Privacy-Utility Tradeoff in Dynamic Spectrum Sharing with Non-Cooperative Incumbent Users

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***Abstract*—Dynamic spectrum access enables opportunistic users (OUs) to access underutilized licensed bands by querying spectrum databases. However, the operational details of the incumbent users may leak to OUs during the query process. Privacy and exclusion zones have been proposed as effective countermeasures to protect the IUs' privacy, while also managing interference. In the case of multiple heterogeneous coexisting IUs, there is an inherent tradeoff between their achieved throughput, which is controlled by the received interference, and the utility provided to OUs, under a fixed privacy constraint. In this paper, we address the problem of maximizing the utility of rational IUs, defined as the weighted sum between the IUs' capacity and compensation from allowing OUs' opportunistic access while meeting the individual IUs' privacy constraints. We formulate the interaction between the heterogeneous IUs as a non-cooperative continuous game and derive the Nash equilibrium that maximizes the utility of each IU. Our simulations show that the NE solution improves the individual utilities of the IUs compared to a joint optimization approach, where the sum of the utilities is maximized while providing more fairness to the IUs.**

I. INTRODUCTION

The fast-growing demand for high speed mobile services and the profit-driven nature of spectrum providers (IUs) make them actively seek new ways to increase their profits. One such a way is Dynamic Spectrum Access (DSA) which allows opportunistic users (OUs) to access underutilized channels of higher-priority incumbent users (IUs) [1]. To enable opportunistic access, spectrum databases store detailed information about IUs' activity, such as their geo-locations, which could be leaked when the database is compromised. Even if the database is well-protected, malicious OUs can issue multiple spectrum access queries to pinpoint the real locations of the IUs using intersection attack [2]. Leakage of such sensitive information affects the location privacy of IUs.

Establishing a privacy zone (PZ) inside an exclusion zone (EZ) has recently attracted significant interest [2]–[4]. This process is shown in Figure 1, where an IU located at x tries to protect the privacy of its location, while managing interference. The IU first randomly perturbs its true location x into a fake one $x^{'}$. Each IU_m announces its PZ as a disk centered at $x^{'}$ with radius r_{1m} that covers x to any OU that queries x to ask about transmitting in location x. In this way, x is indistinguishable from all other locations inside the PZ. To protect all the points inside the PZ from interference, the PZ is surrounded with an EZ of radius r_{2m} [3], inside of which all OUs remain silent. The EZ size scales with the amount of interference that the IU tries to suppress, and PZ size is related to the privacy level that the IU wants to achieve.

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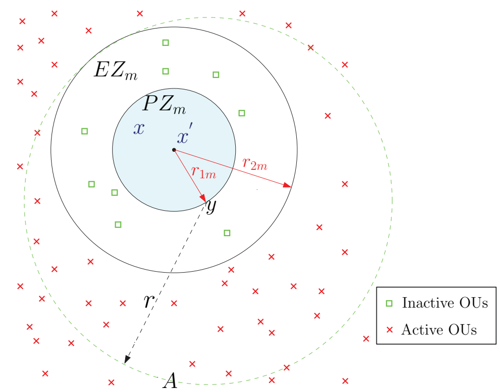

Figure 1: Definition of privacy and exclusion zones.

A centralized architecture, in which a central IU (IU_c) is responsible for deciding the size of the EZ associated with each IU, was first studied in [3]. The authors investigated the tradeoff between OUs' transmission opportunity and IUs' location privacy. Their scheme depends on choosing IU_c at random as the head of an anonymity cluster, which creates a single point of failure and may not achieve privacy if the cluster is too small. In [2], we studied the tradeoff between opportunity, privacy, and interference by designing a PZ inside an EZ for one IU. In [4], we generalized this model to the case of multiple cooperative IUs. In these works, the competition between the heterogeneous IUs was not addressed.

In practical scenarios, multiple IUs selfishly try to maximize their utility. Consider, for example, a commercial area that has multiple competing spectrum operators (IUs) and their deployed micro base stations (OUs) who try to co-exist in the spectra that allow opportunistic access such as the radar and TV bands. Each operator can lease its exclusive spectrum to a group of OUs located in the same area (in exchange for a certain fee). The objective of an IU is to maximize the payments it receives from the OUs and maintain a desired degree of service for its own OUs while limiting the interference caused by them on IUs' transmission.

Consequently, there is a fundamental tradeoff between the IUs' spectrum revenue (from active OUs) and the IUs' trans-

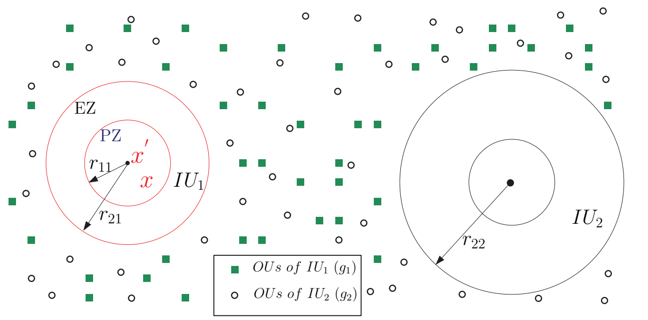

Figure 2: Two IUs, covered by an exclusion zone (EZ) and a privacy zone (PZ), and surrounded by OU groups ($\mathcal{G}_1$, $\mathcal{G}_2$).

mission capacity. For example, if IU_1 in Figure. 2 wants to improve its transmission capacity at the expense of revenue (or to get more privacy), IU_1 would increase the size of its EZ to reduce interference from OUs. This affects the revenue of IU_2, as more of its OUs, located in IU_1's EZ, are forced to be silent. However, if an IU_2 decides to shrink its EZ and improve its revenue, IU_1 would receive more interference from more active OUs. Therefore, having more interference suppression (or more privacy) means having less spectrum revenue, and vice versa, where the parameter that controls the tradeoff is the EZ radius. Given that each IU individually decides its own EZ radius, there will be a conflict of interests between the IUs in terms of privacy and interference. Therefore, an implicit agreement must be reached among IUs so that no IU can benefit from unilaterally deviating.

In this paper, we formulate the problem of maximizing the utility of IUs under privacy constraints as a non-cooperative game, with IUs as players, EZ radii as actions, and the weighted sum between the total payments that an IU gets and IU's capacity, as payoffs. We choose the non-cooperative game because IUs represents separate entities competing for revenue. We prove that the proposed game has a unique Nash equilibrium (NE).

The contributions of this paper are summarized as follows:

1) We propose a non-cooperative game to study the tradeoff between IUs' utilities and their privacy in DSA.
2) For the case of two IUs, we prove that our proposed game has a unique NE, which we obtain numerically by simultaneously solving the best-response equations to find the EZ radii of the two IUs at NE.
3) For the case of multiple IUs, due to the complexity of using the exact method, we use the sub-gradient algorithm to reach a sub-optimal NE and we show that it has a small performance gap with the exact NE.
4) Simulation results show that a) the NE has a small price of anarchy compared with JO solution; b) there is a tradeoff between the PZ size and the IUs' utility; c) NE solution is fairer, has less utility variance, compared with JO solution; d) our work outperforms the work in [3].

To the best of our knowledge, this is the first work to jointly consider interference from OU activity and IU privacy in a distributed multi-operator setting.

II. System Model

A. Spectrum Sharing Architecture and Query Process

We consider two types of spectrum users: incumbent users (IUs), opportunistic users (OUs). We assume a set of M independent IUs who coexist with OUs in the same geographic area. To maintain location privacy, each IU_m determines a circular privacy zone (PZ_m) [5] centered at a fake location x_m'. To limit the interference, each IU_m is surrounded with another circular protection region EZ_m with radius r_{2m}, as shown in Figure 1. Any OU inside the EZ_m remains inactive. Therefore, the IU_m can control the interference it experiences by shrinking and expanding EZ_m. Also, we assume that information such as OUs' transmission power, IUs' PZ radii and λ_m, can be known to all the IUs via a common channel through which the IUs can exchange information.

Each IU_m has a group of OU subscribers, denoted by $\mathcal{G}_m$, that access IU_m's spectrum when IU_m allows it. For mathematical convenience, the subscribers of IU_m are assumed to be uniformly distributed by a Poisson point process [6] with density λ_m. The value of λ_m can be obtained from general location statistical data such as prior query history [7].

Each group of OUs accesses the spectrum opportunistically by querying a spectrum access database (SAS_m) managed by IU_m. The query of an OU $\in \mathcal{G}_m$ includes its location ℓ_i and the channel requested. The IU checks its SAS_m and returns '1' if $\ell_i \notin EZ$ and the requested channel/time is available (granted access), and '0' otherwise (denied access). The query and response process is executed in rounds. IU_m grants channel access to querying OUs at the beginning of every round for a fixed time period. If a longer access period is needed, the OUs repeat the query process. Each IU_m independently decides the query results sent to $\mathcal{G}_m$. This decision is made based on a utility function related to the transmission capacity of the IUs and the payment (spectrum fee) they receive from the OUs. The IU's are assumed to be rational in the sense that they selfishly try to maximize their utilities. This model captures a non-cooperative spectrum management approach without coordination between heterogeneous spectrum providers.

B. Utility of Incumbent Users

Each IU_m independently decides on the sizes of the PZ and the EZ by selecting r_{1m} and r_{2m}, respectively. The value of r_{1m} satisfies a desirable privacy constraint whereas r_{2m} balances the tradeoff between interference at IU_m and transmission opportunity of $\mathcal{G}_m$, for which IU receives payment. The following utility function U_m captures this tradeoff:

$$U_m = \omega_m N_m C_{OU} + C_{IU_m}, \tag{1}$$

where ω_m is weight controls the first term, N_m is the number of OUs $\in \mathcal{G}_m$ that that do not belong to any EZ. We consider payment to be equal to a constant transmission capacity C_{OU} of an OU. Therefore, the total payment is $\omega_m N_m C_{OU}$. The Shannon capacity C_{IU_m} achieved by IU_m is given by:

$$C_{IU_m} = log(1 + \frac{P_m H_m}{\mathcal{I}_m + N}), \tag{2}$$

where P_m is the transmission power of IU_m, N is the noise power, H_m is the combination of the channel impulse response and antennae gains at IU_m's receiver, and $\mathcal{I}_m$ is the cumulative interference experienced at IU_m due to all active OUs

$$\mathcal{I}_m = I_{OU}^m - \sum_{m \neq k}^{M} I_k^s \tag{3}$$

where I_{OU}^m is the total interference from all OU groups to IU_m and $\sum_{m \neq k}^{M} I_k^s$ is the total suppressed interference due to the existence of other EZs. We calculate I_{OU}^k and I_k^s following the model in [2]. The expected interference I_{OU}^k, caused by OUs outside EZ_m, is calculated at a point y on the boundary of PZ_k. The interference effect could be partitioned to circles, where each circle $A = B(y, r)$ is with center y and a different radius r that ranges from $r_{2m} - r_{1m}$ to ∞ (Figure 2). The density of OUs on a circle A is $\lambda = \sum_{m=1}^{M} \lambda_m / A_H$, where λ_ms are independent and, according to [8], they are equivalent to normal distribution in a two-dimensional space. The density λ_m is scaled down by A_H to indicate the actual number of active OUs. For every value of r, A has an average number of OUs = $2\pi r \lambda$ and each OU on this circle produces interference $P_o r^{-\alpha}$, where α is the propagation coefficient and P_o is the OU power and we assume it is constant for all OUs. Therefore, the aggregated interference I_{OU}^m due to all A circles on y can be obtained by integrating over-all the values of r:

$$I_{OU}^m = \int_{r_{2m} - r_{1m}}^{\infty} G P_o r^{-\alpha} \lambda 2\pi r \; dr, \tag{4}$$

where G is the OU's channel gain, and $G = 1$ without loss of generality. According to [4], we calculate I_{a_k} as the interference from the center of EZ_k to the nearest boundary point on PZ_m multiplied by the area of EZ_k. The closed-form expressions of I_{OU}^m and I_{a_k} are given as follows:

$$\begin{aligned} I_{OU}^m &= P_o \lambda \pi \Big[\frac{0.502 \pi r_{1m}^3 - 0.5 r_{1m}^2}{(r_{2m} r_{1m}^3 + r_{2m}^3 r_{1m})} + \frac{2(r_{2m} + r_{1m})^{2-\alpha}}{\alpha - 2} \\ &+ \frac{(1.645 r_{2m} r_{1m} - 0.5 r_{2m}^2) \ln(\frac{r_{2m} - r_{1m}}{r_{1m} + r_{2m}})}{(r_{2m} r_{1m}^3 + r_{2m}^3 r_{1m})} \Big] \end{aligned} \tag{5}$$

$$I_k^s = \frac{\pi \lambda r_{2k}^2}{(d_{mk} - r_{1k})^{\alpha}} \tag{6}$$

where d_{mk} is the distance between IU_m and IU_k.

C. Privacy

IUs aim at maintaining their location privacy by introducing PZs. IUs' privacy can be violated by the OUs curious about the operational details of the IUs. If a traditional EZ is adopted (IU is in the center of EZ without PZ), with many queries, the OUs will be able to accurately estimate the IU location through an "intersection attack" [9]. That is, for every query that returns '0' answer, it means that IU is located within a circle with radius equal to the transmission range of the OU (up to an error margin), centered at the OU's location. We adopt conditional entropy to quantify the uncertainty given the information available to the OUs, i.e., the EZ and PZ uploaded by each IU. Note that the EZ does not provide any additional information about x, since by design, we know that the IU cannot reside in the region between the PZ and EZ. Thus, it is safe to say that for IU_m, where $m \in \{1, \cdots, M\}$, the conditional entropy of $H(X_m | EZ_m, PZ_m) = H(X_m | PZ_m)$:

Definition 1 (Privacy ($\mathcal{P}$)). *Let random variable X_m represents IU_m's location. The privacy $\mathcal{P}$ of IU_m is the $H(X_m | PZ_m)$: the uncertainty of X_m given PZ_m is known.*

When X_m has a uniform prior distribution, $H(X_m | PZ_m)$ is maximized when:

$$P(X_m = x | PZ_m) = \frac{1}{|PZ_m|}, \forall x \in PZ_m, \tag{7}$$

where $|PZ_m|$ is the size of the PZ_m. In other words, by publishing the PZ_m, the IU's location is indistinguishable among all the locations in the PZ_m. Note that $P(X_m = x | PZ_m)$ is inversely proportional to the PZ_m size. Therefore, $H(X_m | PZ_m)$ increases with the PZ_m size.

III. Game Formulation and Equilibrium

A. The Non-cooperative IU game

Assume that IUs are rational users with the main objective of maximizing their payoffs. IUs cannot decide their EZs' radii arbitrarily. If IU_m wants to decrease the interference it receives from the OUs, it can increase its EZ radius, r_{2m}. In this case, the EZ could continue to expand until no OUs can transmit. This decreases the OUs' transmission opportunities, for all OU groups, as well as the revenue of the other IUs. Therefore, IU_m's EZ decision affects the other IUs as well. If an IU increases its EZ radius, OUs from other groups will be silenced which affect the other IUs utilities: 1) the IUs' payments will decrease 2) their communication capacity will sub-linearly increase. If an IU_k solely decides to decrease its EZ, to get more payments from $\mathcal{G}_k$, other IUs will get more payments as well because of the new active OUs. However, they will experience more interference. Therefore, there is a tradeoff between the IUs' utility and interference they experience. The larger the EZ the larger the suppressed interference and less revenue and vice versa. This tradeoff cannot satisfy all the IUs without having an implicit agreement, i.e., equilibrium.

We formulate the interaction between the IUs as a non-cooperative game [10]. Our game consists of the following:

1) *Players*: the IUs.
2) *Actions*: the action of a player IU_m is the EZ radius r_{2m} which has continuous support, i.e., $r_{2m} \in [r_{1m}, d_{mk} - r_{1k}]$, where IU_k is the nearest IU.
3) *Payoffs (Utilities)*: a weighted sum of all the payments that IU_m receives from the OUs and its transmission capacity. The utility of IU_m is written as follows:

$$U_m = \omega_m \lambda_m (\mathcal{A} - \sum_{k=1}^{M} \pi r_{2k}^2) C_{OU} + C_{IU_m}, \tag{8}$$

where $\mathcal{A}$ is the total area which we assume to be very large to decrease the border effect [8]. The first term (T_p) represents the payments that the IU gets from the OUs. This payment is a fee (can be virtual money) that the OU give to the IU in return for granting them spectrum access. We calculate N_m by multiplying λ_m by the EZ-free area $A_f = (\mathcal{A} - \sum_{k=1}^{M} \pi r_{2k}^2)$, where $\sum_{k=1}^{M} \pi r_{2k}^2$ is the sum of all EZ areas. The second term (T_c) represents the transmission capacity of IU_m. We notice that T_p is large, compared to T_c, as it represents the effect of a lot of OUs transmitting data. However, all OUs cannot transmit at the same time due to interference. Therefore, we introduce $\omega_m = \frac{\beta}{\lambda_m A_H}$ as weight to get the actual number of OUs that can simultaneously transmit which equals $\frac{A_f}{A_H}$, where $A_H = 2\sqrt{3}R^2$ is the area of the hexagon that encloses the transmission range circle of an active OU [11], R is the radius at which the OU power reaches P_{min}, and β is a weight that tunes down T_p.

Our game is continuous as its action space has continuous support. Also, the players have complete information which means that IUm knows the PZ radii, the transmission powers, λ_m $\forall m$, etc of the other IUs. Information exchange is attained by a common channel between the IUs. Finally, the game is symmetric because the pay-off functions take the same form.

B. Nash Equilibrium

NE is a stable state of a game involving the interaction of different players, in which no player can increase its payoff by unilaterally changing its action. If the following conditions can be satisfied, a continuous game has at least one NE: (1) the action set is compact and convex; (2) the utility function is continuous and concave [12].

Theorem III.1. *Our proposed game has a NE,*
Proof:

1) The action set is compact as it is closed and bounded. The action set is convex as it is continuous linear range.
2) The utility function is continuous as it has continuous support and it is defined over-all the points. The utility function is concave because: (a) the term $\omega_m \lambda_m \mathcal{A} C_{OU}$ is linear; (b) the term $-\omega_m \lambda_m C_{OU} \sum_{k=1}^{M} r_{2k}^2$ is concave; (c) C_{IU_m} is concave as it is a concave function of a monotonically decreasing function $\mathcal{I}_m$ [13] □

Hence, our game has a NE which can be obtained in two steps. The first step is obtaining the best response of each player IU_m, noted by $BR_m(r_{2m})$, by differentiating each utility function U_m by the decision r_{2m} and equaling it to zero ($\frac{\delta U_m}{\delta r_{2m}} = 0$) which yields:

$$r_{2m} = \frac{1}{\omega_m 2\pi \lambda_i C_{OU}} \frac{-P_m H_m \mathcal{I}_m'}{(N + \mathcal{I}_m)(\mathcal{I}_m + N + H_m P_m)} \tag{9}$$

Note that (9) is a nonlinear ordinary differential equation. Therefore, to simplify (9), we solve the differential equation and the solution [14] is given by

$$\mathcal{I}_m = -N + \frac{P_m H_m K_m}{e^{r_{2m}^2 / 2C_m}}, \tag{10}$$

where $C_m = \frac{1}{\omega_m 2\pi \lambda_m C_{OU}}$ and K_m is an integration constant we obtain from the initial conditions. Consider the initial condition $r_{2m} = r_{1m}$ which causes maximum interference I_{max}^m on PZ's boundary [2]. Then, at the initial condition, we have:

$$K_m = e^{\frac{r_{1m}^2}{2C_m}} \frac{I_{max}^m - \sum_{m \neq k}^{M} I_k^s + N}{I_{max}^m - \sum_{m \neq k}^{M} I_k^s + N + P_m H_m} \tag{11}$$

We can use the simplified version (10) instead of (9) as BR_m. The IUs decide their own r_{1m} based on their privacy preferences and exchange information like r_{1m},λ_m,etc. The second step is jointly solving (12) and (13) which results in a NE point. In other words, IU_m obtains the value of the Nash-optimal r_{2m}^* to operate on. For example, we consider the case of two players IU_1 and IU_2. Then, we use (3) and (6) and (10) with $\alpha = 3$ to get expressions of r_{21} in terms of r_{22} and vice versa as follows:

$$r_{21}^{*^2} = \frac{(d_{12} - r_{11})^3}{(\lambda_1 + \lambda_2) P_o \pi} (N + I_{OU_1(r_{22})} - \frac{K_1 H_1 P_1}{e^{r_{22}^2 / 2c_1}}), \tag{12}$$

$$r_{22}^{*^2} = \frac{(d_{21} - r_{12})^3}{(\lambda_1 + \lambda_2) P_o \pi} (N + I_{OU_2(r_{21})} - \frac{K_2 H_2 P_2}{e^{r_{21}^2 / 2c_2}}), \tag{13}$$

However, solving (12) and (13) simultaneously results in a quintic (5th order) equation which does not have an explicit solution. Therefore, we solve this system of equations numerically by finding an intersection point between them which gives us the NE point (we explain that in Section IV).

This method (NE-exact) gives the exact solution to our game. However, this method is very hard to be used beyond two IUs for the following reasons. First, simultaneously solving (12) and (13) gives a two quintic equations and for three IUs, we have three 15th order equations which doesn't have a closed-form solution. Second, we solve the best response equation by intersection which is a form of exhaustive search and its complexity grows exponentially with the number of IUs Third, NE-exact requires that the IUs exchange a lot of information which increases communication overhead.

C. Nash Equilibrium for Multiple IUs

For the case of more than two IUs, we use a sub-gradient algorithm, which has several merits, we propose using the sub-gradient algorithm (SGA) [15] to solve the problem of multiple IUs which has several merits. First, the computational complexity of SGA is $O(1/\epsilon^2)$ [16] iterations which asymptotically depend on the target tolerance ϵ and not M like NE-exact. Second, the SGA has low overhead as it only requires, at each iteration, the exchange of M EZ values updates.

The NE-SGA method proceeds as follows. First, all the IUs initialize their EZ radii (r_{2m} $\forall m$) by the value of their PZs (r_{1m} $\forall m$). Second, given the actions of the other IUs, each IU_m increases/decreases r_{2m} if the increasing/decreasing action increases its individual utility value, and if not, r_{2m} should stay unchanged. The amount of decrease and increase at iteration t for IU_m is dictated by the value $\delta\nabla(U_m)$, where δ is the step size, ∇ is the gradient of the differentiable utility function U_m. Then, algorithm goes on until the convergence criteria is achieved and NE is reached. The SGA method is summarized in **Algorithm** 1.

The SGA algorithm has been proved to converge to $L^2\delta/2$ gap (L is Lipschitz constant), at most, from the optimal

Algorithm 1: Sub-gradient Algorithm (SGA)

Each IU$_m$ initializes r_{2m}^0 by r_{1m}.
while $|U_m(r_{2m}^t) - U_m(r_{2m}^{t-1})| <= \epsilon$ **do**
  for $m = 1, \cdots, M$ **do**
    - IU$_m$ stores the current value of the EZ radius.
    - IU$_m$ calculates:

$$r_{inc} = r_{2m}^t + \delta\nabla(U(r_{2m}^t)),$$

$$r_{dec} = r_{2m}^t - \delta\nabla(U(r_{2m}^t)),$$

    if $(U_m(r_{dec}), U_m(r_{2m}^t)) \leq U_m(r_{inc})$ **then**
      -$r_{2m}^{t+1} = r_{inc}$
    else
      if $U_m(r_{inc}) \leq U_m(r_{dec})$ **then**
        -$r_{2m}^{t+1} = r_{dec}$
      else
        -$r_{2m}^{t+1} = r_{2m}^t$
      end if
    end if
    -IU$_m$ inform the other IUs about r_{2m}^{t+1}.
  end for
end while

solution and it converges to the optimal solution if $U_m(r_{2m})$ is differentiable, see [16]. Furthermore, in our work, it almost converges to the exact NE in less than 15 iterations. Since implementation shows that SGA has perfect convergence, we choose $\epsilon = 0$ as our stoppage criteria. For the step size δ, we choose it to be around 200 m as this value guaranties the fastest convergence and the nearest utility value to the NE-exact.

IV. Simulation Results

We simulate a system consists of 16 IUs randomly placed on the positions of base stations in a suburban area. If it is not specified, the densities $\lambda_m = 0.01$ OU/m^2, $P_m = 30$ dBm, each OU transmits with constant power $P_o = 20$ dbm, the weights are similar $\omega_m = 10^{-4}$ and $r_{1m} = 500$ m. The game takes place in a large area $\mathcal{A} = 60$ km^2 and the players exist around $\mathcal{A}$'s center to avoid the boundary effect. For NE-exact, the simulation happens as follows. First, for each IU, the best response function is calculated and simplified according to (10) and (11) and the best response equation in (12) and (13) are published. Then, each IU plots (12) and (13) on the range $[r_{1m}, d_{12}]$, where d_{12} is the distance between IU_1 and IU_2 , and find the intersection between the two curves which gives the NE point. For NE-SGA, more than two IUs, we follow the steps of **Algotithm**1 and the parameters provided in Section IV. For the joint optimization (JO) case (only adopted for comparison), all the IUs send their information to an IU$_c$ which solves the following problem to get the optimal global solution r_{2m} $\forall m$.

$$\max_{r_{2m}\ \forall m} \quad \sum_{m=1}^{M} U_m \quad st. \;\; r_{1m} \leq r_{2m} \ \forall m, \tag{14}$$

A. The Exact NE for Two IUs

Figure 3 shows the intersection between the best response equations to get the NE point. When $r_{11} = r_{12}$ and all the parameters are identical, the value of NE point becomes the same for both IUs. For the case when the IUs select different PZ sizes ($r_{11} \geq r_{12}$), increasing the r_{11} causes an increase in the interference on the PZ boundary which results in a super-linear decrease in IU$_1$'s capacity. Therefore, r_{21}^* becomes larger which makes IU$_2$ suffer from payment decrease due to the enlargement of the EZ$_1$. However, this enlargement benefits IU$_2$ in terms of suppressed interference. Therefore, EZ$_2$ would decrease slightly or remain unchanged.

In Figure 4, we show the behaviour of NE against different values of identical PZ radii which results in identical EZs as well. We note that increasing the PZ size causes an increase to EZ's size to cancel the excess interference on PZ's boundary.

Figure 5 shows the utility convergence of NE-SGA and JO (obtained by line search [13]) and compares them to the exact NE, for the case of 2 IUs. We can see that JO performs better than NE-exact and NE-SGA because JO takes into account all the parameters jointly which produces a solution that jointly maximizes the utilities of both IUs. We can see also that there is a small gap between the NE methods and JO. Furthermore, NE-exact and NE-SGA are almost the same with NE-SGA having the merit of less computational complexity. We notice also that for a higher weight value the gap between NE-SGA and the JO becomes smaller and the utility value becomes higher, and the algorithm converges faster. This happens because the payment term T_p becomes higher which means having smaller EZs, a quadratic increase in the payments, and hence more interference which super-linearly decreases the capacity.

B. The Sub-gradient Algorithm for Multiple IUs

Figure 6 shows the convergence of the EZ radius for different schemes($M = 16, \omega = 10^{-4}$). We can see that JO has the smallest EZ as r_{2m} is obtained optimally and the utilities of all IUs are considered jointly. However, NE-SGA waits for the approval of all IUs on the equilibrium decision that triggers no deviation, consequently, convergence takes much more iterations, and more EZ radius, to be reached.

In Figure 7, we plot the average IUs utility versus their number. We can see that average utility decreases as having more IUs requires having more EZs, and hence, less over-all payments from the OUs. We can see also that the gap between NE-SGA and JO becomes bigger as the number of IUs increases. NE-SGA consumes a bigger area to approximates JO and this results in bigger EZs4, as M increases, and hence lower overall utilities for NE-SGA.

Figure 8 shows how the utility changes with different combinations of ω for three IUs with PZs= 1000m, 800$m$and$b$ 500m. We see that having high $\omega = 10^{-4}$ for IUs gives higher utility as T_p dominates the utility. However, if we have low $\omega = 0.5 * 10^{-4}$, for all IUs, T_c dominates the utility, the EZs gets bigger and the overall utility decreases. We notice that JO always does better with the smallest PZ (the first and second groups, third bar) as it produces the

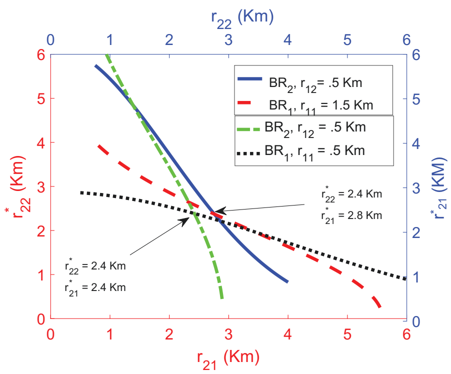

Figure 3: NE point.

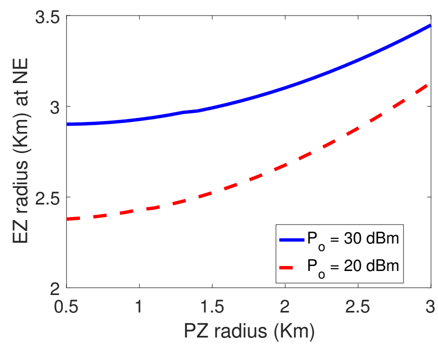

Figure 4: NE point for diff PZ sizes.

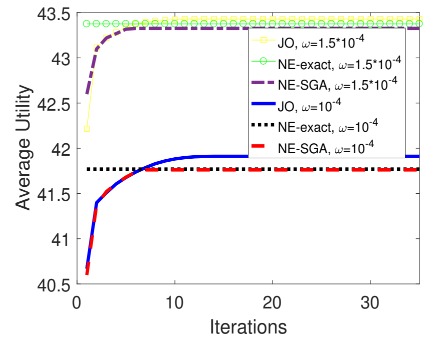

Figure 5: Utility convergence.

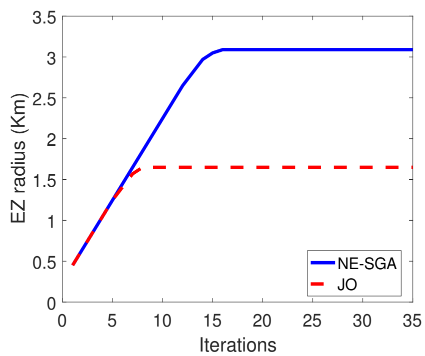

Figure 6: EZ convergence.

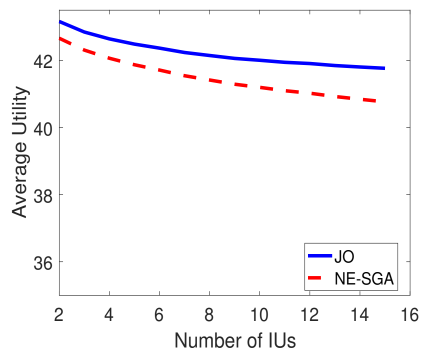

Figure 7: Utility vs. M

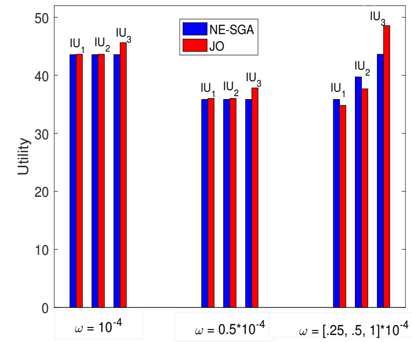

Figure 8: Utility for different ω

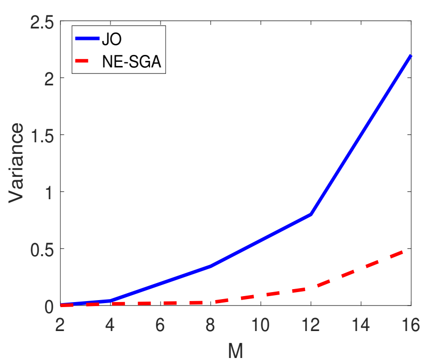

Figure 9: Utility variance vs. M

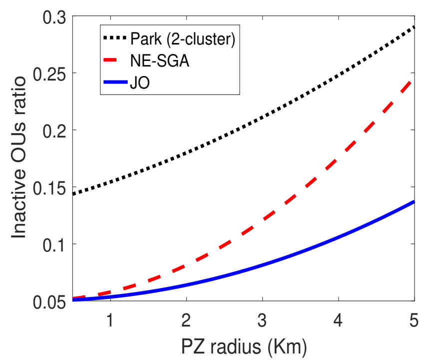

Figure 10: Utility comparison.

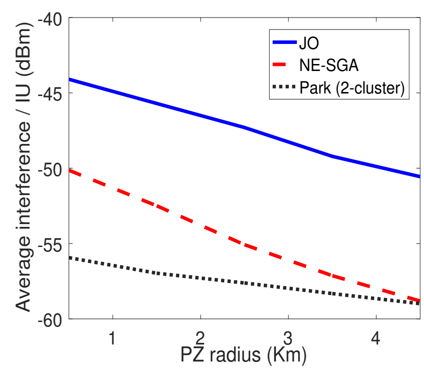

Figure 11: Interference comparison.

smallest EZ and hence better utility. For the third group, we fixed the PZs and had the following weight combination $\omega = [0.25, 0.5, 1] * 10^{-4}$. We can see that as the dependency on T_p decreases (lower ω) NE-SGA performs better as it produces bigger EZs. Note that, although JO could be outperformed in some cases, it has a better over-all utility, but obviously higher variance.

Figure 9 shows the variance of utility values for different number of IUs (we fix the weights and vary the PZs). We can see utility variance increases with the number of IUs. We notice that the utility variance of JO is higher as the EZs are optimized jointly to maximizes the overall sum of the utilities and doesn't take into consideration the differences among the IUs' utilities. However, NE-SGA produces a more homogeneous utility values, which endorses the fairness among the IUs.

C. A comparison with the previous work

In Figure 10 and Figure 11, we compare NE-SGA and JO to the distinguished work done by Park *et. al.* [3]. In [3], each IU has a PZ=EZ (PEZ) and this PEZ is transfigured to add privacy, in case of no clustering (1-cluster). To add more privacy, the IUs are clustered with their proximity of Q IUs (Q-cluster). We can see that Park's scheme consumes a bigger EZs area out of the total area (compared to JO and NE-SGA) which gives higher privacy and more interference suppression in the expense of OUs' utility. On the other hand, NE-SGA has moderate utility loss compared to Park's scheme and better interference suppression compared to JO.

V. Related Works

Early centralized techniques focused on location obfuscation or spatial cloaking [5]. In this technique, the user location is made indistinguishable inside its PZ. Also, K-anonymity [17] was proposed to make the original location indistinguishable among other $K-1$ users. Andres *et al*. [18] proposed geo-indistinguishability, which is a variant of differential privacy [19]. Their mechanism adds two dimensional Laplacian distribution noise to a user's real location before uploading to an untrusted server. In this method, interference is not explicitly calculated or optimized. Finally, cryptographic techniques protect IU and/or OU locations by encrypting them. In [20] Dou *et al*. presented a privacy-preserving SAS design that protects IU's privacy through secure computation on the ciphertext domain based on homomorphic encryption so that the IU's EZ information is hidden from the SAS. Although these schemes achieve location privacy, they don't consider the OUs' interference to the IU. Also, they depend on a central node which compromises the location privacy. The high computation/communication overhead introduced by encryption and decryption is also a concern due to the centrality of the decision.

Game theory has been used in the literature to provide location privacy while ensuring the user's utility. In [21], Liu *et al*. propose a distributed dummy user generation method to grant users control over their own privacy protections. The authors formulate a Bayesian game to analyze the non-cooperative behavior of the users and identify the NE solutions. In [22], Shokri *et al*. propose a methodology that enables the design of optimal user-centric location obfuscation mechanisms respecting each individual user's service quality requirements, while maximizing its location privacy by formulating a Stackelberg Bayesian game. However, these approaches do not consider interference while trying to solve the games. Game theory has been used to study pure DSA problems. In [23], Sengupta *et al*. model the competition between the service providers, over users, using a multiple-bidder auction game and prove the existence of NE. In [24], Rawat *et al*. model the game between spectrum providers and secondary users as a two-stage Stackelberg game and they find the Stackelberg-NE. In [25], Xiao *et al*. the authors propose an inter-operator spectrum aggregation coalition game which allows two or more MNOs to cooperate and share their licensed bands to support a common set of service types. Although these works give promising solutions for spectrum allocation, using game theory, they do not consider privacy as an objective.

VI. Conclusion

In this paper, we formulated a non-cooperative continuous game with complete information to represent the competition between IUs. The NE of this game maximizes the individual utility of each IU, which is the weighted sum between the IU's communication capacity and the revenue it gets from OUs that belong to it. We used numerical simulations to find NE. Simulation results show that the NE-exact and NE-SGA has a very small gap with JO under similar parameters. Also, results show that our work outperforms the work in [3].

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