A Review of Methods for the Geometric Post-Processing of Topology Optimized Models

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Topology optimization (TO) has rapidly evolved from an academic exercise into an exciting discipline with numerous industrial applications. Various TO algorithms have been established, and several commercial TO software packages are now available. However, a major challenge in TO is the post-processing of the optimized models for downstream applications. Typically, optimal topologies generated by TO are faceted (triangulated) models, extracted from an underlying finite element mesh. These triangulated models are dense, poor quality, and lack feature/parametric control. This poses serious challenges to downstream applications such as prototyping/testing, design validation, and design exploration. One strategy to address this issue is to directly impose downstream requirements as constraints in the TO algorithm. However, this not only restricts the design space, it may even lead to TO failure. Separation of post-processing from TO is more robust and flexible. The objective of this paper is to provide a critical review of various post-processing methods and categorize them based both on targeted applications and underlying strategies. The paper concludes with unresolved challenges and future work. [DOI: 10.1115/1.4047429]

Keywords: topology optimization, computational geometry, computer aided design, design optimization, post-processing

1 Introduction

Various design optimization methods are used today to solve engineering problems; these include size, shape, and topology optimization. The focus of this paper is on topology optimization [1–3], which often serves as a starting point for size and shape optimization. Topology optimization (TO) has rapidly evolved from an academic exercise into an exciting discipline with numerous industrial applications. Popular applications include optimization of aerospace and aircraft components [4–9], automotive components [10–12], biomedical devices [13–17], structure design [18–22], compliant mechanisms [23–26], thermofluid applications [27–34], etc.

To illustrate the concepts behind TO, consider the structural problem posed in Fig. 1 where the objective is to find the stiffest topology, i.e., topology with the lowest compliance, within the given design-space for 50% volume fraction.

This can be solved rapidly today, via any of the well-known TO methods [35–44]. A typical optimized topology is illustrated in Fig. 2.

Rapid generation of such optimized designs is particularly beneficial during the early stages of the design process. However, one of the drawbacks of TO is that the optimal topology, such as the one in Fig. 2, is typically extracted as a *faceted (triangulated) model*, from the underlying finite element mesh, independent of any specific TO method. This extraction relies on classic isosurface methods such as marching cubes [45], see Fig. 3.

The faceted models are often of poor quality, non-smooth, dense and lack feature/parametric control. For example, the faceted model in Fig. 3 contains over 25,000 triangles, where most of them are of poor quality. This is often exacerbated in real-world problems. As an illustration, for the TO challenge problem for an upright design, posed during 2020 Topology Optimization Roundtable Conference, Albuquerque [46], millions of elements are necessary to capture critical features. This results in faceted model with millions of triangles (see Fig. 4). Such triangulated models are ill-suited for downstream applications such as prototyping/testing, design validation, and design exploration.

Most TO commercial packages do not have automated tools for post-processing. Post-processing is loosely defined here as the process of converting the faceted TO models into other geometric representations that are more suitable for various downstream applications. Such geometric representations include skeletal representation, simplified triangulated model, NURBS-representation, volume decomposition, and so on. Thus, post-processing strategies can range from simple remeshing to extraction of skeleton and fitting of analytic surfaces. Some of the early commercial packages relied on manual tracing of the TO model for reconstruction, i.e., the faceted models are superimposed over the design space, and the geometry is reconstructed via sketching and Boolean operations. This is laborious and error-prone. However, some commercial systems are beginning to support post-processing with various degree of success. The most common strategy used in commercial systems is surface-based reconstruction (see Sec. 4 for a description). PTC Creo® uses subdivision technique, while Evolve® and Rhino®, was MeshMixer® use Non-Uniform Rational B-Splines (NURBS)-based reconstruction. Fusion 360 Generative Design relies on T-splines to generate multiple watertight computer-aided-design (CAD) models that satisfy designer's requirements. None of these tools efficiently generate a parametric feature-based CAD

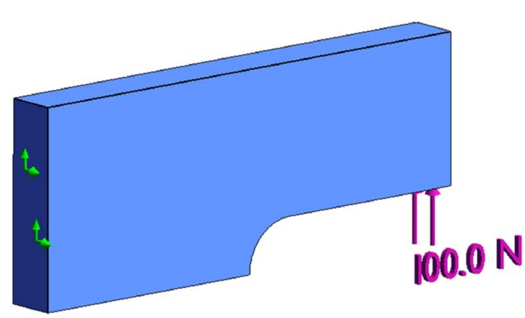

Fig. 1 A structural problem over a design space

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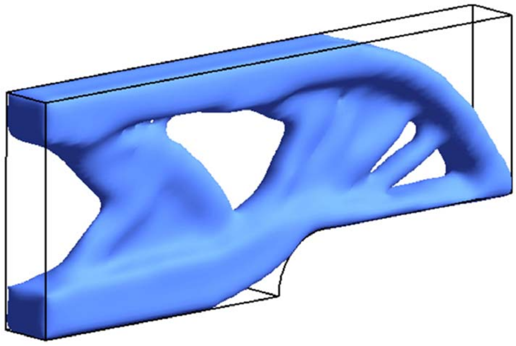


Fig. 2 An optimized topology

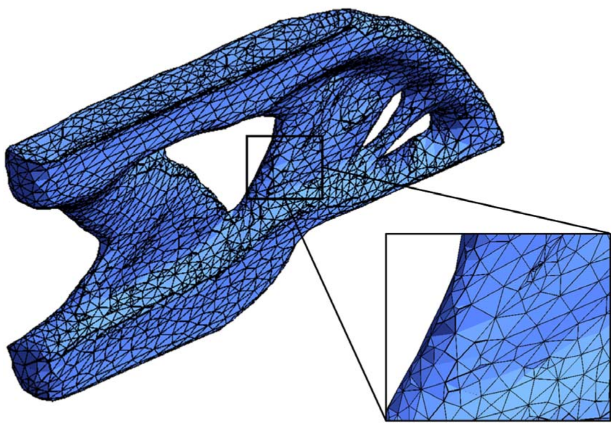


Fig. 3 The faceted representation with noisy and poor quality triangles

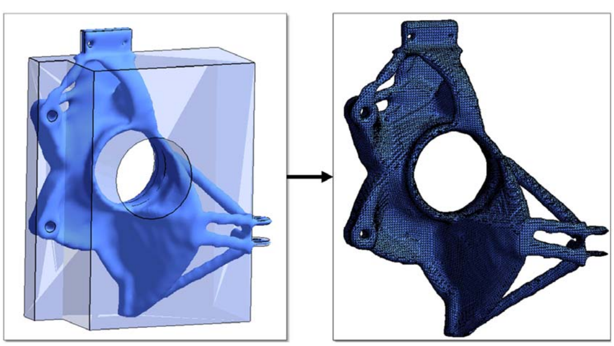


Fig. 4 A TO model with millions of triangles

model that meets all downstream requirements discussed in the subsequent section.

A survey was conducted among users of a free topology optimization service [47], sponsored by the National Science Foundation.[2] One of the questions posed to the users was: *Rank what would you like topology optimization software to include in order of preference?* Five specific choices were provided, with one open choice. Among the 85 responses received, 49% choose: *Generate feature-based CAD model of the optimized design*; see Fig. 5. Lack of automated tools for model reconstruction can be a serious detriment to broader acceptance and proliferation of TO.

Researchers have proposed several strategies and methods to address this challenge. Prior to discussing these strategies, we consider three important downstream applications in Sec. 2 and

[2]http://www.nsf.gov

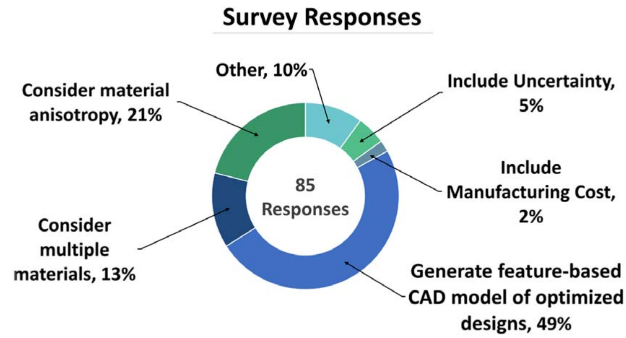


Fig. 5 Results from a survey of TO users

summarize their requirements. Then, in Sec. 3, we consider proposed methods that attempt to meet these downstream requirements by directly incorporating them as constraints in the TO algorithm. These direct methods, however, have limitations. In Sec. 4, we consider post-processing methods that rely on a combination of design rules and computational algorithms. For pedagogical reasons, these are further categorized based on the underlying dimension. Conclusions and future work are discussed in Sec. 5.

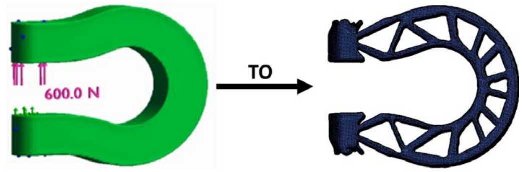
2 Downstream Applications

In this section, we consider three representative downstream applications, namely, *prototyping, validation and (design) exploration*, as illustrated in Table 1. These three applications are representative and not exhaustive. Further, since the requirements for these applications overlap, these are best represented via a Venn diagram as in Fig. 6. For example, "feature control" is essential for design exploration, but not necessary for validation and prototyping. However, "retaining critical features" is essential for all three applications. These requirements are further elaborated below and will be used later to evaluate different post-processing methods and strategies.

2.1 Prototyping. The simplest downstream application is prototyping and testing; the objective is to fabricate the TO model for testing, inspection, and evaluation. A primary requirement is that critical features, edges, and surfaces must be retained for repeatable testing. For example, if a load is applied on a cylindrical feature in the initial design, then this surface will be critical for prototyping and testing. Second, non-critical surfaces must be smooth, both for aesthetic and testing purposes. Finally, the recovered model must meet the constraints of the fabrication process. For example, for conventional milling, tool accessibility is important; for certain additive manufacturing processes, overhang surfaces must be avoided, and so on. However, parametric representation of the model, for example, is not critical for prototyping. For the topology optimized design of clevis from Table 1(*a*), a manufactured part with all critical features is shown in Table 1(*b*).

2.2 Design Validation. The second critical application is design validation where the TO model must be validated through analysis methods such as finite element analysis (FEA). FEA models used within TO are often vastly simplified, for example, they often rely non-conforming voxel mesh to accelerate FEA. To support rigorous FEA-based design validation, retaining critical features is once again important. In addition, one must be able to create a high-quality mesh that conforms to critical surfaces and features. This is more stringent than smoothness requirements for prototyping. Specifically, the recovered model should not contain sharp geometric features that could lead to erroneous simulation results. Finally, the reconstructed model must be functionally equivalent to the TO model in that the behavior of the reconstructed model

Table 1 Typical downstream applications of topology optimized designs

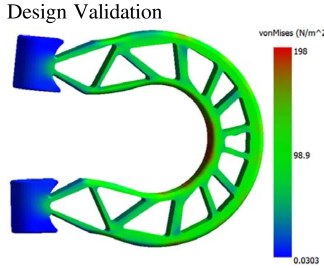


(*a*) *Topology Optimization of Clevis*

Prototyping and Testing

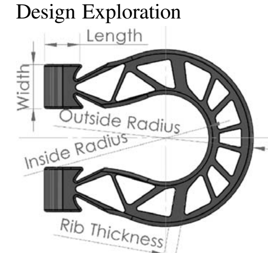
(*b*) *Manufactured Part*

Design Validation



(*c*) *Functionally equivalent design*

Design Exploration



(*d*) Parametric CAD Model

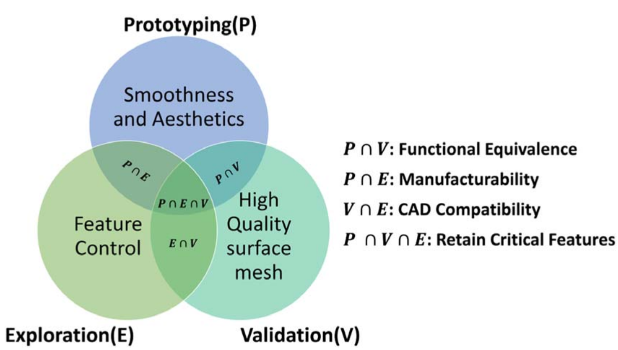


Fig. 6 Requirements of model for different downstream applications

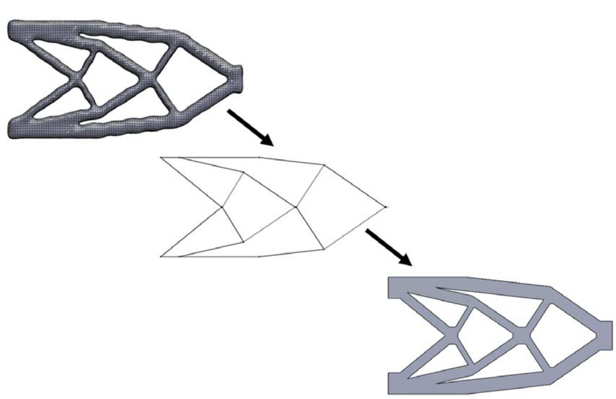


Fig. 7 Geometry reconstruction using skeleton

should not differ significantly from that of the TO model (see Table 1(*c*).

2.3 Design Exploration. The final application, and often the lofty goal, is design exploration and productization. This is the most demanding since the reconstructed model must be (easily) editable by the designer to meet various functional and manufacturing constraints. The model must allow parametric changes (for example: increasing thickness of a strut), suppression/inclusion of features and be compatible with popular CAD packages (see Table 1(*d*).

3 Constrained Optimization

Although the objective of this paper is to survey post-processing methods, we briefly review strategies for imposing downstream requirements directly as constraints within the TO algorithm. This serves two purposes: (1) if the downstream requirements are sufficiently simple, a constraint based TO may be sufficient and (2) to highlight the deficiencies of constraint-based strategies.

Researchers have largely focused on including prototyping and design exploration requirements in TO. We are not aware of strategies to incorporate validation/analysis requirements (for example, high-quality surface mesh) into TO. However, due to the overlap in requirements, many of the techniques discussed below can directly assist in efficient validation. The reader is referred to Ref. [48] for a broader discussion on constrained based TO.

3.1 Prototyping Constrained Topology Optimization. Researchers have proposed several methods to incorporate prototyping, i.e., manufacturing, constraints directly into TO to minimize post-processing. Harzheim and Graf [49,50] provide a review of early work on TO for cast parts. Liu and Ma [51] present a more recent survey on manufacturing focused TO. Zuo et al. [52] incorporated machining constraints, while Li et al. [53] imposed extrusion constraints, and Lui et al. [54] have explored symmetry and pattern repetition constraints in topology optimization. Li et al. [55] incorporated multidirectional molding constraints in TO for cast parts. Vatanabe et al. [56] incorporated constraints such as minimum size, symmetry, extrusion, turning, casting, forging, and rolling into the optimization.

Lui and Ma [57] performed least-square fitting of 2.5D and 3D machining-based features over the evolving boundary, while Groen and Sigmund [58] used homogenization method for generating manufacturable microstructure based designs. Amir et al. [59] proposed an approach for simultaneously satisfying physics based constraints (compliance, volume) as well as kinematics-based constraints (manufacturing, accessibility). There has been significant interest recently in incorporating additive manufacturing (AM) [60] constraints in TO [61]. Doutre et al. [62] compare existing state-of-art tools to obtain CAD models from TO, specifically for AM. Lui and To [63] have used feature fitting on the TO design for additive manufacturing. Leary et al. [64] identify boundaries that require supports in additive manufacturing; these boundaries were then modified to generate support-free structures. Amir and Suresh [65] used topological sensitivity to incorporate AM support structure constraints in TO. Similarly, Mass and Amir [66] and Garaigordobil et al. [67,68] incorporated overhang constraints.

A minimum-member size for additive manufacturing has been used as a constraint in TO by Kwok et al. [69]. Thin features and volume of support structures have been added as constraints by Mhapsekar et al. [70]. Qian [71] added undercut control and minimal overhang angle as constraints in SIMP-based TO.

Similarly, Mezzadri et al. [72] and Matthijs [73] designed self-supporting support structures using TO for additive manufacturing of parts. Chandrasekhar et al. [74] proposed a methodology to incorporate build direction and fiber orientation into a TO formulation for short fiber-reinforced polymers components. Stuben et al. [75] use multiscale TO to generate 2D designs for additive manufacturing. See the work by Lui et al. [76] for an extensive review on TO for AM.

3.2 Design Exploration Constrained Topology Optimization. Next, we consider strategies to include design exploration requirements into TO. Bendsoe and Rodrigues [77] explored the idea of using TO models as a precursor to shape optimization in 2D. The study by Olhoff et al. [78] was one of the earliest to propose CAD-integrated TO to reduce design lead time. Zhou and Wang [79] combined CSG with topology/shape optimization to generate free-form geometric designs. Chen et al. [80] proposed a B-spline-based method for combined shape and topology optimization. Tang and Chang [81] presented an integrated approach to combine topology optimization and shape optimization using B-splines to represent the boundaries. Lin and Chao [82] used image processing to convert the gray-scale results of TO to obtain a parametric geometry in 2D. Zhang and Kwok [83] performed TO over a parametrized 2D mesh obtained by mapping a 3D domain onto a 2D domain. The optimized results are then mapped back to obtain a 3D geometry. Similarly, Christiansen et al. [84] combined shape and topology optimization for 3D structures using explicit shape representation.

Another popular strategy to support design exploration is to directly incorporate design features during TO. Guo et al. [85] and Zhang et al. [86,87] have used moving morphable components to represent the boundaries of TO designs. The size, shape, and orientations of these components are used as variables during topology optimization to generate designs with predefined features. Bell et al. [88] and Norato et al. [89] used parametrically defined bars, while Zhang et al. [86] used parametrically defined bars and plates to obtain TO designs. Lin et al. [90] used NURBS to represent the boundary of features arising during TO. Holes represented by NURBS are inserted in the design domain, and their control points are used as design variables to generate parametrically defined TO geometry. Gao et al. [91] replaced discrete density field by NURBS and then imposed user defined geometric constraints during topology optimization of beams and plates. Zhang et al. [92] traced the topological changes in the geometry using B-Splines to construct free-form shapes. Norato [93] used union of 2D super-shapes to generate free-form geometry.

Da et al. [94] used bi-directional evolutionary structural optimization (BESO) with level set function to generate results with smooth boundaries. Jahangiry and Tavakkoli [95], Kang and Youn [96], Seo et al. [97], and more recently, Gai et al. [98] have used spline-based isogeometric analysis for Topology Optimization. Gao et al. [99] have used density distribution function (DDF) for isogeometric topology optimization to obtain smooth NURBS surface in 2D and 3D.

More recently, machine learning algorithms have been applied toward post-processing of TO models. For example, Sosnovik and Oseledets [100] trained their neural network using image segmentation to obtain final designs from intermediate results of TO, thereby reducing the computational effort. Shen and Chen [101] and Rawat and Shen [102] proposed a conditional generative adversarial network (GAN) to incorporate design constraints such as minimum radius in TO of planar structures. Lei et al. [103] used support vector regression (SVR) and K-nearest-neighbor (KNN) models to predict topology optimized designs.

3.3 Benefits and Limitations. Adding downstream constraints directly into TO eliminate the need for expensive post-processing. Indeed, this may be a practical and viable option in simple scenarios. However, there are several limitations to these strategies:

(1) *Reduced design space:* Adding constraints necessarily reduces the design space, and consequently, the performance of the optimized design.
(2) *Computational challenge:* Adding constraints can significantly increase the cost of TO; furthermore, the optimization may even fail if improper constraints are imposed.
(3) *Lack of generality:* The strategies are often limited in scope; for example, the extension of feature-based strategy to 3D is an open challenge, and not all manufacturing processes can be imposed as a constraint. Further, most methods involve manual intervention and expertise to generate the CAD geometry.
(4) *Lack of flexibility:* Finally, since constraint-based strategies often target a particular application, exploring other options is often not viable once the optimization is complete.

Thus, one must resort to post-processing of TO models, and this is discussed next.

4 Post-processing Strategies

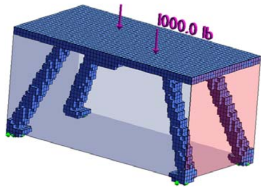
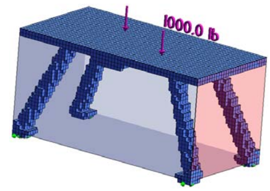
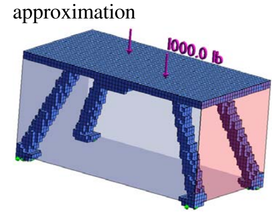
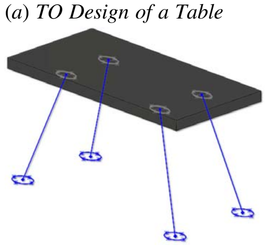
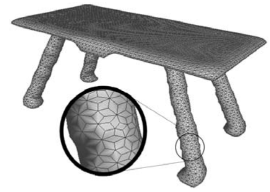
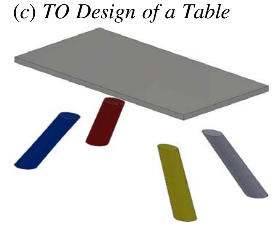
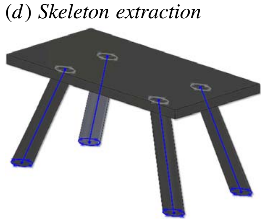
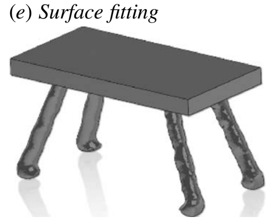
As one can expect, different post-processing strategies fulfill different requirements. For example, if the downstream application is finite element analysis, then post-processing the surface mesh, while imposing geometric and quality constraints may be sufficient. On the other hand, for design exploration, recreating a CAD-compatible parametric model will be necessary and so on.

Post-processing strategies can be classified based on the underlying dimension as in Table 2. Specifically, if the post-processing is based on first extracting a lower-dimensional skeleton, it is classified as 1D. If the strategy relies directly on post-processing the triangulated surface, it is classified as 2D. Finally, if the strategy relies on volume decomposition of the TO model, it is classified as 3D. Similar classification strategies have been proposed by Fabio et al. [104] for reconstruction of geometry from cloud data points and by Thakur et al. [105] for CAD model simplification.

4.1 Skeleton Based (One-Dimensional). As stated earlier, skeleton-based post-processing is largely limited to thin beam-like TO designs (see Table 2(*a*,*d*,*g*)). In addition, two recurring challenges here are as follows: (1) robust handling of junctions where skeletal branches meet and (2) extraction of cross sections.

4.2 Surface Based (Two-Dimensional). The second, and probably the most common, category of post-processing is surface reconstruction. There are three fundamentally different surface-based methods: *remeshing, sub-division, and surface-fitting*. In remeshing, one directly creates an improved triangulation from TO triangulation. In sub-division, a predefined set of rules are used to recreate a discretized surface (triangles and quads) that best fits the original surface. Finally, in surface-fitting, the triangulation is replaced by a parametric surface (such as NURBS) or analytical surface (such as a cylinder). A typical example demonstrating the

Table 2 Proposed classification of geometric post-processing techniques

Classification	Skeletal (1D)	Surface (2D)	Volume (3D)
Underlying Technique	Reconstruction via skeleton	Surface fitting and/or mesh simplification	Volume decomposition and approximation
Reconstruction process	 (*a*) *TO Design of a Table*	 (*b*) *TO Design of a Table*	 (*c*) *TO Design of a Table*
	 (*d*) *Skeleton extraction*	 (*e*) *Surface fitting*	 (*f*) *Volume Decomposition*
	 (*g*) *Sweep cross-section for geometry reconstruction*	 (*h*) *Remesh surface for geometry reconstruction*	 (*i*) *Boolean addition of decomposed volumes*
Strengths	Well suited for beamlike models Applicable to all downstream applications	Relies on popular remeshing methods Applicable to all TO models	Ideal for suppressing small features. Easy to retain critical features
Weaknesses	Handling of junctures	Stitching of gaps, and retaining sharp features	Not suited for complex TO models
	NO suitable for all TO models	Automation	Automation

2D-based approach of geometric post-processing has been shown in Table 2(*b*,*e*,*h*).

4.2.1 Remeshing. Remeshing creates an improved triangulation from a potentially noisy triangulation or sampled (scanned) data [106]. There are two popular methods of remeshing: *implicit and explicit*, and there are several implementations; for example, see PMP [107] and Instant-Meshes [108]. *Implicit remeshing* methods rely on constructing a smooth scalar field from the input triangulation; the scalar field is then used to recreate a high-quality re-triangulation. For example, Kazdhan et al. [109] proposed the Poisson reconstruction method to generate water-tight meshes.

Implicit methods often result in undesirable smoothening of sharp edges. Attene et al. [110] proposed an edge-sharpener algorithm while Nielson [111] used dual marching cubes to recover shape features from the triangulated models. Thomos et al. [112] modified marching cubes tables for topological guarantees. Although implicit methods are robust, numerically stable, and generate water-tight models, they can be computationally expensive and are non-local, i.e., small defects in one region can affect the triangulation globally.

Explicit remeshing methods often rely on Delaunay triangulation of point data [113,114]. Dey and Goswami [115] proposed a water-tight remeshing algorithm. Explicit methods are local and easy to implement but are less stable [116]. Figure 8 illustrates

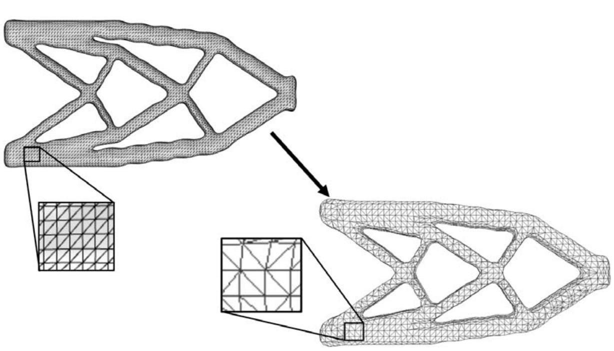


Fig. 8 Remeshing of triangular meshes using screened Poisson surface reconstruction

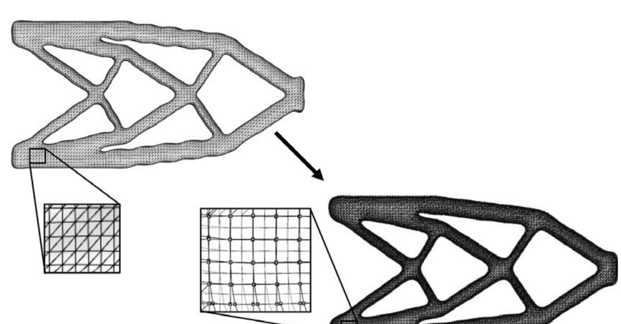


Fig. 9 NURBS surface fitting with control points

remeshing of triangulated surface into a triangular/quad mesh. This reconstruction was performed using Poisson surface reconstruction [109] implemented in Meshlab® v2016.12; the processed geometry is smoother and contains a fewer number of triangles/quads.

4.2.2 Fitting. The objective of surface fitting is to replace the triangulation with either analytical primitives such as planes, spheres, cylinders, etc., or parametric surfaces such as NURBS. The techniques discussed below are often used in the context of scanned data [117] but directly apply to TO post-processing (especially parametric surface fitting). Figure 9 demonstrates smoothing and fitting of the TO model using NURBS. The fitting was performed using Rhino® 6, released in February 2018. Control points generated through surface fitting provide local control over the surface.

Fitting primitives only applies when the underlying surface is analytic. Several methods have been proposed to fit analytic surfaces. Yi and Kim [118] fit basic geometric features such as lines, arcs, circles, fillets, extrusion, and sweep on boundary extracted from a topology optimized design. Reference [119] proposed Globfit algorithm to recover a set of locally fitted primitives. Schnabel et al. [120] proposed an efficient RANSAC algorithm to recover analytic shapes from noisy input models.

In parametric surface fitting, NURBS are often used to fit the triangulation. Joshi et al. [121] created an open source tool that fits a NURBS surface over the mesh using least square fitting. Non-design features are then added manually to the resulting surfaces. Continuity between multiple patches was not discussed. Lui et al. [122] used adaptive B-spline fitting of the surface. The resulting geometry is a smooth parametric model suitable for further shape optimization and targeted for additive manufacturing. Chacon et al. [123] developed a software tool that fits B-Splines on the boundaries of 2D Topology optimized designs and converts them to IGES format for CAD compatibility.

Koguchi and Kikuchi [124] used marching cube based iso-surface extraction algorithm to construct biquartic surface splines. The parametric model preserves all critical features such as flat surfaces and sharp edges. The resulting geometries require further processing to make them manufacturable.

Marsan and Dutta [125] extracted smooth contours layer-by-layer. These contours are then used to fit spline surfaces with C1 continuity. This method works for post-processing of models with holes/branches, but it fails to retain critical features and surfaces. Yoely et al. [126] use B-splines to represent the boundaries of topology optimized designs for generating parametric 2D geometries. Similarly, Zhang et al. [92] make use of closed B-Splines curves to trace optimum topology in 2D geometries.

A common challenge in surface fitting are gaps between surfaces. Various hole-filling approaches have been proposed. Zhao et al. [127] proposed an advancing front method. Branch et al. [128] used a local radial basis function to fill the space with B-spline surfaces. Curless et al. [129] used volumetric diffusion method to fill gaps. Liepa [130] combined remeshing and fairing method to smoothly bridge surface meshes.

4.2.3 Subdivision. Subdivision surfaces were introduced as an alternative to NURBS modeling. A subdivision surface is a representation of smooth surface over a piece wise linear polygon mesh similar to Bezier curve in 2D. A smooth surface is achieved by iterative subdivision scheme, defined by a set of rules. Geometry reconstruction based on subdivision surfaces is illustrated in Fig. 10 using PTC Creo® 6.0.1.0. The subdivision is semi-automated and the surface maintains connectivity with non-design features, while retaining critical surfaces and edges. Catmull-Clark subdivision [131] creates new vertex points using the face points and edge points. These new vertex points are then connected for each quadruple to create new face quadrilaterals. Though this method generates aesthetically pleasing surfaces, planar surfaces are often destroyed. Doo and Sabin [132] subdivision surfaces are created by replacing each vertex with face. The new faces created at the vertices are not necessarily planar. Few other subdivision-based surface generation methods include Loop [133], mid-edge subdivision [134]. Subdivision surfaces offer a high level of user control and can reproduce sharp edges and corners. Despite these advantages, maintaining second-order behavior near singularities is a major challenge for subdivision surfaces, and for complex shapes, it is almost impossible to remove mesh singularities.

Marinov et al. [135] recently used non-uniform rational Catmull-Clark (NURCC) surfaces [136] to convert generative design models to editable B-rep models. The triangular mesh is separated out from the non-design solids and is approximated via NURCC surfaces. Replacing triangular meshes with quad mesh makes it easier for local editing of shapes. Non-design solid geometries are then merged with the NURCC surfaces to construct watertight models. Although the authors use generative design, the same concept could be applied to TO models. This is a significant step toward the automated generation of parametric CAD geometry from TO in product design workflow.

4.3 Volume Based (Three-Dimensional). The primary idea in volume-based post-processing is to reconstruct the model through volume decomposition and Boolean operations. For example, Hsu an Hsu [137] and Shu et al. [138], extract

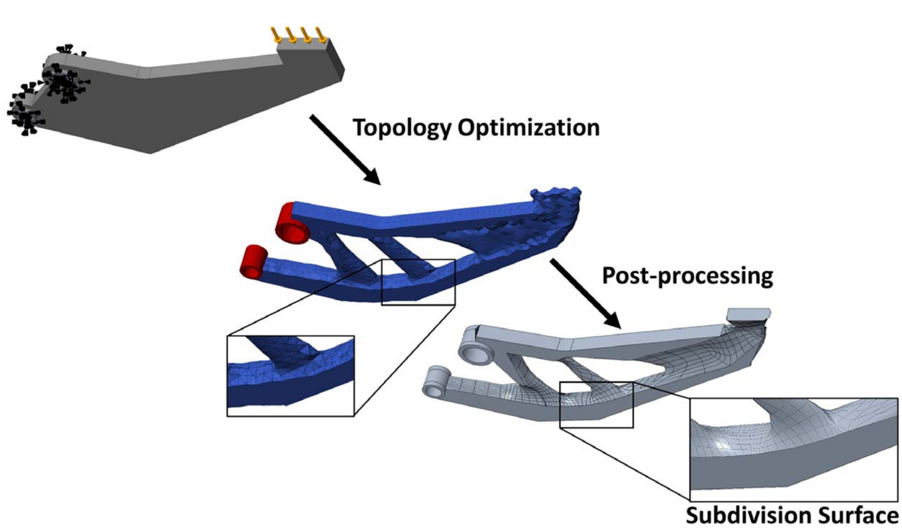


Fig. 10 Geometry reconstruction on TO design with sub-division

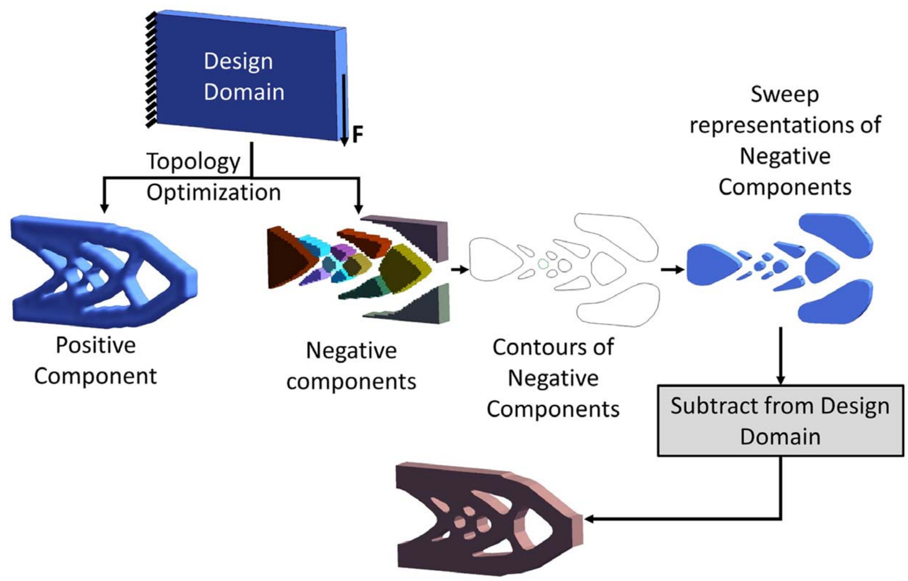


Fig. 11 A classical cantilever beam topology optimization problem with geometry reconstruction

representative cross sections from the topology optimized designs. The boundary points are used as control points to create B-spline boundary curves. Parametric 3D solids are created in a CAD using sweeps through these boundary curves. This method fails if there is a significant difference in the shape/topology between two successive boundary curves. Cuillière et al. [139,140] separate out the non-design features from the design domain. The optimized design is then merged with the non-design features to obtain the final geometry. This method retains critical features from the initial geometry. Connectivity between design and non-design features is a challenge since they are highly dependent on the mesh size. Furthermore, due the use of unstructured mesh, symmetry is lost in the optimized design. Larsen and Jensen [141] used 2D shape template fitting to create sweep geometries. These 3D solid bodies constructed using sweep are subtracted from the initial design domain. The algorithm requires manual intervention to fit different shapes. Recently, Du et al. [142] proposed InverseCSG algorithm to convert 3D models to CSG trees. An example demonstrating the 3D geometric reconstruction using volumetric decomposition has been shown in Table 2(*c*,*f*,*i*).

The methods discussed above work directly on the TO models. Alternately, one can also work with the voids (negative space) as illustrated in Fig. 11. This approach is preferable if the negative components are simpler to approximate than the full TO design. Furthermore, critical features can be easily retained. This post-processing strategy on topology optimized designs is currently being developed as a research tool within Pareto [40]. Volume-based methods are effective only if the TO design can be decomposed into simpler sweep-representable volumes. Furthermore, automatic identification of source/target profiles and sweep path is non-trivial.

5 Conclusions

Topology optimization continues to grow in importance and is being increasingly adopted by the industry to accelerate design. However, one of the roadblocks is the efficient and automated post-processing of topology optimized models for various downstream applications. In this paper, we identified three major applications and their requirements. For simple designs, it may be possible to include downstream requirements as constraints in topology optimization. However, in more complex scenarios, post-processing is unavoidable. Various post-processing strategies were reviewed and classified based on the implicit dimension.

It is evident that research gaps remain. In skeletal-based (1D) methods, computing the cross section, merging of skeletal branches, and handling of pathological cases require significant manual intervention. In addition, skeletal methods largely apply to tubular models.

Surface-based (2D) methods are the most advanced and promising. Among them, triangle-to-quad mesh conversion is the most popular since quad meshes are easier to edit. However, in practice, editing of quad-meshes requires carefully defined geometric constraints. Other challenges include presence of gaps between quad-patches and retaining critical features.

Volume-based methods(3D) require TO models to be decomposed to simpler disjoint volumes. While they offer unique advantages over the other two, we are not aware of robust implementations of 3D methods.

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Conflicts of Interest

There are no conflicts of interest.

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