Do They Understand Your Language? Assess Their Fluency with Vector Representations

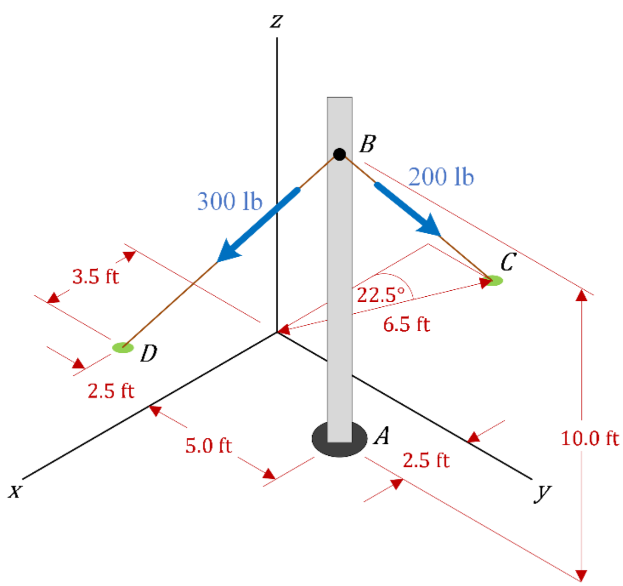
Abstract

In teaching mechanics, we use multiple representations of vectors to develop concepts and analysis techniques. These representations include pictorials, diagrams, symbols, numbers and narrative language. Through years of study as students, researchers, and teachers, we develop a fluency rooted in a deep conceptual understanding of what each representation communicates. Many novice learners, however, struggle to gain such understanding and rely on superficial mimicry of the problem solving procedures we demonstrate in examples. The term representational competence refers to the ability to interpret, switch between, and use multiple representations of a concept as appropriate for learning, communication and analysis. In engineering statics, an understanding of what each vector representation communicates and how to use different representations in problem solving is important to the development of both conceptual and procedural knowledge. Science education literature identifies representational competence as a marker of true conceptual understanding.

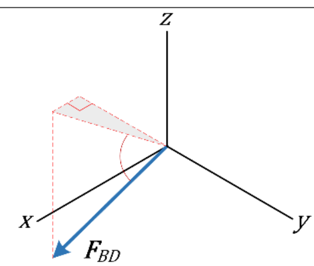
This paper presents development work for a new assessment instrument designed to measure representational competence with vectors in an engineering mechanics context. We developed the assessment over two successive terms in statics courses at a community college, a medium-sized regional university, and a large state university. We started with twelve multiple-choice questions that survey the vector representations commonly employed in statics. Each question requires the student to interpret and/or use two or more different representations of vectors and requires no calculation beyond single digit integer arithmetic. Distractor answer choices include common student mistakes and misconceptions drawn from the literature and from our teaching experience. We piloted these twelve questions as a timed section of the first exam in fall 2018 statics courses at both Whatcom Community College (WCC) and Western Washington University. Analysis of students' unprompted use of vector representations on the open-ended problem-solving section of the same exam provides evidence of the assessment's validity as a measurement instrument for representational competence. We found a positive correlation between students' accurate and effective use of representations and their score on the multiple choice test. We gathered additional validity evidence by reviewing student responses on an exam wrapper reflection. We used item difficulty and item discrimination scores (point-biserial correlation) to eliminate two questions and revised the remaining questions to improve clarity and discriminatory power. We administered the revised version in two contexts: (1) again as part of the first exam in the winter 2019 Statics course at WCC, and (2) as an extra credit opportunity for statics students at Utah State University. This paper includes sample questions from the assessment to illustrate the approach. The full assessment is available to interested instructors and researchers through an online tool.

Introduction

In teaching mechanics, we use multiple representations of vectors to explain concepts and analysis techniques to students. These representations include pictorials, diagrams, symbols, numbers and narrative language. Figure 1 illustrates examples of each type of representation for a typical statics problem involving three dimensional vector analysis. Through years of study as students, researchers, and teachers, we develop a fluency with the language of vector analysis rooted in a deep conceptual understanding of what each representation communicates. Many novice learners, however, struggle to gain and apply such understanding and rely instead on memorizing patterns and the problem solving procedures they see in worked examples [1], [2]. As with most engineering science courses, students arrive in a statics course with prior exposure to many topics including vectors. Their developing conceptual knowledge may contain incorrect information. If instructors can better assess their conceptual knowledge, then they can better target learning activities to address gaps and misunderstandings [3].



(a) Pictorial (problem figure)



(b) Diagram

$$\vec{F}_{BD} = 23.92\hat{\boldsymbol{\imath}} - 179.4\hat{\boldsymbol{\jmath}} - 239.3\hat{\boldsymbol{k}}\ \text{lb}$$

(c) Numeric

$$\vec{F}_{BD} = F_{BD}\frac{\vec{r}_{BD}}{r_{BD}}$$

(d) Symbolic

Cable *BD* exerts a tension force that pulls on point *B* primarily directed down and to the left.

(e) Narrative language

Figure 1. Multiple representations of a force vector in the context of a typical statics problem. Items (a)-(e) depict typical representations used in problem solving and analysis.

In recent years, we have been working on developing activities and manipulatives with the parallel goals of helping students deepen their conceptual understanding of vector representations and strengthen the associated spatial visualization skills [4], [5]. In order to evaluate the effectiveness of these activities, we developed an assessment of vector concepts and representations in the context of their applications in statics that we call the Test of Representational Competence with Vectors (TRCV).

This paper describes our assessment approach and analyzes results from deployment of two versions of the TRCV over two successive terms in three educational contexts. We deployed the first version (TRCV v1.0) in fall 2018 as a timed section of the first course exam in statics courses at both Whatcom Community College (WCC) and Western Washington University (WWU). We made some revisions after this first deployment and administered the revised version (TRCV v2.0) in winter 2019 again as a section of the first exam in the WCC statics course and as an extra credit opportunity for statics students at Utah State University (USU).

Background

Understanding what each vector representation communicates and how to apply it effectively in problem solving is important to the development of both conceptual and procedural knowledge in Statics. These representations form the language we use to teach and learn mechanics. Students generally arrive in statics courses with some prior experience with vector analysis gained in prerequisite math and physics courses, though proficiency with these basic skills is often lacking [6]. Most statics textbooks begin by reviewing these vector concepts, establishing notation conventions, and extending applications to three-dimensional geometries that may be unfamiliar to students. Despite the fundamental importance of vector concepts to understanding statics, they are not included in published statics concept inventories [7], [8]. Vector manipulation; however, is identified as among the most important skills in statics [9].

We have identified the framework of representational competence as useful for thinking about students' conceptual knowledge in statics and with vectors in particular. Kozma and Russel [10] used the term representational competence (RC) in the context of chemistry education research to describe the ability to use multiple representations of a concept as appropriate for learning, problem solving, and communication. While there is still no consensus on RC as a unified theoretical framework [11], the construct is commonly used in the science education literature and is seen as a marker of true conceptual understanding [12], [13], [14]. There is also no consensus on how best to assess students' RC in a domain as evidenced by the diversity of approaches in the literature [15], [16], [17], [18].

Vector concept assessments such as the Test of Understanding Vectors (TUV) and the Vector Evaluation Test (VET) exist in the physics education literature, but these instruments focus exclusively on two-dimensional applications and include dynamics concepts in addition to statics [19], [20], [21]. Nonetheless, we found inspiration in these assessments for strategies to probe conceptual knowledge without requiring students to perform numerical calculations.

Assessment Development

Our goal is to develop a test of representational competence with vectors (TRCV) that students can reasonably complete in 20-25 minutes without the use of a calculator. We chose this length so we could include the assessment as a section in the first 80-minute course exam. The second section of said exam includes more open-ended homework-style problems. We limit the use of statics-specific terminology in an effort to maintain suitability for use of the TRCV as a beginning of course pre-test in future terms and by other users. We chose a multiple-choice format with four alternatives for each question to balance tradeoffs between goals of minimizing the effect of correct guessing, developing plausible distractor choices, and reducing the amount of time required for students to complete the test [22]. We developed distractor choices based on our experience with common student mistakes associated with representation errors.

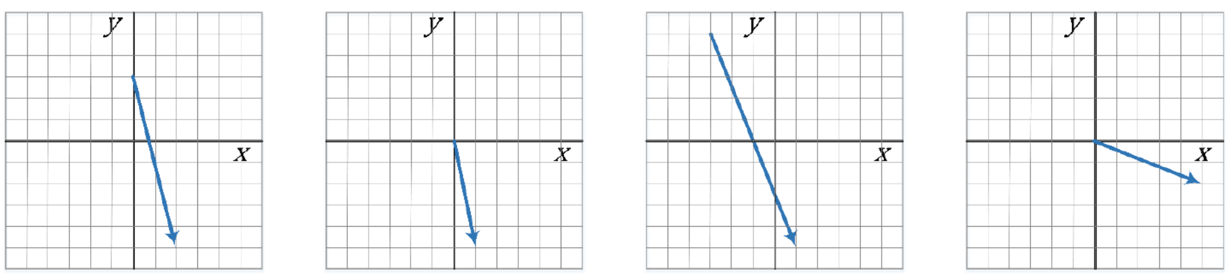
Table 1 summarizes TRCV v2.0 that we administered in winter 2019 after revisions based on the fall deployment experience. Ten multiple-choice questions move through a series of conceptual vector problems. Each question requires interpretation and/or use of at least two different vector representations. The representations listed in the two right columns of Table 1 include Pictorial, Symbolic, narrative Language, Numeric, and Diagrams. Figure 1 above provides examples of each representation in the context of vectors. We do not intend the assessment as a whole to be a comprehensive inventory of vector concepts that are important in statics. Rather, we hope that by sampling relevant concepts and requiring students to make use of multiple representations for each item, we can specifically assess their representational competence in the context of vector analysis. This approach is similar to that taken by Klein et al in developing an RC assessment for kinematics [15].

Table 1. Summary of vector concepts and representations for each item on the TRCV (v2.0). The representations listed in the two right columns include Pictorial, Symbolic, narrative Language, Numeric, and Diagram.

Item	**Relevant Vector Concepts**	**Representations**	
		Question	Answers
1	2D, position vectors, vector addition	P L	S
2	2D, cross product	P L S	L
3	2D, Cartesian components	N	D
4	2D, Cartesian components, vector addition	D N S	L N
5	3D, Cartesian components	N	D
6	3D, Cartesian unit vector, force vector in cable	P S	S
7	3D, cross product	P S	L
8	3D, vector addition, position vectors, Cartesian unit vector	P L	S
9	3D, position vector	P L	S
10	3D, resolving a vector into parallel and perpendicular components	P L	L N

One aspect of the TRCV intended to identify representational difficulties is the scaffolding of analogous problems in both two and three dimensions. Figures 2 and 3 depict items 3 and 5 respectively. These two items both assess whether a student can interpret the relative magnitudes of Cartesian components as a representation of vector direction, but question 5 introduces the third dimension and the associated spatial difficulty of interpreting a three-dimensional diagram. For either question, our experience is that most students, given the option, will gravitate toward entering the component magnitudes into their calculator to compute an angle as a first step in determining a graphical indication of direction. We do not allow calculators; however, so students must reason through the problem by comparing the relative magnitudes of the components and visualizing the direction. The gridlines in problem 3 provide some assistance with scale, but problem 5 forces the student to rely more heavily on spatial visualization and a sense of proportion. We believe this skill of visualizing the direction of 3D vectors expressed in components is important to students' ability to analyze 3D force systems, particularly moments, and has specific application to evaluating the reasonableness of numerical answers in a wide variety of statics problems.

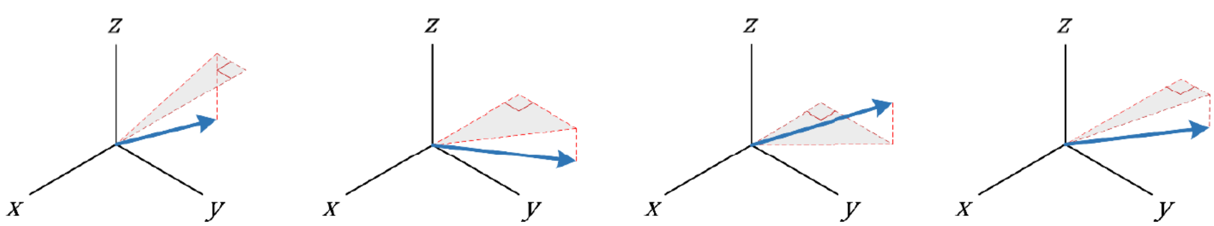
3. The force vector $\vec{\boldsymbol{F}} = 200\hat{\boldsymbol{\imath}} - 500\hat{\boldsymbol{\jmath}}$ (N). Which of the following figures most accurately represents the direction of $\vec{\boldsymbol{F}}$.



(a) (b) (c) (d)

Figure 2. Example 2D question from the TRCV v2.0.

5. A force vector is expressed in Cartesian components as $\vec{\boldsymbol{F}} = -150\hat{\boldsymbol{\imath}} + 100\hat{\boldsymbol{\jmath}} - 50\hat{\boldsymbol{k}}$ (kN). Which of the following figures best represents the direction of $\vec{\boldsymbol{F}}$. Note the shaded triangle in each figure lies in the $\boldsymbol{xy}$ plane.

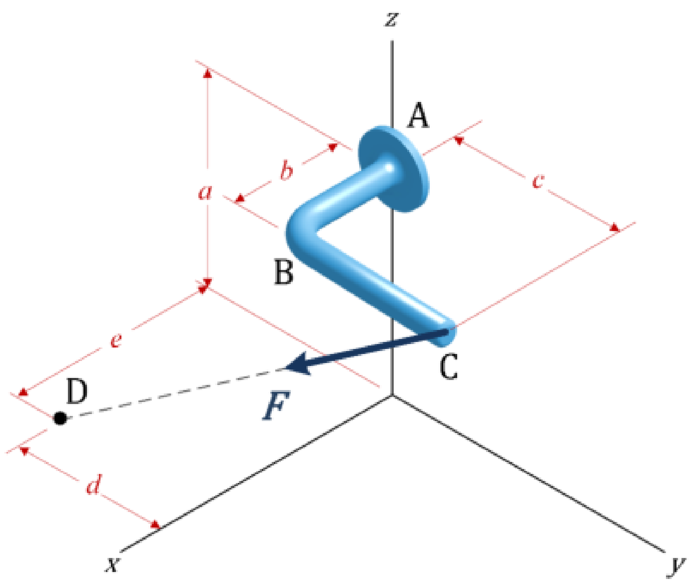


(a) (b) (c) (d)

Figure 3. Example 3D question from the TRCV v2.0 that is analogous to the 2D question example in Figure 2.

Another theme of the TRCV are questions that assess students' ability to interpret geometry information in typical 3D problem figures (pictorial representation) and express that information in formal vector notation (symbolic representation). Others have noted the interpretation of 3D figures as a common point of difficulty in statics [23]. Figure 4 illustrates our approach with item 9 that instructs the student to read a position vector from a 3D figure. This skill is logically important to students' ability to learn effectively from 3D figures illustrating inherently 3D statics concepts (e.g. how the cross product is used to compute a moment) and to solve 3D statics problems in general.

Consider the figure below for questions 8-10. The bar ABC has a 90° bend at B and is attached to a wall in the *yz* plane at A such that segment AB is parallel to the *x*-axis and segment BC is parallel to the *y*-axis. The force $\vec{F}$ acts on point C at the end of the bar and points toward D along line CD. Note the subscripts in the vector notation indicate the points the vector connects. For example, the notation $\vec{r}_{AB}$ indicates the vector pointing from point A to point B.



9. The position vector $\vec{r}_{CD}$ that gives the position of point **D** with respect to point **C** is expressed in components as:

(a) $\vec{r}_{CD} = (e-b)\hat{\imath} - d\hat{\jmath} - a\hat{k}$ (b) $\vec{r}_{CD} = e\hat{\imath} - (c+d)\hat{\jmath} - a\hat{k}$

(c) $\vec{r}_{CD} = (e-b)\hat{\imath} - (c+d)\hat{\jmath} - a\hat{k}$ (d) $\vec{r}_{CD} = (e-b)\hat{\imath} + (c-d)\hat{\jmath} - a\hat{k}$

Figure 4. Example 3D figure interpretation question from the TRCV v2.0.

Results

We administered the assessment in fall 2018 and winter 2019 and conducted some additional data gathering for validity analysis as we describe below.

Fall 2018 Deployment (Version 1.0)

The TRCV v1.0 was part of the first midterm examination in fall 2018 statics courses at both WCC (N = 26) and WWU (N = 33). This version had 12 questions of similar design to the samples from version 2.0 described above. In both administrations, we included the assessment as a closed book and closed note first section of the exam with a firm 25-minute time limit and did not allow students to use calculators. The mean score for the community college students was 67% with a standard deviation of 16%. The mean score for the university students was 69% with a standard deviation of 13%. There is no statistically significant difference between these results ($p = 0.49$), so we have combined the populations for the item analysis presented in Table 2.

Table 2. Item analysis of the TRCV v1.0 administered to 59 students at WCC and WWU in fall 2018. The correct answer is in boldface. NA indicates students did not answer the question.

Item	Description	Difficulty Index	Point-biserial Correlation	A	B	C	D	NA
1	2D position vector components from length and angle given on annotated photo	0.85	0.31	2	7	0	**50**	0
2	Vector addition in 2D from point labels on annotated photo	0.58	0.50	0	23	**34**	2	0
3	Visualizing direction of cross product in 2D from position and force vectors referencing annotated photo and description	0.69	0.63	1	5	12	**41**	0
4	Choosing direction diagram representing given numerical components in 2D	0.97	0.02	0	**57**	1	1	0
5	Vector addition in 2D combining diagram and numeric components	0.92	0.44	**54**	0	2	3	0
6	Visualizing direction of 3D numeric components and expressing answer as a range of angles with the x-direction	0.71	0.28	6	**42**	7	3	1
7	Components of a 3D unit vector from a pictorial with spherical coordinate angles	0.81	0.54	0	**48**	7	3	1
8	Determining magnitude of a position vector from a pictorial with labeled point coordinates	0.78	0.29	5	8	0	**46**	0
9	Direction of a cross product of two vectors on a 3D problem figure. Expressing answer as a range of angles with the y-direction	0.29	0.30	21	17	**17**	4	0
10	Using vector addition to find a unit vector in a 3D problem figure	0.68	0.46	0	9	9	**40**	1
11	Finding a position vector in Cartesian components using dimensions on a 3D pictorial	0.88	0.39	**52**	0	2	4	1
12	Determining direction of the component of a vector perpendicular to a geometric feature. Expressing answer as range of angles with an axis.	0.051	0.27	27	11	**2**	17	2

After the exam, students completed an exam wrapper reflection [24] that provided some useful information for validating the assessment. By examining responses to a prompt for students to self-report the items for which they guessed the answer, we learned that ten students felt like they guessed the correct answer for item 6. This was by far the highest rate of correct guesses for any item (second highest was three correct guesses), indicating this question with moderate difficulty (DI = 0.71) may have been more difficult than these statistics indicate. We also learned by reviewing students' written reflection on their reasons for choosing incorrect answers that many students found the visualization required for items 6, 9, and 12 to be challenging. We expected this to be the case because these questions intend to probe some of the visualization difficulties we observe students having when working with 3D vectors. Finally, the exam wrapper reflections indicated that a significant number of students did not feel they had adequate time to complete the test. In particular, many students did not have adequate time to devote to item 12, which the item analysis found to be by far the most difficult question. The majority of students selected options A and D, both likely choices of a "rushed" student picking an answer choice based on an incomplete or superficial reading of the question prompt.

Assessment Revision

Based on analysis of the fall 2018 results, we made the following modifications to the TRCV to arrive at version 2.0 outlined previously in Table 1 and the source for example items discussed in the assessment development section. The major revisions are as follows. We deleted items 1 (DI = 0.85, $r_{pb} = 0.31$) and 7 (DI = 0.81, $r_{pb} = 0.54$). Both of these items proved relatively easy with student errors sourced primarily to trigonometry mistakes rather than confusion about vector representations. For item 2, we removed the position vector subscripts from the answer choices (e.g, the answer choice $\vec{\boldsymbol{r}}_{BD} = \vec{\boldsymbol{r}}_{BE} + \vec{\boldsymbol{r}}_{ED}$ was replaced with the choice $\vec{\boldsymbol{r}}_{BE} + \vec{\boldsymbol{r}}_{ED}$) so students could not rely on pattern recognition of the subscripts to choose the correct answer. We made item 5 more difficult by introducing the answer format of a range of possible angles in this 2D problem. We used this format in items 6, 9, and 12 in 3D questions in the fall 19 version without providing scaffolding in a 2D analog. We also changed the answer representation for item 6 from a range of possible angles (language) to a diagram as illustrated above in figure 3 (the question moved from number 6 to number 5 in v2.0). We modified the figure for items 10-12 to reduce visual ambiguity and changed from numerical dimensions to variables. For the remaining items, we modified unchosen distractors to make them more compelling and/or made minor edits to the question wording.

Winter 2019 Deployment (Version 2.0)

We deployed TRCV v2.0 in two different statics courses during winter 2019. The WCC deployment (N = 18) was again part 1 of a two part exam with similar format and structure to fall 2018. The mean score at WCC was 71% with a standard deviation of 9.9%. We also deployed the assessment at USU (N = 70) via the Canvas LMS outside of regular class time. The USU test served as an extra credit opportunity offered after the first exam. The mean score at USU was 48% with a standard deviation of 9.8%. The difference in these summary results is statistically significant ($p < 0.0001$). Note that the administration context of the USU deployment was also quite different compared to WCC. Tables 3 and 4 present the item analyses separately. WWU did not participate in administering the TRCV for winter 2019.

Table 3. Item analysis of the TRCV v2.0 administered to 18 students at WCC in winter 2019. The correct answer is in boldface. NA indicates students did not answer the question.

Item	Description	Difficulty Index	Point-biserial Correlation	A	B	C	D	NA
1	Vector addition in 2D from point labels on annotated photo	0.72	0.31	3	**13**	1	1	0
2	Visualizing direction of cross product in 2D from position and force vectors referencing annotated photo and description	0.83	-0.10	2	1	0	**15**	0
3	Choosing direction diagram representing given numerical components in 2D	0.89	0.38	0	2	**16**	0	0
4	Vector addition in 2D combining diagram and numeric components	0.61	0.53	3	2	2	**11**	0
5	Choosing direction diagram representing given numerical components in 3D	0.94	0.03	0	**17**	0	1	0
6	Expressing components of a Cartesian unit vector using point coordinates on a 3D pictorial	0.56	0.10	0	**10**	0	8	0
7	Analyzing 3D pictorial to visualize effect changing a spherical coordinate angle has on cross product direction	0.56	0.32	**10**	0	6	2	0
8	Using vector addition to find a unit vector in a 3D problem figure	0.89	0.21	**16**	2	0	0	0
9	Finding a position vector in Cartesian components using dimensions on a 3D pictorial	0.83	0.63	0	2	**15**	1	0
10	Determining direction of the component of a vector perpendicular to a geometric feature. Expressing answer as range of angles with an axis.	0.28	-0.07	2	4	**5**	6	0

We will not read too much into the item analysis of the WCC deployment because of the small sample size (N = 18), but it is worth discussing the questions with low point-biserial ($r_{pb} < 0.20$). Question 2 (DI = 0.83, $r_{pb} = -0.10$) in v2.0 is the same as question 3 in v1.0 which yielded $r_{pb} = 0.63$. The difficulty index was approximately equal across the two v2.0 deployments, but the problem proved somewhat easier than it did in version 1.0 (DI = 0.69). Looking to the USU results in Table 4 on the next page, we see this same question had $r_{pb} = 0.54$. We interpret the difference in point-biserial largely to be an artifact of the small sample size for WCC.

All but one of the WCC students answered question 5 (DI = 0.94, $r_{pb} = 0.03$) correctly compared to DI = 0.71 at USU. We presented question 5 earlier in figure 3. This question is a rework of question 6 in v1.0 (DI = 0.71, $r_{pb} = 0.28$) in which we changed the representation used for the answer choices as discussed previously. Again we see the USU result for this question (DI =

0.71, r_{pb} = 0.42) to be within acceptable ranges, so the WCC statistics seem likely skewed by the small sample size.

The last question of concern regarding the statistics is question 10. The statistics fall outside desirable ranges for both WCC (DI = 0.28, r_{pb} = -0.07) and USU (DI = 0.10, r_{pb} = 0.17). We modified the problematic item 12 in v1.0 to develop this question, but it still appears to be very difficult for the students and performance does not correlate well with their overall score on the test. We plan further revisions of this item to reduce complexity. Student responses on the exam wrapper at WCC indicated we still had a high rate of guessing and general confusion about what the question is asking.

Table 4. Item analysis of the TRCV v2.0 administered to 70 students at USU in winter 2019. The correct answer is in boldface. NA indicates students did not answer the question.

Item	Description	Difficulty Index	Point-biserial Correlation	A	B	C	D	NA
1	Vector addition in 2D from point labels on annotated photo	0.16	0.44	12	**11**	18	28	0
2	Visualizing direction of cross product in 2D from position and force vectors referencing annotated photo and description	0.81	0.54	7	3	3	**56**	1
3	Choosing direction diagram representing given numerical components in 2D	0.60	0.58	8	20	**42**	0	0
4	Vector addition in 2D combining diagram and numeric components	0.34	0.46	15	11	19	**24**	1
5	Choosing direction diagram representing given numerical components in 3D	0.71	0.42	2	**50**	2	16	0
6	Expressing components of a Cartesian unit vector using point coordinates on a 3D pictorial	0.61	0.23	6	**43**	3	18	0
7	Analyzing 3D pictorial to visualize effect changing a spherical coordinate angle has on cross product direction	0.50	0.39	**35**	10	21	4	0
8	Using vector addition to find a unit vector for direction of a force in a 3D problem figure	0.44	0.39	**31**	24	3	12	0
9	Finding a position vector in Cartesian components using dimensions on a 3D pictorial	0.63	0.56	2	9	**44**	15	0
10	Determining direction of the component of a vector perpendicular to a geometric feature. Expressing answer as range of angles with an axis.	0.10	0.17	13	22	**7**	28	0

As mentioned earlier, the mean score on version 2.0 was significantly higher at WCC compared to USU. Figure 6 on the next page illustrates how this comparison holds true when comparing the difficulty index across the ten items. A higher fraction of the WCC students answered correctly in all cases except for question 6. We speculate the following factors may have contributed to this difference in varying degrees:

- There are likely differences in student motivation connected to the administration of the TRCV as part of an exam versus as an extra credit opportunity. Furthermore, the extra credit was given for participation only rather than connecting the amount of points to the assessment score.
- There could be some evidence here of learning gains students made with the Statics Modeling Kit activities we have been piloting at WCC [4].
- There could be differences in the learning gains in general students are making in the smaller 18-student WCC course versus the large 134-student lecture section at USU.

Since roughly half the students in the USU class chose to sit for this extra credit assessment, we looked into whether the sample of students participating were disproportionately lower performing students. An examination of the students' exam scores indicate this is likely not a significant factor. The average exam 1 score of the course as a whole was 83.3% whereas the average of those who sat for the TRCV was 83.5%.

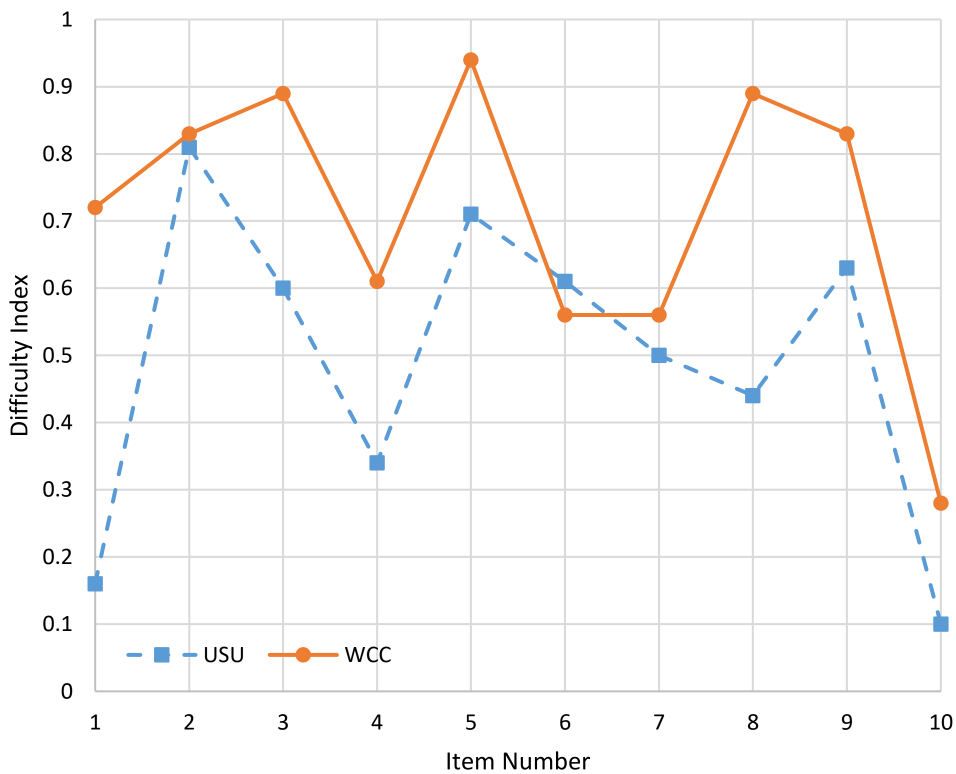


Figure 6. Comparison of the item difficulty scores from the WCC and USU deployments of TRCV v2.0 during winter 2019.

Validity Evidence for TRCV as a Measure of Representational Competence

To validate the TRCV as an assessment of representational competence, we first developed a rubric to score students' unprompted and accurate use of multiple representations in the problem-solving section of the exam (part 2). The rubric assigned a score of 0, 1, or 2 for each representation type (numeric, symbolic, diagram, language) for each of two problems: (1) a particle equilibrium problem; and (2) a 3D vector analysis problem. We assigned a 2 for a representation type if it was applied accurately and effectively as an integral part of the student's problem solving process. We assigned a 1 if the representation was present in the student's work, but somewhat inaccurate or otherwise tangential to the problem solving. We assigned a 0 if the representation was inaccurate, irrelevant, or absent from the work. With four representation types scored for each of two problems, student representation scores could range from 0 to a maximum of 16. Figure 6 shows the students' percentage scores on the TRCV plotted versus their total representation use scores for the two problems. The TRCV and representation use scores show a positive correlation with $R = 0.66$ ($p < 0.001$). We only conducted this analysis for the 26 community college students with v1.0. Unfortunately, problems in the proctoring of v2.0 at WCC prevented us from having a usable data set of student problem solving work to repeat this analysis. Nonetheless, this result from v1.0 indicates our approach in designing this multiple-choice test has potential as a valid measurement of representational competence, but further data collection and analysis is necessary to make a stronger claim.

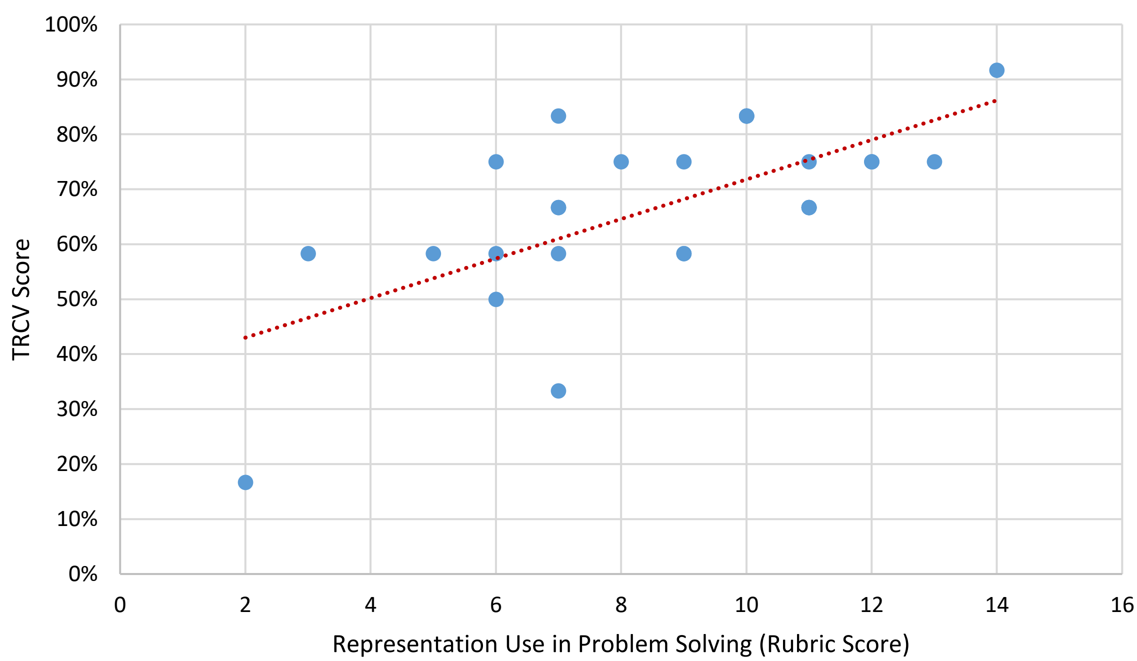


Figure 6. Correlation between students' scores on the TRCV and their spontaneous use of representations in problem solving on the accompanying exam part 2.

Conclusion and Future Work

In conclusion, the TRCV is a multiple-choice test to assess students' conceptual understanding and representational competence with vectors in the context of engineering statics. The test covers basic vector concepts in both two and three dimensions and includes typical statics analysis tasks such as visualizing the direction of a vector expressed in Cartesian components and determining Cartesian unit vectors from problem figures. The assessment approach requires students to interpret multiple representations in working through various vector applications in an effort to specifically-measure representational competence. We administered the test in statics classes at three institutions over two successive terms, with some minor revisions in between. Item analysis indicates the difficulty and discrimination scores are in desirable ranges. Comparison of students' representation use on a problem solving section of the exam to their TRCV scores yielded a positive correlation, indicating this approach has potential to be a valid measure of students' representational competence with vectors.

Going forward, we plan continued refinement of the questions and use of the TRCV as a pre- and post-test to measure potential learning gains from activities specifically targeting students' ability to effectively use and understand vector representations. The test is available to educators and researchers along with a growing library of mechanics-related content in the concept inventory section of the Concept Warehouse (https://jimi.cbee.oregonstate.edu/concept_warehouse/) [25].

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