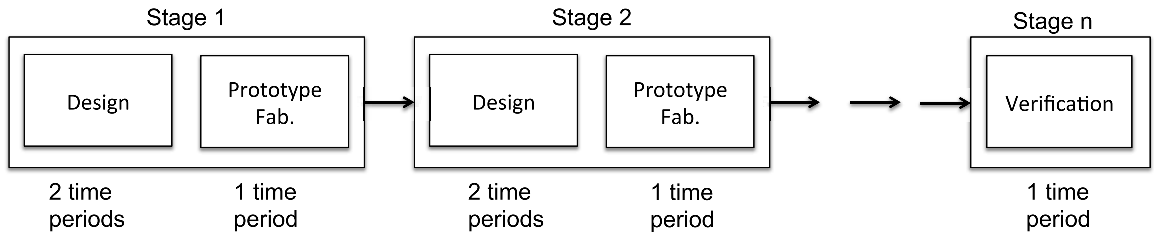

Figure 2. Product development process in semiconductor industry.

of the manufacturing problem are possible, but these constraints capture the essence of our problem: the need to allocate limited factory capacity among revenue-generating products and prototype fabrication for the PDGs which reduces short-term revenue, but is essential to maintaining a viable future revenue stream.

3.1. PDG$_i$ constraints

To describe the constraints for each PDG_i, we propose a simplified model of the product development process in the semiconductor industry, which proceeds in a series of design-test-refine cycles. The development of a particular device consists of a series of stages, each consisting of a number of subtasks linked by linear precedence constraints as shown in Figure 2. Each stage, except the last one, spans three planning periods, with engineering work by the PDG in the first two periods, followed by the fabrication of prototypes incorporating this design work in the third. The engineering work requires specified amounts of engineering resources from the PDG involved. The second task in each stage involves the release of a number of engineering lots into the factory for prototype fabrication. The last stage requires only engineering resources for final verification of the prototypes produced in the previous stage, and can be completed in one time period. Once a stage is started, the PDG must complete it due to the inefficiencies associated with reallocating resources between projects in progress. A product is considered ready for release to manufacturing only when its last development stage is completed. Although more detailed constraints can clearly be added, this simple structure is sufficient to allow us to explore the feasibility of decentralized procedures. The specific PDG_i constraints we consider are given in Appendix A.

4. A Lagrangian relaxation-based iterative combinatorial auction

We now present a Lagrangian Relaxation-based ICA (LR-ICA) framework in which MFG takes the role of auctioneer, while the PDGs behave as utility-maximizing rational agents who follow a myopic best response policy (Parkes, 2001) seeking to maximize the value of their allocation in the current iteration without considering future iterations. The subgradient algorithm obtained by Lagrangian relaxation of the market-clearing constraints serves as the auctioneer's (MFG's) price updating scheme at each iteration of this ICA.

Relaxing the market-clearing constraints (4) and (5) with associated Lagrange multipliers $\beta_{pt},\ p \in P_i, i \in I,\ t = 1, \ldots, T$ and $\lambda_{it}, \forall\ i \in I,\ t = 1, \ldots, T$, we obtain the Lagrangian problem (L1):

$$(\text{L1})\text{Max} \quad \left(\sum_{i=1}^{N} \sum_{t=1}^{T} [\pi_{it}(D_{it} + B_{i,t-1} - B_{it}) - h_{it}U_{it} - r_{it}Q_{it}C_t - b_{it}B_{it}] \right) + \sum_{t=1}^{T} \sum_{i \in I} \sum_{p \in P_i} \beta_{pt}(y_{pt} - Q_{pt}) + \sum_{t=1}^{T} \sum_{i \in I} \lambda_{it}(\bar{a}_{it} - \hat{a}_{it}), \tag{10}$$

$$\text{subject to} \quad MFG \quad Constraints, \tag{11}$$

$$PDG_i \ \ Constraints \ \forall i \in I, \tag{12}$$

$$Q_{pt}, \hat{a}_{it}, \bar{a}_{it} \in [0,1],\ y_{pt} \in \{0,1\}\ \forall p \in P_i,\ i \in I,\ t = 1, \ldots, T, \tag{13}$$

The associated Lagrangian Dual Problem (LD1) can then be written as:

$$(\text{LD1}) \min_{\lambda,\beta \geq 0} \left[\text{Max} \left(\sum_{i=1}^{N} \sum_{t=1}^{T} [\pi_{it}(D_{it} + B_{i,t-1} - B_{it}) - h_{it}U_{it} - r_{it}Q_{it}C_t - b_{it}B_{it}] + \sum_{t=1}^{T} \sum_{i \in I} \sum_{p \in P_i} \beta_{pt}(y_{pt} - Q_{pt}) + \sum_{t=1}^{T} \sum_{i \in I} \lambda_{it}(\bar{a}_{it} - \hat{a}_{it}) \right) \right], \tag{14}$$

$$\text{subject to} \quad MFG \ Constraints, \tag{15}$$

$$PDG_i \ \ Constraints \ \forall i \in I, \tag{16}$$

$$Q_{pt}, \hat{a}_{it}, \bar{a}_{it} \in [0,1],\ y_{pt} \in \{0,1\}\ \forall p \in P_i,\ i \in I,\ t = 1, \ldots, T. \tag{17}$$

The Lagrange multipliers λ_{it} may be viewed as the price PDG_i must pay MFG for a unit of factory capacity in period t. Similarly, β_{pt} can be viewed as the incentive MFG must pay PDG_i to complete the development of product $p \in P_i$ in period t so that MFG can move it into volume manufacturing and generate revenue. The Lagrangian problem (L1) can be decomposed into $|I| + 1$ independent subproblems, one for MFG and one for each of the $|I|$ PDGs as shown in (L2) and (L3) below:

(L2) MFG subproblem

$$\text{Max} \sum_{i=1}^{N}\sum_{t=1}^{T}[\pi_{it}(D_{it}+B_{i,t-1}-B_{it})-h_{it}U_{it}-r_{it}Q_{it}C_t-b_{it}B_{it}] + \sum_{t=1}^{T}\sum_{i\in I}\lambda_{it}\bar{a}_{it} - \sum_{t=1}^{T}\sum_{i\in I}\sum_{p\in P_i}\beta_{pt}Q_{pt}, \tag{18}$$

subject to
MFG Constraints, (19)
$Q_{pt}, \bar{a}_{it} \in [0,1], \ \forall p \in P_i, \ i \in I, \ t = 1, \ldots, T.$

(L3) $\forall \ i \in I$ PDG_i subproblem

$$\text{Max} \ -\sum_{t=1}^{T}\lambda_{it}\hat{a}_{it} + \sum_{t=1}^{T}\sum_{p\in P_i}\beta_{pt}y_{pt}, \tag{20}$$

Subject to :
PDG_i *Constraints* $\forall i \in I$,
$\hat{a}_{it} \in [0,1] \ y_{pt} \in \{0,1\} \ \forall p \in P_i, \ t = 1, \ldots, T.$

In subproblem (L2), MFG maximizes its total contribution, given by its sales revenue from meeting demand and its income from allocating factory capacity to the PDGs, minus the variable costs it incurs in meeting demands and the incentives it pays the PDGs to complete the development of new products. In subproblem (L3), each PDG_i maximizes its contribution, given by the difference between the incentives it receives from MFG for making products available for volume manufacturing and its payments to MFG for the factory capacity necessary to complete its development activities. The solution to the Lagrangian Dual problem (LD1) gives an upper bound on the optimal value of (C1), which is at least as good as the optimal cost of the corresponding LP relaxation. We solve the Lagrangian Dual problem (LD1) using the Deflected Subgradient Method (DSM), a variant of the subgradient method with faster convergence that ensures that there is always an acute angle between the subgradients in consecutive iterations (Guta, 2003).

We define a feasible capacity allocation schedule for PDG_i as a tuple $((\hat{a}_{it})_{t=1,\ldots,T}, (\bar{y}_p)_{p\in P_i})$, where the vector $(\hat{a}_{it})_{t=1,\ldots,T}$ denotes the factory capacity required by PDG_i in each period t to complete the development of all products $p \in P_i$ by period t^* such that $y_{p,t} = \bar{y}_{p,t} = 1 \ \forall t \geq t^*$ and constraints (43)–(55) are satisfied. We define a bid from PDG_i as a capacity allocation schedule that is feasible for its subproblem (L3). Each bid gives the auctioneer some understanding of the capabilities of the PDGs (their private information) by indicating their ability to complete the development of their products in the periods defined by the y_{pt} given the capacity allocation schedule contained in the bid; it is reasonable to assume that PDGs will only submit bids with a delivery schedule that is feasible to their subproblems (L3).

The LR-ICA framework derived from the DSM algorithm is shown in Figure 3. The auctioneer (MFG) initiates the auction by providing all agents with initial values of the Lagrange multipliers, i.e., the price MFG charges the PDGs for capacity in each period, and the incentive MFG is willing to pay each PDG for completing development activities for its products at times desirable to MFG. During the kth iteration, in Step 1a, the auctioneer communicates the current values of the Lagrange multipliers to each PDG_i, who, in Step 1b, solve their subproblem (L3) and calculate a set of bids S_i^k, which is determined by the set of feasible solutions found while solving L3. At this stage of the procedure, a given PDG_i may submit a subset $\tilde{S}_i^k \subset S_i^k$ as their bid set instead of S_i^k to procure an allocation with higher local contribution (20) than it might be able to obtain if it submitted all of them. An extreme case is for each PDG to submit only the solution that maximizes its local objective (20), which from the perspective of the auctioneer amounts to declaring all other bids infeasible.

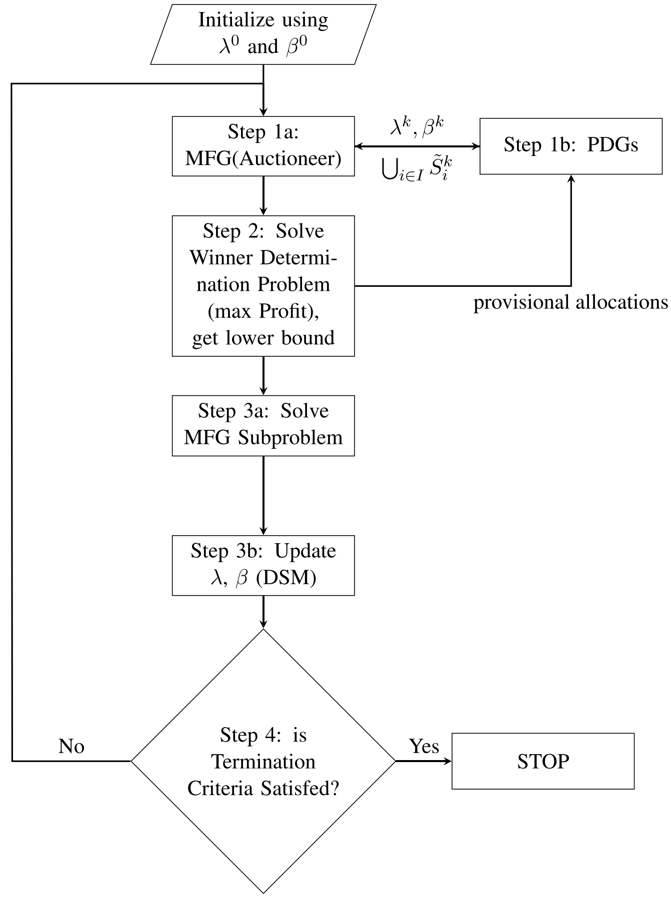

Figure 3. ICA framework based on Lagrangian relaxation.

To address this issue, we assume that the auctioneer retains all the bids from previous iterations and solves a WDP in Step 2 to select at most one bid from among those submitted by each PDG over the entire duration of the auction up to that iteration. Let $\bar{S}_i^k = \cup_{f=1,\ldots,k}\tilde{S}_i^f$ denote the set of all bids submitted by PDG_i up to and including iteration k. The WDP gives a feasible solution to (C1), yielding a lower bound on the optimal total contribution (1) as well as a provisional allocation of factory capacity to the PDGs and provisional new product introduction times to MFG. In Step 3a, MFG solves its subproblem (L2) for the optimal values of Q_{pt}^k and $\bar{a}_{it}^k$ and in Step 3b updates the Lagrange multipliers$[\lambda^k, \beta^k]^T$ in the kth iteration per the DSM as follows:

$$\delta^k = \left\{ -\tau \frac{(s^k)^T d^{k-1}}{\|d^{k-1}\|^2} \text{ if}(s^k)^T d^{k-1} < 0, \text{ 0 otherwise} \right\}, \quad (21)$$

$$d^k = s^k + \delta^k d^{k-1}, \quad (22)$$

$$\gamma^k = \mu_k \frac{UB^k - L^k}{\|d^k\|^2}, \quad (23)$$

$$[\lambda^{k+1}, \beta^{k+1}]^T = \max\{0, [\lambda^k, \beta^k]^T + \gamma^k d^k\}, \quad (24)$$

where $d^0 = 0, 0 \le \tau \le 2, 0 \le \mu_k \le 1$, UB^k is the optimal value of the Lagrangian subproblem (L1), L^k the best lower bound found for (1) and the subgradient $s^k = [[\bar{a}^k - \hat{a}^k]^T, [y^k - Q^k]^T]^T$ in the kth iteration. $\hat{a}^k, \bar{a}^k, Q^k, y^k$ are the optimal values of the decision variables $\hat{a}_{it}, \bar{a}_{it}, Q_{pt}, y_{pt}$ $\forall i \in I, p \in P_i, t = 1, ..., T$ in the optimal solution of the Lagrangian problem in the kth iteration of the DSM. Following Caprara *et al.* (1999), we set $\tau = 1.5$ and update μ_k according to Algorithm 1 in our computational experiments.

If the termination criteria in Step 4 are not satisfied, the algorithm returns to Step 1. Otherwise the auction terminates and the most recent provisional capacity allocations and new product completion time periods become final allocations. The auction terminates when either the total number of iterations or computational time exceed a predefined limit or if the duality gaps falls below a predefined value.

Algorithm 1. Scheme to update μ_k

Initiate with $\mu_0 = 0.1$. Let ϕ_l and ϕ_u be the biggest and the smallest values of the Lagrangian Dual (14) in last p = 20 DSM iterations. Let $\alpha = \frac{\phi_u - \phi_l}{|\phi_l|}$
if $\alpha > 0.01$ **then**
  $\mu_{k+1} \leftarrow 0.5\mu_k$
else if $\alpha < 0.001$ **then**
  $\mu_{k+1} \leftarrow 1.5\mu_k$
else
  $\mu_{k+1} \leftarrow \mu_k$
end if

The WDP solved in the kth iteration of DSM is formulated as a set packing problem (Günlük *et al.*, 2005; Blumrosen and Nisan, 2007) that selects at most one bid for PDG_i from $\tilde{S}_i^k$ such that inventory balance and factory capacity constraints are satisfied while maximizing total contribution, and can be stated as follows:

$$\text{(W1) Max} \quad \sum_{i=1}^{N}\sum_{t=1}^{T}[\pi_{it}\left(D_{it} + B_{i(t-1)} - B_{it}\right) - h_{it}U_{it} - r_{it}Q_{it}C_t - b_{it}B_{it}], \quad (25)$$

subject to

$$U_{it} = U_{i,t-1} + Q_{it}C_t - (D_{it} + B_{i,t-1} - B_{it}) \quad \forall i = 1..., N \quad t = 1, ..., T, \quad (26)$$

$$\sum_{i=1}^{N} Q_{it} + \sum_{i \in I} \bar{a}_{it} \le 1 \quad \forall t = 1, ..., T, \quad (27)$$

$$\sum_{j \in \bar{S}_i^k} \hat{a}_{it}^j \chi_{ij} \le \bar{a}_{it} \quad \forall i \in I \;\; t = 1, ..., T, \quad (28)$$

$$Q_{pt} \le \sum_{j \in \bar{S}_i^k} \hat{y}_{pt}^j \chi_{ij} \;\; \forall \; i \in I, \;\; p \in P_i, \;\; t = 1, ..., T, \quad (29)$$

$$\sum_{j \in \bar{S}_i^k} \chi_{ij} \le 1 \;\; \forall \;\; i \in I,$$
$$U_{it}, B_{it} \ge 0 \quad \forall i = 1, ..., N \quad t = 1, ..., T,$$
$$Q_{pt} \in [0, 1], \;\; \chi_{ij} \in \{0, 1\} \;\; \forall i \in I, \;\; p \in P_i, \;\; j \in \bar{S}_i^k \;\; t = 1, ..., T. \quad (30)$$

where χ_{ij} is a binary variable that takes the value 1 if bid $j \in \bar{S}_i^k$ is awarded to PDG_i and 0 otherwise, $\hat{a}_{it}^j$ is the factory capacity required by PDG_i in period t in bid $j \in \bar{S}_i^k$, and $\hat{y}_{pt}^j$ is a binary parameter that takes the value of 1 if product p is ready for manufacturing in period t in bid $j \in \bar{S}_i^k$ for $p \in P_i$. Constraints (26) and (27) ensure that inventory balance and factory capacity constraints are satisfied. Constraints (28) and (29) ensure that MFG allocates factory capacity to PDG_i and manufactures product $p \in P_i$ as per the bid selected for PDG_i, and constraint (30) that at most one bid is selected for each PDG.

4.1. Private information and the WDP

Submission of a restricted bid set $\tilde{S}_i^k \subset S_i^k$ instead of the full set S_i^k by some PDG_i seeking to improve their local objectives may reduce the auctioneer's ability to capture sufficient knowledge of its capabilities (described by the PDG_i constraints, which are private information to each PDG) to determine an optimal market-clearing solution. Consider two instances $\widehat{W1}$ and $\widetilde{W1}$ of the WDP (W1) based on two bid sets $\hat{S}_i^k$ and $\tilde{S}_i^k$ from PDG_i respectively, such that $\hat{S}_i^k \subseteq \tilde{S}_i^k$ and the bid sets of all other PDGs remain the same in both instances. Then $\widetilde{W1}$ is a relaxation of $\hat{W}1$, with total contribution to the firm at least as great as that of $\hat{W}1$. Thus, more bids from the PDGs will give the auctioneer more knowledge of their constraints (their private information) and therefore their capability to commit to a capacity allocation schedule. This gives the auctioneer greater flexibility in determining a good coordinated solution for the firm that is also feasible to the PDG_i subproblems. Thus, it is clearly in the auctioneer's interest to obtain as many bids as possible from the PDGs, even though it may not be in the PDG's interest to provide them. We show in Appendix B that if the perceived cost of any PDG not receiving any allocation at auction termination are sufficiently high, each PDG will willingly provide the auctioneer with as many bids as possible, provided other PDGs also do so.

5. A CG-ICA

Chapter 1 of Desaulniers *et al.* (2006) describes the dual relationship between Lagrangian Relaxation and CG, noting that both yield the same upper bound on the optimal total contribution (1) to the firm. However, the calculation of such an upper bound using Lagrangian Relaxation requires knowledge of all extreme points of the convex hull of constraints (11)–(13) in the Lagrangian Subproblem (L1), which is impractical due to the NP-hardness of the PDG_i subproblem (L3). Therefore, we use the DSM to determine a set of near-optimal prices for the Lagrangian Dual problem (LD1). For the problem addressed in this article, the LR-ICA with DSM has two favorable properties: (i) it can be easily interpreted as an ICA; and (ii) it is easy to implement and understand.

However, the LR-ICA with DSM suffers from two major drawbacks:

1. As the DSM uses only the optimal solutions to the subproblems to update the Lagrange multipliers (prices) at each iteration, it does not consider any other bids submitted by the PDGs in updating prices. This deprives the auctioneer of information regarding the constraints of the PDGs (their private information), resulting in slower convergence to the optimal upper bound.
2. There is no mechanism in the DSM that elicits new bids that have not been submitted previously from the PDGs in each iteration. The more bids satisfying the PDG's constraints the auctioneer has available, the more information the auctioneer has about the capabilities of the PDG. As the auctioneer retains bids from previous iterations, submission of bids already submitted in previous iterations does not add to the auctioneer's knowledge of the PDGs' capabilities, limiting its ability to quickly generate good coordinated solutions for the firm.

CG-ICA overcomes both these drawbacks by considering all bids submitted by the PDGs while updating prices, and ensuring that no previously submitted bid is resubmitted by a PDG in subsequent iterations. This is accomplished by enforcing provisional budgets specifying the maximum loss each PDG is allowed to incur at the current iteration of the ICA.

Figure 4 outlines the ICA framework derived using CG where θ_i^k denotes the provisional budget of PDG_i in the kth iteration. The auction is initialized with randomly chosen prices and infinite provisional budgets for the PDGs. In Step 1a of the kth iteration the auctioneer communicates the current prices and provisional budgets to the PDGs, who, in Step 1b, solve their subproblems (L3) and submit bids whose local cost, given by the negative of constraint (20), must not exceed the provisional budget. As in LR-ICA, each PDG_i can strategically manipulate the auction by submitting fewer budget-feasible bids than they actually calculate. However, PDGs may be induced to submit multiple budget-feasible bids as described for the LR-ICA in Section 4.1 when the cost of not receiving any allocation is extremely high. The auctioneer retains all bids from previous iterations and in Step 2a, solves the LP relaxation of the Restricted Master Problem to update prices (λ^k, β^k) and the provisional budget θ^k for the next iteration (explained in Section 5.1) in Step 2b. CG-ICA terminates in Step 3 when no PDG submits a bid that is feasible for both the available capacity allocation and their provisional budget; otherwise, the auction returns to Step 1.

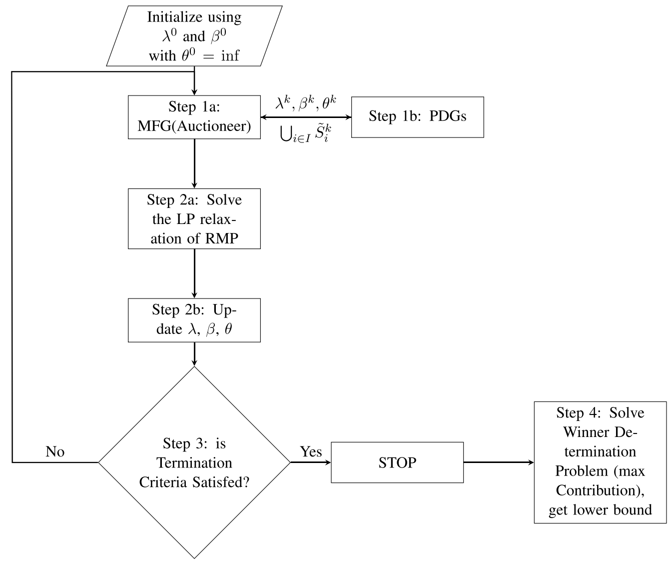

Figure 4. ICA framework based on CG.

Unlike LR-ICA, a lower bound on the optimal total contribution (1) is not required to update prices at each iteration, so in Step 4, the auctioneer solves the WDP (W1) only once after termination to obtain a lower bound on the optimal solution value of the optimal total contribution (1), final capacity allocations and product introduction periods. Since no information except its own bids and the prices it receives from the auctioneer are available to the PDGs at any iteration, they have no knowledge of when the auction will end. Since at the auction termination, PDGs will receive an allocation based on the bids submitted in the ICA, and the cost of not receiving an allocation is perceived by the PDGs to be prohibitively high, the assumption of myopic best response holds and they retain the incentive to submit bids that are likely to result in a successful allocation.

5.1. Restricted master problem and price update

The WDP (W1) can be regarded as the Restricted Master Problem (RMP) derived from the centralized formulation (C1) with bids from the PDGs represented as columns and the *PDG_i Constraints* $\forall i \in I$ as the constraints of $|I|$ pricing subproblems. W1-LP gives the linear relaxation of W1 at the kth iteration of the ICA. However, we modify the inequality (30) to equality in (36) to ensure that prices are updated in a manner that encourages PDGs to submit compatible bids. We define compatible bids as those containing feasible capacity allocation schedules that result in feasible solutions to WDP (W1) where all PDGs receive an allocation (i.e., constraint (30) is tight for all PDGs). The LP relaxation of this WDP is as follows:

$$\text{(W1-LP)}\ \text{Max} \quad \sum_{i=1}^{N}\sum_{t=1}^{T}[\pi_{it}(D_{it} + B_{i,t-1} - B_{it}) - h_{it}U_{it} - r_{it}Q_{it}C_t - b_{it}B_{it}], \tag{31}$$

subject to

$$U_{it} = U_{i,t-1} + Q_{it}C_t - (D_{it} + B_{i,t-1} - B_{it}) \quad \forall i = 1\ldots, N \quad t = 1, \ldots, T, \tag{32}$$

$$\sum_{i=1}^{N} Q_{it} + \sum_{i\in I} \bar{a}_{it} \leq 1 \quad \forall t = 1, \ldots, T, \tag{33}$$

$$\sum_{j\in \tilde{S}_i^k} \hat{a}^j_{it}\chi_{ij} \leq \bar{a}_{it} \quad \forall i \in I \;\; t = 1, \ldots, T, \tag{34}$$

$$Q_{pt} \leq \sum_{j\in \tilde{S}_i^k} \hat{y}^j_{pt}\chi_{ij} \;\; \forall \; i \in I, \;\; p \in P_i, \;\; t = 1, \ldots, T, \tag{35}$$

$$\sum_{j\in \tilde{S}_i^k} \chi_{ij} = 1 \;\; \forall \; i \in I, \tag{36}$$

$$U_{it}, B_{it} \geq 0 \quad \forall i = 1, \ldots, N \quad t = 1, \ldots, T,$$
$$Q_{pt} \in [0, 1], \;\; \chi_{ij} \geq 0 \;\; \forall i \in I, \;\; p \in P_i, \;\; j \in S_i^k \;\; t = 1, \ldots, T. \tag{37}$$

Let

$$\tilde{\lambda}^k = \left(\tilde{\lambda}^k_{it}\right)_{i\in I, \quad t=1,\ldots,T}, \tilde{\beta}^k = \left(\tilde{\beta}^k_{pt}\right)_{p\in P_i, i\in I, \quad t=1,\ldots,T}$$
$$\text{and } \tilde{\theta}^k = \left(\tilde{\theta}^k_i\right)_{i\in I}$$

be the optimal values of the dual variables associated with constraints (34), (35), and (36) respectively, where $\{\tilde{\lambda}^k, \tilde{\beta}^k\} \geq 0$. $\tilde{\theta}^k$ is unrestricted in sign, and can be interpreted as a provisional budget allocated to PDG_i at each iteration k as discussed below. The CG procedure seeks columns (bids) with positive reduced cost at $(\tilde{\lambda}, \tilde{\beta}, \tilde{\theta})$ that are feasible to the PDG_i constraints. Thus, the columns acceptable to the auctioneer in the $(k+1)$th iteration should satisfy

$$-\sum_{t=1}^{T} \tilde{\lambda}^k_{it}\hat{a}_{it} + \sum_{p\in P_i}\sum_{t=1}^{T} \tilde{\beta}^k_{pt}y_{pt} - \tilde{\theta}^k_i > 0 \;\; \forall i \in I. \tag{38}$$

Setting $\lambda^{k+1} = \tilde{\lambda}^k, \beta^{k+1} = \tilde{\beta}^k$ and the provisional budgets for the $(k+1)$th iteration, $\theta^{k+1} = -\tilde{\theta}^k$ in constraint (38), we get

$$-\sum_{t=1}^{T} \lambda^{k+1}_{it}\hat{a}_{it} + \sum_{p\in P_i}\sum_{t=1}^{T} \beta^{k+1}_{pt}y_{pt} > -\theta^{k+1}_i \;\; \forall i \in I, \tag{39}$$

whose-left hand side ensures that all constraints of the optimization model (L3) with objective function (20) are retained for the PDG_i subproblem in CG-ICA. However, (39) implies that:

$$\sum_{t=1}^{T} \lambda^{k+1}_{it}\hat{a}_{it} - \sum_{p\in P_i}\sum_{t=1}^{T} \beta^{k+1}_{pt}y_{pt} < \theta^{k+1}_i \;\; \forall i \in I, \tag{40}$$

suggesting an additional constraint for the PDGs to ensure that only bids whose local total loss, given by the negative of (20), does not exceed the specified provisional budget θ^{k+1}_i can be submitted to the auctioneer, i.e., only columns with positive reduced costs are added to the RMP. Moreover, the dual constraint of W1-LP associated with $\chi_{ij}\forall j \in \tilde{S}_i^k, \;\; i \in I$ at optimality of (W1-LP) is given by:

$$\sum_{t=1}^{T} \tilde{\lambda}^k_{it}\hat{a}^j_{it} - \sum_{p\in P_i}\sum_{t=1}^{T} \tilde{\beta}^k_{pt}\hat{y}^j_{pt} + \tilde{\theta}^k_i \geq 0 \;\; \forall j \in \bar{S}_i^k, \;\; i \in I. \tag{41}$$

Setting $\lambda^{k+1} = \tilde{\lambda}^k, \beta^{k+1} = \tilde{\beta}^k, \theta^{k+1} = -\tilde{\theta}^k$ in Equation (41) we get:

$$\sum_{t=1}^{T} \lambda^{k+1}_{it}\hat{a}^j_{it} - \sum_{p\in P_i}\sum_{t=1}^{T} \beta^{k+1}_{pt}\hat{y}^j_{pt} \geq \theta^{k+1}_i \;\; \forall j \in \bar{S}_i^k, \;\; i \in I, \tag{42}$$

which renders all bids submitted by the PDGs in previous iterations budget infeasible for a provisional budget of θ^{k+1}, ensuring, along with constraint (39), that either bids different from previously submitted ones will be submitted in the next iteration, or none at all. Thus, CG-ICA takes into account all bids submitted by agents up to and including the current iteration while updating prices and provisional budgets, giving the auctioneer a better understanding of the private information of PDGs (specifically, the structure of the PDG constraints that define the capabilities of the PDG to its management, but which are not known to the auctioneer). CG-ICA terminates when either the time limit imposed in the ICA expires or no new budget-feasible bids are submitted by PDGs, implying an optimal solution of the linear relaxation of the RMP.

Since the provisional budgets $\tilde{\theta}^k$ are unrestricted in sign, the provisional budget θ^{k+1}_i for PDG_i can take both positive and negative values. A positive provisional budget $\theta^{k+1}_i > 0$ at iteration $k+1$ can be interpreted as the maximum loss PDG_i can incur from its bids submitted in iteration $k+1$. Moreover, a positive provisional budget implies that the optimal dual variable $\tilde{\theta}^k_i$ is negative. This suggests that, based on the bids submitted by PDG_i until iteration k, the auctioneer can improve the objective of (W1-LP) by reducing the right-hand side of the ith constraint in Equation (36) to zero, i.e., not selecting any combination of bids so far submitted by PDG_i. Thus, a positive provisional budget relaxes the PDG_i subproblem through Equation (40), encouraging PDG_i to submit additional bids in iteration $k+1$. On the other hand, a negative provisional budget $\theta^{k+1}_i < 0$ at iteration $k+1$ represents a penalty paid by PDG_i to the auctioneer for submitting a bid in iteration $k+1$, forcing it to search for a solution whose value offsets the penalty, leading to a bid with positive reduced cost. Moreover, a negative provisional budget implies that the optimal dual variable $\tilde{\theta}^k_i$ is positive. This suggests that, based on the solutions submitted by PDG_i till iteration k, the auctioneer will seek to reduce the optimal objective function value of (W1-LP) by changing the right- hand side of the ith constraint in (36) to zero. This means that PDG_i has submitted some compatible bids in previous iterations, so the auctioneer may not require PDG_i to bid as aggressively as in previous iterations. Thus, a negative provisional budget

Table 2. Parameter values for MFG constraints.

Parameters	*Values*
T	24
I	1,2
N	4
r_{it}	50
h_{it}	$0.1r_{it}$
b_{it}	$20h_{it}$
C_t	5000

Table 3. Demand distribution of new products in *Case 4PDGs*.

PDG	*Demand distribution*
PDG_1	UNIF(1550, 1555)
PDG_2	UNIF(750, 755)
PDG_3	UNIF(1550, 1555)
PDG_4	UNIF(750, 755)

Table 4. Cases in product development process in semiconductor industry.

Cases	*Number of stages*	*Factory capacity requirements*	*Probability*
Scenario 1	3	(500, 100, 0)	0.20
Scenario 2	4	(500, 250, 100, 0)	0.20
Scenario 3	5	(500, 400, 200, 100, 0)	0.60

constrains the PDG_i subproblem through Equation (40), discouraging PDG_i from submitting additional bids in iteration $k+1$.

6. Computational experiments

6.1. Experimental design

Our computational experiments consider problem instances with one MFG unit and either two or four PDGs, denoted by *Case 2PDGs* and *Case 4PDGs*, respectively. In both cases, each PDG develops one new product that will replace an old product currently in volume production. We assume demand for the older products goes to zero when demand for new products are realized, and that all new products have the same introduction deadlines based on the demand forecasts. This will ensure that we generate hard problem instances, as if the development of new products is not complete before their demand is realized, the firm will incur high backordering costs for new products and high opportunity costs for factory capacity as MFG would be left with no product to manufacture. If, on the other hand, the PDGs are allocated too much factory capacity in the initial periods of the planning horizon, the firm will incur large backordering costs for old products, reducing their revenue stream. Although in practice we would expect positive demand for both old and new products in some periods, the current design makes timely completion of new product development by the PDGs especially critical, since the firm cannot receive any revenue if the new product is not ready. Hence, we believe these represent particularly hard test instances for this problem.

We shall use the notation $UNIF(a, b)$ to denote a continuous uniform probability distribution over the interval (a, b). In *Case 2PDGs*, we will refer to the current products as Products 1 and 2 and the new products as Products 3 and 4. PDG_1 develops the new generation of Product 1 (Product 3) with low demand $(UNIF(1600, 1605))$ and high revenue ($175/unit), whereas PDG_2 develops the new generation of Product 2 (Product 4) with high demand $(UNIF(3000, 3005))$ and low revenue($75/unit). All products have the same unit material, inventory holding and backordering costs. The price increments from old products to new generation products are equally likely to be $\{10\%, 15\%, 20\%\}$. Values of other parameters of the MFG constraints are given in Table 2 and are identical for all products.

Case 4PDGs has a similar design to *Case 2PDGs* with two additional PDGs, PDG_3 and PDG_4, that duplicate PDG_1 and PDG_2 of *Case 2PDGs*. The demand for new products in *Case 4PDGs*, shown in Table 3, is adjusted to maintain an average factory utilization of 0.92. These represent quite difficult problem instances where there is limited excess capacity available for MFG to allocate to PDGs; any factory capacity allocated to a PDG is likely to result in lost revenue.

For all PDG_i subproblems, we consider a single engineering resource whose capacity is constant across time periods and is determined using the parameter η. Setting $\eta = 0$ ensures that engineering resource capacity is not constraining, whereas $\eta = 1$ sets the engineering resource capacity to the minimum level required to maintain problem feasibility. During the new product development process, as the engineering resource can be allocated in the first two time periods of a stage (except the last one which spans only one time period), the minimum engineering resource level is set to half of the maximum engineering resource requirement among all stages.

Since the product development process can require different number of stages, we consider three scenarios. The number of stages and factory capacity required for each stage in each scenario is given in Table 4 along with the probability of each scenario occurring. Scenario 1 is the best case scenario that requires the least factory capacity, whereas Scenario 3 is the worst case scenario, with Scenario 2 intermediate between them. We generate different problem instances by assuming that each product development process will require one of these three scenarios with the probabilities shown in Table 4. The probability distribution is chosen such that we emphasize harder problem instances where most PDGs are developing products in Scenario 3.

We obtain multiple bids for each PDG by using the `cplex.populate()` command in the CPLEX MILP solver, which gives all the feasible solutions found in the course of the branch-and-bound algorithm used to solve the subproblems. We consider three different bidding strategies for the PDGs. Under the first, which we shall refer to as *All*, all PDGs submit all bids that they obtain in the process of solving their respective subproblems. Under the *Random* bidding strategy, the PDGs randomly determine the number of bids to be submitted to the auctioneer from their set of feasible capacity allocation schedules. Specifically, the number of bids submitted by a PDG is uniformly distributed between one and the number of feasible bids the PDG has generated. Under *Best*, all PDGs submit their feasible capacity allocation schedules that are optimal to their local objective functions as their bid. We solve 20 random instances for each bidding strategy, each value of $\eta = \{0, 1\}$ with randomization over demand, cases in product development process and per unit revenue increments from old products to new products. As the DSM and CG procedures require optimal subproblem and RMP solutions for convergence, we use the CPLEX LP solver to solve the MFG subproblem and RMP, and the CPLEX MILP solver to solve the PDG subproblems and the WDP to optimality. We execute the computational experiments on an Intel Xeon E5-2680 v2 @ 2.80 GHz processor with 128GB RAM, MATLAB 2017b and CPLEX 12.8.0. We impose a time limit of 3600 seconds for the entire ICA procedure, but if this limit is reached while an iteration is in progress, we allow the iteration to complete. Hence in LR-ICA some CPU times can exceed the 3600 seconds limit, but only by the time required to complete the final iteration.

6.2. Computational results

For the mean and maximum of the performance measures stated in Table 5 for the *All* bidding strategy; Table 6 compares the performances of LR-ICA and CG-ICA. Both ICAs generate near-optimal feasible solutions, as evident from the very low values of Gap_CI in Table 6. It is interesting to note that in *Case 4PDGs*, LR-ICA obtains optimal solutions with values of zero for Gap_CI, while CG-ICA is slightly worse on average. However, CG-ICA constructs markedly better upper bounds in CPU time that is two orders of magnitude faster than LR-ICA for all cases and η values. This is because, unlike LR-ICA, CG-ICA utilizes all the bids submitted by PDGs to update prices, providing the auctioneer with more information regarding the capabilities of PDGs. Moreover, CG-ICA uses the provisional budget of the PDGs to force them to submit different bids in each iteration, which monotonically improves the upper bound. Such a mechanism is absent in LR-ICA, where bids can be repeated in each iteration and the change in upper bound may be non-monotonic. Thus, we focus on the performance of CG-ICA for the rest of this section as it consistently outperforms LR-ICA over two of the three performance measures while constructing feasible solutions of similar quality. We also drop *Case 2PDGs* from subsequent analysis as it provides very easy problems compared with *Case 4PDGs*, as evident from low values of all three performance measures in Table 6.

Table 7, Figure 5, and Figure 6 compare the *All*, *Random*, and *Best* bidding strategies for CG-ICA over the mean and maximum of the performance measures stated in Table 5 for *Case 4PDGs* and both values of η. Mean Gap_CI and Mean Gap_LB decreases from top to bottom for both values of η in Table 7, suggesting that the *All* bidding strategy outperforms *Random*, which in turn outperforms *Best*, with Figures 5 and 6 supporting this observation. This is because more bids from the PDGs give the auctioneer a better understanding of the capabilities of PDGs, giving it an increased flexibility in combining bids and thus in constructing a better feasible solutions

Table 5. Performance measures.

Performance measure	*Definition*
Gap_CI (%)	Relative gap of the best objective function value obtained from DSM or CG from the optimal objective function value of the centralized formulation under compete information (solved using CPLEX MILP solver)
Gap_LB (%)	Relative gap of the best objective function value (lower bound) from the upper bound obtained from DSM or CG
Comp_Time	CPU time in seconds

Table 6. Comparison of LR-ICA and CG-ICA for Case *2PDGs* and *4PDGs* under *All* bidding strategy.

	Mean			*Max*		
	Gap_CI(%)	*Gap_LB(%)*	*Comp_Time (sec)*	*Gap-CI(%)*	*Gap_LB(%)*	*Comp_Time (sec)*
Case 2PDGs ($\eta = 0$)						
LR-ICA	0.011	23.84	2916.88	0.058	64.88	3698.24
CG-ICA	0.002	0.03	31.92	0.011	0.04	46.58
Case 2PDGs ($\eta = 1$)						
LR-ICA	0.0044	22.71	2916.37	0.057	60.64	3675.96
CG-ICA	0.0001	0.02	29.21	0.0011	0.04	45.69
Case 4PDGs ($\eta = 0$)						
LR-ICA	0.00	23.00	3686.77	0.00	83.59	3742.48
CG-ICA	1.00	3.74	59.87	5.81	13.80	80.94
Case 4PDGs ($\eta = 1$)						
LR-ICA	0.00	22.86	3654.09	0.00	83.27	3732.67
CG-ICA	1.07	3.81	59.17	5.25	13.57	90.71

Table 7. Comparison of different bidding strategies in CG-ICA for *Case 4PDGs*.

	Mean			*Max*		
	Gap_CI(%)	*Gap_LB(%)*	*Comp_Time (sec)*	*Gap_CI(%)*	*Gap_LB(%)*	*Comp_Time (sec)*
Case 4PDGs ($\eta = 0$)						
All	1.00	3.74	59.87	5.81	13.80	80.94
Random	1.21	3.96	46.34	6.25	14.33	61.36
Best	1.31	4.10	80.98	8.72	17.90	155.36
Case 4PDGs ($\eta = 1$)						
All	1.07	3.81	59.17	5.25	13.57	90.71
Random	1.29	4.04	48.64	4.38	12.54	66.44
Best	1.49	4.26	78.40	5.37	13.71	149.72

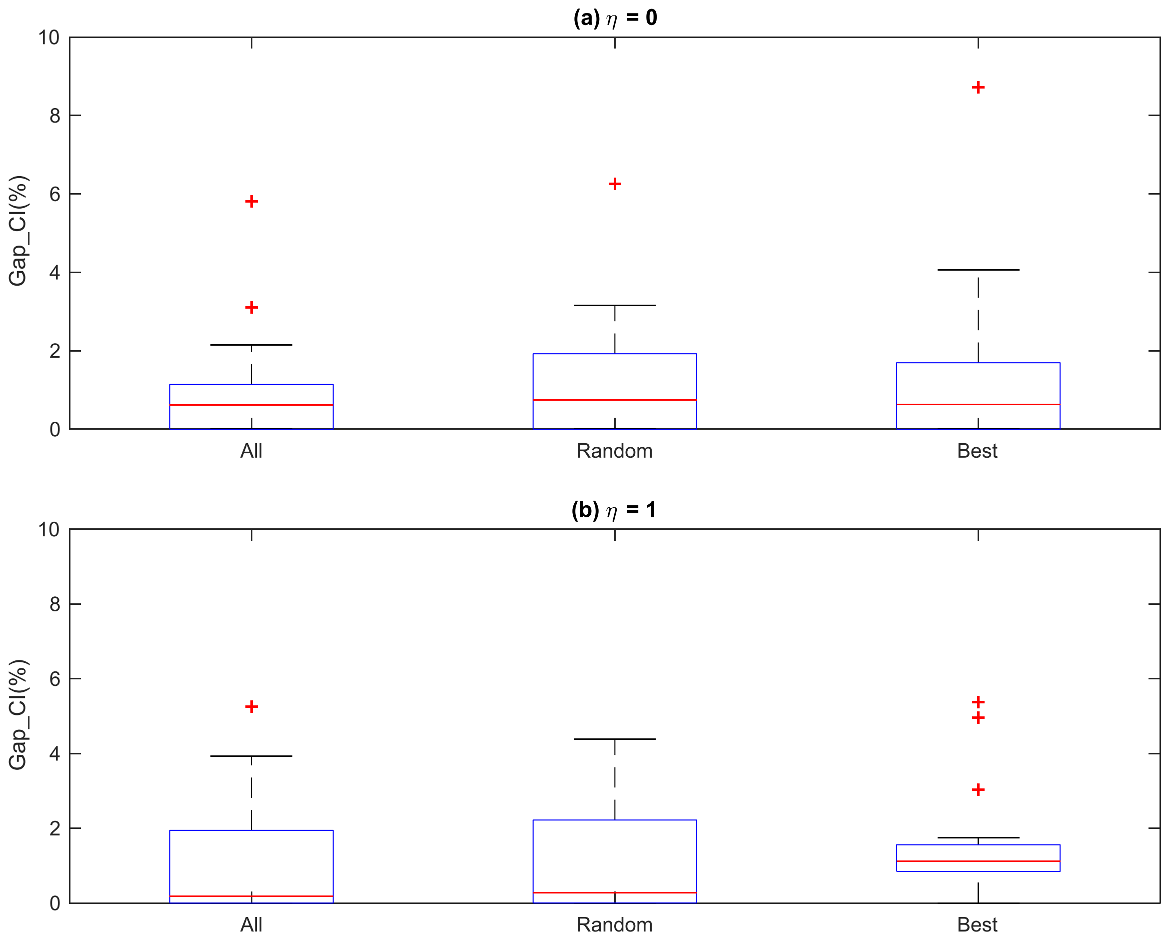

Figure 5. Comparison of three bidding strategies in CG-ICA over Gap_CI for *Case 4PDGs*: (a) $\eta = 0$ and (b) $\eta = 1$.

and upper bounds. Thus, the auctioneer should provide some incentives to PDGs to follow the *All* bidding strategy. We show in Appendix B that if failing to obtain a capacity allocation at auction termination is viewed as being prohibitively expensive for the PDGs, then under the assumption of myopic best response, it is in the best interest of each PDG to submit their complete set of bids.

7. Conclusion

We design two ICA frameworks for coordinating decentralized negotiations over factory capacity in the semiconductor industry using MILP, Lagrangian Relaxation (LR-ICA) and Column Generation (CG-ICA). LR-ICA uses the DSM as the price tâtonnement scheme, whereas CG-ICA uses the all the bids submitted by PDGs to update prices. We show that CG-ICA constructs better quality upper bounds in short CPU times as compared with LR-ICA while generating similar quality feasible solutions. Moreover, it is free from strategic manipulation by PDGs under the assumption of myopic best response if any PDG that cannot procure any factory capacity at auction termination is subject to a heavy penalty.

In our computational experiments we found columns at the termination of CG-ICA to be integer feasible for all instances, but in general, this might not be the case. Therefore, an important direction for future research is to design a CG-ICA that generates integer-feasible solutions for most problem instances. Other future research directions include designing a ICA framework with both MFG and PDGs as agents in the ICA and devising a side payment scheme that makes submission of all bids the optimal strategy for a PDG irrespective of the bids submitted by other PDGs and MFG. The timing of the auctions, i.e., the frequency with which the auction should be conducted in the face of dynamically evolving information, is another question that remains to be addressed. Finally, the schemes proposed in this article assume that agents can obtain optimal solutions for their various subproblems at each iteration of the procedure. How the performance of the procedures might be affected when heuristics are used to produce near-optimal solutions in short CPU times, and how the subproblems might be modified to enhance the performance of procedures using such approximate subproblem solutions, is an important question.

The current article assumes no communication between the individual PDGs except indirectly through the ICA process. In practice, subsets of PDGs might be motivated to communicate among themselves to prepare coordinated bids that would place them in a favorable position relative to others. Examination of effective decentralized solution procedures under such conditions is an important direction for future work. Finally, the extension of the approaches in this article to study stochastic versions of the problem, where different units may have private information about internal sources of uncertainty as well as different assumptions about external uncertainties remains an important long-term goal.

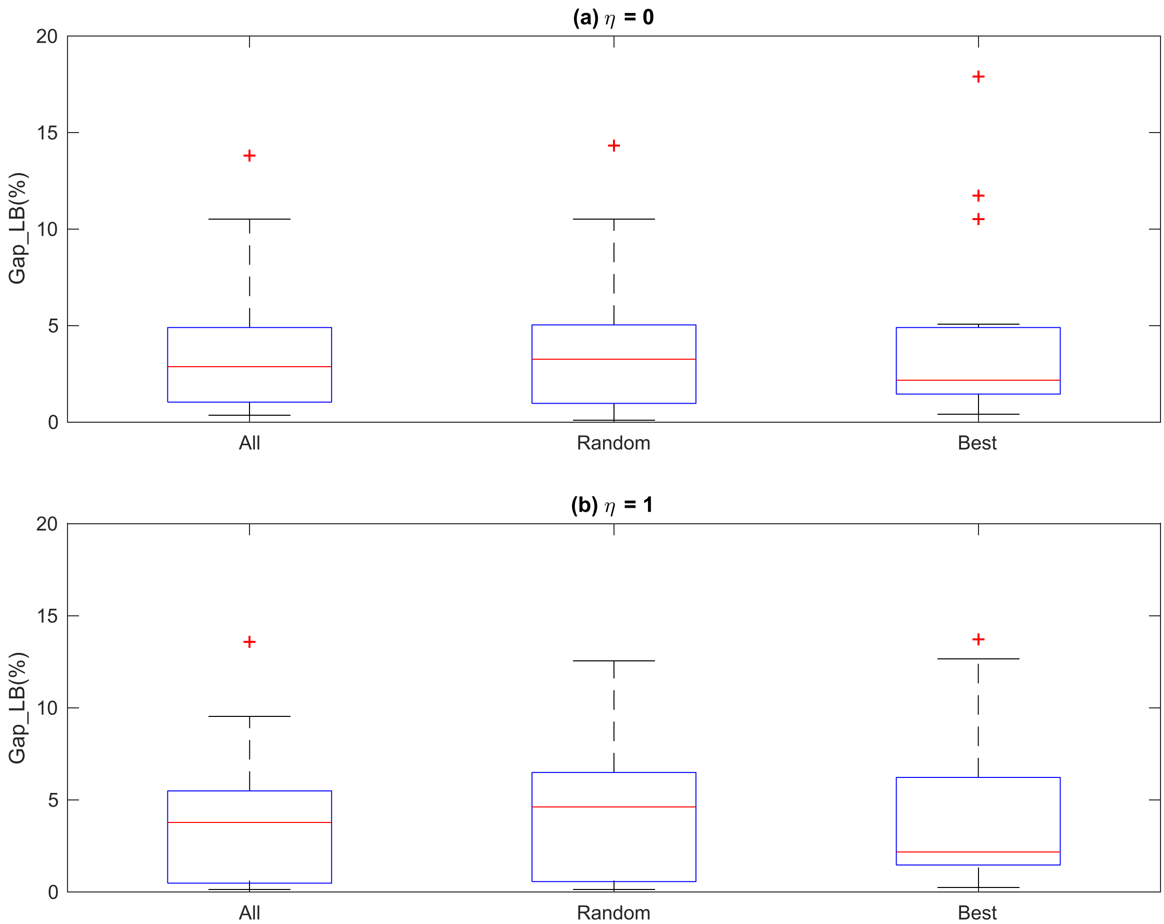

Figure 6. Comparison of three bidding strategies in CG-ICA over Gap_LB for *Case 4PDGs* (a) η =0 and (b) $\eta = 1$.

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Appendices

Appendix A

Detailed formulation of PDG_i constraints

In this section we give a detailed description of the constraints describing the capabilities of each PDG_i using the notation defined in Table 8.

The formulation can then be stated as follows:

$$\sum_{p\in P_i}\sum_{s_p=1}^{\bar{s}_p} H_{prt}^{s_p} \le 1 \quad r=1,\dots,R \quad t=1,\dots,T, \tag{43}$$

$$H_{prt}^{s_p} \le z_{pt}^{s_p} \quad \forall p\in P_i \quad s_p=1,\dots,\bar{s}_p \quad r=1,\dots,R \quad t=1,\dots,T, \tag{44}$$

$$H_{prt}^{s_p} \le z_{p,t+1}^{s_p} \quad \forall p\in P_i \quad s_p=1,\dots,\bar{s}_p-1 \quad r=1,\dots,R-1 \quad t=1,\dots,T-1, \tag{45}$$

$$H_{pRt}^{s_p} \le 1-z_{p,t+1}^{s_p} \quad \forall p\in P_i \quad s_p=1,\dots,\bar{s}_p-1 \quad t=1,\dots,T-1, \tag{46}$$

$$\sum_{t=1}^{T} E_{rt}H_{prt}^{s_p} \ge K_{pr}^{s_p} \quad \forall p\in P_i \quad s_p=1,\dots,\bar{s}_p \quad r=1,\dots,R, \tag{47}$$

$$\sum_{t=1}^{T} z_{pt}^{s_p} = 3 \quad \forall p\in P_i \quad s_p=1,\dots,\bar{s}_p-1, \tag{48}$$

$$\sum_{t=1}^{T} z_{pt}^{\bar{s}_p} = 1 \quad \forall p\in P_i, \tag{49}$$

$$-z_{p(t-1)}^{s_p} + z_{pt}^{s_p} + \frac{1}{t}\sum_{\tau=1}^{t-2} z_{p\tau}^{s_p} \le 1 \quad \forall p\in P_i \quad s_p=1,\dots,\bar{s}_p-1 \quad t=3,\dots,T, \tag{50}$$

$$z_{pt}^{s_p} \le \sum_{\tau=1}^{t-1} z_{p\tau}^{s_p-1} \quad \forall p\in P_i \quad s_p=2,\dots,\bar{s}_p \quad t=2,\dots,T, \tag{51}$$

$$\sum_{s_p=1}^{\bar{s}_p} z_{pt}^{s_p} \le 1 \quad \forall p\in P_i \quad t=1,\dots,T, \tag{52}$$

Table 8. Parameters and decision variables for PDG_i constraints.

Parameters	*Description*
R	Total number of resources with resource R being factory capacity
$\bar{s}_p$	Total number of stages in the development process of product $p \in P_i$
E_{rt}	Capacity of resource r in period t
$K_{pr}^{s_p}$	Total amount of resource r required for the completion of stage s_p of product p
Decision variables	*Description*
$H_{prt}^{s_p}$	Fraction of capacity of resource r allocated to stage s_p product p in period t
$z_{pt}^{s_p}$	binary variable equal to 1 if stage s_p of product p is active in period t and 0 otherwise

$$y_{pt} = \sum_{\tau=1}^{T} A_{t\tau} z_{p\tau}^{\bar{s}_p} \;\; \forall p \in P_i, t = 1, \ldots, T, \tag{53}$$

$$\hat{a}_{it} = \sum_{p \in P_i} \sum_{s_p=1}^{\bar{s}_p - 1} H_{pRt}^{s_p} \;\; \forall t = 1, \ldots, T, \tag{54}$$

$$z_{pt}^{s_p} \in \{0, 1\}, \;\; H_{prt}^{s_p} \in [0, 1] \;\; \forall p \in P_i, \;\; s_p = 1, \ldots, \bar{s}_p - 1, \\ r = 1, \ldots, R, \;\; t = 1, \ldots, T. \tag{55}$$

Constraints (43) are the resource capacity constraints. Constraints (44) ensure that resources are allocated to a stage only when that stage is active, and constraints (45) that engineering resources are allocated in the first two periods of a stage. Constraints (46) ensure that factory capacity is allocated only in last period of a stage, whereas constraints (47) ensure that all stages of all products in P_i are allocated their required resources during the time horizon. Constraints (48) and (49) ensure that all stages except the last one span three time periods and the last stage spans only one. Constraints (50) ensure that once PDG_i starts working on a stage it must be completed, whereas constraints (51) and (52) establish linear precedence constraints between the stages. Lastly, constraints (53) and (54) link the PDG_i subproblem to the centralized formulation (C1), where A is a $T \times T$ lower triangular matrix with all non-zero components equal to one. The resource capacity constraint (43) and project scheduling constraints (44)–(54) make the PDG_i subproblem a special case of the strongly NP-hard Multiple Resource Constrained Project Scheduling Problem (Kolisch *et al.*, 1995). This is again representative of the type of problem that must be solved by a PDG; clearly more detailed, domain-specific constraints may be added.

Appendix B

Bayesian incentive compatibility of proposed ICA scheme

In this section, we show Bayesian Incentive Compatibility for CG-ICA; the proof of the Bayesian Incentive Compatibility of LR-ICA follows the same lines. Let $\bar{\theta}_i^k = |S_i^k|$ denote the number of bids obtained by PDG_i in the course of solving its subproblem associated with the kth iteration of the ICA, which we define as the true type of PDG_i. If PDG_i decides to submit $\theta_i^k < \bar{\theta}_i^k$ bids, let $\tilde{S}_{\theta_i^k}$ denote the set of bids it submits and $g_i^k(\tilde{S}_{\theta_i^k})$ be the probability distribution over all the possible bid sets consisting of θ_i^k bids. We assume that the distribution $g_i^k(\tilde{S}_{\theta_i^k})$ is privately known to PDG_i. The remaining notation in this section is defined in Table 9.

The probability of PDG_i receiving some factory capacity at CG-ICA termination depends on its submitted bid set and on the bids submitted by other PDGs. Therefore, let $\bar{p}_i^k(\theta_i^k)$ denote the expected probability of the ICA ending in iteration k and PDG_i being awarded one of its submitted bids if it submits $\theta_i^k \leq \bar{\theta}_i^k$ bids, that is to say

$$\bar{p}_i^k(\theta_i^k) = \mathbb{E}_{\tilde{S}_{\theta_{-i}^k}} \left(\mathbb{E}_{\tilde{S}_{\theta_i^k}} \left(p_i^k(\tilde{S}_{\theta_i^k}, \tilde{S}_{\theta_{-i}^k}) \right) \right), \tag{56}$$

where $p_i^k(\tilde{S}_{\theta_i^k}, \tilde{S}_{\theta_{-i}^k})$ is the probability of the ICA ending in iteration k and PDG_i getting an allocation if it submits the bid set $\tilde{S}_{\theta_i^k}$ and other PDGs bid $\tilde{S}_{\theta_{-i}^k}$. Expectation is calculated over the random variable $\tilde{S}_{\theta_i^k}$ representing the bid set chosen with θ_i^k bids, which are drawn from $g_i^k(\tilde{S}_{\theta_i^k})$, and $\tilde{S}_{\theta_{-i}^k}$ represents the bid sets submitted by all PDGs except i, which are drawn from some belief distribution in $\mathbb{P}$. Similarly, let $\bar{v}_i^k(\theta_i^k)$ be the expected utility earned by PDG_i through the trading of factory capacity and product introduction time periods if the ICA ends in iteration k and PDG_i receives an allocation if it submits θ_i^k bids.

Table 9. Notations for Bayesian Incentive Compatibility.

Notation	*Description*
$\bar{\theta}_i^k$	True type of PDG_i or total number of bids devised by PDG_i in kth iteration
$\tilde{S}_{\theta_i^k}$	Set of $\theta_i^k (< \bar{\theta}_i^k)$ bids submitted by PDG_i
$\bar{t}_i(\theta_i^k)$	Expected payment for PDG_i if it submits θ_i^k bids
θ_{-i}^k	Revealed types of all other PDGsexcept i
$\tilde{S}_{\theta_{-i}^k}$	Set of all bids submitted by all PDGs except i associated with their types θ_{-i}^k
$U_i(\theta_i^k)$	Expected utility earned by PDG_i if it submits θ_i^k bids in iteration k
$\mathbb{P}$	A set of distributions used by each PDG to devise their belief distribution of $\tilde{S}_{\theta_{-i}^k}$ and is assumed to be common knowledge

When MFG assumes the role of auctioneer, we model the expected payment paid by PDG_i for not getting a factory capacity allocation by setting:

$$\bar{t}_i(\theta_i^k) = M\left(1 - \bar{p}_i^k(\theta_i^k)\right), \tag{57}$$

where M is the very large penalty paid by each PDG_i for not getting an allocation at auction termination and $(1 - \bar{p}_i^k(\theta_i^k))$ the expected probability of PDG_i of not getting an allocation if it submits θ_i^k bids. Thus, the expected utility of PDG_i with θ_i^k bids is:

$$U_i(\theta_i^k) = \bar{v}_i(\theta_i^k)\bar{p}_i(\theta_i^k) - \bar{t}_i(\theta_i^k) \tag{58}$$

$$\Rightarrow U_i(\theta_i^k) = \bar{v}_i(\theta_i^k)\bar{p}_i(\theta_i^k) - M\left(1 - \bar{p}_i^k(\bar{\theta}_i^k)\right) \quad \text{from (57)} \tag{59}$$

$$\Rightarrow U_i(\theta_i^k) = \left(\bar{v}_i(\theta_i^k) + M\right)\bar{p}_i(\bar{\theta}_i^k) - M. \tag{60}$$

Following the Mechanism Design literature for linear environments (Mas-Colell *et al.*, 1995), we assume that the types of PDGs are statistically independent, i.e., their joint density distribution can be written as the product of their marginal distributions. By Equation (60), we know that improving $U_i(\theta_i^k)$ involves increasing $\bar{p}_i(\theta_i^k)$. We have the following results:

Proposition 1: *If $(\bar{S}_{\bar{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k})$ denote the complete set of bids for all PDGs with $(\bar{\theta}_i^k, \bar{\theta}_{-i}^k)$ being their true types in the* k*th iteration of CG-ICA then $p_i^k(\tilde{S}_{\tilde{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k}) \leq p_i^k(\bar{S}_{\bar{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k}) \;\forall \tilde{\theta}_i^k < \bar{\theta}_i^k, \;\; i \in I$ if WDP is solved optimally as the allocation rule.*

Proof. Let $\mathbb{X}_{\tilde{S}_{\tilde{\theta}_i^k}}$ be the set of feasible solutions for the WDP at iteration k if PDG_i submits $\tilde{S}_{\tilde{\theta}_i^k}$ and all other PDGs submit $\bar{S}_{\bar{\theta}_{-i}^k}$. If PDG_i gets an allocation when WDP is solved optimally over $\mathbb{X}_{\tilde{S}_{\tilde{\theta}_i^k}}$, it will definitely get an allocation when WDP is solved over $\mathbb{X}_{\bar{S}_{\bar{\theta}_i^k}}$ as $\mathbb{X}_{\tilde{S}_{\tilde{\theta}_i^k}} \subset \mathbb{X}_{\bar{S}_{\bar{\theta}_i^k}}$ for $\tilde{S}_{\tilde{\theta}_i^k} \subset \bar{S}_{\bar{\theta}_i^k}$. Moreover, if PDG_i doesn't get an allocation then submitting all its $\bar{\theta}_i^k$ bids will not make its probability of getting an allocation worse. Thus, $p_i^k(\tilde{S}_{\tilde{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k}) \leq p_i^k(\bar{S}_{\bar{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k})$.

In Proposition 1, we show that with WDP solved optimally as an allocation rule and bids of all other PDGs remaining unaltered, the probability of PDG_i getting an allocation is maximized when it submits the complete set of bids. Proposition 2 is a generalization of Proposition 1 where we show that expected probability of PDG_i getting an allocation is maximized when it submits its complete set of bids at kth iteration of ICA.

Proposition 2:

$$\bar{p}_i^k(\theta_i^k) \leq \bar{p}_i^k(\bar{\theta}_i^k) \;\; \forall \theta_i^k \leq \bar{\theta}_i^k \;\; \forall i \in I.$$

Proof. From Proposition 1, $\forall i \in I$ we get:

$$p_i^k(\tilde{S}_{\tilde{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k}) \leq p_i^k(\bar{S}_{\bar{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k}) \tag{61}$$

$$\Rightarrow \sum_{\tilde{S}_{\theta_i^k} \in \bar{S}_{\theta_i^k}} p_i^k(\tilde{S}_{\tilde{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k}) g_i^k(\tilde{S}_{\theta_i^k}) \leq \sum_{\tilde{S}_{\theta_i^k} \in \bar{S}_{\theta_i^k}} p_i^k(\bar{S}_{\bar{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k}) g_i^k(\tilde{S}_{\theta_i^k}) \tag{62}$$

$$\Rightarrow \sum_{\tilde{S}_{\theta_i^k} \in \bar{S}_{\theta_i^k}} p_i^k(\tilde{S}_{\bar{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k}) g_i^k(\tilde{S}_{\theta_i^k}) \le p_i^k(\bar{S}_{\bar{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k}) \sum_{\tilde{S}_{\theta_i^k} \in \bar{S}_{\theta_i^k}} g_i^k(\tilde{S}_{\theta_i^k}) \tag{63}$$

$$\Rightarrow \mathbb{E}_{\tilde{S}_{\theta_i^k}}\left(p_i^k(\tilde{S}_{\bar{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k})\right) \le p_i^k(\bar{S}_{\bar{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k}) \tag{64}$$

$$\Rightarrow \bar{p}_i^k(\theta_i^k) = \mathbb{E}_{\bar{S}_{\bar{\theta}_{-i}^k}}\left(\mathbb{E}_{\tilde{S}_{\theta_i^k}}\left(p(\tilde{S}_{\bar{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k})\right)\right) \le \mathbb{E}_{\bar{S}_{\overline{\theta^k}_{-i}}}\left(p_i^k(\bar{S}_{\bar{\theta}_i^k}, \bar{S}_{\bar{\theta}_{-i}^k})\right) = \bar{p}_i^k(\bar{\theta}_i^k) \tag{65}$$

$$\Rightarrow \bar{p}_i^k(\theta_i^k) \le \bar{p}_i^k(\bar{\theta}_i^k) \ \forall \theta_i^k \le \bar{\theta}_i^k. \tag{66}$$

Equation (64) follows from the observation that $\sum_{\tilde{S}_{\theta_i^k} \in \bar{S}_{\theta_i^k}} g_i^k(\tilde{S}_{\theta_i^k}) = 1$ and the second equality in Equation (65), because $\bar{\theta}_i^k$ consists of only one set of bids.

Definition 1. Bayesian Incentive Compatibility of CG-ICA (Mas-Colell *et al.*, 1995): CG-ICA with WDP solved optimally as an allocation rule is said to be Bayesian Incentive Compatible (BIC) if submitting their complete set of bids, i.e., revealing their true types is in Bayesian Nash Equilibrium for each PDG.

In other words, solving WDP optimally as an allocation rule in CG-ICA is BIC if it ensures that the expected utility of PDG_i is maximized when it submits its complete set of bids i.e. $\forall i = 1, \ldots, I$:

$$U_i(\bar{\theta}_i^k) = \bar{v}_i(\bar{\theta}_i^k)\bar{p}_i(\bar{\theta}_i^k) - \bar{t}_i(\bar{\theta}_i^k) \ge \bar{v}_i(\theta_i^k)\bar{p}_i(\theta_i^k) - \bar{t}_i(\theta_i^k) \ \forall \theta_i^k \le \bar{\theta}_i^k. \tag{67}$$

Moreover, an outcome in Bayesian Nash Equilibrium ensures that no PDG has an incentive to misrepresent its type as long as other PDGs submit their bids associated with their true types (Mas-Colell *et al.*, 1995). We now show that, under the assumption of myopic best response, the CG-ICA which solves WDP optimally as an allocation rule is BIC, if the proposed high penalty scheme is implemented. Myopic best response assumes that each PDG seeks to obtain the best result it can in the current iteration without considering the possibilities of future iterations. This is a reasonable assumption due to the complexity of the valuation problem by which the PDGs generate their bids.

Theorem 1. *If the penalty for not receiving an allocation in CG-ICA at termination is sufficiently large, specifically if*

$$M \ge \operatorname*{Max}_{\theta_i^k \le \bar{\theta}_i^k}\left\{\frac{\bar{v}_i(\theta_i^k)\bar{p}_i(\theta_i^k) - \bar{v}_i(\bar{\theta}_i^k)\bar{p}_i(\bar{\theta}_i^k)}{\bar{p}_i(\bar{\theta}_i^k) - \bar{p}_i(\theta_i^k)}\right\}, \tag{68}$$

then, under the assumptions of myopic best response, CG-ICA with WDP solved optimally as an allocation rule is BIC.

Proof. From (68) we get:

$$M \ge \frac{\bar{v}_i(\theta_i^k)\bar{p}_i(\theta_i^k) - \bar{v}_i(\bar{\theta}_i^k)\bar{p}_i(\bar{\theta}_i^k)}{\bar{p}_i(\bar{\theta}_i^k) - \bar{p}_i(\theta_i^k)} \forall \theta_i^k \le \bar{\theta}_i^k \tag{69}$$

$$\Rightarrow M\left(\bar{p}_i(\bar{\theta}_i^k) - \bar{p}_i(\theta_i^k)\right) \ge \bar{v}_i(\theta_i^k)\bar{p}_i(\theta_i^k) - \bar{v}_i(\bar{\theta}_i^k)\bar{p}_i(\bar{\theta}_i^k) \ \forall \theta_i^k \le \bar{\theta}_i^k \tag{70}$$

$$\Rightarrow \bar{v}_i(\bar{\theta}_i^k)\bar{p}_i(\bar{\theta}_i^k) + M\bar{p}_i(\bar{\theta}_i^k) \ge \bar{v}_i(\theta_i^k)\bar{p}_i(\theta_i^k) + M\bar{p}_i(\theta_i^k) \ \forall \theta_i^k \le \bar{\theta}_i^k \tag{71}$$

$$\Rightarrow \bar{v}_i(\bar{\theta}_i^k)\bar{p}_i(\bar{\theta}_i^k) + M\bar{p}_i(\bar{\theta}_i^k) - M \ge \bar{v}_i(\theta_i^k)\bar{p}_i(\theta_i^k) + M\bar{p}_i(\theta_i^k) - M \ \forall \theta_i^k \le \bar{\theta}_i^k \tag{72}$$

$$\Rightarrow \bar{v}_i(\bar{\theta}_i^k)\bar{p}_i(\bar{\theta}_i^k) - M\left(1 - \bar{p}_i(\bar{\theta}_i^k)\right) \ge \bar{v}_i(\theta_i^k)\bar{p}_i(\theta_i^k) - M\left(1 - \bar{p}_i(\theta_i^k)\right) \forall \theta_i^k \le \bar{\theta}_i^k \tag{73}$$

$$\Rightarrow \bar{v}_i(\bar{\theta}_i^k)\bar{p}_i(\bar{\theta}_i^k) - \bar{t}_i(\theta_i^k) \ge \bar{v}_i(\theta_i^k)\bar{p}_i(\theta_i^k) - \bar{t}_i(\bar{\theta}_i^k) \forall \theta_i^k \le \bar{\theta}_i^k, \tag{74}$$

which is the condition for BIC as per Equation (67) and we get Equation (73) as $\bar{p}_i^k(\theta_i^k) \le \bar{p}_i^k(\bar{\theta}_i^k) \ \forall \theta_i^k \le \bar{\theta}_i^k$ due to Equation (66). Thus, the ICA we have derived using CG with WDP ensures that under the assumptions of myopic best response, if each PDG faces very high penalty (at least stated in (68)) for not procuring an allocation at auction termination then, submitting its complete set of bids is its weakly dominant strategy given other PDGs truthfully submit their complete set of bids.