Exploring student facility with “goes like” reasoning in introductory physics

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Covariational reasoning—reasoning about how changes in one quantity relate to changes in another quantity—has been examined extensively in mathematics education research. Little research has been done, however, on covariational reasoning in introductory physics contexts. We explore one aspect of covariational reasoning: “goes like” reasoning. “Goes like” reasoning refers to ways physicists relate two quantities through a simplified function. For example, physicists often say that “the electric field goes like one over r squared.” While this reasoning mode is used regularly by physicists and physics instructors, how students make sense of and use it remains unclear. We present evidence from reasoning inventory items which indicate that many students are sense making with tools from prior math instruction that could be developed into expert “goes like” thinking with direct instruction. Recommendations for further work in characterizing student sense making as a foundation for future development of instruction are made.

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I. INTRODUCTION

A perhaps unexpected byproduct of the COVID-19 pandemic is renewed clarity on how challenging it is for many to conceptualize the exponential function. This is certainly not novel; Albert Bartlett famously stated "The greatest shortcoming of the human race is our inability to understand the exponential function"[1]. This has become a public issue in the face of the coronavirus epidemic. Headlines such as "What Does Exponential Growth Mean in the Context of COVID-19?,"[2] "The Exponential Power of Now,"[3] and "Is Poor Math Literacy Making It Harder For People To Understand COVID-19 Coronavirus?"[4] have put conceptualization of function on the national stage.

It is evident that quantitative literacy—the set of skills that support the use of mathematics to describe and understand the world—is important, and lacking, in the United States today. Quantitative literacy has many facets, including reasoning about signed quantities, proportional reasoning and *covariational reasoning*—conceptualizing change in one quantity with respect to change in another quantity—and has been studied across both Mathematics and Physics Education Research [5–7]. Introductory physics, a broadly-required college course with a focus on quantifying and modeling nature, is an excellent place to address this need.

This paper describes a study of students' covariational reasoning in physics contexts. It contributes to the work in mathematics education, as well as to closing a gap in Physics Education Research (PER), where it has been shown that reasoning in physics contexts is different from reasoning in purely mathematical ones [8, 9]. We focus on one expert-like facet of physics covariational reasoning: "goes like" reasoning, which refers to the use of proportionality to illustrate relationships between two quantities (i.e. "The force *goes like* one over r squared") [10]. We present some of the ways an expert might use this reasoning, and preliminary results from two items on the Physics Inventory for Quantitative Literacy (PIQL) that suggest students have some productive resources and emergent abilities with "goes like" reasoning from prior math courses that are not yet coordinated with resources about physical quantities in some contexts. Recommendations for future work and instruction are made.

II. QUANTITATIVE LITERACY IN MATH AND PHYSICS

Proportional reasoning—reasoning about ratio as a quantity—has been identified as critical for success in physics by physics educators and in PER. Early PER focused on identifying specific reasoning difficulties such as the tendency to use additive, rather than multiplicative, strategies and the tendency of physics students to manipulate mathematical formalism without understanding the physical meaning of the associated quantities and operations [11–13]. By the early 1980's, studies in PER had begun to systematically document and extend this body of work by using individual demonstration interviews to explore student understanding of velocity as the ratio $\Delta x/\Delta t$ and acceleration as the ratio $\Delta v/\Delta t$ [14–16]. More recent work has examined the relationship between basic reasoning ability, including proportional reasoning, and the learning of physics content [17].

Work on the role and challenge of proportional reasoning in physics contexts has included attention to scaling and functional reasoning [13]. We build on this body of work by integrating the language of covariational reasoning established by Research in Undergraduate Mathematics Education (RUME) community [6, 18–22]. Covariation encompasses all functions that relate two or more quantities and considers multiple ways that one can think about those relationships. For example, one can consider discrete covariation (if the radius is doubled, what happens to the electric field at a point?), or continuous covariation (how does the field change smoothly as the radius is increased?) [6]. We suggest that proportional reasoning is a subset of covariational reasoning, focused specifically on linear relationships and using ratios that have meaning as a single entity (such as velocity and acceleration).

Physics educators regularly identify "thinking like a physicist" as a goal of introductory physics. In a 2019 study of the ways in which experts use covariational reasoning to solve introductory physics problems by Zimmerman, Olsho, Boudreaux, Loverude, and White Brahmia, it was noted that physics experts use functional reasoning by employing the "$\propto$" symbol or phrases like "goes like" to illustrate relationships in statements like Area $\propto r^2$, Force goes like $1/r^2$, etc. [10] This kind of "goes like" expert thinking is used to represent a wide variety of simplified relationships between quantities, and can be viewed through the lens of conceptual blending where physics experts are subconsciously blending together their reasoning about physics concepts and their reasoning about mathematical functions [23].

In his work on proportional reasoning, Arons asserts that the capacity for scaling and functional reasoning will not necessarily develop spontaneously [13]. Indeed, the need for curricular intervention is evident from the current literature. What is less clear is what resources and emergent abilities students *do have* regarding quantitative literacy prior to physics instruction, and what educators can do to build upon these skills to develop quantitative reasoning in their students.

III. EXPERT "GOES LIKE" REASONING

"Goes like" reasoning refers to simplifying and making sense of the covariational relationship between two changing quantities as a single function that illustrates the behavior of an evolving system. For example, consider a classic introductory physics problem: a ball thrown from a cliff. An expert might reason that if the ball's initial height is increased, the final speed of the ball will also increase. They might reason further that the final speed of the ball "goes like" the square root of the height. Here, "goes like" reasoning allows the expert to focus on the functional form of the relationship between two changing quantities, and to ignore any constants or pre-factors. This is a form of physics covariational reasoning as it describes how one quantity changes with respect to another in a simplified, functional way and in turn allows for efficient problem solving, as the expected behavior of the system can be quickly and clearly illustrated.

Expert use of "goes like" reasoning is the result of a conceptual blend between their facility with the mathematical functions involved and the physics content knowledge that enables experts to relate physical phenomena to those functions [23]. Zimmerman et. al found that physics graduate students have strong associations between certain routinely used physics quantities that allow them to make inferences about relationships between quantities in a given problem, termed "compiled relationships" [10]. This simplifies problems to those they can solve more efficiently, or to which they may already know the answer from experience [10]. Unlike novices, someone with substantial experience with physics is able to make claims such as "This problem involves a potential, which goes like $1/r$" or "This looks like scattering, so I expect it to be an exponential."

Novice physics students, in our experience, often also have useful compiled relationships that model real world contexts, and we consider these to be resources for physics learning. Many of these associations seem to evolve from prior math instruction and we see emerging evidence that students may resort to solving a physics problem by "doing math," meaning that the absence of a conceptual blend is an obstacle for effective problem solving. For example, where experts may associate circular motion with sinusoidal curves, introductory physics students may more readily associate trigonometric functions with right triangles.

This led us to wonder what resources students in an introductory physics course are using to relate two quantities, and whether they include "goes like" reasoning. In particular, we asked: do students enter introductory physics with reasoning about functional behavior already formed and ready to be applied from math courses? In addition, do their "goes like" resources improve after instruction in a physics class, where instruction typically takes the form of experts modeling their reasoning and discussing it in lecture? To answer these questions, we probed students' covariational reasoning using items from an inventory currently in development: the Physics Inventory for Quantitative Reasoning (PIQL) [7, 24].

IV. ASSESSING "GOES LIKE" REASONING

The PIQL is a reasoning inventory that measures fundamental aspects of mathematical reasoning that are ubiquitous in physics modeling. The instrument began with items targeting proportional reasoning and reasoning about signed quantities and has since grown to include items related to covariational reasoning more broadly. During its development, the PIQL has been administered over several years in a 3-quarter calculus-based introductory physics course at a large research university in the Pacific Northwest. It is given at the start of each of the quarters (Phys 121, 122, and 123), such that it serves as a pretest for each term and we are able to measure how students improve during the course of instruction. In this paper, we report on the results of two items from 918 students responses (N=326 in 121, N=309 in 122, and N=283 in 123) from the Winter 2020 Administration (PIQL 20W), which was given in-person during 50-minute class sessions.

In this paper, we focus on the results of two PIQL items aimed at assessing covariational reasoning: *Flag of Bhutan* and *Ferris Wheel*. Flag of Bhutan has been a validated item on the assessment for several years, originally taken with permission from previous work on proportional reasoning [25]. We will reframe the problem and results using the language of covariational reasoning and "goes like" thinking. Ferris Wheel is a new item on the PIQL, drawn from previous work on expert covariational reasoning in mathematics and physics [10, 26]. We will provide some analysis of the results from PIQL 20W as well as some insights on student "goes like" reasoning from recent validation interviews. Other aspects of quantification and PQL are also involved in these responses, but will not be discussed in this paper.

A. Flag of Bhutan

In the Flag of Bhutan question, students are asked what aspects of the flag would be larger by a factor of 1.5 if the length and the width were both increased by a factor of 1.5 (see Fig. 1). This item was originally designed as a scaling assessment to measure student facility with both linear and non-linear relationships, as some answer choices depend linearly on length and width (such as the length of the dragon's backbone, or the distance around the edge of the flag) and the answer choice "the amount of cloth needed to make the flag" depends on length times width [25]. While scaling was considered a facet of proportional reasoning at the time, it was understood by the researchers that scaling with non-linear functions is notably different than scaling with linear relationships. We believe that this question can be re-examined in the context of discrete covariation and "goes like" thinking.

One of the challenges of the Flag of Bhutan item as written on the PIQL is that it is a multiple-choice/multiple-response (MCMR) question. Thereby, its score is low compared to other items on the PIQL because these items are scored dichotomously for comparison with other multiple-choice/single-response items [27]. However, the nature of the item does not fully account for the significantly low number of completely correct responses (26% of all students) and this rate does not change significantly throughout the introductory sequence (25% in 121, 25% in 122, and 31% in 123) suggesting that students do not improve with instruction. The benefit of MCMR items is that the percentage of students that give partially correct answers—the student chooses *at least* one correct response and no incorrect responses—tells a more nuanced story. 74% of all students choose a combination of

FIG. 1. The Flag of Bhutan. The prompt for this item asks students to **select all of the following quantities** that are larger by a factor of 1.5 when the length and width of the flag are both increased by a factor of 1.5: (a) The distance around the edge of the flag, (b) the amount of cloth needed to make the flag, (c) the length of the curve forming the dragon's backbone, (d) the diagonal of the flag, and (e) none of these. Students are prompted to choose all answer choices that apply. We believe the correct answers to be (a), (c) and (d).

(a), (c), and/or (d) which suggests that most students have some productive resources about the functional relationship between length, width, and area that could be built upon.

By examining the partially correct responses, we can infer what kinds of resources students may be using. Only 24% of students do not choose (a), which we interpret to mean that the majority of students have facility with direct linear relationships (in this case, perimeter to length and width). In contrast, 55% of students do not choose (c) and 43% of students do not choose (d), suggesting that some students may not have yet developed the resources about more complex functional relationships, such as $\sqrt{l^2 + w^2}$, or those that do not have a known functional relationship, such as the dragon's backbone, even if the result is linear. Students do, however, tend to choose (c) and (d) together (66% of students either chose both or neither). This indicates that while the diagonal of the flag can be described by a geometric function and the backbone cannot, the majority of students are able to realize they have the same dependence on length and width. These early signs of "goes like" reasoning could be developed by coordinating the demonstrated student facility of linear functions to more complex relationships with direct instruction.

B. Ferris Wheel

Ferris Wheel asks students to choose an equation that represents how the height of a Ferris wheel cart changes as a function of the total distance it has traveled (see Fig. 2). This item was developed for the PIQL to directly assess "goes like" reasoning, inspired by the 2017 Hobson and Moore study on covariational reasoning in which expert mathematicians were given an animated version of a Ferris Wheel and asked to produce a graph that relates the total distance traveled by the cart and the height of the cart [26]. Variants of the Ferris Wheel problem have been used thoroughly and are well-validated in mathematics education research, and the Hobson and Moore study was recently replicated with physics experts where it was observed that physics experts reasoned differently from the mathematicians [10, 28–31]. In particular, physics experts demonstrated "goes like" reasoning and strong compiled relationships between the circular motion presented in the animation and trigonometric functions: "the height goes like a trig function" [10].

In developing Ferris Wheel for the PIQL, we were interested to see if students also held compiled relationships, and if they were using "goes like" reasoning to solve the problem. When reformatting the item as multiple choice, we chose distractors based on common covariational relationships in physics (i.e. exponential growth). We also chose distractors based on compiled relationships students may have including the Pythagorean theorem, which students associate with triangles in interviews and in open-ended version of other PIQL items, and an expression containing the circumference, which students associate with circles.

This item appears to be considerably less challenging than Flag of Bhutan, as 58% of all students answer correctly on PIQL 20W, however it is not an MCMR question so it cannot be compared directly [27]. While the majority of students in this sample answered correctly, incorrect answer choices provide insight into what productive resources students that do not answer correctly are using. The most common incorrect choices were (d) and (a) from Figure 2, with an answer rate of 25% and 15% respectively across all three courses. As before, the answer rate does not change significantly during the course of instruction. These results suggest that the "circumference-like" distractor and the "Pythagorean-like" distractor are appealing to a significant portion of the student population. We interpret these answer choices as early "goes like" reasoning—these are functions that are familiar to students from prior math classes, and have been fruitful in past experiences reasoning about circles and triangles. We suggest these resources could be built upon and coordinated with reasoning about circular motion through direct instruction to develop the "goes like" reasoning experts demonstrate between circular motion and trigonometry.

Student interviews were conducted as part of the validation process for new items on the PIQL. Semi-structured, individual think-aloud interviews were conducted with six introductory physics students at another public university in the Pacific Northwest. Ferris Wheel was given as shown in Figure 2, and students were selected on a volunteer basis. We do not claim that these two institutions represent identical populations; they often have slightly different average scores on PIQL assessment items, and fewer than half of those interviewed arrived at the correct answer. Therefore, the interviews provide some details into what resources students that do not yet have facility with circular motion may be using.

The Circumference-Like Distractor, answer choice (d), was highly appealing to nearly all students interviewed, citing it as familiar and associating circumference with total distance traveled: "I'd say (d) because its the only one that has $2\pi R$ in there, which is the, essentially, the circumference formula." Indeed, nearly all students interviewed began by defining the total distance traveled by the circumference,

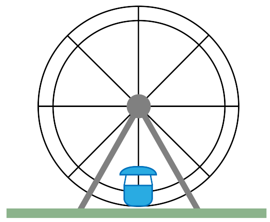

FIG. 2. Ferris Wheel. The prompt for this item asks students to identify which expression correctly identifies how h, the height of the cart, directly changes with s, the distance traveled by the rider, where the radius of the Ferris wheel is given by R_0: (a) $h(s) = \sqrt{s^2 + R_0^2}$, (b) $h(s) = R_0 \exp(s/R_0)$, (c) $h(s) = R_0 - R_0 \cos(s/R_0)$, (d) $h(s) = s^2/(2\pi R_0)$

and many returned to this definition throughout their problem solving process. We recognize this as a form of quantitative reasoning—students demonstrate a strong compiled relationship between distance and circumference. They also tend to connect total distance traveled (a quantity that changes with time) with the circumference of one revolution (a quantity that is fixed), meaning that the students interviewed did not spontaneously consider the total distance *as it is changing* when using circumference to solve the problem.

The Pythagorean-like Distractor, answer choice (a), was also of significant interest to those interviewed. Every student interviewed verbally labelled this option as "Pythagorean," and many students drew an accompanying triangle, demonstrating a strong compiled relationship between the expression itself and triangular geometry. Some students used this understanding to recognize quickly that the Pythagorean approach would not work, one stating, "(A) is the Pythagorean theorem, but that doesn't make sense because that's linear distance." We interpret this as the student recognizing that the Pythagorean theorem uses linear distances, and the total distance traveled is not linear. Another student debated about the correctness of (a), stating, "This is like the Pythagorean theorem... if we do it like this, [the student draws a triangle with the hypotenuse representing total distance] I guess you could estimate [the total distance] as being a straight line." Here, we infer the student is using a method often taught in calculus courses to approximate curves as linear to make sense of the linear quantities in a circular context. Both of these students demonstrate the use of strong sense-making about linear quantities in the context of circular motion using resources from prior mathematics instruction.

The Trigonometric Answer, choice (c), was challenging for students due to its high level of complexity. Students puzzled over how to draw the corresponding triangle, "cosine gives me s over R_0...so they're saying the radius is the hypotenuse. How can that be?" We interpret this as the student sense-making about how the mathematical expression is connected to the physical representation, early evidence of student conceptual blending. Another student was unique among those interviewed in drawing a connection between the unit circle and answer choice (c), recognizing that "θ is equal to arc length over the radius...the radius should be the hypotenuse because the radius is the one thing that is measured throughout the circle." The student makes the connection of the angle as a ratio, and uses their conceptualization of the unit circle to identify the corresponding triangle.

These patterns suggest that the students interviewed have a variety of strong sense-making resources about the circumference formula, geometric approaches to the Pythagorean theorem, and early signs of conceptual blending between physical representations and mathematical formula. Students may not yet coordinate these resources completely correctly with circular motion, however, both in terms of time evolution and the connection between the formalism and the physical representation. Notably, those that did answer correctly during the interviews arrived at their answer by plugging in points. In particular, students focused on physically significant points, for example, the bottom and top of the Ferris Wheel where the height is at a minimum or maximum. This kind of problem solving— choosing physically relevant points to better understand the behavior of the system—has been identified as an expert-like behavior [10]. However, students uniformly did so as a last effort, suggesting they may not see the expert-like nature of this approach.

V. CONCLUSIONS

The results of Ferris Wheel and Flag of Bhutan demonstrate that while students have difficulty with physics "goes like" reasoning, they illustrate productive resources that could be used to develop physics covariational reasoning. Responses to Flag of Bhutan show that students have strong "goes like" reasoning about linear relationships, and those to Ferris Wheel demonstrate that students have strong compiled relationships regarding right triangles and the Pythagorean theorem, and circles and circumference. However, without direct instruction it is challenging for students to spontaneously coordinate complex functions like $\sqrt{l^2 + w^2}$ to linear behavior and their reasoning about trigonometry to circular motion. It is important for instructors to recognize their own expert habit of thinking in terms of a small, finite number of preferred functions in physics, while students emerge from math courses without that framework. We recommend instructors consider including explicit instruction to help students develop a recognition of our preferred functions to facilitate "goes like" thinking. As this is preliminary work, additional studies are needed to more deeply understand both student and expert covariational reasoning in physics.

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