DYNAMIC SYSTEM IDENTIFICATION FOR GUIDANCE OF STIMULATION PARAMETERS IN HAPTIC SIMULATION ENVIRONMENTS

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ABSTRACT

We develop a dynamic system identification model to identify relationships among simultaneously recorded electroencephalography (EEG), electromyography (EMG) and force signals measured from 12 participants performing haptic interactions with 3D printed surfaces having different textures. In the first stage, we solve for the maximum likelihood (ML) parameter vector of a parsimonious integrated vector autoregression model (VAR) to estimate the latency between endogenous time variables, utilizing a grid search over the log likelihood scores. In the second stage, we explore the modality dependencies between synchronized EEG, EMG and haptic interactions by training parsimonious VAR models of the same structure. We use our knowledge of signal latency, lag orders and modality dependencies to predict EEG and haptic forces from any provided different combination of EEG, EMG and force measurements. In our future work, this model will guide external stimulation parameters for haptic interaction simulation in scenarios, including teleoperations and virtual environments.

***Index Terms*—** haptic feedback, BCI, EEG, linear dynamic system identification, auto-regressive endogenous non-stationary sequence modeling.

1. INTRODUCTION

Haptic devices are commonly used to present limited sense of touch (vibration, motion, force or temperature), in order to improve spatial cognition and presence in simulated environments. [1, 2, 3]. The sense of touch isn't only critical to improve psychomotor performance in virtual object manipulation tasks, such as minimally invasive surgery training [4], laparoscopy [5], interventional radiology procedures [6], trajectory tracking and spatial navigation [7, 8, 9], but also it conveys information about emotions such as fear [10] and mood changes [11, 12].

Despite the presence of different measurement methods of spatial presence, and their utilization in evaluating the sensory feedback [13], these measures haven't been employed as design metrics to achieve specific levels of spatial presence. That is, there is currently no closed-loop control of sensory stimulations that couple the spatial presence measures to adaptive adjustment of haptic stimulation levels. Our goal is to develop a brain computer interface (BCI) based on closed-loop haptic stimulation framework guided by the changes in the brain activity measured through EEG. EEG has been shown to successfully and reliably measure the spatial presence of a user in an immersive VR (virtual reality) environment [13]. To achieve this goal our initial step is to identify a relationship/model among simultaneously measured contact forces (measured through force transducer), muscle activity (measured through EMG) and brain activity (measured through EEG). This paper presents our preliminary results on our system identification approach to identify the above mentioned relationship among the three modalities.

More specifically, we utilize an integrated vector autoregressive model for linear dynamic system identification with multiple endogenous time series. For the analysis of multivariate time series, it is one of the easy to use and flexible models and extends the univariate autoregressive model to dynamic multivariate time series. It regards the values that a particular variable has assumed in a specific period as realizations of all random variables generates the time series data for each variable through an underlying stochastic process [14].

In practice, latency between haptic force interactions, EEG, and EMG stand as the key challenge in modeling system identification for BCI in a simulated environment. Hence, before proceeding with the design and implementation of a linear system identification model, we explore the parameters of latency by utilizing a grid search among the Bayesian Information Criterion (BIC) scores of VAR models over a region linearly spaced for different latency parameters between haptic force interactions-EMG and EMG-EEG. The underlying presumption in this search is that there is a time delay between haptic interactions, stimulation of touch and pressure receptors in the skin, and brain's response to the sensory information. Besides, there is latency added to that by the data acquisition hardware. Then, we search for the lag order that provides the lowest BIC score to employ a parsimonious model that predicts the future samples of the multivariate time data and explore the modality dependencies.

978-1-7281-0824-7/19/$31.00

Fig. 1: **Experimental Setup**. The participant is tapping the texture mounted on the force transducer.

2. EXPERIMENTAL SETUP

In this experiment, we have a total of 18 conditions: a combination of surface texture (flat, medium rough and rough), speed level (0.5, 1, and 2 Hz) and movement type (rub and tap). Within a condition, the participant is instructed to rub or tap the chosen surface multiple times at a specific speed. Each complete rub or tap movement is considered a trial. These trials are segmented using the normal contact force component.

The experimental setup is shown in Fig.1. In order to avoid any visual or auditory distracting factors, the subjects were asked to look at a black screen presented on the computer, while sitting in a quiet room and rest their right arm on the table as shown in Fig.1. Three surfaces with different levels of roughness were securely attached to a force transducer, which was fixed on a table. The measured contact forces and EEG were recorded synchronously, while the participant was rubbing or tapping each surface for one minute. The 18 conditions were randomized for each participant and there was one minute of rest after each condition.

A set of three textures with different levels of roughness (flat, medium rough, and rough surfaces) have been generated using MATLAB and fabricated with 3D printing. The power spectral density of each surface is given by :

$$\phi(|k|) = \begin{cases} C, & \text{if } k_l <= |k| <= k_r. \\ C(\frac{|k|}{k_r})^{-2(1+H)}, & \text{if } k_r <= |k| <= k_s. \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

where C is the roughness amplitude, k_l, k_r, k_s are the lower roll-off and upper cutoff wave numbers and H is the Hurst roughness exponent. Fig.2 shows both the medium rough (H = 0.5, C = $10 * 10^{10}$, k_l = k_r = 16, k_s = 64) and rough (H = 0.5, C = $10 * 10^{10}$, k_l = k_r = 32, k_s = 256) surfaces used in this study.

2.1. Data Acquisition

EEG was recorded from 12 right-handed healthy participants according to the 10-20 system from 14-channels, using electrodes placed over the frontal and somatosensory cortex focusing around the sensorimotor integration regions, respectively (F3, F4, FC3, FC4, C1, C3, C5, CZ, C2, C4, C6, CP1, CPZ and CP2). IRBs approved by the authors' institutions are used for recruitment and obtaining written consents from the participants. The left mastoid was used as a reference. In this study, two g.USBamp (from g-tec) amplifiers were used, one for EEG data acquisition and one for force data. Recorded EEG data were digitized with 1200 Hz sampling rate. EEG signals were filtered using a 4th order notch filter with corner frequencies of 58 and 62 Hz, and an 8th order bandpass filter with corner frequencies of 2 and 62 Hz.

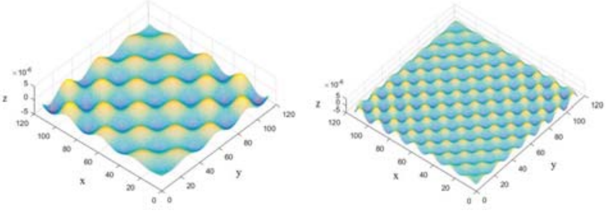

Fig. 2: **The power spectral density** of the medium rough and rough 3D printed surfaces respectively.

A force and torque transducer (NANO17 F/T transducer, ATI Industrial Automation, USA) was used to record the force data. The force data was then transferred to the analog inputs of the g.USBamp amplifier, and sampled at 1200 Hz. Two amplifiers are connected to each other to enable synchronization across EEG and force data. Besides, both EEG and force data were synchronized to each condition through a digital trigger. The presentation software (Psychtoolbox) is used in MATLAB to send triggers to the two amplifiers to mark the "go" cue for each condition. Then, the time stamp for each trigger is saved along with the acquired data. These triggers are then used to segment both the EEG and force data per condition.

3. PROPOSED METHOD

Given y as the multivariate time signal with K endogeneous variables (signals from 14 EEG and 4 EMG channels and force measurements from x, y and z dimensions) and a sample size of T (observations from more than 10 thousand trials) for each of the K variables, v as the intercept terms, A_i as the coefficient matrices for $i = 1, \dots, p$, and u_t as the independent and identically distributed fundamental innovation, a VAR model of order p is denoted as,

$$y_t = v + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t$$
$$y_t \in \mathbb{R}^{K\times 1},\ \ v \in \mathbb{R}^{K\times 1},\ \ A_i \in \mathbb{R}^{K\times K}\ \ \forall i,\ \ u_t \in \mathbb{R}^{K\times 1}$$
$$\mathrm{E}(u_t) = 0, \quad \mathrm{E}(u_t u_\tau^{'}) = \begin{cases} \Omega & t = \tau \\ 0 & t \neq \tau \end{cases}$$

$\Omega \in \mathbb{R}^{K\times K}$ denotes the covariance matrix and it is positive semidefinite.

(1)

A VAR(p) model can be written more compactly as,

$$
\begin{aligned}
&Y = BZ + U, \quad \text{where:} \\
&Y := (y_1, \dots, y_T) \in \mathbb{R}^{K\times T} \\
&B := (v, A_1 \dots, A_p) \in \mathbb{R}^{K\times(Kp+1)} \\
&Z_t := (1, y_{t-1}, \dots, y_{t-p}) \in \mathbb{R}^{(Kp+1)\times 1} \\
&Z := (Z_0, \dots, Z_{T-1}) \in \mathbb{R}^{(Kp+1)\times T} \\
&U := (u_1, \dots, u_T) \in \mathbb{R}^{K\times T}
\end{aligned} \tag{2}
$$

Given the observed random variables y_t and unobserved Gaussian noise $U \sim \mathcal{N}(0, I_T \otimes \Omega)$, VAR model employs system identification using T total number of observations, conditioned on the first p observations [14]. The samples of the endogenous multivariate time data are drawn from a Gaussian distribution at each time index,

$$
\begin{aligned}
&f_{Y_t|Y_{t-1},\dots,Y_{t-p}(y_t|y_{t-1},\dots,y_{t-p}}; \theta) = \\
&\mathcal{N}(v + A_1 y_{t-1} + \dots + A_p y_{t-p}, \Omega) = \\
&(2\pi)^{-\frac{K}{2}} |\Omega^{-1}|^{\frac{1}{2}} \exp(-\frac{1}{2}(y_t - BZ_t)^{'} \Omega^{-1}(y_t - BZ_t)), \\
&\text{for a likelihood parameter vector defined as,} \\
&\theta = (v^{'}, \mathrm{vec}(A_1)^{'}, \mathrm{vec}(A_2)^{'}, \dots, \mathrm{vec}(A_p)^{'}, \mathrm{vech}(\Omega)^{'})^{'}.
\end{aligned} \tag{3}
$$

Recursively, conditional on the $y_0, \dots, y_{t-p}$, the product of the single conditional densities returns the likelihood function for the full sample,

$$
\begin{aligned}
&f_{Y_T,\dots,Y_1|Y_0,\dots,Y_{t-p}} = \prod_{t=1}^{T} f_{Y_t|Y_{t-1},\dots,Y_{t-p}}(y_t|y_{t-1},\dots,y_{t-p}; \theta) \\
&= \prod_{t=1}^{T} \frac{(2\pi)^{-K/2}}{|\Omega|^{1/2}} \exp\left(-\frac{(y_t - BZ_t)^{'}\Omega^{-1}(y_t - BZ_t)}{2}\right)
\end{aligned} \tag{4}
$$

Hence, the log likelihood function is,

$$
\begin{aligned}
\ell(\theta|y_t) = &-\frac{KT}{2}\ln(2\pi) + \frac{T}{2}\ln|\Omega^{-1}| \\
&-\frac{1}{2}\sum_{t=1}^{T}(y_t - BZ_t)^{'}\Omega^{-1}(y_t - BZ_t)
\end{aligned} \tag{5}
$$

In order to minimize the negative of the concentrated log likelihood function and determine the ML estimates of v, A_i and Ω, the system of first order partial derivatives is needed.

Lemma 1 *If* $\mathbf{w}$ *does not depend on* $\mathbf{A}$ *and* $\mathbf{A}$ *is symmetric,*

$$
\frac{\partial \mathbf{w}^{\mathbf{T}}\mathbf{A}\mathbf{w}}{\partial \mathbf{w}} = \mathbf{2Aw} \tag{6}
$$

To take the derivative w.r.t. B and equate to zero we make use of Lemma 1,

$$
\begin{aligned}
&\frac{\partial}{\partial B} l(\theta|y_t) = \sum_{t=1}^{T} \Omega^{-1}(BZ_t - y_t) = 0 \\
&\text{Since } \Omega \text{ is positive semidefinite, } TBZ_t - \sum_{t=1}^{T} y_t = 0 \\
&\hat{B}_{\mathrm{ML}} = \frac{1}{T}\sum_{t=1}^{T} y_t Z_t^{+}, \;\; Z_t^{+}: \text{Moore-Penrose inverse of } Z_t.
\end{aligned} \tag{7}
$$

Deriving the ML estimate for the covariance matrix, requires the following trick in Lemma 2.

Lemma 2

$$
\frac{\partial}{\partial \mathbf{A}} \mathbf{x}^{'}\mathbf{A}\mathbf{x} = \frac{\partial}{\partial \mathbf{A}} \mathrm{Tr}[\mathbf{x}^{'}\mathbf{x}\mathbf{A}] = [\mathbf{x}\mathbf{x}^{'}]^{'} = \mathbf{x}^{''}\mathbf{x}^{'} = \mathbf{x}\mathbf{x}^{'} \tag{8}
$$

We rewrite log likelihood function to compute the derivative w.r.t. Ω^{-1},

$$
\begin{aligned}
l(\theta|y_t) &= \mathrm{C} + \frac{T}{2}\log|\Omega^{-1}| \\
&- \frac{1}{2}\sum_{t=1}^{T} \mathrm{Tr}[(y_t - BZ_t)(y_t - BZ_T)^{'}\Omega^{-1}] \\
\frac{\partial}{\partial \Omega^{-1}} l(\theta|y_t) &= \frac{T}{2}\Omega - \frac{1}{2}\sum_{t=1}^{T}(y_t - BZ_t)(y_t - BZ_t)^{'}
\end{aligned} \tag{9}
$$

Equating this to 0 and solving for Ω yields,

$$
\begin{aligned}
&T\Omega - \sum_{t=1}^{T}(y_t - BZ_t)(y_t - BZ_t)^{'} = 0 \\
&\hat{\Omega}_{\mathrm{ML}} = \frac{1}{T}\sum_{t=1}^{T}(y_t - \hat{B}_{\mathrm{ML}}Z_t)(y_t - \hat{B}_{\mathrm{ML}}Z_t)^{'}
\end{aligned} \tag{10}
$$

We use the data from 6 participants for all 18 conditions across all channels of EEG, EMG and force to learn the ML parameters of a VAR model and data from other 6 participants to test how well the trained model fits on the observations. In training a VAR model, it is important to ensure stationarity of all time variables and avoid choosing a high lag order (p), since the number of parameters grow very fast with high lag order. First, we transform nonstationary variables to stationary ones before their VAR involvement by differencing the series, which is called integration process [14, 15]. Then, we select the lag order, based on BIC scores of the fitted models. The other methods that are commonly used to determine the lag order and can be applied to this domain are: i) plotting the autocorrelation and crosscorrelation functions for the individual residual series to see if there is any obvious autocorrelation left in the residuals and ii) starting with a sufficiently large lag order and successively testing lower lag orders until observing a significant performance decay [16].

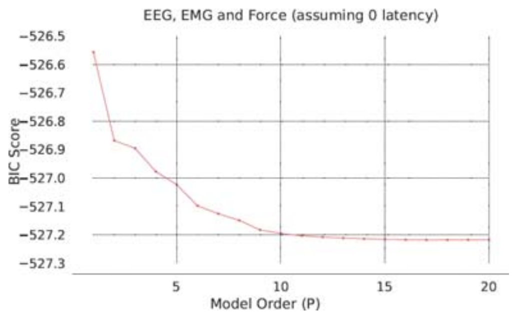

Fig. 3: **BIC scores for model orders ranging between 1-20**, provided EEG, EMG, and haptic force interactions as system input, assuming there is no latency between driving series.

In conjunction with latency minimization routine, we make inferences about the modality dependency on different input feature sets, hence in the experiments K was tested for all different combinations of 14 EEG, 4 EMG and 3 force measurements. For instance, we train our VAR model to predict current EEG from: only past EEG ($K = 14$), both EEG and force ($K = 17$), both EEG and EMG ($K = 18$) and all ($K = 21$), to investigate which features are more informative.

4. RESULTS AND DISCUSSION

Since choosing lag order unnecessarily large is extremely memory consuming, increases the computational overhead and reduces the forecast precision of the model [17], we begin with finding the exact order of data generation process under the assumption that all endogenous variables are provided and data from all these variables are time aligned, as demonstrated in Fig.3. BIC suggests to select the VAR order choice that acquires the lowest one step ahead mean squared error (MSE) with optimal number of ML estimators. The lowest BIC score is achieved at $p = 17$.

The method we employ for the elimination of latency between endogenous time series that are shifted in time is exhaustively searching through a manually specified subset of a hyperparameter space. We utilize grid search over a hyperparameter space of 400 EMG samples recorded after haptic force stimuli and 400 EEG samples recorded after EMG measurements, both linearly spaced by 50 samples to reduce the cost of computation, as illustrated in the first heatmap of Fig.5. In this search, the lowest BIC score, which also corresponds to the highest log likelihood, is acquired when the latency between haptic force-EMG is 200 samples, and EMG-EEG is 400 samples. We zoom in this search space centered at (425, 225), 25 samples backward and forward in time across both axis and reduce the linear spacing to 10 samples, as illustrated in the second heatmap of Fig.5. Based on our analytical investigation of exact latency synchronization, we find the optimum latency between haptic force stimuli-EMG: 200 samples (167 ms), and EMG-EEG: 400 samples (400 ms). Although latency times vary among users and data

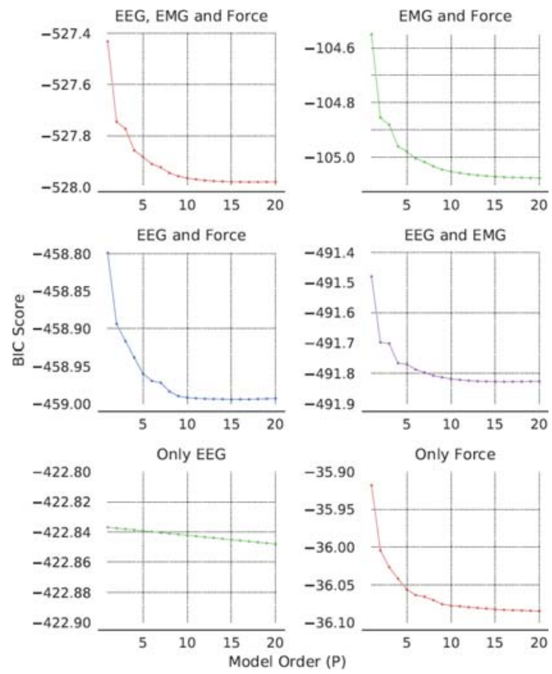

Fig. 4: **Model order selection stage** for each different combination of system input series based on BIC scores, after synchronizing input series through the latency responses evaluated.

acquisition hardware, knowledge of the approximate latency is of considerable practical importance before modeling system identification. Following the time alignment process, we repeat the lag order selection for different modalities as shown in Fig.4.

After latency elimination, we observe lower BIC scores and hence higher log likelihood, provided all integrated model variables, yet model converges at the same lag order. Additionally, we see EEG is highly nondeterministic and highly correlated with the EEG at the one previous time sample that increasing the lag order doesn't have a significant impact on its forecast precision. This is also proven in the first box plot of Fig.6, which we see the percentage error distributions of EEG predictions realize a marginal increase, when EMG and force data become available. On the contrary, percentage error distributions of force improve, when EEG and EMG time data are passed in, as illustrated in the second box plot of Fig.6. We translate our knowledge of latency, optimum lag orders and modality dependencies into future data generation objective. In Fig.7, provided stationarized and synchronized EEG, EMG, and haptic interactions, we demonstrate actual measurements and one-step-ahead predictions modeled by VAR(p=17) for all of the EEG channels and force measurements from z-dimension in a time window of 300 ms (360 samples).

Our model exhibits good performance in one-step-ahead prediction tasks. To attain a longer prediction horizon, a

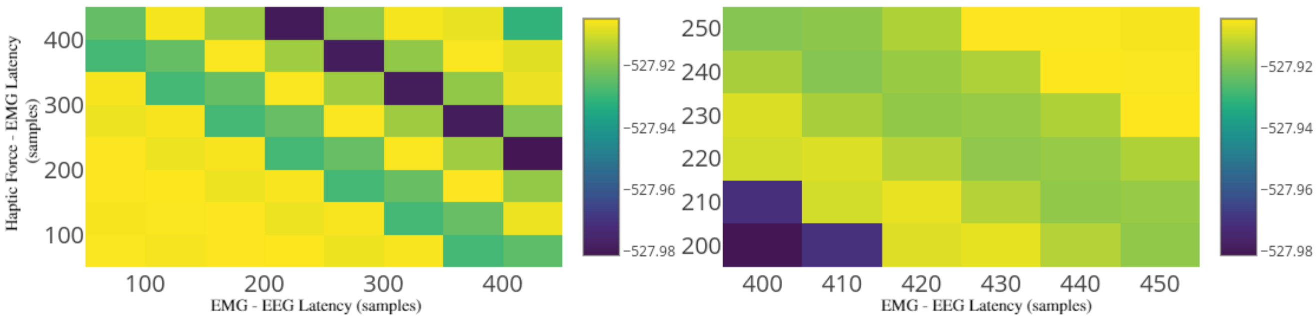

Fig. 5: **Heatmaps for BIC scores at different haptic force-EMG and EMG-EEG latencies** reveal which number of lags between the input series has potentially the most significant effect on the performance of the system identification. We utilize grid search to estimate the latency responses that minimize BIC, consequently maximize log likelihood.

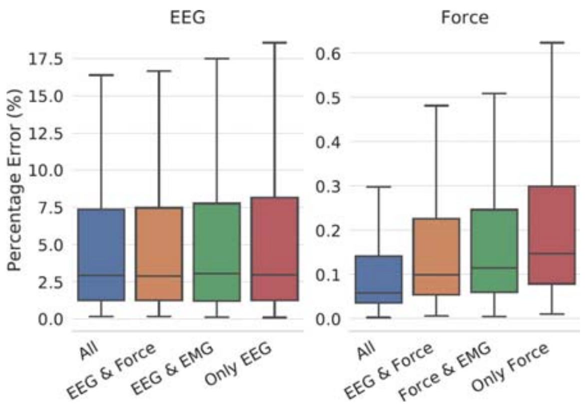

Fig. 6: **Goodness-of-fit.** Percentage error distributions of the predicted EEG and haptic force interactions for each different combination of system input series averaged across all the channels and all the participants in the test set.

procedure known as multi-step-ahead prediction, we feed the model's output back to the input regressor for a fixed but finite number of time steps [18]. As the prediction horizon tends to infinity, system identification becomes a dynamic modeling task, in which the VAR model acts autonomously, recursively emulating the dynamic behavior of the system that generated the time series. Since the reverse characteristic polynomial of VAR process has all roots outside the complex unit circle, it is stable and multi-step-ahead predictions never diverge to infinity.

The major drawback of the proposed linear model is that the mean, variance, and autocorrelations of the original series aren't constant in time, even after detrending. Increasing the number of integration steps helps with stationarizing the time data, but at the cost of losing long term temporal dependencies. We also conjecture that using a fixed lag order is a bottleneck in improving the precision of predictions. In future, we will develop a nonlinear model, capable of exploiting long term temporal dependencies and adaptively extracting relevant endogenous variables such that bidirectional EEG-to-force and force-to-EEG translations can be generated.

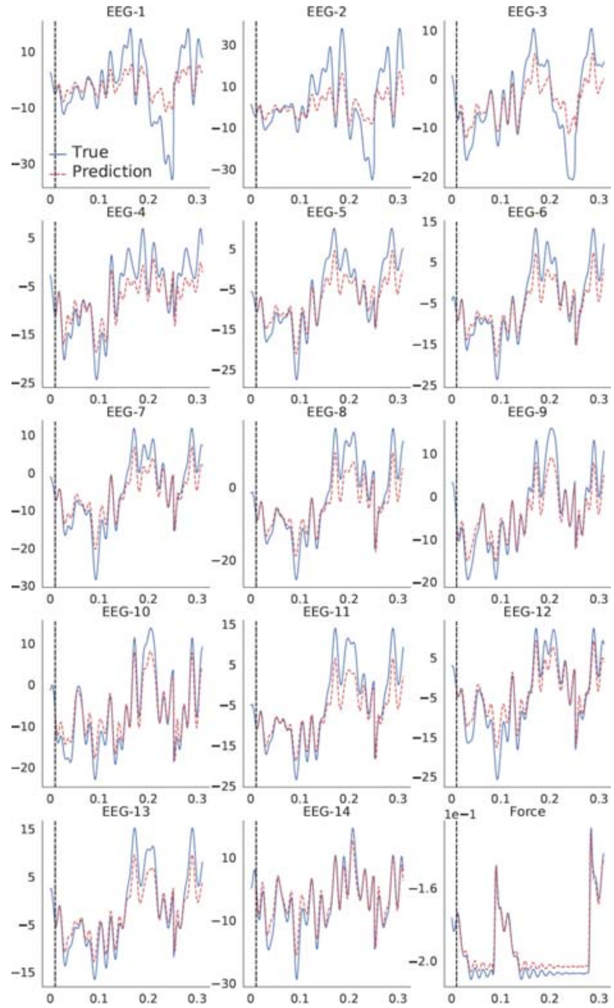

Fig. 7: Actual measurements and one-step-ahead predictions for all of the EEG (μV) channels and force (V) measurements from z-dimension in a time window of 300 ms (360 samples). Samples before the dotted line, marked at time 0, are the first 17 samples observed and there is no prediction within this time frame.

5. CONCLUSION

We implement a maximum likelihood approach for fitting a multivariate VAR model to different combination of EEG, EMG, and force signals simultaneously recorded during haptic stimulation. Specifically, we estimate the lag order of the model and the latency between data from different modalities by optimizing the BIC score of the fitted model. Then: (i) we employ the proposed method to quantify the dependencies among the different modalities, and (ii) test the capability of the proposed method in one-step-ahead prediction.

Guidance of stimulation parameters in haptic interaction simulations has the potential of playing a critical role in VR/AR platforms by offering an extra dimension to a 3D environment and allowing a feeling of true immersion in those environments. Integrated VAR approach we presented estimate the latency between endogenous variables, explore modality dependencies and yield a data prediction process. The proposed linear dynamic system identification model can be deployed to BCI based VR/AR systems where the role of BCI is to record EEG signals, process them to extract relevant features, and classify mental states in order to generate commands, and the role of VR/AR system is to render a virtual environment and provide a meaningful haptic feedback to user.

6. ACKNOWLEDGEMENTS

This project was supported by NSF grants IIS-1717654 and IIS-1715858.

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