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DRAFT: GRAPH PARTITIONING TECHNIQUE TO IDENTIFY PHYSICALLY INTEGRATED DESIGN CONCEPTS

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ABSTRACT

This study proposes a graph partitioning method to facilitate the idea of physical integration proposed in Axiomatic Design. According to the physical integration concept, the design features should be integrated into a single physical part or a few number of parts with the aim of reducing the information content, given that the independence of functional requirements is still satisfied. However, no specific method is suggested in the literature for determining the optimal degree of physical integration of a design artifact. This is particularly important with the current advancement in Additive Manufacturing technologies. Since additive manufacturing allows physical elements to be integrated, new methods are needed to help designers evaluate the impact of the physical integration on the design success. The objective of this paper is to develop a framework for determining the best way that functional requirements can be assigned to different parts of a product.

Keywords: Physical Integration, Axiomatic Design, Additive Manufacturing, Graph Partitioning

1. BACKGROUND

Axiomatic Design (AD) was developed by MIT mechanical engineering professor Num P. Suh in 1976 and was the first to coin the idea of independence of functional requirements. The primary focus of AD is on mapping the problem into several domains (e.g. customer domain, functional domain, physical domain, and process domain), to enable designers to check the axioms and select the best design solution [1]. The first step in designing a system is to define a set of FRs. The minimum set of independent requirements that the design should satisfy is considered the set of FRs. The next step is to map the set of FRs into the physical domain, or a set of Design Parameters (DPs). Once DPs are determined based on design embodiment principles, designers consider the process domain and identify the Process Variables (PVs). PVs often act as constraints in the system, since designers are not free to change the existing manufacturing processes [2].

Based on the philosophy that good designs share the same characteristics regardless of their physical nature or their domain of application, Suh attempted to root the engineering design

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process in two main axioms- (1) Independence Axiom and (2) Information Axiom. According to the independence axiom, FRs (which represent the goals of a design) must remain independent. To satisfy FRs, a set of DPs is chosen. According to the Independence axiom, DPs must be chosen such that the independence of FRs is maintained [3]. Based on the independence axiom, if one of DPs failed, not all functional requirements would be affected. The Independence axiom is based on the concept of changing multi-input/multi-output systems into a set of one-input/one-output systems to maintain the independence of FRs. The aim of information axiom is to minimize the complexity in the system or the information content [3].

What is important in AD is that the design derived from the mapping process must satisfy the Independence Axiom, meaning that the FRs should be satisfied independently with a set of DPs. AD uses design matrices to relate FRs to DPs and represents the design using a set of equations. What makes Axiomatic Design powerful is that it provides a quantitative approach to the formation of normative theories of design [4]. The relationship between the FRs and the DPs is characterized as follows:

$$\{FR\}=[A]\{DP\} \qquad (1)$$

Where each element of Matrix A, Aij, connects a component of the FR vector to a component of the DP vector [5]. The characteristics of design matrix A determine the degree in which the proposed design satisfies the Independence Axiom (Figure 1). For example, a diagonal matrix is an ideal matrix, where each FR is independently satisfied by one corresponding DP (uncoupled design). In the case of a full matrix (coupled design), the design violates the Independence Axiom since the change of any DP has an impact on all FRs. The independence axiom is particularly useful in the case of multi-objective optimization problems, due to the fact that each FR is independently satisfied by a set of design variables [6].

	Uncoupled design	Decoupled design	Coupled design
Design Matrix	$\begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}$	$\begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$	$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

Figure 1. Three different forms of design matrices

Since its origination in the late 1970s, Axiomatic Design has been the point of attention in the academic research, has been used widely across many disciplines, and has been taught internationally as part of engineering curricula [7][8]. In fact, Axiomatic design is known one of the most important engineering developments of the last century [9]. So far, 10 international conferences on Axiomatic Design have been held in countries around the world, with the last one in Xi'an, China, September 21-24, 2016. In addition to the field of engineering design, AD has impacted a wide range of practices in other disciplines including but not limited to: healthcare delivery systems [10], software design [11], production scheduling [12], manufacturing system design [13], supplier selection [14], interactive art [15], decision science [16], and additive manufacturing [17]. There are, however, several flaws in Axiomatic Design, including the point that there is no structured method available for generating design matrices based on the axioms and the two axioms do not sufficiently capture all that is needed in the design (e.g., human aspects of design [18], consumer preference, market demand [19], manufacturing considerations, and the potential to force a preference structure on designers [20]).

However, one main challenge about AD is that the concept of coupled design is very confusing to practitioners. Often designers believe that a simple design is a good design. From this belief, we may conclude that a coupled design in which one DP satisfies multiple FRs is preferred [5]. However, the Independence Axiom does not mean that the DPs must be independent nor that each DP must correspond to a separate physical part. For example, a bottle-can opener is designed to satisfy two FRs and has more than 10 DPs, but has only one piece (Figure 2). It should be noted that the concept of physical integration is completely different from modular design. Module is defined as a part or a group of parts that can be dismantled from the product in a non-destructive way as a unit [21][22]. Ishii et al. [23] have referred to modular design as minimizing the number of functions per part. According to Ulrich and Eppinger [24] the most modular design is one in which each function is implemented by exactly one module or subassembly and there are limited interactions between modules.

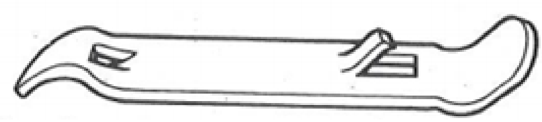


Figure 2. Bottle-can opener: An example of a physically integrated device that satisfies two functional requirements [25].

However, the idea behind *physical integration* or physical coupling is to integrate more than one FR in a single component, as long as FRs remain independent. Therefore, physical integration reduces the design complexity (at least in the physical domain). *While designers are in favor of physical integration, there is no normative approach on how to achieve physical integration using scientific engineering design techniques.*

Graph theory algorithms are widely used in making design decisions [27,28,29,30,31,32]. In this paper we have used a graph partitioning approach to solve the design problem. Researchers have adopted this method to arrive at design conclusions [33,34].

The objective of this paper is to provide some background information about the concept of physical integration and open a new venue for determining the best level of physical integration, particularly for additively manufactured parts that have less manufacturing constraints in terms of geometry and shape. The graph theory helps us in finding the best pair of FRs that can be combined to achieve a more feasible design.

2. PROPOSED METHOD: GRAPH PARTITIONING APPROACH

We can correspond each of the functional requirements with a vertex. Two functional requirements are adjacent in the graph, if they have common design parameters. Each edge between two functional requirements can be labeled by a number (say a number between 1 and 10), where the value of the number implies how strong the desire to have those two functional requirements in different parts.

Now suppose we aim to design a product with a fixed number of parts. We want to see what the optimal way for us is to partition the functional requirements into different parts. Here is its graph theory approach.

Let G be a graph whose edges have weights in {1,..., 10}, i.e. there exists a function w: $E(G) \rightarrow \{1,\ldots,10\}$. For a partition P = (V1,...,Vk) of the vertex set of G, we define the penalty of P to be the sum of the weights of bad edges, where bad edges are the edges within a part. $Penalty\ (G,P) = \sum_{e\ is\ an\ edge\ inside\ a\ part} w(e)$. We aim to find the best way we can partition a vertex set of G into k parts in such a way that the penalty is as small as possible.

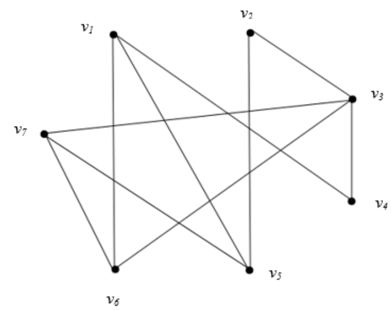
To do this, we first randomly partition V(G) into k parts, say V1,…,Vk. Now we apply the following algorithm until it stops. Let V(G)={v1,…,vn}. Start from vertex v1. At each step look at the vertex vi. Let Sj(vi) be the summation of the weights of the edges that join vi to the part Vj. Suppose minj{Sj(vi)} = Sk(vi). If the vertex vi does not belong to Vk, relocate vi to part Vk and start the algorithm again. Otherwise continue to vertex vi+1. The algorithm stops when vn belongs *to* the part with minimum Sj(vn).

3. NUMERICAL EXAMPLE

In this section, first we provide a numerical example to show the way a design matrix or its graph equivalent is partitioned to determine the assignment of functional requirements (edges or nodes of the graph) to a fix number of parts.

Figure 3 provides a numerical example in which we attempt to optimally partition seven functional requirements into three parts.

The graph shown in Figure 3 is an illustration of the theory. The purpose is to partition seven functional requirements represented as the edges of the graph into 3 parts V_1, V_2 &V_3. The $w(v_iv_j)$ defined several constraints or preferences with respect to these six edges.



$w(v_1v_6) = 1$
$w(v_1v_4) = 2$
$w(v_3v_7) = 4$
$w(v_5v_7) = 5$
$w(v_6v_7) = 7$
$w(v_2v_5) = 3$
$w(v_3v_4) = 9$
$w(v_2v_3) = 2$
$w(v_1v_5) = 8$
$w(v_3v_6) = 4$

Figure 3. Initial graph G (the equivalent of design matrix) and the corresponding weight of each arc

The scores represent the relation between the corresponding functional requirements. Higher scores represent that functional requirements are independent of each other and cannot be integrated. In this particular example, the above-mentioned values in Figure 3 are assigned randomly to check the algorithm. In Section 4, a practical design is discussed to obtain the best possible design integration considering a set of requirements and constraints.

Figure 4 shows six different steps of the algorithm and Table 1 lists the summation of weights for the first step. Similarly, the same approach is implemented for the remaining 5 steps.

Each functional parameter is assigned to the part based on the summation of weights. The value of zero indicates that the FR can remain in that part. The higher the value of the summation of weights, they are moved to the other parts and the algorithm is run again. This is repeated till we obtain a minimum summation value for all the parts.

Table 1 shows the value for the first four iterations. As shown, the first FR v_1 is assigned to part V1. If you look at v_2 , It has a value of 2 in V1 and 0 in V2, so we prefer to have it in part V2. Similarly, we obtain the lowest value for every FR and finally integrate them to the part with the lowest summation value.

Table 1. Steps to assign FRs to parts by summation of weights

$S_1(v_1)$=0	$S_1(v_3)$= 2
$S_2(v_1)$=2	$S_2(v_3)$= 9
$S_3(v_1)$=9	$S_3(v_3)$= 8
So v_1 remains in V_1.	So we move v_3 to V_1
$S_1(v_2)$= 0	$S_1(v_2)$= 2
$S_2(v_2)$= 2	$S_2(v_2)$= 0
$S_3(v_2)$= 3	$S_3(v_2)$= 3
So v_2 remains in V_1	So v_2 remains in V_2

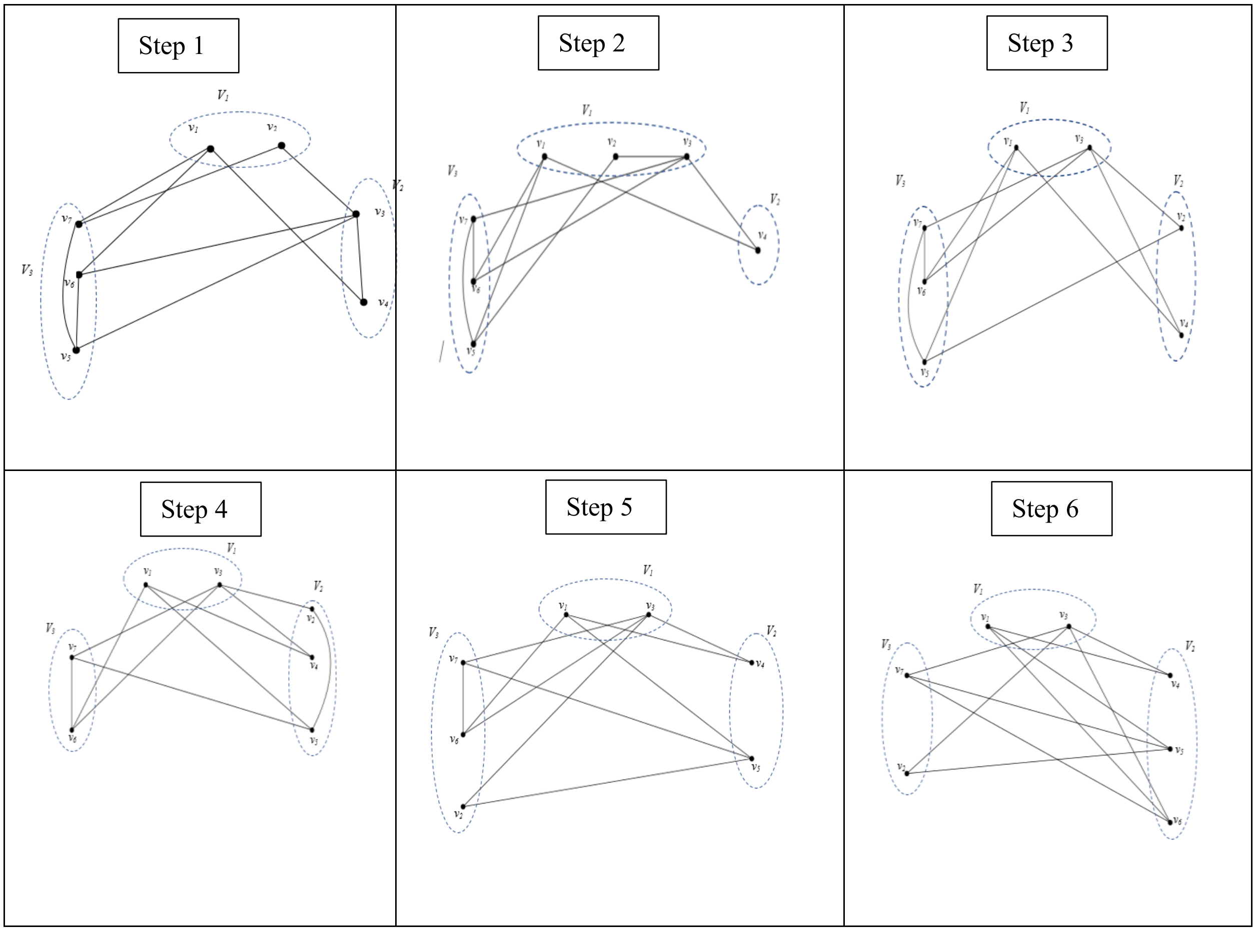


Figure 4. Steps involved in optimal partitioning of the FRs among parts.

4. EXAMPLE OF PENCILE DESIGN

This section explains an example of a mechanical pencil. First, the FRs and the DPs of the pencil are defined. DSM matrix is formed for the proposed design.

Figure 5 illustrates the different parts of a mechanical pencil. It mainly consists of a body, lead reservoir tube, eraser, and lead sleeve [35]. It has 8 parts assembled together to get the product.

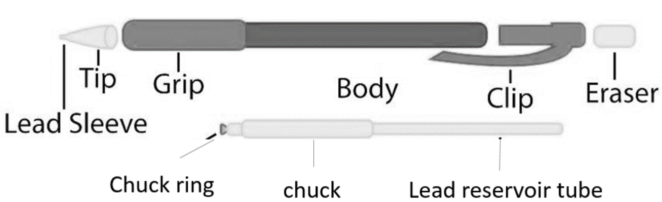


Figure 5. Currently used design for a Mechanical Pencil [35].

The following functional requirements are defined for the mechanical pencil:

FR1- Erasing
FR2- Lead storage
FR3 – Eraser storage
FR4- Advance lead
FR5- Support lead while using
FR6- Position lead in place (lead sleeve)
FR7- Grip for comfort.
FR8- Clip to hold.
FR9- Body must accommodate all parts

Next, in order to explore the design concept, the design parameters need to be defined.

DP1- Eraser
DP2- Opening for eraser
DP3-Cylinder with stopper
DP4- Spring lead advancer for the lead movement(spring)
DP5- Chuck to hold lead
DP6- External grip

DP7-Chuck opening to accomdate lead of different size(chuck ring)
DP8- Clip design(integrated to push button)
DP9- Body geometry

The design matrix [A] in [FR]=[A][DP] shows the relations between the given set of functional requirements and the design parameters. The design matrix for the mechanical pencil is defined in Equation 2.

$$\begin{pmatrix} FR1 \\ FR2 \\ FR3 \\ FR4 \\ FR5 \\ FR6 \\ FR7 \\ FR8 \\ FR9 \end{pmatrix} = \begin{pmatrix} X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & X & X & 0 & 0 & 0 & 0 & 0 & 0 \\ X & X & X & X & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & X & X & 0 & X & 0 & 0 \\ 0 & 0 & 0 & X & X & 0 & X & 0 & 0 \\ 0 & 0 & 0 & 0 & X & 0 & X & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & X & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & X & 0 \\ X & X & 0 & 0 & 0 & X & 0 & X & X \end{pmatrix} \begin{pmatrix} DP1 \\ DP2 \\ DP3 \\ DP4 \\ DP5 \\ DP6 \\ DP7 \\ DP8 \\ DP9 \end{pmatrix} \quad (2)$$

If we look at FR4, advancement of the lead, the spring (DP4), chuck (DP5), and chuck ring (DP7) work together when we push the button, the lead is transferred from the lead reservoir tube through the lead sleeve. When the push button is pressed, the chuck goes past the chuck ring and the lead falls through, and when the button is released the jaws close to hold the lead. They retract back into the chuck ring thereby holding the lead in place. Similarly, FR6, the position of lead is dependent on chuck design (DP5) and chuck ring (DP7).

The DSM indicates the design is either uncoupled or decoupled. The current design does not satisfy independence axiom; each individual functional requirement is not satisfied by fully independent physical components or subsystems. A decoupled or uncoupled design for the mechanical pencil is essentially difficult to achieve, as many of the design parameters are reused for multiple functions. So make it satisfy the independence axiom, we have to integrate design features in a single physical part if FRs can be independently satisfied in the proposed solution [36].

Per the concept of physical integration, the objective is to satisfy the list of functional requirements with minimum number of parts. We will look at two cases in this section. Case 1 in which FRs are assigned to 5 parts and Case 2 in which FRs are assigned to 4 parts.

4.1. Case 1: 5-part Design

In this case we are integrating the design to satisfy two or more FRs with a single physical part. So this design is reduced into 5 parts by the concept of physical integration by clubbing the functional requirements together.

We have a strong desire to have FR1 (eraser), FR6 (sleeve), and FR8 (clip) in separate parts in our design. Therefore, when modeling the corresponding graph, we make vertices FR1, FR6, and FR8 adjacent to all other vertices of the graph.

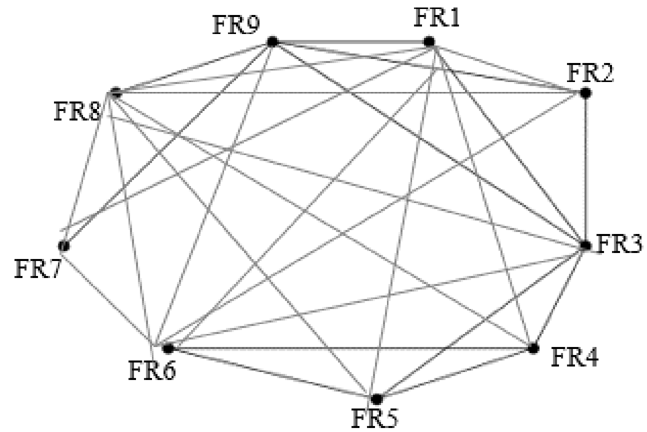


Figure 6: The initial graph of design matrix

Let us assume the initial parts be {FR1}, {FR6}, {FR8}, {FR2, FR3, FR4}, {FR5, FR7, FR9}.

Table 2: the weight of each arc considered for the graph of Figure 6 based on technical constraints and manufacturability requirements

w(FR1-FR3) = 10	*w*(FR1-FR5) = 10
w(FR1-FR9) = 10	*w*(FR1-FR6) = 10
w(FR2-FR3) = 2	*w*(FR1-FR7) = 10
w(FR2-FR9) = 5	*w*(FR1-FR8) = 10
w(FR3-FR4) = 1	*w*(FR6-FR2) = 10
w(FR3-FR5) = 2	*w*(FR6-FR3) = 10
w(FR3-FR9) = 10	*w*(FR6-FR7) = 10
w(FR4-FR5) = 2	*w*(FR6-FR8) = 10
w(FR4-FR6) = 10	*w*(FR6-FR9) = 10
w(FR5-FR6) = 10	*w*(FR8-FR2) = 10
w(FR7-FR9) = 5	*w*(FR8-FR3) = 10
w(FR8-FR9) = 10	*w*(FR8-FR4) = 10
w(FR1-FR2) = 10	*w*(FR8-FR5) = 10
w(FR1-FR4) = 10	*w*(FR8-FR7) = 10

In the above table we have labeled all edge incident to FR1, FR6, FR8 by 10. This represent the technical constraints that designer often have. Now let's apply the algorithm. We randomly partition the vertices of the graph into 5 parts.

After applying the algorithm, the final answer obtained {FR1}, {FR6}, {FR8}, {FR2, FR3, FR4, FR7}, {FR5, FR7, FR9}.

In this case, the penalty is not zero like the numerical example. The minimum penalty is 2+1=3, because FR2, and FR3 are in the same parts and FR3 and FR4 are in the same parts. The algorithm suggested FR7 to be integrated either with FR2, FR3 & FR4 or with FR5 & FR9.

Based on the algorithm results, it seems that several FRs are grouped and can be put in same parts. One example such proposed 5-part design is illustrated in Figure 7. The FRs and DPs are also integrated to form a design matrix with 5 parameters. The Chuck, chuck ring, spring and the lead reservoir tube are integrated into a single part. This satisfies FR2 (lead storage), FR4 (advance lead), FR5 (support lead while using). The body design is integrated together as one, it has the opening to accommodate eraser, it integrates the clip and the grip with the body thereby satisfying FR3 (eraser storage), FR7 (grip for comfort), FR8 (clip to hold). The other two parts are tip with lead sleeve and eraser.

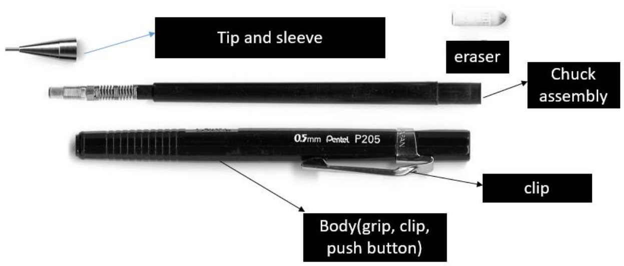


Figure 7: 5-part design-modified from [38]

The functional requirement for this particular mechanical pencil can be summarized as follow:

FR1- Erasing
FR2- Lead chuck
FR21- lead storage
FR22-Advance lead
FR23- Support lead while using
FR3 – Body
FR31- grip for comfort
FR32-Eraser storage
FR4- Position lead in place (lead sleeve)
FR5-Clip to hold.

Further to explore the design concept the design parameters need to be defined.

DP1- Eraser
DP2-Body design
DP21- Opening for eraser
DP22-External grip
DP3-Chuck design
DP31-Cylinder with stopper
DP32- Spring lead advancer for the lead movement(sprinng)
DP33-Chuck opening to accomdate lead of different size(chuck ring)
DP4-Lead sleeve design
DP5-Clip design

The new design matrix is as follow:

$$\begin{pmatrix} FR1 \\ FR2 \\ FR3 \\ FR4 \\ FR5 \end{pmatrix} = \begin{pmatrix} X & 0 & 0 & 0 & 0 \\ 0 & 0 & X & X & 0 \\ 0 & X & 0 & 0 & 0 \\ 0 & 0 & X & X & 0 \\ 0 & 0 & 0 & 0 & X \end{pmatrix} \begin{pmatrix} DP1 \\ DP2 \\ DP3 \\ DP4 \\ DP5 \end{pmatrix} \quad (3)$$

We should note that we do not necessarly need to redefine the design matrix and the original design matrix with 9 FRs can be used in the analysis.

4.2. Case 2: 4-part Design

Now, consider a different set of requirements. Suppose that designers are interested in designing a four-part product in which FR1 must be in a separate part in our design due to some technical constraints (e.g. material selection, shared design parameters). Therefore, when modeling the corresponding graph, we make vertex FR1 adjacent to all other vertices of the graph. Figure 8 explains all conditions that we want to consider for this case. Since we have the strong desire that FR1 be in a separate part, we label all edge incident to FR1 by 10. Our aim here is to design this product in such a way that it has 4 parts. Now let's apply the algorithm. We randomly partition the vertices of the graph into 4 parts.

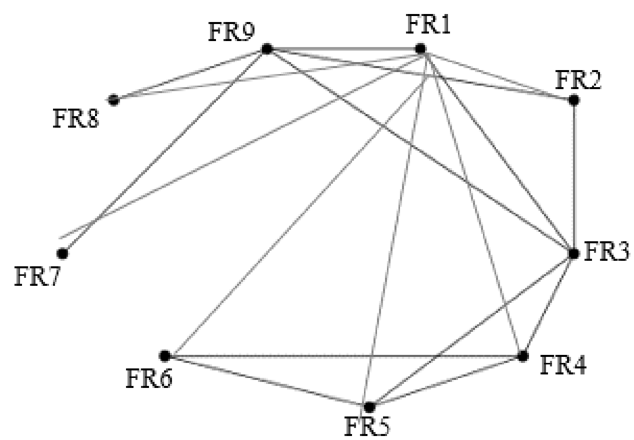


Figure 8: Initial graph for the proposed design

So let the initial parts be {FR1}, {FR2, FR5}, {FR3, FR4}, {FR6, FR7, FR8, FR9}. This design consideration was proposed from an additive manufacturing perspective. These factors were adopted from a logical perspective to reduce the number of parts. The idea of this consideration was based on a design proposed by Scott [37] who successfully printed a 3D pencil with four moving parts. His design can be printed using white polypropylene material with a water soluble gel-like support structure. He also suggested the pencil body must have a geometry with lot of holes to facilitate easy removal of material from the support structure in a 3D printer.

After applying the algorithm, the final answer will be {FR1}, {FR3, FR6, FR7, FR8}, {FR2, FR5}, and {FR4, FR9}. we can integrate multiple FRs and can be printed. So this design considers the FR1 (erasing) as a separate part and rest can be integrated in any way.

Table 4: weight of each arc considered for the graph in Figure 8

w(FR1-FR3) = 10	*w*(FR5-FR6) = 2
w(FR1-FR9) = 10	*w*(FR7-FR9) = 5
w(FR2-FR3) = 2	*w*(FR8-FR9) = 2
w(FR2-FR9) = 5	*w*(FR1-FR2) = 10
w(FR3-FR4) = 1	*w*(FR1-FR4) = 10
w(FR3-FR5) = 2	*w*(FR1-FR5) = 10
w(FR3-FR9) = 10	*w*(FR1-FR6) = 10
w(FR4-FR5) = 2	*w*(FR1-FR7) = 10
w(FR4-FR6) = 1	*w*(FR1-FR8) = 10

In Table 4, we have labeled all edge incident to FR1 by 10 since we want to include FR 1 as a separate part. In this particular example, FR 1 can be excluded from analysis as well, however we have included in as inputs to the algorithm to show the application of the model for the cases in which such constraints exist. Our aim here is to design this product in such a way that it has 4 parts. We have included the FR1 also in the algorithm to show, the independent FRs does not affect the result of proposed algorithm. The FR is assigned to the separate part and the algorithm runs to integrate other FRs among the parts.

After applying the algorithm, the final answer obtained is {FR1}, {FR3, FR6, FR7, FR8}, {FR2, FR5}, and {FR4, FR9}. So this design is devoid of any penalty.

It appears that we can satosfy the list of our FRs with only four parts. One example of such designs in when the body design is modifed to accomdate the lead positioning, clip and grip[39] (as shown in Figure 9). This design is inspired from screw mechanism, on rotation the back enters inside the hollow body of the pencil and facilitates the movement of lead. The dentents in the screw piece clicks into place every turn, on roation cockwise the lead moves forward and viceversa. The lead sleeve helps us in holding the lead (prevents sliding).

The functional requirement for this particular mechanical pencil can be summarized as follows:

FR1- Erasing
FR2- Lead chuck
  FR21- lead storage
  FR22-Advance lead
  FR23- Support lead while using
FR3-Back press button
  FR31-Eraser storage
  FR32-prevent lead falling out.
FR4– Body
  FR41-Clip to hold
  FR42- grip for comfort

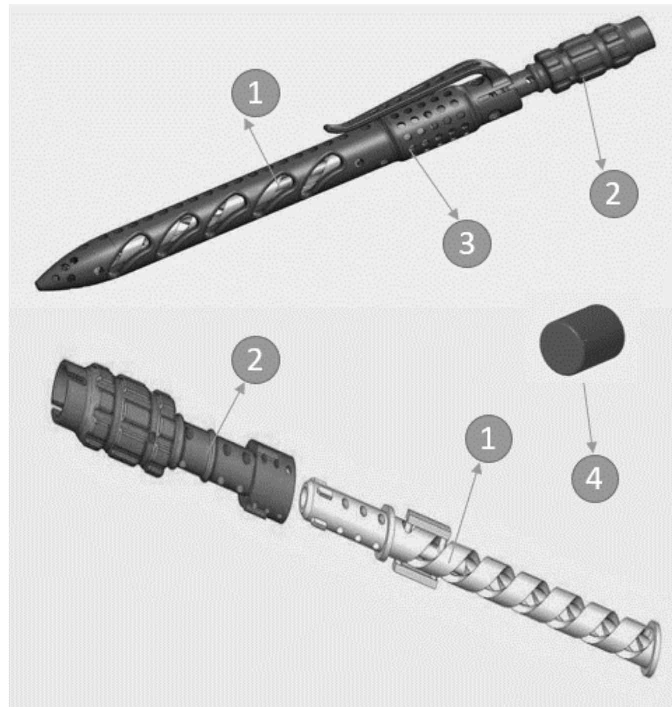


Figure 9: An example of a 4-part design, gotten from [39] (1) Screw piece (2) Back of pencil (3) Pencil Body (4) Eraser

Further to explore the design concept the design parameters need to be defined.

DP1-Eraser
DP2- Screw piece
  DP21- lead advancer
  DP22-cylinder with stopper.
DP3-Back of pencil
  DP31-opening for eraser.
  DP32-accomdate screw motion
DP4-Body design
  DP41-clip design
  DP42-grip design

The design matrix for the new design is:

$$\begin{pmatrix} FR1 \\ FR2 \\ FR3 \\ FR4 \end{pmatrix} = \begin{pmatrix} X & X & 0 & 0 \\ 0 & X & X & 0 \\ 0 & 0 & X & 0 \\ 0 & 0 & 0 & X \end{pmatrix} \begin{pmatrix} DP1 \\ DP2 \\ DP3 \\ DP4 \end{pmatrix} \tag{4}$$

We have analyzed 3 cases, by the concept of physical integration, more than one FR in a single component is integrated together.

The original design was made of 8 parts and had 8 FRs, the proposed design has 5 parts- 5 FRs and 4 parts- 4FRs, where these 9 FRs are integrated together.

Table 5 shows the comparison of these three cases in terms of the number of design parts and number of parts.

Table 5: The comparison of the 3 resulting designs from graph partitioning method.

Case	Design parameters	Number of parts
Original design	9	8
Proposed design-1	5	5
Proposed design-2	4	4

If we look at the two proposed design, the 4-part design seem to work better. With the emerging 3D printing technology, parts can be integrated and printed from a CAD file. So the concept of physical integration or simple design that often results to lower cost and higher quality is feasible through AM. For example, the 4-part design has different set of FRs integrated together but the pencil works the way it is intended to. The penalty for this design was also zero. So this gives an edge over the 5-part design.

Figure 10 shows the overall procedure of the proposed analysis. We can repeat the process from Step 2, defining the number of products and then fix the edges according to our needs and a set of design constraints, and check the feasibility of the proposed design. This step can be repeated till we get the least number of parts with consideration on the FRs to be independent.

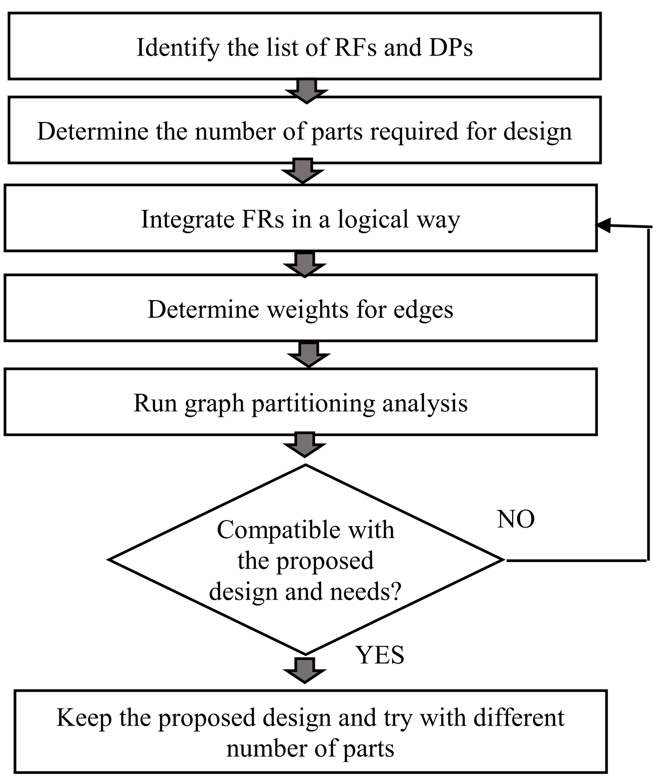


Figure 10: The overall procedure of the analysis

5. CONCLUSION AND FUTURE WORK

This research deals with analyzing the concept of physical integration originated in axiomatic design field. partitioning graph partitioning method is proposed for deciding the best pairs of FRs that can be physically integrated in a single part. The proposed method is employed for an example of designing a pencil, which initially is made of 8 parts. It has been shown that the number of parts can be reduced to 4 and 5 parts where all the FRs are independent and serves it purpose. Depending on the manufacturing constraints, cost and other technical considerations, designers may select one design over the other.

This research can be extended in several ways. The algorithm can be extended to determine the optimum assignment of FRs while satisfying the independence of the FRs as one of the main principles of Axiomatic Design. In addition, the information content of each design alternative can be calculated as another factor to be added to the algorithm. Furthermore, the proposed method can be extended to determine the optimal number of parts needed to satisfy the pre-defined set of FRs. The proposed method is one step towards developing methods that can help designers define the optimal degree of physical integration for design alternatives.

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