Can We Improve Information Freshness with Predictions in Mobile Crowd-Learning?

Zhengxiong Yuan[†]   Bin Li[‡]   Jia Liu[†]

[†]Dept. of Computer Science, Iowa State University

[‡]Dept. of Electrical and Computer Engineering, University of Rhode Island

Abstract—The rapid growth of mobile devices has spurred the development of crowd-learning applications, which rely on users to collect, report and share real-time information. A critical factor of crowd-learning is information freshness, which can be measured by a metric called age-of-information (AoI). Moreover, recent advances in machine learning and abundance of historical data have enabled crowd-learning service providers to make precise predictions on user arrivals, data trends and other predictable information. These developments lead to a fundamental question: Can we improve information freshness with predictions in mobile crowd-learning? In this paper, we show that the answer is affirmative. Specifically, motivated by the age-optimal Round-Robin policy, we propose the so-called "periodic equal spreading" (PES) policy. Under the PES policy, we first reveal a counter-intuitive insight that the frequency of prediction should not be too often in terms of AoI improvement. Further, we analyze the AoI performances of the proposed PES policy and derive upper bounds for the average age under i.i.d. and Markovian arrivals, respectively. In order to evaluate the AoI performance gain of the PES policy, we also derive two closed-form expressions for the average age under uncontrolled i.i.d. and Markovian arrivals, which could be of independent interest. Our results in this paper serve as a first building block towards understanding the role of predictions in mobile crowd-learning.

I. INTRODUCTION

In recent years, the rapid growth of mobile devices (e.g., smartphones, tablets, wireless sensors, etc.) has spurred the development of crowd-learning applications, which rely on users to collect, report and share real-time information for a set of points of interest (PoIs). Such applications include, but are not limited to, real-time gasoline price information sharing (GasBuddy [1]), real-time traffic states (Google Waze [2]), WiFi hotspots searching (WiFi Finder [3]), etc. Although crowd-learning has become increasingly popular, its future prospect heavily hinges upon a performance metric termed *information freshness*, which is also known as "age-of-information" (AoI) in the research community. Ensuring crowd-learned information freshness is critical because fresh information retains existing users and attracts new users to participate, which in turn improves the information freshness and creates a positive feedback loop. Meanwhile, recent advances in machine learning and abundance of historical data collected by pervasive mobile devices have enabled crowd-learning service providers to make precise predictions on user

arrivals, data trends and other predictable information. These developments lead to several fundamental open questions: 1) Can we improve information freshness with predictions in mobile crowd-learning? 2) If the answer to 1) is "yes", how to exploit predictions to achieve better AoI performance? 3) What are the bounds and limits of prediction-assisted AoI performance in mobile crowd-learning?

However, analyzing crowd-learning AoI performance with predictions faces the following challenges: First, there is a lack of analytical model that takes predictions into consideration in mobile crowd-learning in the literature. Second, the inter-actions between arrival patterns, real-time information states and their predictions are highly complex, where changes in one factor would significantly affect the others. Third, as will be shown later, there exists a long-range coupling among different prediction windows, which significantly increases the difficulty in analyzing the AoI performance.

As a starting point, in this paper, we focus on a single-PoI system with predictable arrivals (up to a window size into the future). In this setting, we address the above challenges and obtain several fundamental results on understanding the role of predictions in mobile crowd-learning. The main results and contributions of this paper are summarized as follows:

- First, we introduce an analytical model for a single-PoI crowd-learning system with finite-range predictable arrivals, which takes into account the strong coupling between the stochastic user arrivals and the AoI of the data. In this setting, motivated by the age-optimal Round-Robin policy, we propose the so-called "periodic equal spreading" (PES) policy, which reshapes the arrivals in such a way that the inter-arrival times are nearly equalized.

- Then, under the PES policy, we first consider the problem of choosing an appropriate prediction period, which is referred to as "step size" in this paper. Towards this end, we reveal a surprising insight that the prediction frequency should not be made too often in terms of AoI improvement.

- Finally, we analyze the AoI performance of the PES policy and establish upper bounds for the average age under i.i.d. and Markovian arrivals, respectively. In order to evaluate the AoI performance gain of the PES policy, we also derive two closed-form expressions for the average age under uncontrolled i.i.d. and Markovian arrivals, which could be of independent theoretical interest.

Collectively, our results in this paper serve as a first building
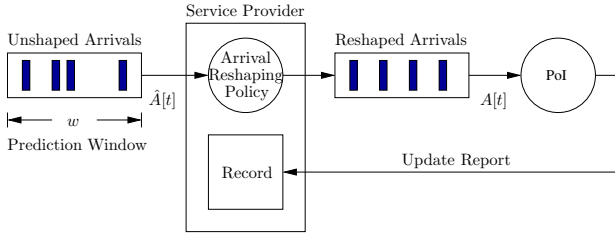
Fig. 1: A single-PoI predictive crowd-learning system.

block towards understanding the role of predictions in mobile crowd-learning. The remainder of this paper is organized as follows: Section II reviews related work. Section III presents system model and problem statement. Section IV introduces the PES policy. Section V studies the AoI performance for i.i.d. and Markovian arrivals, respectively. Section VI illustrates numerical results and Section VII concludes this paper.

II. RELATED WORK

As a new performance metric, AoI has recently attracted increasing attention from the information theory, signal processing, and communications communities due to its close connections and yet clear distinctions from queueing delay. These key differences between AoI and queueing delay have sparked intense research in, e.g., real-time sampling and remote estimation trade-off [4], [5], joint source-channel coding exploitation [6], [7], caching [8], [9], optimization algorithms for AoI minimization [10], [11], age-based scheduling [12], [13], just to name a few. However, research on AoI in mobile crowd-learning remains in its infancy. The most related work to this paper is [14], where the authors proposed a new dynamic model that captures the most essential features of many mobile crowd-learning systems with selfish users. Based on this analytical model, they considered a linear reward mechanism and investigated the AoI performance under selfish user behaviors measured by price-of-anarchy (PoA). We note that our work differs from [14] in the following key aspects: i) The model in [14] does not consider any predictions. In comparison, our focus in this paper is to explore the impacts of predictions in mobile crowd-learning; ii) While the goal in [14] was to evaluate the AoI performance of the linear reward mechanism, the emphasis of this paper is to design an arrival reshaping policy based on predictions to improve AoI performance; iii) Unlike the model in [14] that only considered i.i.d. Bernoulli arrivals, we further consider a more challenging Markovian arrivals process. Because of these key differences, the results in this paper are all new.

III. SYSTEM MODEL AND PROBLEM STATEMENT

Consider a mobile crowd-learning system with one PoI as shown in Fig. 1. The PoI could represent, e.g., a road intersection, a parking garage, a WiFi hot spot, a gas station, etc. We consider a time-slotted system. In each time slot t, the PoI holds some real-time state information (e.g., congestion level, parking rate and space, gas price, etc.) that is time-varying and to be sampled by the arriving users. A service provider (i.e., a crowd-learning-based information/data analytics platform)

relies on randomly arriving users to sample and report the state of the PoI.

We assume that in every time-slot t, the service provider can *accurately* predict a window of future user arrivals, which is of w time-slots[1]. Although the natural arrivals of the users follow some underlying stochastic process, we assume that the arrival pattern of the users can be *reshaped* by the service provider through some reward/incentive mechanism. In other words, the reward/incentive provided by the service provider is sufficiently high so that all users are fully cooperative and willing to change their arrival times. We assume that the time-slot duration is sufficiently short so that there is at most one user arriving in any given time-slot. We use $A[t]$ and $\hat{A}[t]$ to denote the reshaped and unshaped arrival in time-slot t, respectively. Here, $A[t] = 1$ ($\hat{A}[t] = 1$) represents that there is a reshaped (unshaped) user arrival at the PoI in time-slot t; otherwise, $A[t] = 0$ ($\hat{A}[t] = 0$) means if there is no reshaped (unshaped) arrival in time-slot t.

The service provider maintains a record for the PoI. We use $\Delta[t]$ to denote the *age* (freshness) of the recorded information in time-slot t, which is defined as $\Delta[t] = t - U[t]$, where $U[t]$ is the most recent update time for the PoI. We assume that every user will report the real-time state information when he/she arrives at the PoI. Clearly, under a reshaped arrival process $\{A[t]\}_{t \geq 0}$, the AoI process $\{\Delta[t]\}_{t \geq 0}$ evolves as follows:

$$\Delta[t+1] = \begin{cases} \Delta[t] + 1, & \text{if } A[t] = 0; \\ 0, & \text{if } A[t] = 1. \end{cases} \quad (1)$$

In this paper, we consider both i.i.d. (independent and identically distributed) and Markovian *unshaped* arrivals. We also assume that the PoI serves exactly one user if there is any. As a result, there is no queueing effect at the PoI. With the above system setting, a fundamental question is: *Given a prediction window of size w, how could we design an arrival reshaping policy to change the inter-arrival times of the users, so that the information freshness of the PoI can be improved?* Answering this question constitutes the rest of the paper.

IV. ARRIVAL RESHAPING POLICY DESIGN

In this section, we take a first step to answering the fundamental question in Section III by proposing an arrival reshaping policy called "Periodic Equal-Spreading" (PES). Towards this end, we first discuss the motivation and rationale behind the PES policy in Section IV-A, which is followed by the formal presentation of the general PES policy in Section IV-B. Then, we will discuss the impact of a key parameter called "step size" on the performance of the PES policy in Section IV-C.

A. Motivation and Rationale behind the Policy Design

Before formally stating our PES policy, it is insightful to take a look at the rationale behind this policy. Our PES policy is motivated by the fact that the periodic reshaped arrivals of the users enable us to approximately sample the PoI evenly in

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[1]The impacts of prediction errors will be left for our future studies.

temporal domain in a spirit similar to the Round-Robin scheme in the spatial domain, which is known to be age-optimal in the case with multiple PoIs (see [14]). Consider the single-PoI system shown in Fig. 1 with bursty arrivals. For example, the arrivals follow the pattern that every three consecutive arrivals are followed by three time slots that have no arrivals, i.e., the arrival sequence is "111000111000 . . .", where '1' denotes that a user arrives at the PoI and '0' denotes no user arrival. In comparison, consider an alternative "even" arrival sequence "101010101010 . . .," which has the same arrival rate. Suppose that the initial age of the PoI is 0. The age evolution processes of both sequences are shown in Fig. 2.

We can see that the average and peak ages of the bursty-arrival sequence are twice and three times as high as those of the even-arrival sequence, respectively. Indeed, it can be shown that the average and peak ages are both minimized when user arrivals are equally-spaced between each other in the temporal domain. This insight inspires us to propose the PES policy in Section IV-B, which spreads the foreseen arrivals within the limited prediction window to generate nearly equally-spaced arrivals to decrease average and peak ages.

B. The Periodic Equal Spreading Policy

The basic idea of the PES policy is that, periodically, given an arrival sequence that is predicted within a window of size w into the future, the PES policy reshapes the arrivals in such a way that the inter-arrival times are (nearly) equalized. The PES policy is stated in Algorithms 1.

Algorithm 1: Periodic Equal-Spreading (PES) Policy .

Initialization:
1. Choose a step size value $s \in \{1, \ldots, w\}$. Let $i = 1$.

Main Loop:
2. In the i-th time-slot, observe the current time-slot and predict the future $w - 1$ time-slots to obtain the vector $\hat{\mathbf{a}}_i$ that contains the sequence of arrivals foreseen in the i-th time-slot, i.e., $\hat{\mathbf{a}}_i \triangleq \left[\hat{A}[i], \ldots, \hat{A}[i+w-1] \right]^\top \in \{0, 1\}^w$. Let $n_i = \|\hat{\mathbf{a}}_i\|_1$, where $\|\cdot\|_1$ denotes the ℓ_1 norm.
3. Perform "equal spreading" on $\hat{\mathbf{a}}_i$ using Algorithm 2 to obtain a reshaped arrival sequence $\mathbf{a}_i$. Let $i = i + s$ and go to Step 2.

The "equal spreading" subroutine used in Algorithm 1 is stated as follows:

Algorithm 2: Equally spreading the predicted arrivals in $\hat{\mathbf{a}}_i$.

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1. Given the prediction window size w and the length of the predicted arrival sequence $n_i = \|\hat{\mathbf{a}}_i\|_1$, compute $b_i = (w - n_i) \mod (n_i + 1)$ and $k_i = \lfloor (w - n_i)/(n_i + 1) \rfloor$, where $\lfloor x \rfloor$ denotes the maximum integer that is not greater than the real number x.
2. Generate $(n_i + 1 - b_i)$ zero-valued sequences of length k_i and b_i zero-valued sequences of length $k_i + 1$.
3. Shuffle these sequences uniformly at random and insert a "1" element (i.e., an arrival) between every pair of consecutive intervals. Return the reshaped sequence as $\mathbf{a}_i$.

The intuition of Algorithm 2 is that, in order to reshape the predicted arrivals to be equally spread, we need to equalize the inter-arrival times. If the number of arrivals n_i and arrival prediction window size w are given, the number of time slots with no arrivals is $w - n_i$. Thus, one only needs to distribute these $w - n_i$ slots of no arrivals into $n_i + 1$ groups. Let k_i and b_i be the quotient and remainder of $(w - n_i)/(n_i + 1)$, i.e., $w - n_i = k_i(n + 1 - b_i) + (k_i + 1)b$. In other words, $(w - n_i)$ zeros could be partitioned into $(n + 1 - b_i)$ zero-valued sequences of length k_i and b_i zero-valued sequences of length $(k_i + 1)$.

An Example of Equal Spreading: Consider a predicted arrival sequence "000011." In this case, we have $w = 6$ and $n_i = 2$, which entails $b_i = (6 - 2) \mod (2 + 1) = 1$ and $k_i = \lfloor (6 - 2)/(2 + 1) \rfloor = 1$. According to Line 2 of Algorithm 2, we generate two zero-valued sequences of length 1 (i.e., "0") and one zero-valued sequences of length 2 (i.e., "00"). Shuffling the zero-valued sequences uniformly at random and inserting a "1" between every pair of adjacent zero-valued sequences could yield "010100", "010010" or "001010." We can see that any of these reshaped sequences is more even than the original unshaped arrival sequence. □

In fact, it can be shown that the reshaped arrival sequence resulted from Algorithm 2 is the "most even" one in the sense that the lengths of the zero-valued sequences in $\mathbf{a}_i$ produced by Algorithm 2 have the minimum variance. We state this insight as follows (proof details are relegated to Appendix A):

Proposition 1 (Most Even Reshaping). *With two natural numbers M and N such that $M \geq N$, define an $(N + 1)$-partition of M as a set $\mathcal{X}_{N+1} \triangleq \{X_1, \ldots, X_{N+1}\}$, where all X_i's are natural numbers and satisfy $\sum_{i=1}^{N+1} X_i = M$. Let $k = \lfloor M/N \rfloor$ and $b = M \mod N$. Then, any $(N + 1)$-partition of M with $(N + 1 - b)$ k-valued elements and b $(k + 1)$-valued elements, denoted as $\mathcal{X}_{N+1}^*$, has the minimum variance in all $(N + 1)$-partitions.*

C. The Impact of Step Size

Given the PES policy, one important question immediately arises: *How to pick a good step size* (cf. Step 1 in Algorithm 1)? A closer look at the PES policy reveals that it bears close resemblance to the classic MPC method (model predictive control, a.k.a. receding horizon control [15]) when the step size $s = 1$. Specifically, the controller in the MPC method computes control/optimization decisions over a finite future time horizon, but only implements the current time-slot and then computes control/optimization decisions in the next time-slot again. It has been widely observed that, although being a heuristic, the MPC method has excellent empirical performance [16]. Therefore, one may tend to choose $s = 1$ in our PES policy. Surprisingly, in what follows, we will show that the "MPC step size" ($s = 1$) is a poor choice for our PES policy in terms of the AoI performance.

An Example of the MPC Fallacy: Suppose that the original arrival sequence is "001000", and $w = 3$. When $s = 1$, the reshaping under MPC method is shown in Fig. 3, where bold
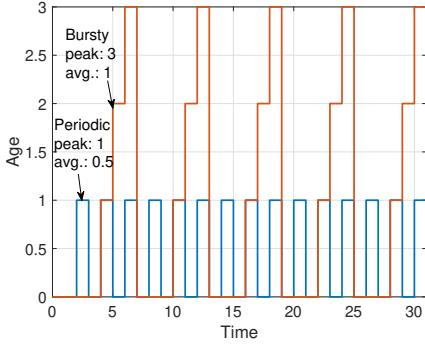
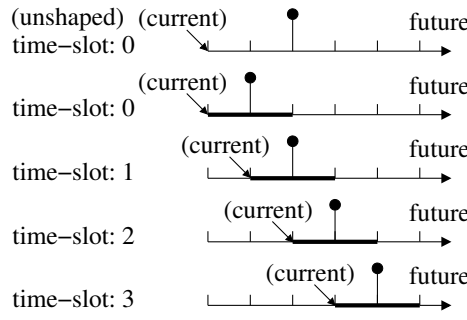Fig. 2: Age performance with bursty and periodic arrivals.

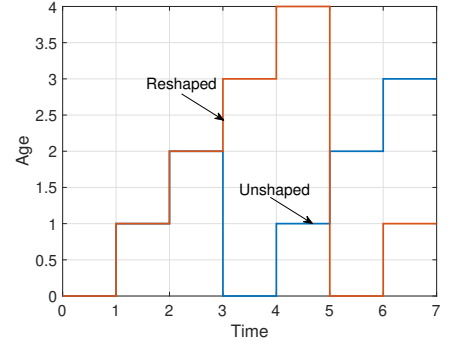Fig. 3: An example of the PES policy with step size $s = 1$.

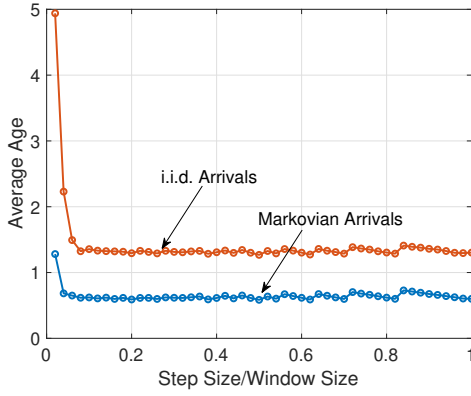Fig. 4: Age performance with MPC when $s = 1$ and $w = 3$.

Fig. 5: The effects of step size.

line segments and dotted bars denote prediction windows and arrivals, respectively. We can see that the PES policy places the future arrival at the center of the current prediction window when $s = 1$. This effectively creates a "pushing" effect, which delays the sampling time of the PoI and leads to worse AoI performance. The comparison of the age performance between the unshaped and the reshaped arrival sequences is shown in Fig. 4. It is obvious that the average age and peak age of the reshaped arrival sequence are larger than that of the unshaped arrival sequence when $s = 1$. □

Now that knowing $s = 1$ is not preferable, it remains to choose an optimal step size $s \in \{2, \ldots, w\}$. Unfortunately, determining an optimal step size is hard. Particularly, when $s < w$, due to the tight coupling and long-range dependence between prediction windows, it is intractable to characterize the effect of the step size on AoI in a closed-form expression. Fortunately, extensive experiments show that there exists a "phase transition" with respect to the step size. The effects of step size on an i.i.d. arrival sequence with $p = 0.3$ and a two-state Markovian arrival sequence with transition rate being 0.3 are shown in Fig. 5. The window size in the previous two examples are 50. We can see that when $s/w < 0.1$, the average performance is poor. However, once the step size is sufficiently large ($s/w \geq 0.1$), the age performance is *insensitive* with respect to s. This phenomenon occurs consistently in all of

our experiments. Hence, in what follows, we set $s = w$ for analytical tractability.

V. PERFORMANCE EVALUATION

In this section, we analyze the age performances of PES policy under both i.i.d. Bernoulli arrivals and Markovian arrivals. For this purpose, we introduce two key notations, D_i and X_i^j, where D_i denotes the number of time-slots from the time of the last seen arrival to the beginning of the i-th prediction window, and X_i^j is the length of the j-th zero-valued subsequence within the i-th prediction window. Fig. 6 is an illustration of D_i and X_i^j.
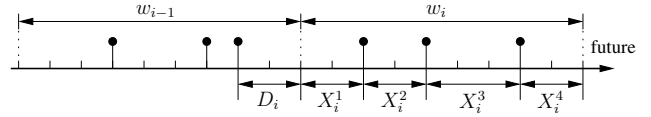

Fig. 6: An illustration of D_i and X_i^j.

A. Independent and Identically Distributed Bernoulli Arrivals

In this subsection, we consider the simpler case with i.i.d. Bernoulli arrivals. The results of the i.i.d. Bernoulli arrivals are not only interesting in their own rights, they also serve as a foundation for the more complex Markovian arrival case. We first analyze the age performance without using any reshaping policy as a baseline.

Theorem 1 (Age of Unshaped i.i.d. Bernoulli Arrivals). *Without any reshaping policy, the expected average age over an i.i.d. Bernoulli arrival sequence with arrival probability p can be computed as:*

$$\mathbb{E}[\bar{\Delta}] = \frac{1}{p} - 1. \tag{2}$$

Proof. First, we introduce a lemma (see Lemma 1 of [17]), which is useful to prove the stated result:

Lemma 1. *For any arrival process for which the steady-state distribution exists, it holds that*

$$\mathbb{E}[\bar{\Delta}] = \frac{1}{2}\left(\frac{\mathbb{E}[X^2]}{\mathbb{E}[X]} - 1\right), \tag{3}$$

where X is the length of the inter-arrival time.

When the arrivals follow the i.i.d. Bernoulli-p distribution, X is a geometrically distributed random variable. Thus, we have $\mathbb{E}[X] = \frac{1}{p}$ and $\mathbb{E}[X^2] = \text{Var}[X] + \mathbb{E}[X]^2 = \frac{1-p}{p^2} + \frac{1}{p^2} = \frac{2-p}{p^2}$. Plugging them into (3), we have $\mathbb{E}[\bar{\Delta}] = \frac{1}{p} - 1$. $\qquad \square$

Due to the complex coupling between the arrivals in different prediction windows, it is hard to analyze the exact mean age performance even under the i.i.d. arrival case. Instead, we provide the following tight upper bound (which is also validated through simulations).

Theorem 2 (Age Upper Bound of the PES Policy: i.i.d. Arrivals). *With PES policy and $s = w$, the upper bound of the expected average age over an i.i.d. Bernoulli arrival sequence with arrival probability p can be computed as:*

$$\mathbb{E}[\bar{\Delta}] \leq \left(\frac{1 - (1-p)^{w+1}}{pw} - \frac{1}{w} \right) \left(\frac{w+1}{2} + \limsup_{I \to \infty} \frac{\sum_{i=1}^{I} \mathbb{E}[D_i]}{I} \right) + \frac{wp+1}{8w}, \tag{4}$$

where

$$\mathbb{E}[D_i] = \sum_{k=0}^{i-1} (1-p)^{wk} \left\{ \left(kw - 1 + \frac{1}{p} \right) [1 - (1-p)^w] - w(1-p)^w \right\} + (1-p)^{wi}(iw), \forall i \geq 1. \tag{5}$$

Proof. If the interval length between two arrivals is X, then the age sequence during this interval is $0, 1, 2, \cdots, X$. It follows that the age sum of this interval is $X(X+1)/2$. We then obtain the age sum of a window by adding the age sum of each interval within that window. Note that there may be an initial age at the beginning of each window, which only affects the age in the very first interval within that window. Recall that D_i is the distance from the last arrival to the beginning of the i-th window, the initial age of the i-th window is D_i. Then the age sum of the i-th window can be computed as:

$$\Delta_i^w = \frac{1}{2} \sum_{j=1}^{n_i+1} X_i^j (X_i^j + 1) + D_i X_i^1, \quad \forall i \geq 1, \tag{6}$$

where n_i is the number of arrivals in the i-th window, X_i^j is the length of the j-th interval of the i-th window. For convenience, we will omit "$\forall i \geq 1$" for the following equations.

As described in Algorithm 2, $w - n_i = k_i(n_i + 1) + b_i = k_i(n_i + 1 - b_i) + (k_i + 1)b_i$, which implies that there are $n_i + 1 - b_i$ intervals of length k_i and b_i intervals of length $k_i + 1$ in the i-th window. Thus, we have:

$$\frac{1}{2} \sum_{j=1}^{n_i+1} X_i^j (X_i^j + 1) = \frac{(n_i + 1 - b_i)k_i(k_i+1) + b_i(k_i+1)(k_i+2)]}{2}$$
$$= \frac{(n_i+1)(k_i+1)}{2} \left[\frac{2(w - n_i)}{n_i + 1} - k_i \right]. \tag{7}$$

We denote the right hand side of (7) as $f(k_i)$, which is a quadratic function with the axis of symmetry being $\frac{w-n_i}{n_i+1} - \frac{1}{2}$.

Since $k_i = \left\lfloor \frac{w-n_i}{n_i+1} \right\rfloor$, we have $\frac{w-n_i}{n_i+1} - 1 < k_i \leq \frac{w-n_i}{n_i+1}$. Hence, $f(k_i) \leq f(\frac{w-n_i}{n_i+1} - \frac{1}{2})$. Then we have:

$$\frac{1}{2} \sum_{j=1}^{n_i+1} X_i^j (X_i^j + 1) \leq \frac{w+1}{2} \frac{w-n_i}{n_i+1} + \frac{1}{8}(n_i+1). \tag{8}$$

Note that we shuffle the inter-arrivals uniformly at random in our algorithm, the expectation of X_i^j resulted from the shuffling is $\frac{w-n_i}{n_i+1}$. When $s = w$, D_i is independent of X_i^1. Hence, it follows that:

$$\mathbb{E}[D_i X_i^1] = \mathbb{E}[D_i]\mathbb{E}[X_i^1] = \mathbb{E}[D_i]\mathbb{E}\left[\frac{w-n_i}{n_i+1} \right]. \tag{9}$$

Combining (8) and (9), we further have:

$$\mathbb{E}[\Delta_i^w] \leq \frac{w+1}{2} \mathbb{E}\left[\frac{w-n_i}{n_i+1} \right] + \mathbb{E}[D_i]\mathbb{E}\left[\frac{w-n_i}{n_i+1} \right] + \frac{1}{8}(\mathbb{E}[n_i]+1). \tag{10}$$

Note that,

$$\mathbb{E}[D_i] = \sum_{k=0}^{i-1} P(n_{i-1}, n_{i-2}, \cdots, n_{i-k} = 0, n_{i-1-k} > 0) \Big\{ kw + \mathbb{E}\left[\frac{w-n_i}{n_i+1} \Big| n_i > 0 \right] \Big\} + P(n_{i-1}, n_{i-2}, \cdots, n_0 = 0)(iw). \tag{11}$$

If the arrivals are i.i.d., then n_i and n_j are independent if $i \neq j$. It follows that $P(n_{i-1} = 0) = (1-p)^w$, $P(n_{i-1} \geq 0) = 1 - (1-p)^w$, and $\mathbb{E}[X_i^j | n_i > 0] = \mathbb{E}[\frac{w-n_i}{n_i+1} | n_i > 0]$, $\forall i, j$. Then, plugging the above back to (11), we have:

$$\mathbb{E}[D_i] = \sum_{k=0}^{i-1} (1-p)^{wk} [1 - (1-p)^w] \left(kw + \mathbb{E}\left[\frac{w-n_i}{n_i+1} \Big| n_i > 0 \right] \right) + (1-p)^{wi}(iw). \tag{12}$$

From (10) and (12), we can see that, to obtain an upper bound of $\mathbb{E}[\Delta_i^w]$, we only need to calculate $\mathbb{E}[n_i]$, $\mathbb{E}[\frac{w-n_i}{n_i+1}]$, and $\mathbb{E}[\frac{w-n_i}{n_i+1} | n_i > 0]$. Towards this end, note that, $s = w$ and the arrivals are i.i.d., the following equalities hold:

$$\mathbb{E}[n_i] = wp, \tag{13}$$

$$\mathbb{E}\left[\frac{1}{n_i+1} \right] = \sum_{k=0}^{w} \frac{1}{k+1} P(n_i = k)$$
$$= \sum_{k=0}^{w} \frac{1}{k+1} \binom{w}{k} p^k (1-p)^{w-k}$$
$$= \frac{1}{(w+1)p} [1 - (1-p)^{w+1}], \tag{14}$$

$$\mathbb{E}\left[\frac{1}{n_i+1} \Big| n_i > 0 \right] = \sum_{k=1}^{w} \frac{1}{k+1} \frac{P(n_i = k)}{1 - P(n_i = 0)}$$
$$= \frac{1}{1 - (1-p)^w} \left[\frac{1-(1-p)^{w+1}}{(w+1)p} - (1-p)^w \right]. \tag{15}$$

It then follows that,

$$\mathbb{E}\left[\frac{w - n_i}{n_i + 1} \right] = \mathbb{E}\left[\frac{w+1}{n_i+1} - 1 \right] = \frac{1}{p}[1 - (1-p)^{w+1}] - 1, \tag{16}$$
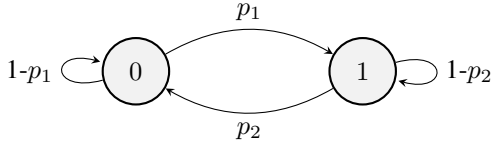
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Fig. 7: The Gilbert-Elliot model.

$$\mathbb{E}\left[\frac{w-n_i}{n_i+1}\Big|n_i>0\right]=\mathbb{E}\left[\frac{w+1}{n_i+1}-1\Big|n_i>0\right]$$
$$=\frac{w+1}{1-(1-p)^w}\left[\frac{1-(1-p)^{w+1}}{(w+1)p}-(1-p)^w\right]-1. \quad (17)$$

Plugging (17) into (12) yields (5). Then, plug (13) and (16) into equation (10), we obtain

$$\mathbb{E}[\Delta_i^w]\leq\Big(\frac{1-(1-p)^{w+1}}{p}-1\Big)\Big(\frac{w+1}{2}+\mathbb{E}[D_i]\Big)+\frac{wp+1}{8}. \quad (18)$$

The upper bound of overall average age can be calculated as:

$$\mathbb{E}[\bar{\Delta}]\leq\limsup_{I\to\infty}\frac{\sum_{i=1}^I\mathbb{E}[\Delta_i^w]}{wI}. \quad (19)$$

Therefore, with (18) and (19), we arrive at (4). This completes the proof. $\square$

B. Markovian Arrivals

For the Markovian arrivals, we consider the Gilbert-Elliot model, which is a two state Markov chain. As shown in Fig. 7, States 0 and 1 represent "no arrival" and "an arrival has occurred," respectively. Also, p_1 and p_2 are the state transition probabilities, i.e., the transition matrix P is:

$$P=\begin{bmatrix}1-p_1 & p_1\\ p_2 & 1-p_2\end{bmatrix}. \quad (20)$$

Similar to Section V-A, we first establish the following result for unshaped Markovian arrivals as a baseline.

Theorem 3 (Age of Unshaped Markovian Arrivals). *Without any reshaping policy, the expected average age over a Markovian arrival sequence following the Gilbert-Elliot model can be computed as:*

$$\mathbb{E}[\bar{\Delta}]=\frac{p_2}{p_1(p_1+p_2)}. \quad (21)$$

Proof. Similar to the proof of Theorem 1, we will use the Lemma 1 of [17]. Thus we need to know the first and second moments of the inter-arrival times. For Markovian arrivals, the inter-arrival time is equivalent to the recurrence time for State 1. Let us denote the stationary distribution vector π as $[\pi_0,\pi_1]$, where π_i $(i=0,1)$ is the stationary probability of being at State i. We also use m_{ij} to denote the expected first passage time from State i to State j, and $m_{ij}^{(2)}$ denotes its second moment. Then the first moment of the recurrence time for State 1 (m_{11}) can be represented as $1/\pi_1$. To calculate $m_{11}^{(2)}$, we need the following lemma (see [18, Corollary 2.4.2]).

Lemma 2. *The matrix of the second moments of the first passage time can be computed as:*

$$M_d^{(2)}=2M_d(\Pi M)_d-M_d, \quad (22)$$

where Π is a 2×2 matrix with each row being the stationary distribution π, $M=[m_{ij}]$ is the matrix of the first moments of the first passage times, $M_d=[\delta_{ij}m_{ij}]$ ($\delta_{ij}=1$, if $i=j$, 0, otherwise) is a diagonal matrix with elements being the diagonal elements of M and $M_d^{(2)}=[\delta_{ij}m_{ij}^{(2)}]$.

Next we are going to calculate $m_{11}^{(2)}$ with Lemma 2. For stationary distribution π, we have

$$\pi=\pi P, \quad (23)$$

$$\pi_0+\pi_1=1. \quad (24)$$

With (20), (23) and (24), we can derive:

$$\pi\triangleq[\pi_0,\pi_1]=\Big[\frac{p_2}{p_1+p_2},\frac{p_1}{p_1+p_2}\Big]. \quad (25)$$

It then follows that:

$$\Pi\triangleq\begin{bmatrix}\pi_0 & \pi_1\\ \pi_0 & \pi_1\end{bmatrix}=\begin{bmatrix}\frac{p_2}{p_1+p_2} & \frac{p_1}{p_1+p_2}\\ \frac{p_2}{p_1+p_2} & \frac{p_1}{p_1+p_2}\end{bmatrix}, \quad (26)$$

$$m_{00}=\frac{1}{\pi_0}=\frac{p_1+p_2}{p_2},m_{11}=\frac{1}{\pi_1}=\frac{p_1+p_2}{p_1}, \quad (27)$$

$$m_{01}=p_{01}+(1+m_{01})p_{00},m_{10}=p_{10}+(1+m_{10})p_{11}. \quad (28)$$

With (27) and (28), we have:

$$M=\begin{bmatrix}\frac{p_1+p_2}{p_2} & \frac{1}{p_1}\\ \frac{1}{p_2} & \frac{p_1+p_2}{p_1}\end{bmatrix},M_d=\begin{bmatrix}\frac{p_1+p_2}{p_2} & 0\\ 0 & \frac{p_1+p_2}{p_1}\end{bmatrix}. \quad (29)$$

Plugging (26) and (29) into (22), we then derive:

$$M_d^{(2)}=\begin{bmatrix}\frac{p_1+p_2}{p_2}+\frac{2p_1}{p_2^2} & 0\\ 0 & \frac{p_1+p_2}{p_1}+\frac{2p_2}{p_1^2}\end{bmatrix}, \quad (30)$$

which implies that $m_{11}^{(2)}=\frac{p_1+p_2}{p_1}+\frac{2p_2}{p_1^2}$. Then with Lemma 1, $\mathbb{E}[\bar{\Delta}]=\frac{1}{2}\big(\frac{m_{11}^{(2)}}{m_{11}}-1\big)=\frac{p_2}{p_1(p_1+p_2)}$. This completes the proof. $\square$

Next, we state the average age upper bound of the PES policy under Markovian arrivals.

Theorem 4 (Age Upper Bound of the PES Policy: Markovian Arrivals). *With PES policy and $s=w$, the upper bound of the expected average age over a Markovian arrival sequence (at steady state) following the Gilbert-Elliot model can be computed as:*

$$\mathbb{E}[\bar{\Delta}]\leq\frac{w+1}{2w}\mathbb{E}\left[\frac{w-n_i}{n_i+1}\right]+\frac{\mathbb{E}[n_i]+1}{8w}$$
$$+\sqrt{\mathbb{E}\left[\Big(\frac{w-n_i}{n_i+1}\Big)^2\right]}\limsup_{I\to\infty}\frac{1}{Iw}\sum_{i=1}^I\sqrt{\mathbb{E}[D_i^2]}, \quad (31)$$
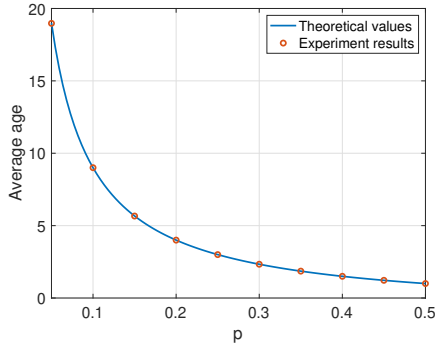
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Fig. 8: AoI performance for different i.i.d. Bernoulli arrivals without any reshaping policy.
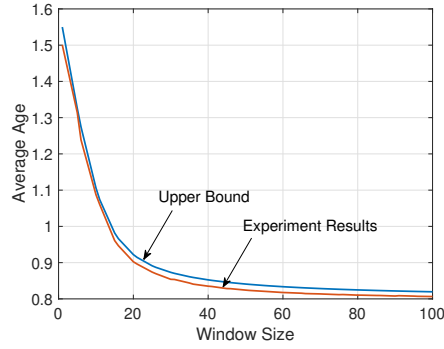

Fig. 9: AoI performance under the PES policy with respect to window size for i.i.d. Bernoulli arrivals ($p = 0.4$).
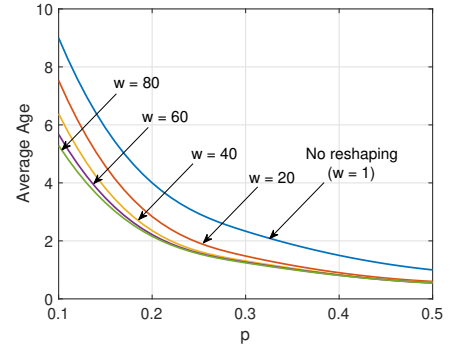

Fig. 10: AoI performance comparison for i.i.d. Bernoulli arrivals with varying prediction window sizes.
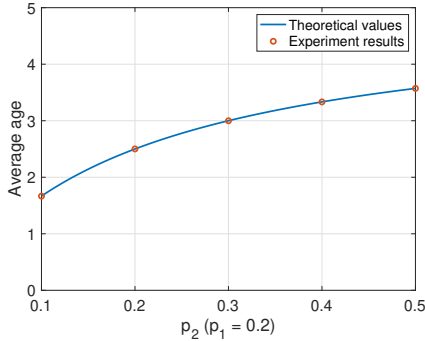

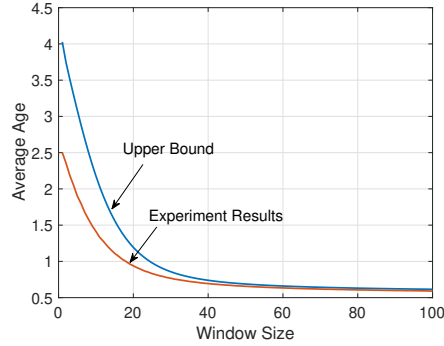Fig. 11: AoI performance for different Markovian arrivals without any reshaping policy.

Fig. 12: AoI performance under the PES policy with respect to window size for Markovian arrivals ($p_1 = 0.2$, $p_2 = 0.3$).
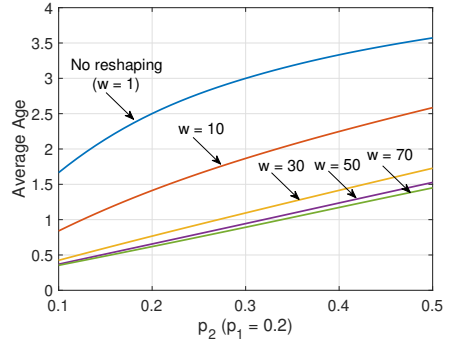

Fig. 13: AoI performance comparison for Markovian arrivals with varying prediction window sizes.

where

$$\mathbb{E}[D_i^2] = \sum_{k=0}^{i-1} P(n_{i-1}, n_{i-2}, \cdots, n_{i-k}=0, n_{i-1-k}>0)\Big\{(kw)^2$$
$$+ 2kw\mathbb{E}\Big[\frac{w-n_i}{n_i+1}\Big|n_i>0\Big] + \mathbb{E}\Big[\Big(\frac{w-n_i}{n_i+1}\Big)^2\Big|n_i>0\Big]\Big\}$$
$$+ P(n_{i-1}, n_{i-2}, \cdots, n_0 = 0)(iw)^2. \tag{32}$$

Proof. Similar to the proof of Theorem 2, Eqs. (6) to (8) also hold for Markovian arrivals. However, for Markovian arrivals, D_i is dependent of X_i^1, which means that Eq. (9) does not hold. To address this issue, by Cauchy-Schwarz inequality, we have:

$$\mathbb{E}[D_i X_i^1] \le \sqrt{\mathbb{E}[D_i^2]\mathbb{E}[(X_i^1)^2]} = \sqrt{\mathbb{E}[D_i^2]\mathbb{E}\Big[\Big(\frac{w-n_i}{n_i+1}\Big)^2\Big]}. \tag{33}$$

Thus, combining (8) and (33) yields:

$$\mathbb{E}[\Delta_i^w] \le \frac{w+1}{2}\mathbb{E}\Big[\frac{w-n_i}{n_i+1}\Big] + \frac{1}{8}(\mathbb{E}[n_i]+1)$$
$$+ \sqrt{\mathbb{E}[D_i^2]\mathbb{E}\Big[\Big(\frac{w-n_i}{n_i+1}\Big)^2\Big]}. \tag{34}$$

By the definition of D_i, we have the stated result in (32). Lastly, plugging (34) into (19) leads to the final result stated in (31). This completes the proof. $\square$

VI. NUMERICAL RESULTS

In this section, we conduct simulations to verify the age performance under the PES policy for i.i.d. Bernoulli arrivals and Markovian arrivals in a single-server system. In the following simulations, we generate $1,000$ arrival sequences of length $100,000$ for each trial uniformly at random.

A. Independent and Identically Distributed Bernoulli Arrivals

First, to confirm the results in Theorem 1, we evaluate the average age performance without reshaping and the results are shown in Fig. 8. We can see that the experimental results perfectly match our theoretical predictions in Theorem 1.

Then, we evaluate AoI performance under the PES policy with respect to window size for i.i.d. Bernoulli arrivals to verify Theorem 2 and the results are shown in Fig. 9 (we only show the case with $p = 0.4$ due to space limitation). Out experimental results show that the upper bound stated in Theorem 2 is tight for window sizes ranging from one to 100.

Finally, we evaluate the AoI performance under the PES policy with respect to the prediction window size ($w = 1$ corresponds to the no-reshaping case). As shown in Fig. 10, under the PES policy, the AoI performance is significantly better compared to that of the no-reshaping case. Also, the AoI performance improves as w gets large, which makes intuitive sense because larger w implies better prediction. However, we

also note a diminishing return effect: the AoI improvement becomes increasingly marginal as w gets large.

B. Markovian Arrivals

First, to confirm the results in Theorem 3, we evaluate the average age performance without reshaping and the results are shown in Fig. 11. Again, the experimental results perfectly match our theoretical predictions in Theorem 3.

Then, we evaluate AoI performance under the PES policy with respect to window size for Markovian arrivals to verify Theorem 4 and the results are shown in Fig. 12 (we only show the case with $p_1 = 0.2$, $p_2 = 0.3$ due to space limitation). Out experimental results show that the upper bound stated in Theorem 4 is valid. Moreover, the upper bound becomes sharper as w increases. We note that the looseness of the upper bound for small w values is mainly due to the approximation error of the Cauchy-Schwarz inequality.

Finally, we evaluate the AoI performance under the PES policy with respect to the prediction window size ($w = 1$ corresponds to the no-reshaping case). As shown in Fig. 13, with reshaping, the AoI performance gain is even more pronounced compared to that of the i.i.d. Bernoulli arrival cases. Again, the AoI performance improves as w gets large in the Markovian arrival cases. Interestingly, the same diminishing return effect also occurs for Markovian arrivals.

VII. Conclusion

In this paper, we strived to understand the impacts of predictions on information freshness over a single-PoI system. To answer this question, we first introduced a single-PoI system model that takes into account the essential features of predictive mobile crowd-learning. Based on this model and motivated by the fact that periodic arrivals have better AoI performance than bursty arrivals, we proposed an arrival reshaping policy called "periodic equal spreading" (PES) to generate nearly equally-spaced arrivals to decrease average and peak ages. To analyze the AoI performance of the PES policy, we considered two types of arrivals: i.i.d. Bernoulli and Markovian arrivals. For each type of arrivals, we first derived a closed-form expression for the average age without reshaping. Then we established upper bounds for the average age under the PES policy. Numerical results match our analysis well. We know that the research on AoI in predictive mobile crowd-learning remains an under-explored area and many problems are still wide open. Future directions include extensions to multi-PoI systems, consideration of prediction errors, and further predictions on real-time PoI state information processes.

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Appendix A
Proof of Proposition 1

Proof. Note that, $M = (N+1)k + b = (N+1-b)k + b(k+1)$. Thus $\mathcal{X}_{N+1}^*$ is a $(N+1)$-partition of M. Let $\mathcal{Y}_{N+1}$ and $\mathcal{Z}_{N+1}$ be the partitions of $k(N+1)$ and b, respectively. It is evident that partitioning M into $(N+1)$ parts is equivalent to partitioning $k(N+1)$ and b into $(N+1)$ parts. Then we have $X_i = Y_i + Z_i, i = 1, 2, \ldots, N+1$. It follows that $\mathrm{Var}(\mathcal{X}_{N+1}) = \mathrm{Var}(\mathcal{Y}_{N+1} + \mathcal{Z}_{N+1}) = \mathrm{Var}(\mathcal{Y}_{N+1}) + \mathrm{Var}(\mathcal{Z}_{N+1}) + 2\mathrm{Cov}(\mathcal{Y}_{N+1}, \mathcal{Z}_{N+1})$. Note that $\mathrm{Cov}(\mathcal{Y}_{N+1}, \mathcal{Z}_{N+1}) = 0$ since $\mathcal{Y}_{N+1}$ and $\mathcal{Z}_{N+1}$ are independent. Thus, $\mathrm{Var}(\mathcal{X}_{N+1}) = \mathrm{Var}(\mathcal{Y}_{N+1}) + \mathrm{Var}(\mathcal{Z}_{N+1})$. Let $\mathcal{C}_X$ be the collection of all possible $\mathcal{X}_{N+1}$'s, and $\mathcal{C}_Y$, $\mathcal{C}_Z$ be the collections of all possible $\mathcal{Y}_{N+1}$'s and $\mathcal{Z}_{N+1}$'s, respectively. Then we have:

$$\min_{\mathcal{X}_{N+1} \in \mathcal{C}_X} \mathrm{Var}(\mathcal{X}_{N+1}) = \min_{\substack{\mathcal{Y}_{N+1} \in \mathcal{C}_Y, \\ \mathcal{Z}_{N+1} \in \mathcal{C}_Z}} (\mathrm{Var}(\mathcal{Y}_{N+1}) + \mathrm{Var}(\mathcal{Z}_{N+1}))$$
$$= \min_{\mathcal{Y}_{N+1} \in \mathcal{C}_Y} \mathrm{Var}(\mathcal{Y}_{N+1}) + \min_{\mathcal{Z}_{N+1} \in \mathcal{C}_Z} \mathrm{Var}(\mathcal{Z}_{N+1}).$$

Then the minimum variance of $\mathcal{Y}_{N+1}$ can be achieved when $Y_i = k, \forall i$, in which case $\mathrm{Var}(\mathcal{Y}_{N+1}) = 0$. In addition, since $b < N + 1$, the minimum variance of $\mathcal{Z}_{N+1}$ can be achieved when there are b 1's and $(N - b)$ 0's. Thus, $\mathcal{X}_{N+1}^*$ achieves the minimum variance. $\square$