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Research paper

The design of coiling and uncoiling trusses using planar linkage modules

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ABSTRACT

This paper presents the design of planar trusses constructed from a sequence of four-bar linkages that guide a one degree-of-freedom coiling movement of the truss from a linear deployed configuration to a coiled circular or coiled spiral stowed configuration. For a given number of modules in the truss, the dimensions of each module are identical for the circular stowed configuration, and are varied by a scale factor for the spiral stowed configuration. We present how to determine the dimensions of the linkage modules and provide example applications.

1. Introduction

This paper presents the design of planar trusses that can move from a linear deployed configuration to a stowed configuration by coiling into either a circle or a spiral. This is an approach to the design of deployable structures that is different from the existing methods based on scissor-hinged structures and linear grids, Escrig and Valcarel [1]. By assembling a truss constructed from general four-bar linkage modules, we obtain 12 bar, 24 bar, 34 bar, and 100 bar trusses that move smoothly from a linear deployed configuration to a coiled stowed configuration. Our approach eliminates the singular configurations typical of scale changing deployable systems and therefore results in an inherently more stable structural system. Fig. 1 is a physical model of our curling 12 bar truss. In this model, the adjacent bars are alternated up and down, so there is no interference for this deployable truss. Fig. 2 illustrates how this truss moves from the linear deployed configuration to the stowed coiled configuration. This coiling movement is new, and, in what follows, we show how to design the four-bar linkage modules that guide the coiling and uncoiling movement of these deployable structures.

2. Literature review

A survey of expandable space structures is presented by Escrig and Valcarel [1], who show how scissor-hinged mechanisms provide scalable structures in one, two and three dimensions. The systematic design of scissor-like elements for deployable structures is presented by Gantes, et al. [2], who show how to achieve a planar slab and a circular arch that deploy by changing scale from a compact stowed configuration to an expanded deployed configuration. You and Pellegrino show that scissor-hinged elements can be assembled along the sides of a polygon so that it can deployed from a compact assembly to an expanded polygon. General scale-changing linkages are described in Bai et al. [3], also see Choe et al. [4].

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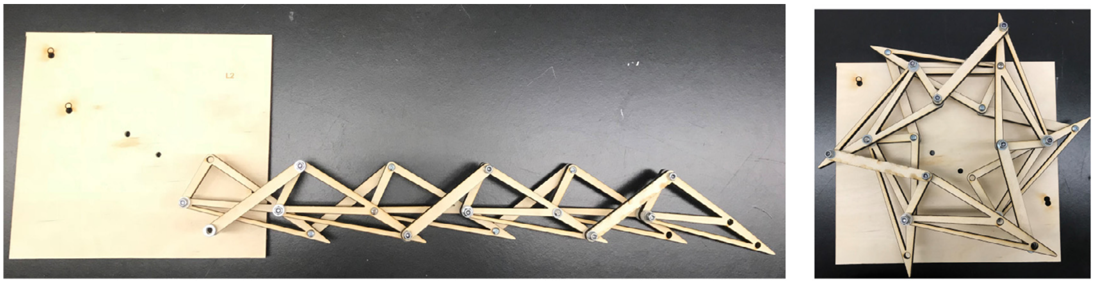
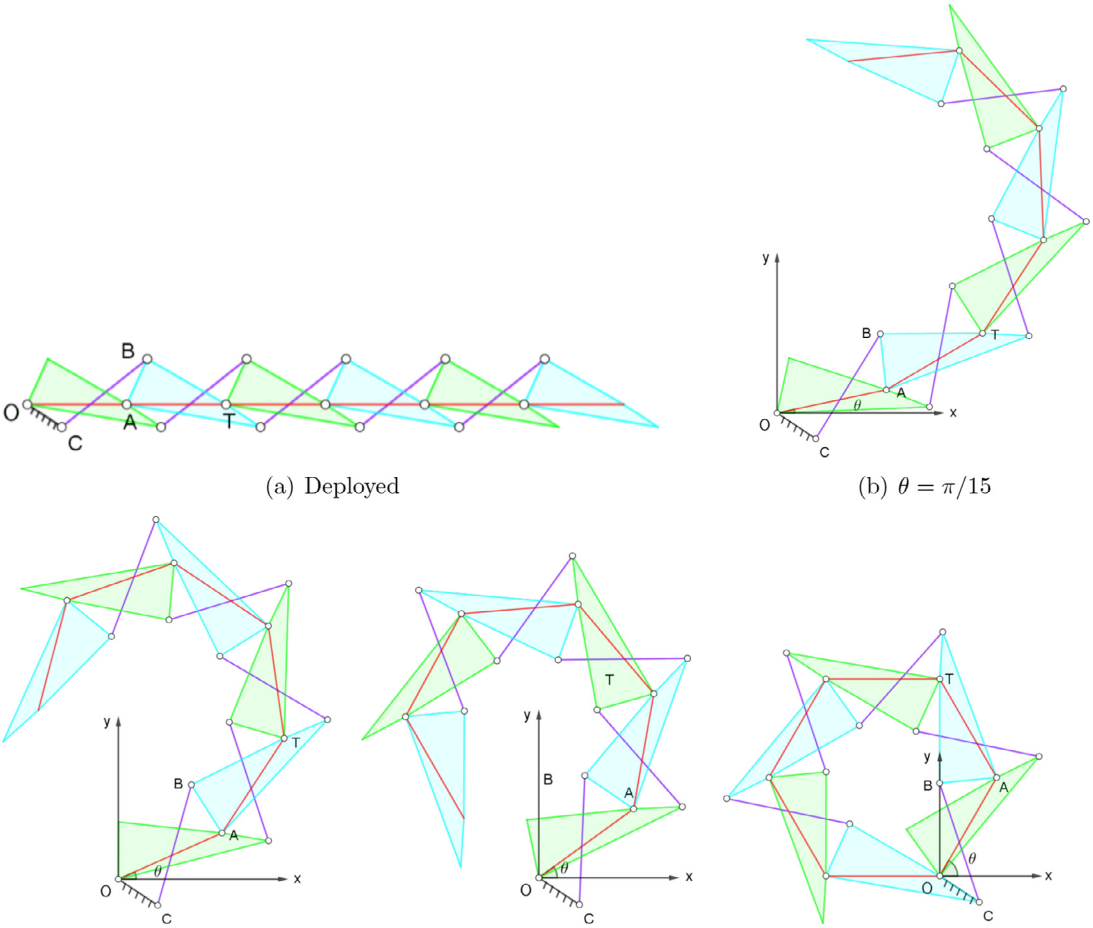


(a) Deployed (b) Stowed

Fig. 1. A prototype truss that unfurls into the linear configuration and coils into a hexagon.



(a) Deployed (b) $\theta = \pi/15$

(c) $\theta = 2\pi/15$ (d) $\theta = 3\pi/15$ (e) $\theta = 5\pi/15$

Fig. 2. A truss constructed from six linkage modules. The rotation θ of the segment OA relative to the horizontal axis drives the movement of all the linkage modules. The result is the truss curls into a hexagon.

Maden et al. [5] provide a survey of the geometric principles and design methods for deployable mechanisms based on scissor mechanisms. Recent work on the use of scissor-link mechanisms by Kaveh and Abedi [6] presents a stowed package that expands to form a barrel vault, which is another example of a radially expanding structure. Patel and Ananthasuresh [7] use scissor mechanisms to design a range of radially expanding shapes.

The use of other mechanisms rather than scissor-hinge mechanisms to design deployable mechanisms can be found in Lu et al. [8]. They obtain a radially expanding structure using interlocked Bennet linkages to form spatial scissor mechanisms. Another approach is the use of parallelogram linkages by St-Onge and Gosselin [9] to provide the linear expansion of the vertices of a polygon. Lu et al. [10] use Hoeken's linkage, which is an approximate straight-line mechanism, to de-

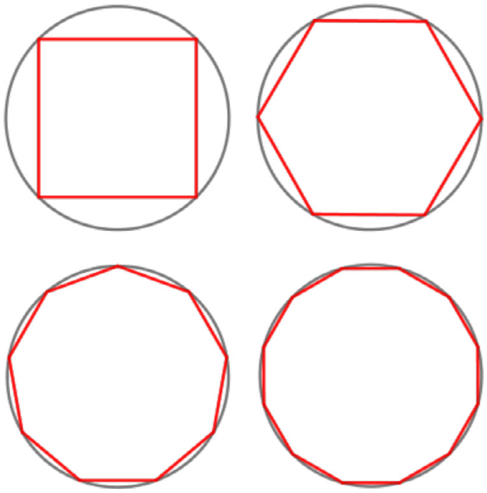


Fig. 3. A circle and an inscribed regular quadrilateral, hexagon, nonagon and dodecagon.

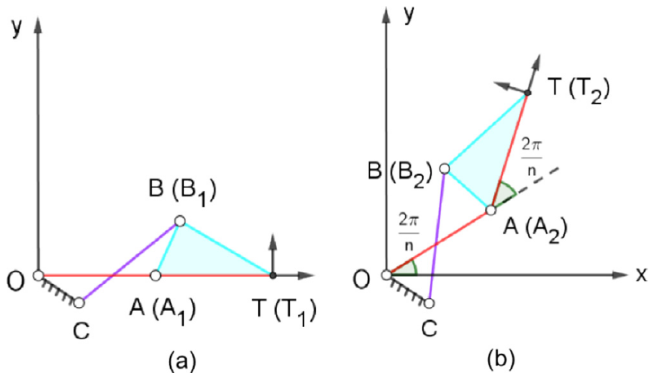


Fig. 4. Two positions of the four-bar linkage module.

sign a linearly expanding deployable structure. Morgan et al. [11] use folded spherical origami linkages to obtain expanding deployable structure.

In this paper, we introduce a new deployable structure that moves between deployed and stowed configurations by coiling and uncoiling rather than by expanding or changing scale. This deployable structure does not rely on scissor-hinged elements or special linkages that provide linear movement. Instead, a general four-bar linkage is designed to provide the required local change in angle. This linkage is reproduced to form a truss with the property that each of the modules rotate together causing the structure to coil and uncoil with one degree of freedom. This movement is distinctly different from the linear and radial expansion typical of other deployable structures. In what follows we present the design process and example applications.

3. Truss with a circular stowed configuration

In order to stow the truss in a circle, adjacent sides along the length of the truss must rotate from a relative angle of $\theta = \alpha = 0$ to $\theta = \beta = 2\pi/n$, where n is the number of the polygon that approximates the circle. Fig. 3 shows examples of $n = 4, 6, 9, 12$-sided regular polygons that we can use for our deployable structures. The angle β is the exterior angle of the regular polygon that we use to design the truss.

Let **OA** and **AT** define two links along the deployed truss and let $|\mathbf{OA}| = |\mathbf{AT}| = a = b = L/n$, where L is the length of the truss. The links **OA** and **AT** form an RR serial chain (R denotes a revolute joint) that moves relative to a fixed frame F located at **O**. Our goal is to design a coupler link **CB** so that constrains the RR chain, so an input rotation of **OA** of $\alpha = 2\pi/n$ yields the rotation of **AT** by the same amount $\beta = 2\pi/n$, see Fig. 4.

For the first linkage module, we consider the reference frame M in the end-effector of the RR chain that has its origin at **T** and its x-axis directed along **AT**. The homogeneous transformations from the reference frame M to the fixed frame F in

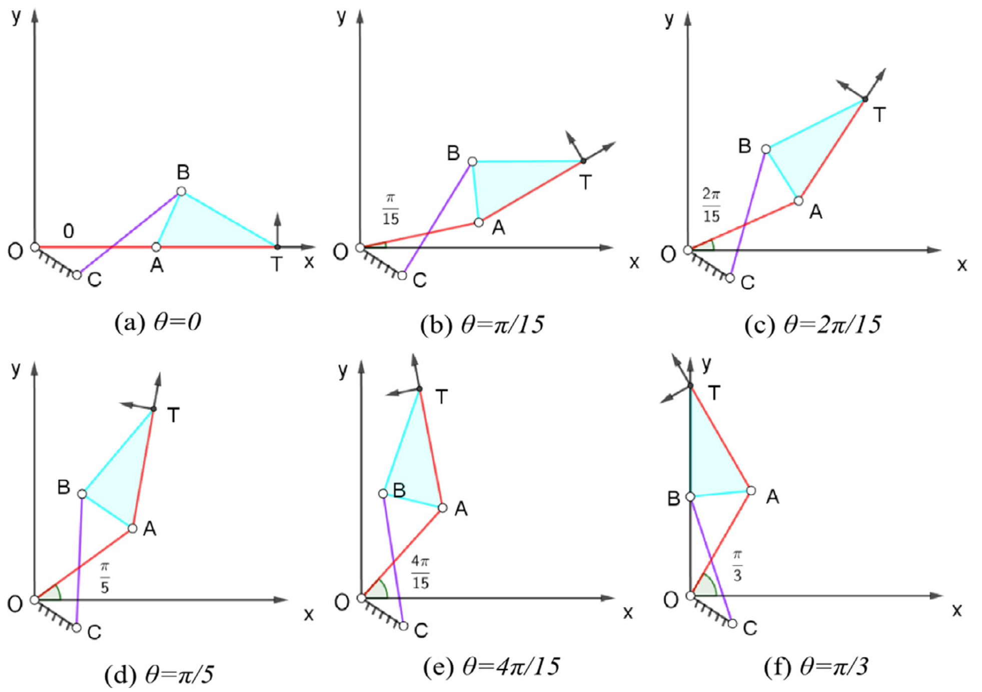


Fig. 5. The positions of the four-bar linkage module when the input angle is equal to 0, $\pi/15$, $2\pi/15$, $\pi/5$, $4\pi/15$, $\pi/3$, respectively.

the deployed and stowed configurations can be given by,

$$\mathbf{K}_1 = \begin{bmatrix} 1 & 0 & a+b \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{K}_2 = \begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & a\cos\alpha + b\cos(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) & a\sin\alpha + b\sin(\alpha+\beta) \\ 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

We seek the coordinates of a fixed pivot **C** in the frame F, and of a moving pivot **b** in the frame M, such that coordinates of the moving pivot in F, $\mathbf{B}_1 = (x, y, 1)$ and $\mathbf{B}_2 = \mathbf{K}_2\mathbf{K}_1^{-1}\mathbf{B}_1$, satisfy the constraints,

$$\begin{aligned} (\mathbf{B}_1 - \mathbf{C}) \cdot (\mathbf{B}_1 - \mathbf{C}) &= K^2, \\ (\mathbf{B}_2 - \mathbf{C}) \cdot (\mathbf{B}_2 - \mathbf{C}) &= K^2, \end{aligned} \tag{2}$$

where K is a constant that defines the length of the link **CB**. This is known as two-position synthesis of the four-bar linkage **OABC**, McCarthy and Soh [12] or McCarthy [13].

3.1. Truss with six linkage modules

In order to design the linkage modules for a truss that has a regular hexagon as its stowed configuration, we set the length of the links of the RR chain to be $a = b = 10$cm. The input and output angles are $\alpha = \beta = 2\pi/6 = \pi/3$. We substitute these values into Eq 2, and subtract the first equation from the second to obtain,

$$\mathcal{C}_6 : u(3x + \sqrt{3}y - 20) - v(\sqrt{3}x - 3y) - 10(x + \sqrt{3}y - 10) = 0 \tag{3}$$

where $\mathbf{B_1} = (x, y)$ and $\mathbf{C} = (u, v)$ are the unknown coordinates that define the coupler link **CB**. There are many solutions to this equation that we examined to ensure smooth movement between the task positions. We selected the solution that yields the four-bar linkage given by

$$\mathbf{O} = (0, 0), \quad \mathbf{A} = (10, 0), \quad \mathbf{B}_1 = (12.08, 4.59), \quad \text{and} \quad \mathbf{C} = (3.47, -2.30). \tag{4}$$

This yields the four-bar linkage module shown in Fig. 5. Assembling six of these linkage modules in series yields the deployable truss shown in Figs. 1 and 2.

This procedure yields the design of a one degree-of-freedom 12 bar mechanism that guides a truss from a linear configuration to a circular configuration.

3.2. Truss with 12 linkage modules

In order to design the linkage modules for a truss that has a regular dodecagon (12 sides) as its stowed configuration, we set the length of the links of the RR chain to be $a = b = 10$cm, as we did for the hexagon. The input and output angles

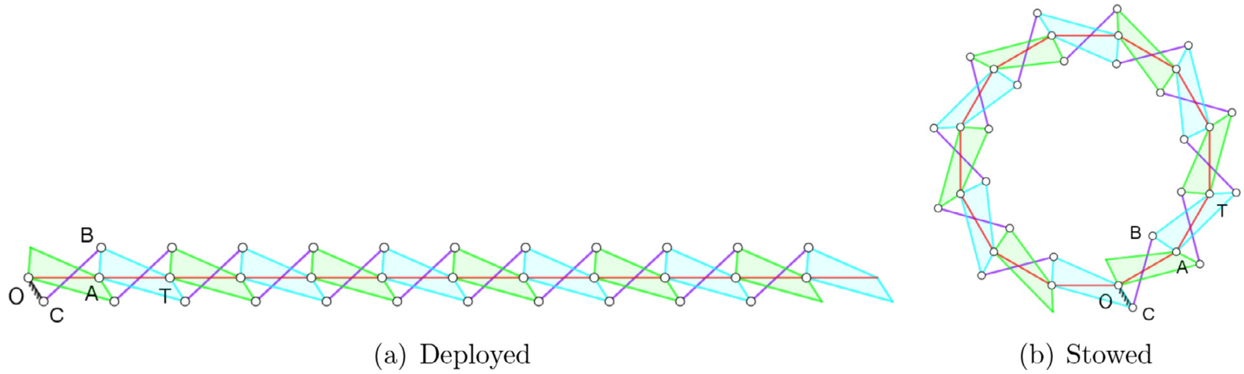


Fig. 6. The deployed and stowed configurations of a truss that coils into a dodecagon.

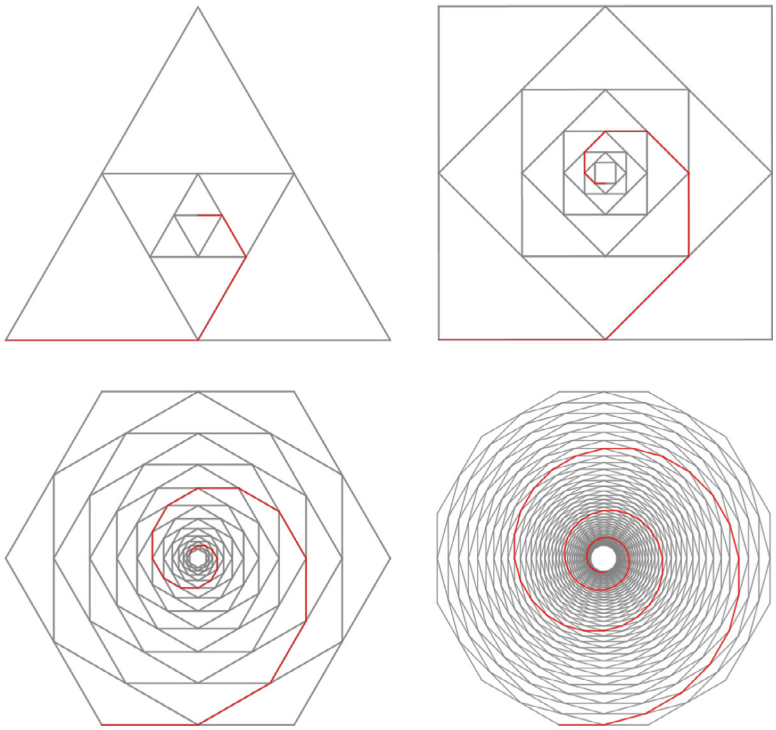


Fig. 7. The regular triangle spiral, quadrilateral spiral, hexagon spiral and dodecagon spiral.

now become $\alpha = \beta = 2\pi/12 = \pi/6$. We substitute these values into Eq 2, and subtract the first equation from the second to obtain,

$$\mathcal{C}_{12} : u(x + \sqrt{3}y + 10 - 10\sqrt{3}) - v(\sqrt{3}x - y + 10 - 10\sqrt{3}) + 10((\sqrt{3} - 2)x - y + 20 - 10\sqrt{3}) = 0, \tag{5}$$

where $\mathbf{B_1} = (x, y)$ and $\mathbf{C} = (u, v)$ are the unknown coordinates that define the coupler link **CB**. There are many solutions to this equation, we selected the solution that yields the four-bar linkage,

$$\mathbf{O} = (0, 0), \quad \mathbf{A} = (10, 0), \quad \mathbf{B}_1 = (10.30, 4.24), \quad \text{and} \quad \mathbf{C} = (2.15, -3.35). \tag{6}$$

Assembling 12 of the linkage modules in series yields the deployable truss shown in Fig. 6.

This procedure yields the design of a one degree-of-freedom 24 bar mechanism that guides a truss from a linear configuration to a circular configuration.

3.3. Length ratio for a circular stowed configuration

In order to compare the space occupied by the truss when it is in the deployed and stowed configurations, we introduce the ratio μ_n of the length of the truss when deployed and the diameter of the circumscribed circle of the regular polygon

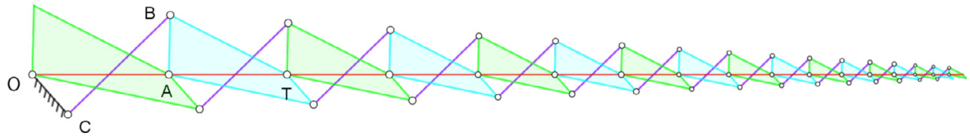


Fig. 8. The deployed position of a hexagon spiral truss with $k = 17$ linkage modules.

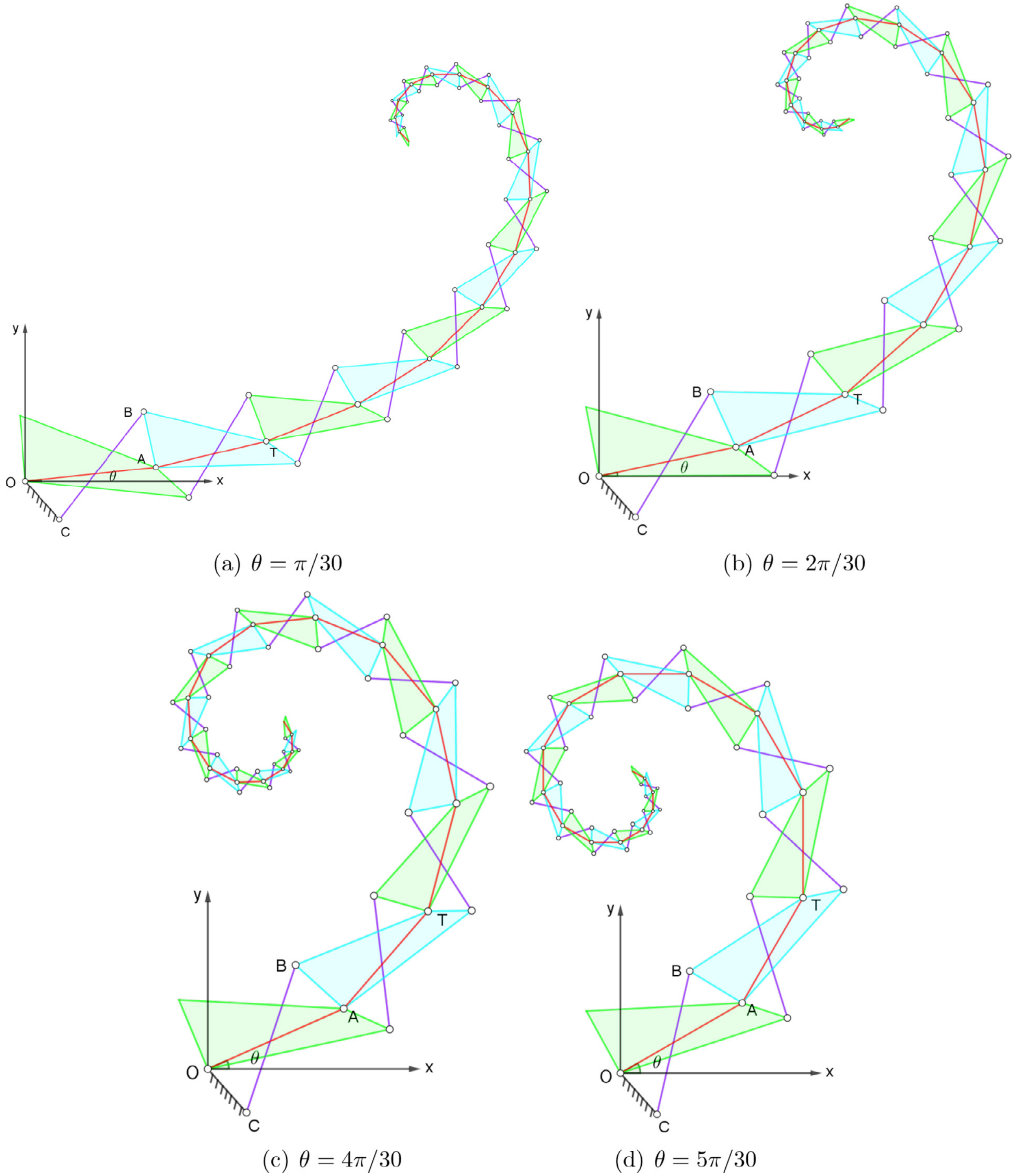


Fig. 9. The configurations of the hexagon spiral truss as it coils from deployed to stowed configurations.

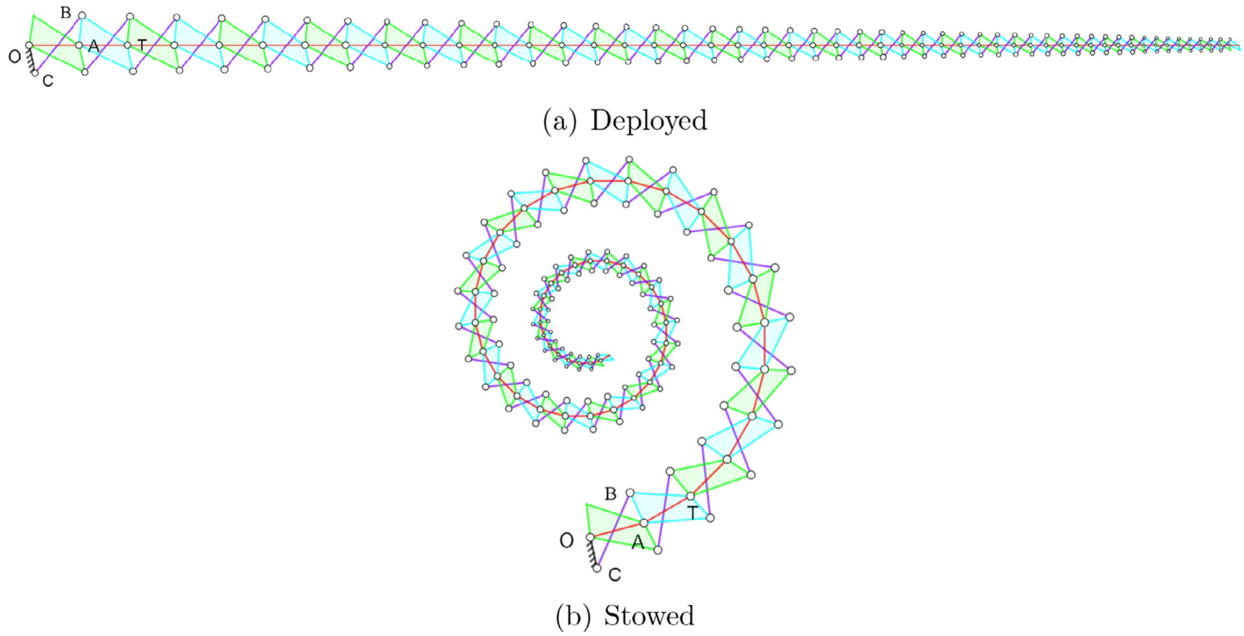


Fig. 10. The deployed and stowed configurations of a dodecagon spiral truss constructed with $k = 50$ linkage modules.

that it forms when it is stowed. This yields the relationship,

$$\mu_n = \frac{D\sin(\frac{\pi}{n})\cdot n}{D} = n\sin(\tfrac{\pi}{n}), \tag{7}$$

where D denotes the diameter of the circumscribed circle of the n-sided regular polygon. As the number of the sides of the inscribed polygon increases so does this length ratio. When n becomes large this length ratio becomes,

$$\lim_{n\to\infty}\mu_n = \lim_{n\to\infty} n\sin(\tfrac{\pi}{n}) = \pi. \tag{8}$$

4. Truss with a spiral-shaped stowed configuration

In this section, we use two position synthesis to design linkage modules that guide a truss from a linear deployed configuration into a polygonal spiral configuration. A polygonal spiral is a self-similar curve that is an approximation to the logarithmic spiral, see Sandefur [14] or Weinsstein [15]. There are a number of variations on the polygonal spiral, we focus on curves obtained from the sequence of inscribed regular polygons obtained by connecting the midpoints of the sides of one polygon to obtain the next. The polygonal curve is generated by connecting one side of one polygon to a side of the next inscribed polygon. Fig. 7 shows polygonal curves generated by inscribed regular triangles, quadrilaterals, hexagons and dodecagons.

In order to stow the truss in a polygonal spiral, adjacent sides must rotate from a relative angle of $\theta = \alpha = 0$ to $\theta = \beta = \pi/n$, where β is the one-half of the exterior angle of the n-sided regular polygon that generates the spiral. Let **OA** and **AT** define the RR serial chain that forms the first linkage module. The fixed frame F is located at **O** and the moving frame M has its origin at **T** and its x-axis directed along **AT**.

The RR chain for the first linkage module has links of length $|\mathbf{OA}| = a$ and $|\mathbf{AT}| = b = a\cos(\pi/n)$. The positions of the moving frame M in the deployed and stowed configurations are given by the same transformations defined in (1). As we did in the previous section, we seek the coordinates of a fixed pivot **C** in F, and of a moving pivots $\mathbf{B}_1$ and $\mathbf{B}_2$ in F, such that the distances between these points are equal in both configurations, that is so

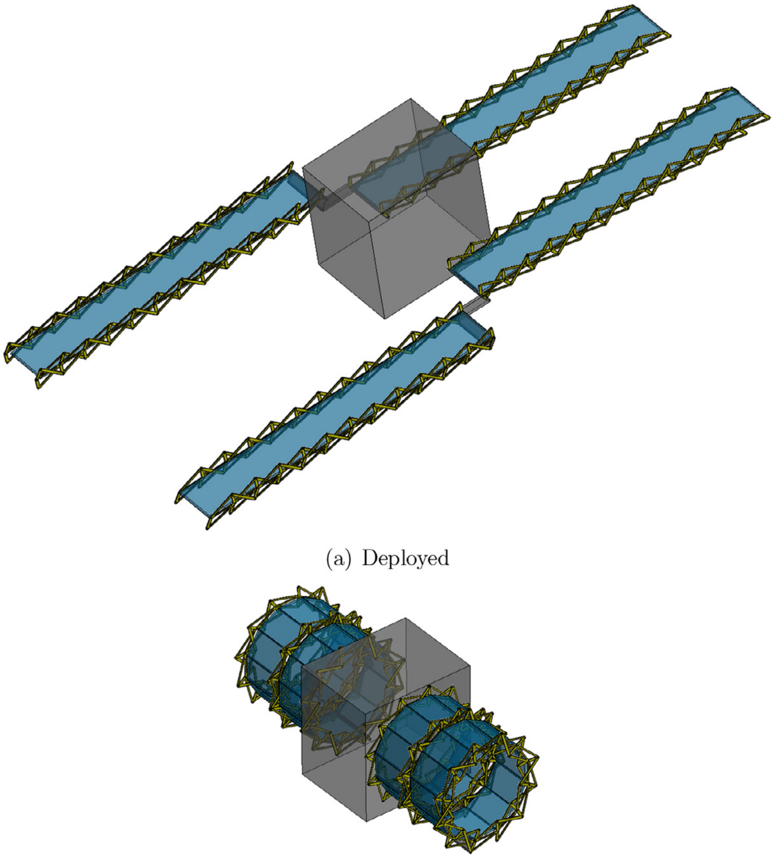
$$|\mathbf{B}_1 - \mathbf{C}| = |\mathbf{B}_2 - \mathbf{C}|. \tag{9}$$

This condition yields the design Eq. (2).

The number of linkage modules, k, in a polygonal spiral truss is independent of the number of sides n of the polygon that generates the spiral. Because each subsequent linkage module must be scaled by the fraction $\mu = \cos(\pi/n)$, the length of the kth link is,

$$L_k = a\cos^{(k-1)}(\pi/n). \tag{10}$$

Thus, the acceptable size of this last link determines the number of modules k.



(a) Deployed

(b) Stowed

Fig. 11. The deployed and stowed configurations of four pairs of dodecagon spiral trusses that deploy antenna arrays or solar panels.

4.1. Spiral generated by hexagons

In order to design the first linkage module for a hexagon spiral truss, we set the length of the links of the RR chain to be $|\mathbf{OA}| = 10$cm and $|\mathbf{AT}| = 10\cos(\pi/6)$cm. The input and output angles are $\alpha = \beta = \pi/6$. We substitute these values into Eq 2, and subtract the first equation from the second to obtain,

$$\mathcal{S}_6 : u(x + \sqrt{3}y + 10 - 10\sqrt{3}) - v(\sqrt{3}x - y + 10 - 10\sqrt{3}) + 10(20 - 10\sqrt{3} - (2 - \sqrt{3})x - y) = 0, \tag{11}$$

where $\mathbf{B_1} = (x, y)$ and $\mathbf{C} = (u, v)$ are the unknown coordinates that define the coupler link **CB**. There are many solutions to this equation, we selected the solution that yields the four-bar linkage,

$$\mathbf{O} = (0, 0), \quad \mathbf{A} = (10, 0), \quad \mathbf{B}_1 = (10.10, 4.36), \quad \text{and} \quad \mathbf{C} = (2.60, -2.93). \tag{12}$$

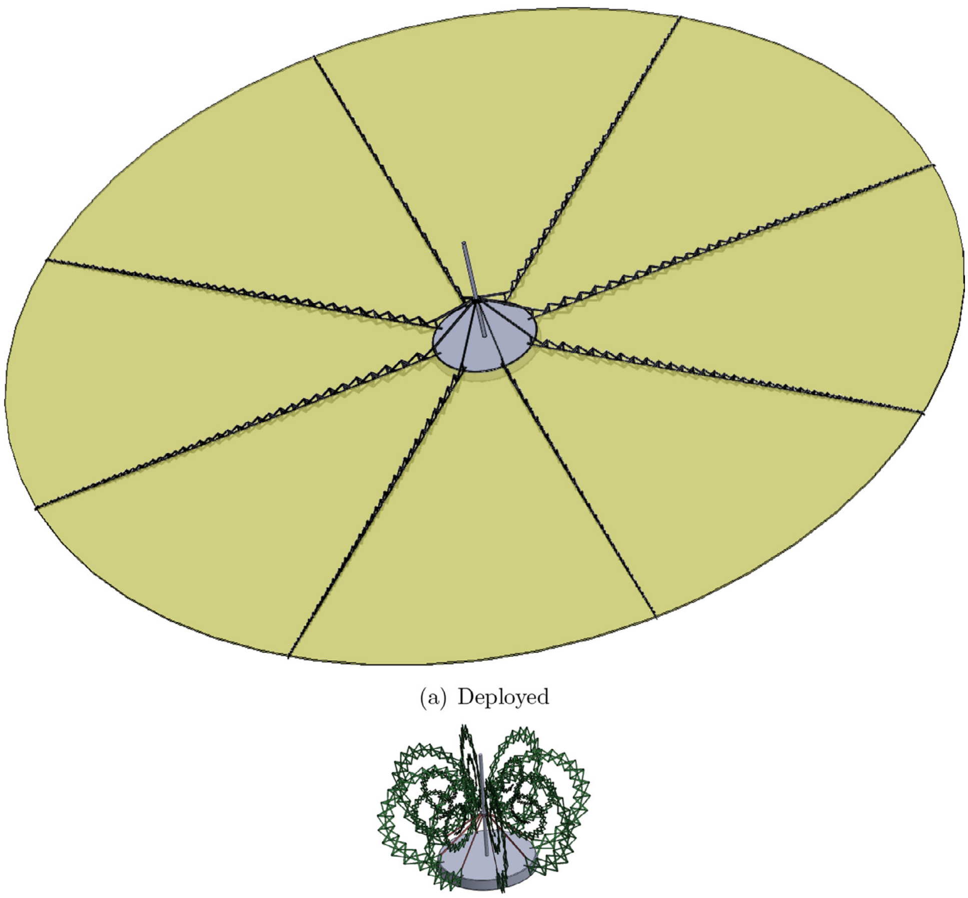
Fig. 8 shows the linear deployed position of a hexagon spiral truss that has $k = 17$ linkage modules. The dimensions of each module is simply scaled from the dimensions of the first linkage module. Fig. 9 shows how the truss curls into a polygonal spiral as the input crank rotates from $\theta = 0$ to $\theta = \pi/6$.

This procedure yields the design of a one degree-of-freedom 34 bar mechanism that guides a truss from a linear configuration to a spiral configuration.

4.2. Spiral generated by dodecagon

In order to design the first linkage module for a dodecagon spiral truss, we set the length of the links of the RR chain to be $|\mathbf{OA}| = 10$cm and $|\mathbf{AT}| = 10\cos(\pi/12)$cm. The input and output angles are $\alpha = \beta = \pi/12$. Substitute these values into Eq 2, and subtract the first equation from the second to obtain,

$$\begin{aligned}\mathcal{S}_{12} : {} & u((2 - \sqrt{3})x + y - 5(\sqrt{2} - 2\sqrt{3} + \sqrt{6})) - v(x - (2 - \sqrt{3})y - 5(2 + \sqrt{2} \\ & -\sqrt{6})) + 5((-4 + \sqrt{2} + \sqrt{6})x + (\sqrt{2} - \sqrt{6})y + 10(4 - \sqrt{2} - \sqrt{6})),\end{aligned} \tag{13}$$



(a) Deployed

(b) Stowed

Fig. 12. The deployed and stowed configurations of six supporting trusses for a star-shade. Each truss stows as a dodecagon spiral with 50 linkage modules.

where $\mathbf{B_1} = (x, y)$ and $\mathbf{C} = (u, v)$ are the unknown coordinates that define the coupler link **CB**. Of the many solutions to this equation, we selected the one that yields the four-bar linkage,

$$\mathbf{O} = (0, 0), \quad \mathbf{A} = (10, 0), \quad \mathbf{B}_1 = (10.56, 5.96), \quad \text{and} \quad \mathbf{C} = (1.20, -5.56). \tag{14}$$

Fig. 10 shows the spiral stowed configuration and the linear deployed configuration of a dodecagon spiral truss that has $k = 50$ linkage modules.

This procedure yields the design of a one degree-of-freedom 100 bar mechanism that guides a truss from a linear configuration to a spiral configuration.

4.3. Length ratio for a spiral-shaped stowed configuration

In order to compare the space occupied by the truss when it is in the deployed and stowed configurations, we introduce the ratio σ_{nk} of the length of the truss when deployed and the diameter of the circumscribed circle of the regular polygon of the polygonal spiral that it forms when it is stowed. This yields the relationship,

$$\sigma_{nk} = \frac{\sum_{i=1}^{k} \frac{D}{2} \sin(\frac{\pi}{n}) \cos^{i-1}(\frac{\pi}{n})}{D} = \sum_{i=1}^{k} \frac{1}{2} \sin(\tfrac{\pi}{n}) \cos^{i-1}(\tfrac{\pi}{n}). \tag{15}$$

When k tends to infinity,

$$\lim_{k\to\infty} \sigma_{nk} = \lim_{k\to\infty} \frac{1}{2} \frac{\sin(\frac{\pi}{n})(1-\cos^{m}(\frac{\pi}{n}))}{1-\cos(\frac{\pi}{n})} = \frac{sin(\frac{\pi}{n})}{2(1-cos(\frac{\pi}{n}))}. \tag{16}$$

When both k and n tend to infinity,

$$\lim_{k,n\to\infty} \sigma_{nk} = \infty. \tag{17}$$

It can be known that as the number of the sides of the inscribed polygon and the number of linkage modules increase so does this length ratio. And the length ratio can be infinity theoretically.

5. Example applications

In this section, we describe example application for these deployable trusses. Those with a circular deployed configuration can be used as the supporting mechanisms for a planar antenna array or solar panels. Fig. 11 shows the stowed and deployed configurations of panel on a satellite or equivalent structure. In this case, we use four pairs of dodecagon trusses. The trusses form a pair of circular cylinders on each side of the structure when stowed, and deploy to form a pair of planar structures in opposite directions.

An interesting application for the polygonal spiral truss is to support a Starshade that is used to shield the light from a star in the search for exoplanets. Webb et al. [16] present an interesting design for the Starshade that includes deployable trusses and origami folded elements. We propose to use eight trusses that deploy from a dodecagon spiral with 50 linkage modules. The one degree-of-freedom movement of these deployable mechanisms provides a reliable deploying operation from a compact package. Fig. 12(a) shows the deployed configuration of the eight dodecagon spiral trusses which support the component of shielding the light. Fig. 12(b) shows the folded configuration of the eight dodecagon spiral trusses.

6. Conclusions

This paper presents the design of deployable trusses that move between a linear deployed configuration to a circular or polygonal spiral stowed configuration by coiling and uncoiling movements. This is different from existing deployable structures that rely on scissor-hinge mechanisms and other special linkages to provide expansion, or changes in scale, to move between the stowed and deployed configurations. Our deployable truss is constructed from a series of four-bar linkage modules that provide local rotational movement in a way that causes the entire truss to coil and uncoil in a one degree-of-freedom movement between the stowed to the deployed configuration. The design equations yield many solutions for these linkage modules, and we present example designs with applications to deployable antenna and solar panel arrays, as well as for deployable trusses that support a star-shade for astronomical observation of exoplanets.

Besides, we are working on the actuation methods of these types of mechanisms. Future research will explore distributed and coordinated actuation, such as cables and pulleys for larger systems, and shape memory alloy for smaller systems.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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