Constraining the External Capture to the ^{16}O Ground State and the E2 S Factor of the $^{12}C(\alpha,\gamma)^{16}O$ Reaction

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The $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction is one of the most crucial reactions in nuclear astrophysics. The E2 external capture to the ^{16}O ground state (GS) has not been emphasized in previous analyses but may make a significant contribution to the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ cross section depending on the value of the GS asymptotic normalization coefficient (ANC). In the present work, we determine this ANC to be 337 ± 45 fm $^{-1/2}$ through the $^{12}\text{C}(^{11}\text{B},^{7}\text{Li})^{16}\text{O}$ reaction using a high-precision magnetic spectrograph. This sheds light on the existing large discrepancy of more than 2 orders of magnitude between the previously reported ANC values. Based on the new ANC, we experimentally constrain the GS external capture and show that through interference with the high energy tail of the 2^+ subthreshold state, a substantial enhancement in the GS $S_{E2}(300)$ factor can be obtained (70 ± 7 keV b) compared to that of a recent review (45 keV b), resulting in an increase of the total S factor from 140 to 162 keV b, which is now in good agreement with the value obtained by reproducing supernova nucleosynthesis calculations with the solar-system abundances. This work emphasizes that the external capture contribution for the ground state transition cannot be neglected in future analyses of the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction.

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Introduction.—The nuclear reactions in stars are responsible for the formation of most of the naturally occurring elements. Tens of thousands of nuclear reactions can participate in a specific nucleosynthesis scenario, but only a small fraction of these reactions have a strong impact on the overall chemical evolution of the elements. One reaction of particular relevance is $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$. This reaction, together with the 3α process, determines the absolute abundance of carbon and oxygen that is the fundamental basis for all organic chemistry and for the evolution of biological life in our Universe [1–3]. Great efforts have been made in the past several decades that further our understanding of this fundamental reaction, but most estimates still find that we are far from the uncertainty of better than 10% required by stellar models [1,4]. To date, all direct measurements have been performed at energies higher than $E_{\rm c.m.}=891~{\rm keV}$ (see Refs. [5-7] and references therein) because of the extremely low cross section resulting from the small Coulomb penetrability at low energies. At temperatures of helium burning, the corresponding energy is 300 keV where the cross section is estimated to be on the order of 10^{-17} b. This is about 5 orders of magnitude below the sensitivity achieved by the most advanced measurements. Therefore, achieving a reliable extrapolation of the cross section from such higher energies to the Gamow window has been a long-standing challenge. The phenomenological R matrix [8–10] has long been the method used to extrapolate the cross section from the higher observed energies down to the astrophysical ones and it remains so in the latest state-of-the-art analyses.

Recently, it has been emphasized in the review by deBoer *et al.* [3] that the contribution from the high energy tail of the 2^+ subthreshold state and that of the external capture to the ground state (GS) interfere with one another and result in a similar energy dependence over the region of the currently available experimental data. This means that in any *R*-matrix fit, the GS asymptotic normalization coefficient (ANC) and the 2^+ subthreshold state ANC will

be highly correlated fit parameters. For example, Sayre *et al.* [11] demonstrated that the presently available *E*2 capture data can be well reproduced given a large enough value for the GS ANC. However, to do so requires an ANC for the 2^+ state that is substantially larger than those determined by the precise sub-Coulomb transfer reactions [12,13]. Currently, reported experimental values of the GS ANC range from 13.9 ± 2.4 to $3390 \text{ fm}^{-1/2}$ [11,14–16]. In light of this large discrepancy between GS ANC values and the consistent values for the 2^+ ANC determined through sub-Coulomb transfer reactions, a value of 58 fm $^{-1/2}$ for the GS ANC was adopted in that work [3].

In this Letter, we shed light on these discrepancies by reporting a GS ANC of ^{16}O with an uncertainty of 13% using the $^{12}\text{C}(^{11}\text{B},^7\text{Li})^{16}\text{O}$ transfer reaction for the first time. Based on the *R*-matrix fit parameters obtained by deBoer *et al.* [3], we make new *R*-matrix calculations to estimate the effect of our newly determined GS ANC. We find that it has a substantial impact on the extrapolation of the low energy $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ *S* factor, increasing the GS *E*2 *S* factor at 300 keV from 45 to 70 ± 7 keV b.

Experiment.—The ¹²C(¹¹B, ⁷Li)¹⁶O experiment was carried out at the HI-13 tandem accelerator national laboratory of the China Institute of Atomic Energy (CIAE) in Beijing, China. The experimental setup and procedures are similar to those previously reported [17–20]. A ¹¹B beam with an energy of 50 MeV was delivered and utilized to measure the angular distribution of the ¹²C(¹¹B, ⁷Li)¹⁶O reaction leading to the GS of ¹⁶O. The beam current was measured by a Faraday cup connected to a calibrated charge integrator. A self-supporting natural carbon target was used. Previously we used the Rutherford scattering cross sections on a Au target to evaluate the systematic uncertainty except for the ¹²C target thickness. In order to calibrate and monitor the thickness of the target, ¹¹B + ¹²C elastic scattering [17] was measured repeatedly during the experiment. The thickness of the target was determined to be $80 \pm 4 \,\mu \text{g/cm}^2$ and no obvious carbon buildup was found. When calibrating the target thickness with elastic scattering, the experimental setup was not altered from that of the present experiment measurements. Thus, the systematic uncertainties, comprised of the beam charge collection efficiency, the acceptance of the Q3D magnetic spectrograph, and the transport efficiency have already been included in the uncertainty of target thickness. The reaction products were separated and focused by the Q3D magnetic spectrograph and detected by a two-dimensional position sensitive silicon detector (X1) fixed at the focal plane. The two-dimensional position information from X1 enables the products emitted into the acceptable solid angle to be completely recorded, and the energy information was used to remove the impurities with the same magnetic rigidity. As an example, Fig. 1 displays the particle identification diagram of ^7Li at $\theta_{\text{lab}} = 8^\circ$ from the $^{12}\text{C}(^{11}\text{B}, ^7\text{Li})^{16}\text{O}_{\text{g.s.}}$

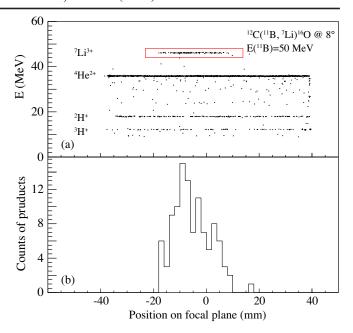


FIG. 1. Focal-plane position spectrum of ${}^7\mathrm{Li}$ at $\theta_{\mathrm{lab}} = 8^{\circ}$ from the ${}^{12}\mathrm{C}({}^{11}\mathrm{B},{}^7\mathrm{Li}){}^{16}\mathrm{O}$ reaction. (a) Two-dimensional spectrum of energy vs focal-plane position. (b) Spectrum gated by the ${}^7\mathrm{Li}$ events in (a). The alpha, deuterons, and tritons are produced by the multibody breakup of the incident ${}^{11}\mathrm{B}$ ions on the ${}^{12}\mathrm{C}$ target.

reaction. In Fig. 2, we display the angular distribution of the ¹²C(¹¹B, ⁷Li)¹⁶O reaction leading to the GS of ¹⁶O.

Analysis.—The ANC for the ^{16}O GS is extracted by normalizing finite-range distorted wave Born approximation (DWBA) calculations to the experimental data. The DWBA calculations are made with the computer code FRESCO [21]. Model parameters required in these calculations include the optical model potentials (OMPs) for the entrance and exit channels, the core-core ($^7\text{Li} + ^{12}\text{C}$) interaction, the binding potentials for the ($^{11}\text{B} = \alpha + ^{7}\text{Li}$) and ($^{16}\text{O} = \alpha + ^{12}\text{C}$) systems, and the ANC for the ^{11}B GS. We determine the OMPs for the entrance channel, the exit

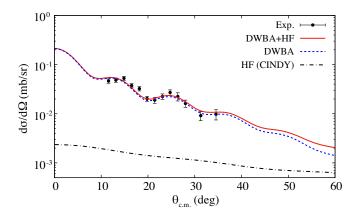


FIG. 2. Angular distribution of the $^{12}\text{C}(^{11}\text{B}, ^{7}\text{Li})^{16}\text{O}$ reaction leading to the GS of ^{16}O . The black dashed-dotted line denotes the compound nucleus contribution.

channel, and the core-core interaction with a single-folding model [22,23] and the binding potential for the ($^{11}\text{B} = \alpha + ^7\text{Li}$) system with a method similar to our previous work [17]. The $^7\text{Li} + ^{16}\text{O}$ elastic scattering data are taken from Schumacher *et al.* [24]. The uncertainties coming from these parameters are reevaluated with the present data. The ANCs for the ^{11}B GS are taken to be 117 ± 8 and 63 ± 4 fm $^{-1/2}$, respectively, for $3S_0$ and $2D_2$ components from Shen *et al.* [18], where the $^7\text{Li}(^6\text{Li}, d)^{11}\text{B}$ angular distribution was measured and analyzed.

The binding potential parameters $(r_0 \text{ and } a)$ for the $(\alpha + {}^{12}\mathrm{C})$ system are constrained by a minimum- χ^2 fitting to the present experimental ¹²C(¹¹B, ⁷Li)¹⁶O_{g,s.} angular distribution. The resulting parameters are $r_0 = 1.00$ and a = 0.65 fm. We investigate the dependence of the extracted ANC on these potential parameters within the range of radius r_0 (0.98–1.015) and diffuseness a (0.57– 0.71 fm) selected by the minimum- $\chi^2 + 1$ principle (see, e.g., Ref. [25]). The impact of this change on the ANC is found to be 7.5%, indicating a good peripheral nature for the ¹²C(¹¹B, ⁷Li)¹⁶O transfer reaction. We also use another typical method to constrain the binding potential by reproducing the root-mean-square (rms) radius of the α -cluster wave function, as reported in our previous works [17,18,20]. The rms radii of ${}^{4}\text{He}$, ${}^{12}\text{C}$, and ${}^{16}\text{O}$ are taken to be 1.47 ± 0.02 [26], 2.481 ± 0.080 [27], and 2.631 ± 0.080 0.061 fm [27], respectively. This method confirms the minimum- χ^2 constraint by yielding consistent r_0 and aalthough a larger uncertainty is found when propagating the errors of these radii.

The compound nuclear (CN) calculation is performed using the Hauser-Fesbach (HF) code CINDY [28], which has been applied in our previous work [17]. The calculation requires the optical potentials for the entrance and exit channels, which are kept the same as those in the DWBA calculation described above. The contribution from the CN process is found to be small (less than 3% on the GS ANC). The DWBA and CN calculations for the ¹²C(¹¹B, ⁷Li)¹⁶O_{g,s.} reaction are shown in Fig. 2. One sees that the DWBA calculation reasonably reproduces the experimental data, which presents strong evidence of the direct nature of the ¹²C(¹¹B, ⁷Li)¹⁶O reaction at this energy. The ANC for the ¹⁶O GS is extracted to be $337 \pm 45 \text{ fm}^{-1/2}$ by normalizing the DWBA calculation to the experimental angular distribution after the subtraction of the CN contribution. The uncertainty for the GS ANC mainly results from the OMPs for the entrance and exit channels (1.4% and 0.9%), the binding potentials for the (${}^{11}B = \alpha + {}^{7}Li$) and (${}^{16}O = \alpha + {}^{12}C$) systems (1.9% and 7.5%), the ANC of ${}^{11}B$ (10.2%), the target thickness (2.5%) and the statistics (2.3%).

Four independent investigations, in addition to the present work, have been performed previously to study the GS ANC. Adhikari and Basu [14] found a very small ANC of 13.9 ± 2.4 fm^{-1/2} by analyzing the ¹⁶O breakup

TABLE I. Present ANC of the ¹⁶O GS and other available results in the literature.

Reference	ANC (fm ^{-1/2})	Method
Adhikari (2009) [14]	13.9 ± 2.4	¹⁶ O + Pb breakup
Morais (2011) [16]	3390 (WS1)	$^{12}\text{C}(^{16}\text{O}, ^{12}\text{C})^{16}\text{O}$
	1230 (WS2)	
	750 (FP)	
Sayre (2012) [11]	709	R matrix
Adhikari (2017) [15]	637 ± 86	$^{12}\text{C}(^{7}\text{Li}, t)^{16}\text{O}$
Present	337 ± 45	$^{12}C(^{11}B, ^{7}Li)^{16}O$

on ²⁰⁸Pb. Subsequently, they updated the GS ANC to be $637 \pm 86 \text{ fm}^{-1/2}$ via the $^{12}\text{C}(^{7}\text{Li}, t)^{16}\text{O}$ reaction using silicon detector telescopes [15]. Morais and Lichtenthäler [16] investigated the GS ANC by analyzing the elastic transfer reaction of ¹²C(¹⁶O, ¹²C)¹⁶O. They derived the GS ANC to be 750, 1230, and 3390 $fm^{-1/2}$ using three sets of the binding potential, and claimed that such a significant sensitivity is probably due to the fact that this reaction cannot be considered a peripheral reaction. In addition, Sayre et al. [11] included the E2 external capture in their Rmatrix fit to the E2 capture data and found the GS ANC to be 709 $fm^{-1/2}$. In Table I we list the ANC values from the present work and from the literature sources mentioned above. It is well known that the most important region to extract the ANC reliably is at most forward angles where the pole mechanism dominates [29]. The previous measurements [15,16] presented the transfer reaction angular distributions at wide angles, however they lack sufficient data at the most forward angles. This work focuses on the measurement of the transfer reaction angular distribution at most forward angles by using a high-precision magnetic spectrograph, and thus determines the GS ANC value with an uncertainty of 13% due to the constraint on the binding potential with the minimum- χ^2 fitting to the present experimental data and the peripheral nature of the ¹²C(¹¹B, ⁷Li)¹⁶O reaction.

R-matrix calculations.—The full implications of our new determination of the GS ANC in ¹⁶O will require a full *R*-matrix reevaluation similar to that presented in deBoer *et al.* [3] that is beyond the scope of the present work. In order to make an initial estimate of its effect, *R*-matrix calculations have been performed based on those reported in deBoer *et al.* [3] using the code AZURE2 [9,30]. In that work, a smaller value of the GS ANC of 58 fm^{-1/2} was adopted considering the value of 13.9 ± 2.4 fm^{-1/2} obtained by Adhikari and Basu [14] and by the precise and consistent values of the 2^+ ANC reported by Brune *et al.* [12] [(1.14 \pm 0.10) \times 10⁵ fm^{-1/2}] and Avila *et al.* [13] [(1.22 \pm 0.07) \times 10⁵ fm^{-1/2}].

In the present work the GS ANC is found to be significantly larger than that adopted in deBoer *et al.* [3].

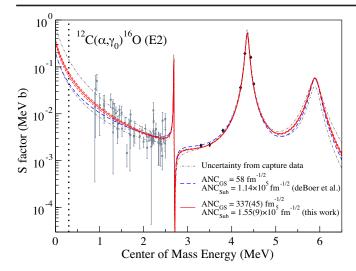


FIG. 3. Comparison of *R*-matrix calculations for the GS E2 component of the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reaction. The blue dashed line indicates the best fit *S* factor from deBoer et~al. [3], while the red solid and dashed lines indicate that of the present work as described in the text. The black data points represent those of Schürmann et~al. [37], while the lower energy data of Fey [5], Schürmann et~al. [37], Assunção et~al. [38], Ouellet et~al. [39], Kunz et~al. [40], Redder et~al. [41], Roters et~al. [42], Makii et~al. [43], and Plag et~al. [44] are indicated by gray points. The vertical dotted line denotes the representative energy of astrophysical interest of $E_{c.m.}=300$ keV. The gray dashed-dotted lines indicate the range of uncertainty given only by the constraint of the E2 reaction data.

However, the GS ANC and 2⁺ subthreshold state ANC are highly correlated R-matrix fit parameters. That is, if the value of one is increased (decreased) the other can be increased (decreased) to produce a nearly identical S factor over the region of the experimental data. Only at the lowest energies of the observed data does the S factor begin to diverge, and over these energies the experimental uncertainties are large in comparison as shown in Fig. 3. The uncertainty in the low energy S factor extrapolation, based only on the constraint of the E2 capture data, is indicated in Fig. 3. This presents a challenge to future low energy GS E2 S factor measurements, to reach a level of precision where the data can better differentiate between these two reaction components. For example, proposed measurements using the inverse ${}^{16}\text{O}(\gamma,\alpha){}^{12}\text{C}$ [31] and $^{16}O(e, e'\alpha)^{12}C$ [32] reactions estimate such improved levels of uncertainty [33,34] and future direct measurements at underground laboratories like JUNA [35] and LUNA [36] will also aim to greatly reduce the uncertainty in the low energy S factor.

When our value of 337 ± 45 fm^{-1/2} is used for the GS ANC, a nearly identical reproduction of the *S* factor compared to that given in deBoer *et al.* [3] can be obtained by increasing the 2⁺ subthreshold ANC to $(1.55 \pm 0.09) \times 10^5$ fm^{-1/2} as shown in Fig. 3. As summarized in Fig. 4,

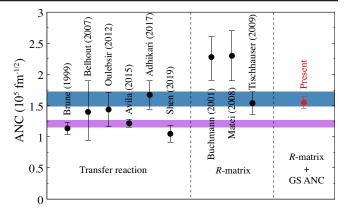


FIG. 4. Comparison of recent values of the 2^+ subthreshold ANC in $^{16}{\rm O}$ obtained from R-matrix fits to $^{12}{\rm C}(\alpha,\alpha)^{12}{\rm C}$ data [47], $^{12}{\rm C}(\alpha,\gamma)^{16}{\rm O}$ 6.92 MeV cascade data only [48] or global fitting [49], none of which include constraints on the ANCs from transfer measurements, or transfer reaction measurements using $^{12}{\rm C}(^7{\rm Li},t)^{16}{\rm O}$ [12,13,15,45], $^{12}{\rm C}(^6{\rm Li},d)^{16}{\rm O}$ [12,46], and $^{12}{\rm C}(^{11}{\rm B},^7{\rm Li})^{16}{\rm O}$ [17] reactions. The red point indicates the value deduced from the present GS ANC of $^{16}{\rm O}$ and the low energy E2 capture data as discussed in the text. The blue and purple bands represent the weighted average of the ANCs from the R-matrix analyses including the present result and the one of the ANCs from six measurements of transfer reactions, respectively. See Table XIII of deBoer et~al. [3] for comparisons to additional measurements that lack uncertainty estimates.

this value for the 2^+ subthreshold ANC is significantly larger than the precise sub-Coulomb transfer reaction values obtained by Brune *et al.* [12] [$(1.14 \pm 0.10) \times 10^5 \text{ fm}^{-1/2}$] and Avila *et al.* [13] [$(1.22 \pm 0.07) \times 10^5 \text{ fm}^{-1/2}$] [and more recently by Shen *et al.* [17], (1.05 ± 0.14) $\times 10^5 \text{ fm}^{-1/2}$] but is consistent with the transfer measurements of Belhout *et al.* [45], Oulebsir *et al.* [46], and Adhikari *et al.* [15], where larger uncertainties are reported. Further, at very low energies, there is a substantial enhancement to the *E2 S* factor, rising to a value of $70 \pm 7 \text{ keV}$ b at 300 keV compared to the value of 45 keV b found in deBoer *et al.* [3].

The value that our GS ANC implies for the 2^+ subthreshold ANC is also consistent with the most recent R-matrix analyses where the 2^+ ANC was extracted from $^{12}\text{C}(\alpha,\alpha)^{12}\text{C}$ scattering data [47,50]. In addition, it was noted in deBoer et~al. [3] that there was a significant tension between the larger value of the 2^+ ANC favored by the R-matrix fit to the scattering data versus those values reported in the sub-Coulomb transfer measurements of Brune et~al. [12] and Avila et~al. [13].

Another point of interest is that the E2 capture data that most constrain the correlated values of the GS ANC and the 2^+ subthreshold ANC in the R-matrix fit are the three lowest energy data points of Schürmann $et\ al.\ [51]$. These off-resonance data have significantly smaller uncertainties than any of the lower energy E2 data [5,37-44]. It should also be noted that the interference solution for the narrow above threshold 2^+ state of Sayre $et\ al.\ [11]$ has been

adopted (as also in deBoer et al. [3]), and that the above conclusions are somewhat dependent on this choice.

Given these results, the choice of the recent review of deBoer et al. [3] to rely solely on the sub-Coulomb transfer reaction for the 2+ ANC may be neglecting a large component of the GS E2 S factor uncertainty where S(300) was estimated to be 45 keV b and the total S factor was given as $140 \pm 21_{(MC)} ^{+18}_{-11 (model)}$ keV b. Our results reinforce the tension between the scattering data [47,50] and the sub-Coulomb transfer measurements for the 2⁺ ANC as shown in Fig. 4. Further, if the larger value of the E2 S factor of this work is combined with the E1 and cascade transition S factors from deBoer et al. [3], the total value of S(300) becomes 162 keV b. The new result is in agreement with the value of $170 \pm 20 \text{ keV b}$ [4] obtained by reproducing supernova nucleosynthesis calculations with the solar-system abundances and the value of 161 \pm $19_{(stat)^{+8}_{-2(syst)}}$ keV b reported by Schürmann $\it{et~al.}$ [51].

One of the basic assumptions of the R-matrix "best fit" of deBoer et al. [3] was that the ANCs of the 2⁺ subthreshold state given by Brune et al. [12] and Avila et al. [13] were precise and accurate. The model assumptions discussed in deBoer et al. [3] then explored the range of uncertainties when those basic assumptions were relaxed. Our new results highlight the growing discrepancy between ANCs determined from Coulomb transfer studies and those determined through other methods. Therefore, to fit into the uncertainty framework established in deBoer et al. [3], we recommend an increase in the upper model uncertainty of that work from 18 to 28 keV b, resulting in an updated estimate for S(300) of $140 \pm 21_{(MC)}{}^{+28}_{-11 (model)}$ keV b. It should be emphasized that the underlying uncertainty is really a bi-model distribution, but a quantitative mapping of this distribution would require a reevaluation of the global R-matrix analysis, which is beyond the scope of the present work.

Conclusion.—In this Letter we present a new determination of the GS ANC in ¹⁶O using the ¹²C(¹¹B, ⁷Li)¹⁶O transfer reaction for the first time. With this new value for the GS ANC, we perform R-matrix calculations illustrating the large impact that external capture has on the $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reaction which results in a substantially increased uncertainty over that given in a recent review [3]. This highlights the correlation between the GS ANC and the 2⁺ subthreshold state ANC and points to a growing discrepancy of the 2⁺ ANCs with different methods. This work finally finds a substantial increase for the GS E2 S factor from 45 [3] to 70 ± 7 keV b. The total S factor is then increased from 140 [3] to 162 keV b, which is in good agreement with the value of $170 \pm 20 \text{ keV} \text{ b}$ [4] from the supernova nucleosynthesis calculations by reproducing the solar-system abundances and the value of $161 \pm 19_{(stat)}^{+8}_{-2(syst)} \text{ keV b}$ reported Schürmann et al. [51].

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