Single-Bit Millimeter Wave Beam Alignment Using Error Control Sounding Strategies

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***Abstract*—Millimeter wave technology is an essential component of most solutions that address the coverage and throughput demands of next-generation cellular networks. To overcome the high propagation losses however, it is necessary to deploy large antenna arrays for spatial localization of energy by beamforming. This imposes a significant communication overhead, especially when channel reciprocity does not hold. In this work, we study the problem of successive one-bit feedback-assisted beam alignment. We exploit the sparse nature of the millimeter wave channel to pose the beamforming problem as a questioning strategy. We consider both adaptive (*closed-loop*) and non-adaptive (*open-loop*) channel sounding techniques which are robust to erroneous feedback signals caused by noisy quantization. In the adaptive case, we formulate new sounding signals by drawing a parallel with the well known Ulam's problem. In the non-adaptive case, the beams are designed in accordance to an open-loop code. We demonstrate that multiple paths can also be resolved by using ideas from group testing. Finally, we show the efficacy of our proposed techniques via simulations.**

***Index Terms*—Millimeter wave, beamforming, Ulam's problem, Channel estimation, Antenna Arrays, Closed-Loop**

I. Introduction

It is estimated that by 2021, there will be up to 1.5 billion wireless devices with cellular connections [2]. The current efforts for 5G standardization have thus proposed for use of frequencies in the 20 to 100 GHz range commonly referred to as millimeter wave (mmWave) frequencies [3]. The millimeter wave spectrum affords extremely wide channel bandwidths (up to 1 GHz) which would provide the necessary capacity increase and enable high data rates as envisioned in 5G.

Communication at mmWave frequencies suffers from higher isotropic path loss, attenuation due to rain, reduced diffraction around obstacles, sparse scattering, and sensitivity to blockages [4]–[6]. It is thus necessary to use a large number of antennas to synthesize highly directional beams with high beamforming gain. Spatial localization of energy will require selection of a high-dimensional beamformer at the transmitter.

Transparent beam sounding is an important feature in LTE, specifically enabling advanced beamforming and coordinated multipoint [7]. One way this is accomplished is by precoding the pilot or the reference signal the same way as the accompanying data. The beamforming operation at the transmitter is then open to implementation and remains oblivious to the UE.

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This work deals with the problem of feedback-assisted selection of beams for communication between two nodes operating in the mmWave band. The goal is to pick a good beam to maximize the desired performance metric while minimizing the time and resources to do so. It is well known from MIMO theory that the optimal beam achieving maximum data throughput is a function of the channel realization. Maximum spectral efficiency is obtained when the beam picked is aligned to the channel subspace [8]. Unfortunately, the current channel realization (CSI) is not available to the transmitter apriori. The receiver must provide some auxiliary channel information in the feedback to help the transmitter ascertain the optimal beam. In legacy cellular systems, a known pilot or reference signal is sent out from each antenna element [9]. The receiver then estimates the individual per-antenna gains and conveys it back to the transmitter. Due to a large number of antennas in a millimeter wave MIMO system however, the multi-dimensional channel vectors impose a large communication overhead rendering per-atenna sampling inefficient.

Several codebook-based techniques that allow CSI acquisition without explicit channel vector estimation for mmWave use have been proposed in the literature [10]–[15]. The transmitter is equipped with a finite codebook of beamformers. In exhaustive sampling, each beam in the codebook is sounded once and the receiver feeds back the index of the best beam after all beams have been sounded. In hierarchical sampling, the transmitter and receiver jointly determine the best beam pair of a relatively coarse resolution which is further refined in successive stages. This involves the receiver feeding a locally optimal index back to the transmitter to ascertain the beams for subsequent channel sounding. This is usually accomplished by designing hierarchical subcodebooks containing beams of varying resolution.

Another popular approach is the so-called *compressive beam alignment* approach, where the channel entries are compressed into a few linear measurements using random beamforming vectors and fed back to the transmitter. In [16], [17] for example, the sounding or sensing beams are generated by applying quantized random i.i.d. phase shifts across antenna elements. The path gains and angles of departure in the downlink are then estimated by exploiting the spatial sparsity of the millimeter wave channel. Many other beam alignment strategies leveraging tools from compressed sensing have been studied extensively (see for eg. [18], [19]). These approaches typically require phase coherence between measurements meaning that the receiver needs to report both the signal magnitude and phase information in the feedback link. Alignment with magnitude only measurements is explored in [20].

In this work, we consider a model where the receiver conveys only *one bit* of information per channel sounding about the optimal beam. This ACK/NACK type of feedback can be a function of the decoded channel sounding sequence or determined via simple thresholding. This model enables transparent beam sounding - the UE does not see the actual number of transmit antennas and is not required to be informed of the beamformers used at the transmitter. Rough beam alignment using *very* low-resolution feedback (such as ARQ) may prove to be important for standardization. The system setup is shown in Figure 1.

Our contributions can be summarized as :

1) We demonstrate that the highly directional nature of the millimeter wave channel allows us to interpret channel sounding as a questioning strategy. The sounding beams correspond to *questions* (about the channel) while the feedback bits correspond to *answers*.
2) We investigate both adaptive (or closed-loop) and non-adaptive (or open-loop) beam alignment algorithms in this framework. In the non-adaptive algorithm, all beams to be sounded are pre-selected and are designed corresponding to a chosen error control code.
3) In the adaptive case, where the beams are selected 'on the fly', we show that the beam alignment problem ties closely with Ulam's problem well known in computer science literature [21]–[23]. We formulate new sounding signals by exploiting this connection. We then study the sounding time gap between adaptive and non-adaptive beam alignment techniques via simulations.
4) The questioning interpretation of channel sounding is also useful when multiple paths are present. Using tools developed in group testing [24]–[27], we design new sounding signals that enable the transmitter to identify the dominant channel directions. The beam for communication is then selected by training only on these directions.
5) The quantization of channel state information in a real system is noisy. This could be due to beam imperfections, fading, channel noise or interference. The alignment algorithms we propose are designed to be resilient to noisy quantization.

A preliminary version of this work was presented in 2018 [1]. The authors in [28] built upon our work to propose a non-adaptive resource-efficient design only for the case of one path. An open-loop channel estimation technique inspired by linear block codes in a different setting was described in [29]. In [30], the authors develop new techniques for quantitative group testing, and note that this may be useful for simultaneous sensing of multiple users when the number of users within a sounding beam is available as feedback to the transmitter.

The rest of the paper is organized as follows. In Section II, we explain the system model and state the problem we wish to solve. Section III and IV discuss open-loop and closed-loop beam alignment algorithms respectively when a single path dominates. Section V deals with the case of multipaths. Simulation studies are presented in Section VI. Finally, concluding remarks are presented in Section VII.

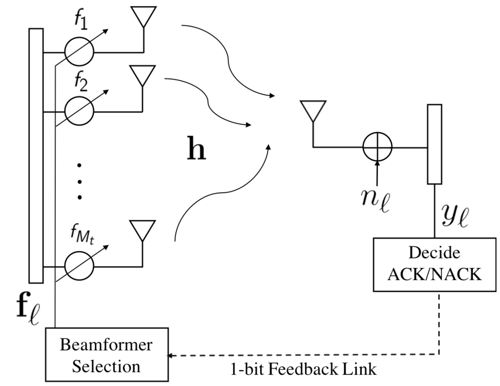

Figure 1. System Model for a Millimeter wave MISO system with 1-bit feedback considered in this paper.

Notation: All vectors unless stated are column vectors and their ℓ^2-norm is represented by $\|.\|$. $\mathcal{CN}(m, \sigma^2)$ represents a circularly symmetric complex Gaussian random variable with mean m and variance σ^2. $\mathbf{a}^*$ denotes the conjugate transpose of the vector $\mathbf{a}$. The hamming distance between two binary vectors $\mathbf{x}$ and $\mathbf{y}$ is denoted by $\mathcal{H}(\mathbf{x}, \mathbf{y})$. $\mathbb{1}(.)$ denotes the indicator function. $\lceil\ \rceil$ denotes the ceiling function while $\lfloor\ \rfloor$ is the floor function. The set of complex numbers is denoted by $\mathbb{C}$ and the set of natural numbers by $\mathbb{N}$. $GF(q)$ is the finite field with q elements where q is some power of a prime.

II. System Model

Consider a multiple-input single-output (MISO) millimeter wave communication system with M_t antennas at the transmitter. The receiver is assumed to have a single omni-directional antenna. The methods discussed are applicable to a multi-antenna receiver, but this is beyond the scope of the present article. The system setup is shown in Figure 1. A number of different beamforming architectures have been proposed in literature (see for eg. [18], [31], [32]). Sounding schemes proposed in this work can be adapted to any given architecture.

To accomplish beam alignment, we assume that the transmitter sends a training sequence or reference signal. The receiver (or user) then processes this known signal. After processing, we model the symbol received by the receiver on the ℓ^{th} sounding interval as

$$y_\ell = \sqrt{M_t}\mathbf{h}^*\mathbf{f}_\ell + n_\ell \quad (1)$$

where $\mathbf{f}_\ell \in \mathbb{C}^{M_t}$ is the beamforming vector picked by the transmitter in the ℓ^{th} sounding, $\mathbf{h} \in \mathbb{C}^{M_t}$ describes the channel and $n_\ell \sim \mathcal{CN}(0, 1/\rho)$ is the noise term. ρ is the post processed SNR after the channel sounding sequence is match filtered. To restrict the total power at the transmitter, $\mathbf{f}_\ell$ is constrained to be unit norm.

The millimeter wave channel is characterized by large coherent bandwidths and a sparse scattering environment. We thus adopt a ray-based narrow-band channel model [10]. The multi-path propagation delays are suppressed and the channel $\mathbf{h} \in \mathbb{C}^{M_t \times 1}$ with p paths is modeled as

$$\mathbf{h} = \sum_{i=1}^{p} \alpha_i \mathbf{a}(\theta_i) \quad (2)$$

where θ_i corresponds to the angle of departure (AoD) for the i^{th} path, $\alpha_i \sim \mathcal{CN}(0,1)$ is its complex channel gain, and $\mathbf{a}(\theta_i) \in \mathbb{C}^{M_t \times 1}$ represents the beam steering vector for direction θ_i in the transmitter's array manifold. Due to its highly directional nature, the mmWave channel has only a few dominant paths. The channel model in (2) with a single dominant path reduces to $\mathbf{h} = \alpha \mathbf{a}(\theta_0)$. For pedagogical reasons, we will assume a uniform linear array (ULA) at the transmitter. Extensions to planar arrays is possible. For a ULA, the unit norm beam steering vector is

$$\mathbf{a}(\theta) = \frac{1}{\sqrt{M_t}} \left[1 \; e^{j2\pi\beta \sin\theta} \; e^{j2\pi(2)\beta \sin\theta} \cdots \; e^{j2\pi(M_t-1)\beta \sin\theta} \right]^T \tag{3}$$

where β is the ratio of inter-antenna spacing to wavelength.

Suppose that the desired coverage area is $\mathcal{I} = [\theta_{min}, \theta_{max}]$. The transmitter chooses an appropriate collection of T mutually disjoint intervals or bins that covers $\mathcal{I}$, labeled 1 through to T. One possible choice is uniform partitioning where all intervals are picked to have equal length. A non-uniform partitioning may be used if the base station has some prior knowledge of where the user is located. For example, it may choose finer intervals in regions where the user is more likely to be to achieve greater beam directionality. For the purpose of beam alignment, the transmitter is equipped with a beam set denoted as $\mathcal{C} = \{\mathbf{a}_1, \mathbf{a}_2, \cdots \mathbf{a}_T\}$, where each beam $\mathbf{a}_i$ is designed to span the interval with label i. To test if the user is in a region $S \subset \{1, 2, 3, \cdots T\}$, the transmitter chooses the beamformer $\mathbf{f}$ as a normalized linear combination of beams in S according to

$$\mathbf{f}_S = \frac{\sum_{i \in S} \mathbf{a}_i}{\|\sum_{i \in S} \mathbf{a}_i\|}. \tag{4}$$

As noted previously, we consider a model where the receiver provides only *one bit* of information per channel sounding about the best beam. The received symbol y_ℓ is quantized to a single bit r_ℓ and fed back to the transmitter according to some rule

$$r_\ell = \Gamma_\ell(y_\ell). \tag{5}$$

One possible choice is to set $r_\ell = \mathbb{1}(|y_\ell|^2 > \gamma_\ell)$ where $\mathbb{1}(.)$ is the standard indicator function and γ_ℓ is the chosen threshold on the ℓ^{th} channel sounding.

The channel-normalized beamforming gain corresponding to a beamforming vector $\mathbf{f}$ is defined to be

$$A(\mathbf{f}) = \frac{|\mathbf{h}^* \mathbf{f}|^2}{\|\mathbf{h}\|^2}. \tag{6}$$

After receiving N bits of feedback over the N sounding intervals denoted $\{r_j\}_{j=1}^N$, the transmitter wishes to select

$$\mathbf{f}_{opt} = \arg\max A(\mathbf{f}). \tag{7}$$

In the case of one dominant path for example, if all bits r_ℓ were reliable and no prior information was available, the optimal strategy for the transmitter is to use a simple binary search like algorithm by successively refining the search beam width by half until the user is located to the desired beam resolution. However due to noisy quantization, some of the bits received in the feedback could be inconsistent with the receiver's actual location. We then say that these bits are in error or are erroneous. This is depicted in Fig. 2.

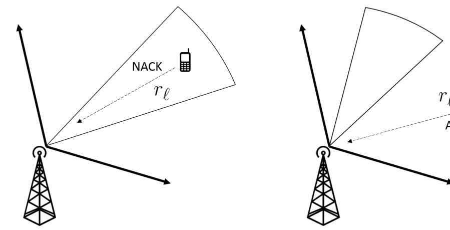

Figure 2. Down-link noise, beam imperfections and other factors can cause the ACK/NACK feedback to be in error.

Since the feedback link carries only a single bit per feedback interval, channel sounding can be interpreted as a *questioning strategy* with yes/no answers (ACK or NACK). However, quantization error can cause some of the yes/no answers to be incorrect, which equates to lies in questioning. The general problem of searching over a finite set under different error models is a well studied problem in theoretical computer science [21]–[23], [33]. The actual number of erroneous bits in a real system is a random quantity. We thus follow a worst-case design philosophy in that our algorithms have garaunteed resilience against a given maximum number of errors.

The beam $\mathbf{f}_\ell \in \mathbb{C}^{M_t \times 1}$ picked in the ℓ^{th} sounding can be described mathematically as

$$\mathbf{f}_\ell = \mathcal{F}(\mathcal{C}, r_1, \cdots, r_{\ell-1}) \qquad \ell = 1, 2, \cdots, N, \tag{8}$$

a function of the beam set $\mathcal{C}$ and the previously received feedback bits $\{r_i\}_{i=1}^{\ell-1}$. This type of beam selection is referred to as closed-loop or adaptive channel sounding. The base station keeps track of bits received in the feedback to select subsequent beams in an 'online' manner.

An alternative approach with far less complexity is to sound beams agnostic to the received bits. Mathematically,

$$\mathbf{f}_\ell = \mathcal{F}(\mathcal{C}) \qquad \ell = 1, 2, \cdots, N. \tag{9}$$

In other words, $\mathbf{f}_\ell$ is not a function of $\{r_i\}_{i=1}^{\ell-1}$. All beamforming codewords $\{\mathbf{f}_\ell\}_{\ell=1}^N$ are designed 'offline' before the sounding process even begins. We refer to such a strategy as open-loop or non-adaptive channel sounding. Sections III and IV discuss open-loop and closed-loop channel sounding techniques respectively for the case of one dominant path. Channel sounding for multi-path is dealt with in Section V.

It is clear from Fig. 2 that an erroneous bit r_ℓ received in the feedback corresponds to a 'lie' in the questioning interpretation. In closed-loop channel sounding, since $\mathbf{f}_\ell$ is selected as a function of $\{r_i\}_{i=1}^{\ell-1}$, an erroneous bit will change the subsequent beams that are sounded. On the other hand, in open-loop channel sounding, the effect of erroneous bits is felt only in post processing. For beamforming vector $\mathbf{f}$, its normalized spatial pattern as a function of the physical angle $\theta \in [-90^\circ, 90^\circ]$ is characterized as

$$G_{\mathbf{f}}(\theta) = |\mathbf{a}(\theta)^* \mathbf{f}|^2. \tag{10}$$

III. Open-Loop Channel Sounding

A. Algorithm Description

This section describes non-adaptive techniques for beam alignment when only one path dominates. Without loss of generality, assume $T = 2^K$. We label each of the 2^K bins in the coverage area with K bits according to a one-to-one mapping

$$\varphi : \{1, 2, 3, \cdots, T\} \longmapsto \{0, 1\}^K, \tag{11}$$

One choice of φ in (11) is simply to use the standard decimal to binary representation. This is illustrated for the case of $T = 8$ in Fig. 3. Let $\mathbf{m}_j$ denote the vector label for bin j and $\mathbf{m}_{dp}$ be the label that corresponds to the dominant path. This is to say the optimal beam from the codebook maximizing (6) is $\mathbf{a}_{\varphi^{-1}(\mathbf{m}_{dp})}$.

Notice in Fig. 3 that the first bit of the vector label for each of the bins in the second half of the search region is 1. In other words, a beam sounded to search the second half of the desired coverage area say $\mathbf{f}_1$ can be mapped to a question of the form : "Is the first bit of $\mathbf{m}_{dp}$ equal to 1?". After $\mathbf{f}_1$ is sounded, the receiver has access to the symbol y_1 according to (1) which is quantized to a single bit r_1 and fed back to the transmitter. Then, $r_1 = 0$ corresponds to a no answer while $r_1 = 1$ corresponds to a yes answer. Due to the fact that quantization is noisy, some of the r_i, $i = 1, 2, \cdots, N$, could be inconsistent with the direction of the dominant path. Hence, K *questions*, one for each bit, are not enough.

Suppose we assume that no more than L bits received at the transmitter are erroneous. It is then clear that picking the optimal beam is equivalent to determining K information bits (of the label $\mathbf{m}_{dp}$) in the presence of up to L errors. This is the classical problem of error control coding.

Let $\mathbf{G} = [\mathbf{g}_1\mathbf{g}_2 \cdots \mathbf{g}_N]$ be the generator matrix of a linear (N, K, d_{min}) block code over $GF(2)$ where N is the code length, K is the code's dimension and d_{min} is the minimum distance between codewords. Denote by S_ℓ the bins to be sounded in the ℓ^{th} sounding. Under the non-adaptive strategy, we select S_ℓ according to

$$S_\ell = \{j : \mathbf{m}_j\mathbf{g}_\ell = 1\}, \tag{12}$$

where the corresponding beam $\mathbf{f}_\ell$ is given by (4). The transmitter thus picks the sounding signals in accordance with the columns of the generator. Only those regions whose labels when XORed corresponding to the generator's ℓ^{th} column return 1 are picked on the ℓ^{th} channel sounding. Fig. 3 illustrates the shapes of the beams to be sounded for a $(6, 3, 3)$ code. The first 3 columns of the generator form the identity matrix and translate to bit-by-bit questions, one for each bit. Since the 4^{th} column is $[1\ 1\ 0]^T$, $\mathbf{f}_4$ is chosen as a linear combination of beams corresponding to labels $(010), (011), (100)$ and (101). The related *question* is, "Is the XOR of the first two bits in $\mathbf{m}_{dp}$ equal to 1?"

From (12), if all bits $\{r_i\}_{i=1}^N$ are consistent with the direction of the dominant path, we would have

$$r_\ell = \mathbf{I}(\varphi^{-1}(\mathbf{m}_{dp}) \in S_\ell) = \mathbf{m}_{dp}\mathbf{g}_\ell. \tag{13}$$

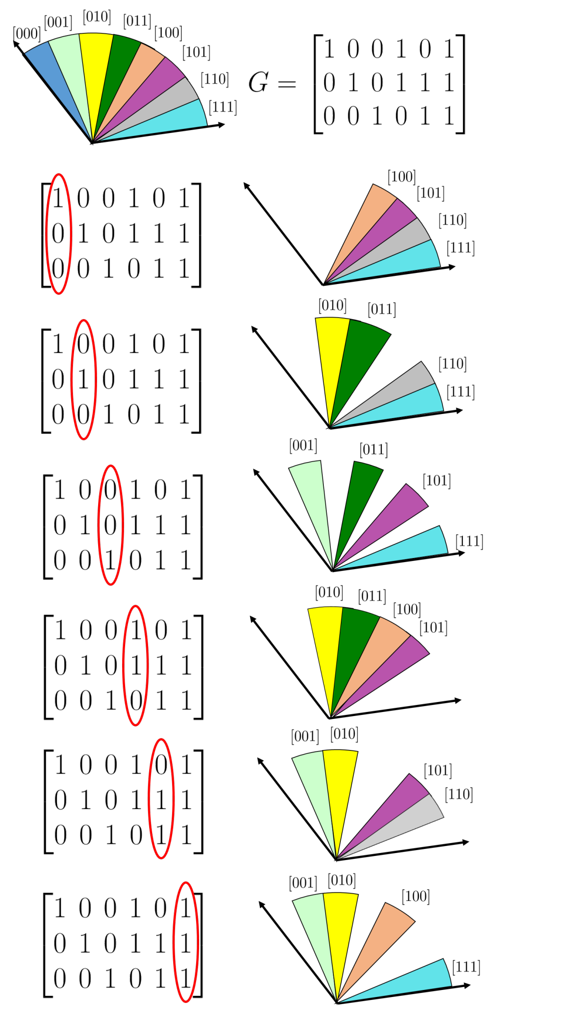

Figure 3. Assume $T = 2^3 = 8$, $L = 1$, N=6. The generator matrix used for the (6,3,3) code is G. The regions to be sounded are shown and correspond to the columns of the generator.

After N beams are sounded then, the received bit vector $\mathbf{r} = (r_1, r_2, \cdots, r_N)$ can be interpreted as a distorted version of the codeword $\mathbf{m}_{dp}\mathbf{G}$, corrupted due to the noisy quantization.

At the end of channel sounding, standard decoding methods from coding theory literature are applied to decode the received codeword $\mathbf{r}$ into the binary label $\hat{\mathbf{m}}$ for the dominant path. The transmitter selects

$$\mathbf{f}_{sel} = \mathbf{a}_{\varphi^{-1}(\hat{\mathbf{m}})} / \|\mathbf{a}_{\varphi^{-1}(\hat{\mathbf{m}})}\|. \tag{14}$$

Since a linear code with $d_{min} \geq 2L + 1$ is resilient against up to L errors [34], the code in Fig. 3 can determine the optimal beam even if one of the six bits received in the feedback is in error. It should be noted that one can also use non-linear codes for designing the sounding scheme.

B. Analysis

The generator matrix $\mathbf{G}$ for any (N, K, d_{min}) linear block code can be converted into what is called the systematic form by Gaussian elimination [34], meaning it has the form

$$\mathbf{G} = [\mathbf{I}_K |\ \mathbf{P}] \tag{15}$$

where $\mathbf{G}$ is of size $K \times N$ and $\mathbf{I}_K$ is the $K \times K$ identity matrix. This implies that the first K beams sounded at the transmitter in open-loop channel sounding can always be made to correspond to K individual bit level questions. We shall see that a parallel observation also holds in the case of closed-loop channel sounding.

One natural goal for beam alignment is to minimize the total sounding time and correspondingly the feedback overhead N. Any bounds known for open-loop codes are useful to characterize the trade-offs between N, the desired beam resolution (relates to K) and the desired degree of error correction L. Given K and L, we look for a $(N, K, 2L+1)$ code with the smallest codeword length N possible. A lower bound on N is due to the celebrated sphere-packing bound

$$2^K \left(\sum_{j=0}^{L} \binom{N}{j} \right) \leq 2^N. \tag{16}$$

The minimum sounding time with an open-loop beam alignment algorithm is the smallest N for which (16) holds.

The well-known Singleton bound [34] states that for any arbitrary block code, $d_{min} \leq N - K + 1$. Thus, if N and K are fixed, the maximum number of erroneous feedback bits L that are guaranteed to be corrected satisfies $L \leq \frac{N-K}{2}$. Codes meeting this bound are called Maximum Distance Separable (MDS) codes. These have been used extensively in data storage systems due to their excellent error correction capabilities. We can leverage them for robust open-loop beam alignment.

IV. Closed-Loop Channel Sounding

In this section, we describe selection of beams as a function of previously received feedback bits as in (8).

A. Preliminaries

The famous mathematician S.M Ulam in his autobiography *'Adventures of a Mathematician'* [35] posed the following question: What is the minimal number of yes/no questions that one needs to determine an unknown number between one and a million if at most one or two of the answers may be lies? The generalization of this problem to distinguish between T numbers with at most L lies has since been extensively studied in the computer science literature [21]–[23] and is popularly called *Ulam's problem*.

We can think of an adaptive channel sounding strategy for the case of one dominant path as Ulam's game between the communicating nodes. The unknown number corresponds to the bin index containing the dominant path. The channel sounding relates to questioning and the erroneously received bits relate to lies during questioning. As before, we have a total of T bins covering the region of interest $[\theta_{min}, \theta_{max}]$ and assume that no more than L bits received at the transmitter are in error.

Berlekamp was the first to develop an analytical framework to analyze Ulam's problem [36] which we outline in IV-B. The general idea is to assign a negative vote to bins that disagree with the received bit in a given sounding iteration. As more beams are subsequently sounded, the 'incorrect' bins hopefully accumulate enough votes and are eventually discarded until only the correct one remains.

Suppose that the transmitter sounds a region $S \subset \{1, 2, 3, \cdots T\}$. If it receives an ACK, the bins in S^c are each assigned a negative vote and if a NACK is received, the bins in S are each assigned a negative vote. Denote A_i to be the collection of bin numbers that have received i negative votes so far. In other words, A_i contains bin numbers with a disagreement tally of i. Since we assume a maximum of L erroneous bits, any bins receiving more than L negative votes can be discarded.

The transmitter's knowledge at any point in the sounding process can thus be summarized by a collection of $L+1$ disjoint sets $\{A_0, A_1, \cdots A_L\}$. An example of how these sets evolve as the channel sounding progresses is shown in Fig. 4

B. Berlekamp's Analysis

Since the sets change only by assignment of negative votes, it is enough to work with their cardinalities. Let x_j denote the cardinality of A_j and define the *n-state* to be the integer sequence $\underline{x} = (x_0, x_1, \cdots x_L) \in \mathbb{N}^{L+1}$. The integer n refers to the number of times that the transmitter is allowed to sound the channel from that point onward. With a total budget of N sounding signals then, the initial state is the N-state and the final state is the 0-state.

On sounding beams with labels in the set $S \subset \{1, 2, 3, \cdots T\}$, we define $U_i = S \cap A_i$ representing channel sounding as the vector $\underline{u} = (u_0, u_1, \cdots u_L)$, where $u_i = |U_i|$. This is equivalent to partitioning each set A_i into disjoint subsets U_i and V_i of sizes u_i and v_i and testing if the user is in the region $\bigcup_{j=0}^{L} U_j$. The initial state (the N-state) is $(T, 0, 0, \cdots 0) \in \mathbb{N}^{L+1}$.

In this formulation, the goal is to devise a strategy with minimal sounding time so that at the end of channel sounding, only one of the sets A_j is non-empty. What one would like is for the final 0-state to look like one of the following : $(1, 0, 0, \cdots, 0)$, $(0, 1, 0, 0, \cdots, 0)$, $(0, 0, 1, 0, \cdots, 0)$, $\cdots$, $(0, 0, 0, 0, \cdots, 1)$. Note also that not all sets can come up empty as that would imply more than L lies have occurred, a violation of the rules.

Berlekamp introduced the concept of "volume" for states. The volume of a n-state $\underline{x} = (x_0, x_1, \cdots x_L)$ is defined to be [36]

$$V_n(\underline{x}) = \sum_{i=0}^{L} x_i \sum_{j=0}^{L-i} \binom{n}{j}.$$

This definition is intuitively the total number of ways in which lies could possibly be distributed; for each of the x_i elements in the set A_i, there are $\binom{n}{j}$ arrangements of j erroneous feedback bits in the n remaining probes, where j takes any value from 0 to $L - i$. Berlekamp proved the following theorems:

Theorem 1. *[36] (Conservation of Volume) Let $\underline{x}$ be any non trivial n-state, and let $\underline{y}$ and $\underline{z}$ be the $(n-1)$-states that result*

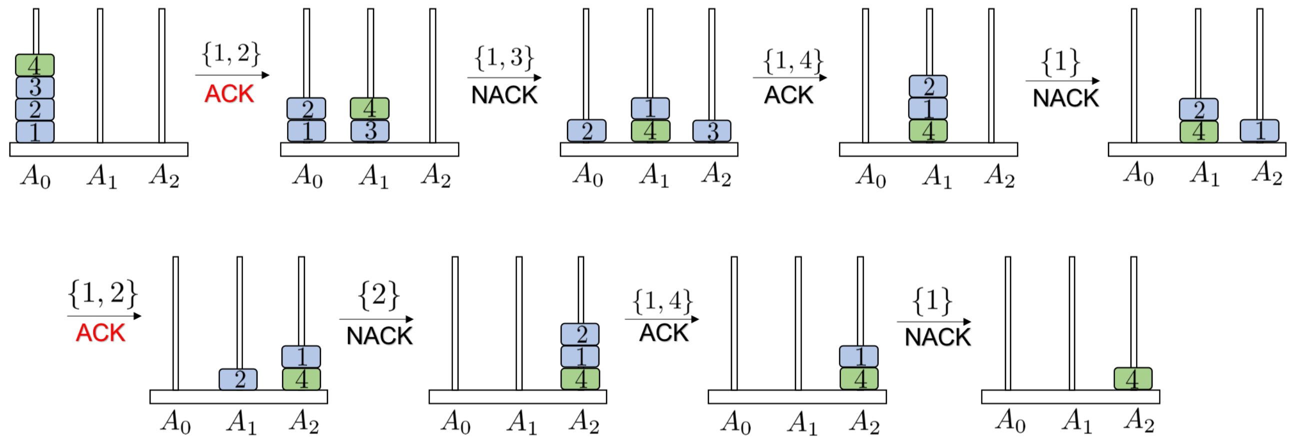

Figure 4. An example tracking the complete evolution of states for $K = 2$ and $L = 2$ assuming that the correct bin is the one labelled 4. The query regions are shown within braces at each step. The answers that are erroneous lies are colored red. The question at each step was selected by solving the integer program in (23) exactly. At the end of $N = 8$ questions, only the correct bin remains.

from it following a probe that corresponds to ACK and NACK respectively. We then have

$$V_n(\underline{x}) = V_{n-1}(\underline{y}) + V_{n-1}(\underline{z}).$$

The above theorem is a simple consequence of Pascal's combinatorial identity and the definition of volume. In words it states, no matter the question selected, volumes of the resulting ACK-state and the NACK-state add up to the volume of the state at which the question was asked.

Theorem 2. *[36] (Volume Bound) If the current n-state $\underline{x}$ is such that n sounding signals are sufficient to determine the dominant path, then $V_n(\underline{x}) \leq 2^n$.*

The initial state is $(T, 0, 0, \cdots 0) \in \mathbb{N}^{L+1}$ with volume $T\left(\sum_{j=0}^{L}\binom{N}{j}\right)$ by definition. If the transmitter has a sounding strategy that determines the bin number corresponding to the dominant path with no more than N sounding signals, the volume bound implies

$$\left(\sum_{j=0}^{L}\binom{N}{j}\right) \leq \frac{2^N}{T}. \tag{17}$$

Thus, (17) gives the a lower bound on the minimum sounding time to adaptively determine the optimal beam, no matter how the lies are distributed.

C. Adaptive Selection Of Sounding Signals

An examination of (17) reveals that this bound is identical to the sphere packing bound for open-loop codes in (16). For certain values of N, L and T, perfect error correcting codes exist that meet this bound exactly and *adaptation provably offers no benefits*. But such codes are extremely limited since it is known that any non-trivial perfect code over a finite field has the same code parameters as the Hamming code or the Golay code [34].

The Volume Bound together with the conservation of volume reveals the optimal regions to sound at any stage of the beam alignment algorithm. Suppose that the transmitter in the $(N - \ell)$ state picks $\mathbf{f}_\ell$ to sound beams with labels in a set $S_\ell \subset \{1, 2, 3, \cdots T\}$ according to (4). Theorems 1 and 2 imply that S_ℓ must be selected so that the resulting states that correspond to ACK and NACK have nearly the same volume. This idea of splitting into equal halves is

If in a current n-state $(x_0, x_1, \cdots x_L)$, all $x'_j s$ are even, then the selection $\underline{u} = \left(\frac{x_0}{2}, \frac{x_1}{2}, \cdots \frac{x_L}{2}\right)$ results in the two $(n-1)$-states corresponding to ACK and NACK having equal volume, for any n. Thus any sounding strategy that is optimal begins with the same sounding signals, which is to pick half the number of elements in each of the $A'_i s$ successively as long as all terms in the state sequence are even. This observation is in parallel to (15). The first batch of optimal sounding signals for both adaptive and non-adaptive algorithms are simply individual bit-level questions. A simple induction argument with Pascal's identity gives the following Lemma.

Lemma 1. *Suppose that the initial state is $(T, 0, 0, \cdots 0) \in \mathbb{N}^{L+1}$. The resulting state after q beams are optimally sounded is*

$$\left(\frac{T}{2^q}\binom{q}{0}, \frac{T}{2^q}\binom{q}{1}, \cdots, \frac{T}{2^q}\binom{q}{L}\right) \tag{18}$$

as long as 2^q divides T. If $T = 2^K$, the resulting state after K beams are optimally sounded is

$$\left(\binom{K}{0}, \binom{K}{1}, \cdots, \binom{K}{L}\right). \tag{19}$$

Proof. We use an induction argument. The proposition is clearly true for $q = 1$. Since 2 divides T, the optimal beam is that which sounds half of the desired coverage region. The resulting state after the first beam is sounded is then $\left(\frac{T}{2}, \frac{T}{2}, \cdots, 0\right)$. Suppose that the proposition holds for $q = k - 1$ and that 2^k divides T. The current state is then $\left(\frac{T}{2^{k-1}}\binom{k-1}{0}, \frac{T}{2^{k-1}}\binom{k-1}{1}, \cdots, \frac{T}{2^{k-1}}\binom{k-1}{L}\right)$. The optimal beam to sound at this stage is to pick half the bins in each set. The resulting state by assigning votes and by Pascal's identity is $\left(\frac{T}{2^k}\binom{k}{0}, \frac{T}{2^k}\binom{k}{1}, \cdots, \frac{T}{2^k}\binom{k}{L}\right)$. $\square$

The difficulty then in designing an optimal strategy is that eventually, some terms in the resulting states will be odd. It is clear that the leading terms in a state contribute most to the volume. Hence, even a unit difference between the respective leading terms of the Yes-state and the No-state will cause a large difference in their volumes. To compensate for this difference, the rest of the terms will have to be split unevenly.

A dumb strategy is to set $u_j = \left\lfloor \frac{x_j}{2} \right\rfloor$ for each $j = 0, 1, \cdots, L$ until a stage where each set $A_j, j = 0, 1, \cdots, L$ has at most one bin. Using a result from [33], we obtain an upper bound on the sounding time under this strategy.

Lemma 2. *[33] Denote $x_i(q)$ as the number of bins at level i after q probes. If we use the strategy described above, then $\forall q \geq 0$ and $j \leq q$*

$$\sum_{i=0}^{j} \left(x_i(q) - \frac{T}{2^q} \binom{q}{i} \right) \leq j + 1 \tag{20}$$

Theorem 3. *Denote the minimum sounding time given by the volume bound to be N_{vol}. The sounding time with the dumb strategy is no more than $N_{vol} + L^2 + L + 1$.*

Proof. After q beams are sounded, Lemma 4 implies

$$\sum_{i=0}^{L} x_i(q) \leq \sum_{i=0}^{L} \frac{T}{2^q} \binom{q}{i} + L + 1. \tag{21}$$

Choose the smallest $q = q_{min}$ such that

$$\sum_{i=0}^{L} \frac{T}{2^q} \binom{q}{i} < 1 \tag{22}$$

An inspection with equation (17) reveals either $q_{min} = N_{vol}$ or $q_{min} = N_{vol} + 1$. (the latter holds when (17) is satisfied with equality) Thus after q_{min} probes, sets A_0 through A_L collectively contain at most $L+1$ bins. Since each probe will push at least one bin ahead, the claim holds. □

While this strategy is extremely simple, a penalty of $L^2 + L + 1$ may not be tolerable especially when L is large. We instead look for a direct attack. If $\underline{x} = \underline{u} + \underline{v}$ where $\underline{u}$ is the probe that reduces the n-state $\underline{x}$ to either the $(n-1)$-yes-state $\underline{y}$ or the $(n-1)$-no-state $\underline{z}$, we see that

$$\left| V_{n-1}(\underline{y}) - V_{n-1}(\underline{z}) \right| = \left| \sum_{j=0}^{L} \binom{n-1}{L-j} (2u_j - x_j) \right|. \tag{23}$$

The optimal choice of $\underline{u} = (u_0, u_1, \cdots u_L)$ is then to minimize (23). Fig. 4 demonstrates the complete sequence of states for $K = 2$ and $L = 2$ where $\underline{u}$ is always selected optimally.

The optimization problem of minimizing (23) is an integer linear program and known to be NP-hard. A commonly used technique to handle these type of problems is to first relax the integer constraints and solve the corresponding linear program. The solutions obtained are then rounded to integers using methods like branch and bound or cutting planes. A comprehensive discussion of different solution techniques can be found in [37]. Alternately, the authors in [38] suggest a greedy-like approach to minimize (23) by accumulating one term at a time. This is summarized as Algorithm 1.

Table I
SOUNDING TIME COMPARISON FOR ADAPTIVE VS NON-ADAPTIVE ALIGNMENT

K\L	1	2	3	4	5	6
1	3	5	7	9	11	13
2	5	8	11	14	17	20
3	6	9(10)	12(13)	15(17)	18(20)	21(24)
4	7	10(11)	13(14)	16(19)	19(22)	22(26)
5	9	12(13)	15	18(20)	21(23)	24(27)
6	10	13(14)	16(17)	19(22)	22(25)	25(29)

The function ChooseU $(A_0, A_1, \cdots A_L, n)$ chooses the region $[U_0, U_1, \cdots U_L]$ to test, given the current n-state.

Algorithm 1 ChooseU $(A_0, A_1, \cdots A_L, n)$ [38]

$p, q \leftarrow 0$ ▷ Initialise
for $i \leftarrow 0$ to L **do**
  $\Delta \leftarrow \left| \left(p + \binom{n-1}{L-i} u_i \right) - \left(q + \binom{n-1}{L-i} (x_i - u_i) \right) \right|$
  Choose $U_i \subseteq A_i$ to minimise Δ
  $p \leftarrow p + \binom{n-1}{L-i} u_i$
  $q \leftarrow q + \binom{n-1}{L-i} (x_i - u_i)$
end for
return $S = \bigcup_{j=0}^{L} U_j$

We are now ready to describe the adaptive channel sounding strategy. The sounding time budget N is fixed before. The transmitter maintains $\{A_0, A_1, \cdots A_L, n\}$ for an appropriately chosen value of L. The role that parameter L plays is that any bin receiving more than L negative votes is discarded during the sounding process. On testing a region and receiving an ACK/NACK, the sets are updated by assignment of votes. Having received bits r_1 through r_j in the feedback link, the transmitter picks $S_{j+1} =$ ChooseU $(A_0, A_1, \cdots A_L, N - j)$. Thus in sounding interval $j+1$, transmitter sounds the beam

$$\mathbf{f}_{j+1} = \frac{\sum_{i \in S_{j+1}} \mathbf{a}_i}{\|\sum_{i \in S_{j+1}} \mathbf{a}_i\|}. \tag{24}$$

The most likely angular region corresponding to the path angle in the channel is the one with the least number of negative votes. Thus at the end of channel sounding, the transmitter picks A_j with the smallest index j such that it is non-empty. If A_j contains only one bin number $p \in \{1, 2, 3, \cdots T\}$, we set $\mathbf{f}_{sel} = \mathbf{a}_p$. If not, we set $\mathbf{f}_{sel} = \mathbf{a}_q$ where q is randomly chosen from A_j.

In Table 1, we compare the (worst case) sounding times for adaptive and non-adaptive beam alignment. Non-adaptive alignment is implemented with the shortest length code known for the given parameters. A table of best known codes is in [39]. The adaptive alignment is implemented via the greedy sub-optimal Algorithm 1. Table cells with a single entry are cases where the total sounding time for both algorithms coincide meaning that adaption does not provide any benefit. As a general trend, the gap between sounding time for the two techniques gets larger as either K or L increase.

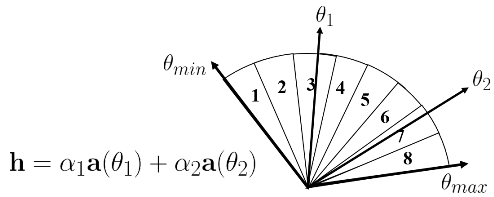

Figure 5. The coverage area is split into $T = 8$ regions and the beamset $\mathcal{C} = \{\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_8\}$. In this example, there are two paths that correspond to directions θ_1 and θ_2 respectively. We thus have $B = \{3, 7\}$ assuming that each beam $\mathbf{a}_i$ directs energy only in the sector labelled i.

V. Channel Sounding for Multipath

We now consider the effect of multipaths between the transmitter receiver pair. The channel model is

$$\mathbf{h} = \sum_{i=1}^{p} \alpha_i \mathbf{a}(\theta_i) \tag{25}$$

with parameters as defined in (2). As before, the desired coverage area is covered by T angular regions or bins. The beam alignment algorithm is split into two phases. In the first phase, the channel is sounded N times and the N bits received in the feedback are post-processed to identify the set $B \subset \{1, 2, \cdots, T\}$ of bin indices that correspond to path directions that are active. See Fig. 5 for an example. In the second phase, the transmitter sounds beams along directions specified in B to determine the optimal beam. For pedagogical reasons, we assume in this section that all of the beams in the beamset $\mathcal{C}$ are idealized beams whose gain patterns are constant in their intended support and zero elsewhere (see Fig. 8).

A. Preliminaries

The beam alignment algorithms described for the single path case can no longer be applied to the multi-path scenario in a straightforward manner. In the case of open-loop beam alignment, each bin was assigned a message label $\mathbf{m}_j \in \{0, 1\}^K$, $j = 1, 2, \cdots, T$. A suitable code with the generator matrix $\mathbf{G}$ was then chosen for beam selection. Suppose that there are two strong paths that correspond to bins with labels say $\mathbf{m}_1$ and $\mathbf{m}_2$. Even with perfect feedback (which means no bits in the feedback are inconsistent), the transmitter after channel sounding receives $\mathbf{r} = (r_1, r_2, \cdots, r_N)^T$ given by

$$\mathbf{r} = \mathbf{m}_1\mathbf{G} \vee \mathbf{m}_2\mathbf{G}, \tag{26}$$

where $\vee$ is the component-wise logical OR operation. With access to only $\mathbf{r}$ at the end of channel sounding, the identities of individual paths are completely lost and they cannot be resolved without imposing some additional constraint on $\mathbf{G}$.

In the adaptive algorithm, negative votes were assigned to individual regions with the goal of picking out one dominant path. Suppose that there are two dominant paths and we decompose $\{1, 2, \cdots, T\} = S_1 \cup S_2$ where S_1 and S_2 each contain one path. For example in Fig. 5, we could take $S_1 = \{1, 2, 3\}$ and $S_2 = \{4, 5, 6, 7, 8\}$. If the transmitter now sounds either $\mathbf{f}_{S_1}$ or $\mathbf{f}_{S_2}$ as in (4), all bins in one of these sets are assigned a negative vote. Thus, the notion of a disagreement tally is obscured.

We instead model channel sounding with one-bit of feedback as a *noisy group testing* problem with d defectives [25]–[27]. Group testing was originally introduced during World War II to detect the presence or absence of syphilitic antigen in a blood sample from a large population of samples in as few tests as possible [24]. Blood samples containing the antigen are called positives or defectives and are far fewer in number compared to the total population size. The main idea is to test groups of samples (called *pools*) together rather than test each one individually. A group testing algorithm is adaptive if the successive pools to be tested depend on the the outcomes of previously tested pools. However, much of research in this area is focused on non adaptive algorithms where all pools to be tested are decided beforehand. This is primarily since adaptive algorithms are sequential by nature and hence incur high latency, while the tests in a non-adaptive algorithm can be implemented in parallel when used for applications like blood testing. We consider here a noisy variant of the problem where the outcome of a test may be erroneous.

All vectors we deal with in this section have 0-1 entries. We say that a vector $\mathbf{p} = [p_1, p_2, \cdots, p_N]^t$ *contains* vector $\mathbf{q} = [q_1, q_2, \cdots, q_N]^t$ denoted $\mathbf{q} \preceq \mathbf{p}$, if $q_i \leq p_i$ for all $i = 1, 2, \cdots, N$. We associate channel sounding to a binary $N \times T$ test matrix $\mathbf{Z} = \{\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_T\}$ with a 1 in position (i, j) if j^{th} bin is sounded on the i^{th} channel sounding and 0 otherwise. In other words, the i^{th} row of $\mathbf{Z}$ completely specifies the bins that are sounded in the i^{th} channel sounding interval. The corresponding beamformer $\mathbf{f}_i$ is selected as

$$\mathbf{f}_i = \frac{\sum_{j=1}^{T} \mathbf{a}_j \mathbb{1}(\mathbf{z}_j(i) = 1)}{\|\sum_{j=1}^{T} \mathbf{a}_j \mathbb{1}(\mathbf{z}_j(i) = 1)\|}. \tag{27}$$

Here, $\mathbf{z}_j(i)$ refers to the i^{th} entry in the column vector $\mathbf{z}_j$. An example of the shapes of beams to be sounded is illustrated in Fig. 6. We can also think of the columns of $\mathbf{Z}$ as the individual binary vector labels or *codewords* assigned to each bin.

B. Perfect Feedback

First, suppose that all of the bits $\{r_i\}_{i=1}^{N}$ are consistent with the actual path directions. In other words, none of the bits received at the transmitter are incorrect. By definition, the rows of $\mathbf{Z}$ indicate the regions where energy is directed in a particular sounding. Thus, if the bins that correspond to the p path directions have labels $\mathbf{z}_{i_1}, \mathbf{z}_{i_2}, \cdots, \mathbf{z}_{i_p}$ (columns of $\mathbf{Z}$), we would have

$$\mathbf{r} = \bigvee_{j=1}^{p} \mathbf{z}_{i_j} \tag{28}$$

where $\bigvee$ represents the component-wise logical OR operation of column vectors. In practice, we may not know p exactly and instead assume $p \leq d$. (28) reveals how $\mathbf{Z}$ should be designed. In principle, all we need is the component-wise logical OR of every d or less columns of $\mathbf{Z}$ to be a unique vector to determine which path directions are active. However, even if we had such a $\mathbf{Z}$, determining the set B would involve an exhaustive search over a total of $\sum_{i=1}^{d} \binom{T}{i}$

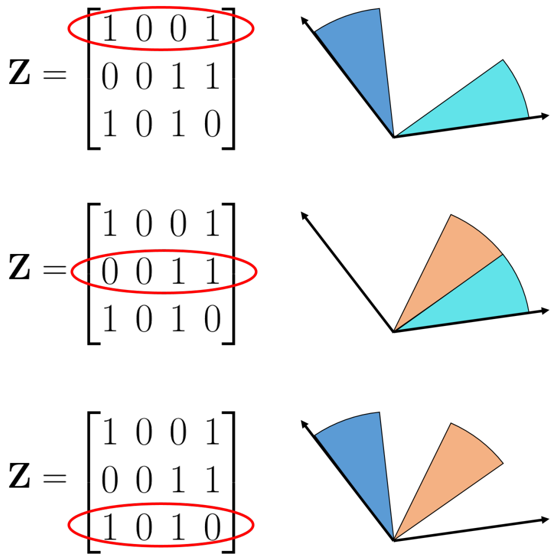

Figure 6. Shapes of beams to be sounded for given design matrix $\mathbf{Z}$.

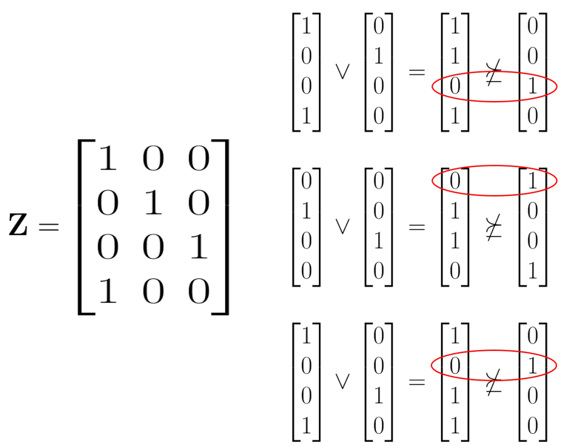

Figure 7. An example of a 2-disjunct matrix $\mathbf{Z}$. Clearly, no individual column contains any other column. Further, the logical OR of any 2 columns does not contain the third due to the violations that are marked in red.

possibilities. This would make the scheme impractical from an implementation perspective. To overcome this, it is common in the group testing literature to enforce the following additional structure on $\mathbf{Z}$ [25].

Def: *A matrix $\mathbf{Z}$ is said to be d-disjunct if the component-wise logical OR of any d or less columns does not contain any other column.*

A 1-disjunct matrix is one where no column is contained in another. An example of a 2-disjunct matrix is illustrated in Fig. 7. If $\mathbf{Z}$ were d-disjunct, it suffices to iterate over each of the T columns once and check the ones that are contained in the received bit vector $\mathbf{r}$ to decide which bins correspond to active path directions. This greatly reduces the implementation complexity and involves only T vector comparisons. Mathematically,

$$B = \{k \mid \mathbf{z}_k \preceq \mathbf{r}\}. \tag{29}$$

C. Imperfect Feedback

As noted before, the quantization of the received symbol at the receiver is noisy. As a result, the transmitter only has access to a corrupted version of $\mathbf{r}$ in (28), say $\mathbf{r}'$. If $\mathbf{n}$ is a 0-1 noise sequence, 1 indicating an erroneously received bit, we have

$$\mathbf{r'} = \left(\bigvee_{j=1}^{p} \mathbf{z}_{i_j}\right) \oplus \mathbf{n} = \mathbf{r} \oplus \mathbf{n} \tag{30}$$

where $\oplus$ is the addition operation over $GF(2)$. Even a single error in the received bit vector $\mathbf{r}$ can cause a failure if we used only a d-disjunct matrix. To be resilient against up to e errors, the logical OR of one set of d (or less) columns should be at least $2e+1$ away in hamming distance from that of another set of d (or less) columns. We thus need special kind of d-disjunct matrices.

Def: *A d^e-disjunct matrix $\mathbf{Z}$ is a d-disjunct matrix with the following property: given any $d+1$ columns with one designated, there are at least $e+1$ rows with a 1-entry in the designated column and a 0-entry in the others.*

Theorem 4. *A d^e-disjunct binary matrix $\mathbf{Z}$ can identify up to d multipaths correctly against up to $\lfloor \frac{e}{2} \rfloor$ erroneous feedback bits.*

Proof. Let $\mathbf{P} = \{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_{|\mathbf{P}|}\}$ and $\mathbf{Q} = \{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_{|\mathbf{Q}|}\}$ each be a set of d or less columns from $\mathbf{Z}$ i.e. $|\mathbf{P}|, |\mathbf{Q}| \leq d$. Denote component-wise OR of columns in $\mathbf{P}$ and $\mathbf{Q}$ as

$$\tilde{\mathbf{p}} = \bigvee_{i=1}^{|\mathbf{P}|} \mathbf{x}_i \quad \text{and} \quad \tilde{\mathbf{q}} = \bigvee_{j=1}^{|\mathbf{Q}|} \mathbf{y}_j \tag{31}$$

Choose a column column $\mathbf{c} \in \mathbf{Q} \setminus \mathbf{P}$. Since $\mathbf{Z}$ is d^e-disjunct, $\mathbf{c}$ contains a 1-entry in $e+1$ rows where all columns $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_{|\mathbf{P}|}$ have 0-entries. Thus, $\mathcal{H}(\tilde{\mathbf{p}}, \tilde{\mathbf{q}}) \geq e+1$. □

The property of d^e-disjunctness in addition to providing error tolerance also allows for a very simple decoding strategy whose runtime is linear in the dimensions of the test matrix (Section V-D). However, deterministic construction of such matrices is a non-trivial endeavor and a subject of intensive research. We refer the interested reader to [25], [26], [40] for specific design methodologies and a general overview on group testing literature.

In a parallel line of thought, many authors have advocated for random constructions of test matrices. One particularly simple construction is where each individual entry in the test matrix is sampled i.i.d. from a Bernoulli distribution and has been shown to perform well in practice [41]. In our simulations, we consider both kinds of designs and compare their performances under different decoding algorithms. It was shown in [42] that optimal adaptive measurements provide no gain over Bernoulli i.i.d designs in the asymptotic regime for either perfect or noisy feedback, provided that the number of defectives d grows slowly with the number of items T. We will thus only consider non-adaptive designs for the multi-path case.

D. Recovering Path Directions

Suppose that $\mathbf{Z}$ is d^e-disjunct. The matrix $\mathbf{Z}$ with binary entries completely describes the sounding signals at any stage of the algorithm. It is designed to identify up to certain number of paths with the desired error tolerance. Specifically, it can provably identify up to d paths and correct up to $\lfloor \frac{e}{2} \rfloor$ erroneously received bits. As before, the transmitter is equipped with a beam set $\mathcal{C}$. Sounding progresses according to design matrix $\mathbf{Z}$ and the one bit ACK/NACK responses are collected in a vector $\mathbf{r}^{'}$. The active path directions then need to be inferred by decoding $\mathbf{r}^{'}$ described in algorithm 2. A straightforward proof of correctness is in [43]. Note that its time complexity scales as $O(NT)$ and it returns a set B of bin indices that correspond to path directions. Since one does not know the number of errors that can occur apriori, we shall also consider other decoding methods in our simulations.

Algorithm 2 Decoding Multipath($\mathbf{Z}, \mathbf{r}^{'}$)

$B \leftarrow \emptyset$ ▷ $\mathbf{Z}$ is d^e-disjunct
for $i \leftarrow 1$ to T **do** ▷ size of $\mathbf{Z}$ is $N \times T$
  $C(\mathbf{z}_i, \mathbf{r}^{'}) \leftarrow \text{card}\left(\{j \mid \mathbf{z}_i(j) = 1 \text{ and } r^{'}_j = 0\}\right)$
  if $C(\mathbf{z}_i, \mathbf{r}^{'}) \leq \lfloor \frac{e}{2} \rfloor$ **then**
    $B \leftarrow B \cup \{i\}$
  end if
end for
return B

E. Selection Of Beamformer

For the case of multipath, we allow the transmitter to choose its beam for communication as a linear combination of a small subset of beams from the beamset. Suppose that the beamset $\mathcal{C} = \{\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_{M_t}\}$ consists of M_t orthogonal beams. Having recovered the set $B = \{i_i, i_2, \cdots, i_m\}$ with a group testing approach, the beamformer $\mathbf{f}$ chosen at the transmitter is a linear combination of beams $\mathbf{a}_{i_1}$ through $\mathbf{a}_{i_m}$. The transmitter estimates the complex weights to be applied to these beams before they are summed by training only on these beams. Due to channel sparsity, $|B| \ll |\mathcal{C}|$. As a concrete example, suppose $B = \{2, 5\}$. By sending out training beacons on $\mathbf{a}_2$ and $\mathbf{a}_5$, the transmitter obtains a good estimate of $\gamma_2 = \mathbf{a}_2^* \mathbf{h}$ and $\gamma_5 = \mathbf{a}_5^* \mathbf{h}$. The beamformer is then selected to be $\mathbf{f}_{sel} = \frac{\gamma_2 \mathbf{a}_2 + \gamma_5 \mathbf{a}_5}{\|\gamma_2 \mathbf{a}_2 + \gamma_5 \mathbf{a}_5\|}$.

VI. Simulation Results

We present simulation studies to validate our proposed algorithms. It is convenient to define the spatial frequency variable $\psi = 2\pi\beta \sin(\theta) = \pi \sin(\theta)$, assuming $\beta = \frac{1}{2}$. We consider full $180°$ beamforming ($\theta_{min} = -90°$, $\theta_{max} = 90°$) and the beam set at the transmitter $\mathcal{C} = \{\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_{M_t}\}$ has the form

$$\mathbf{a}_i = \frac{1}{\sqrt{M_t}} e^{-j\frac{(M_t-1)}{2}\psi_i} \left[1 \; e^{j\psi_i} \cdots \; e^{j(M_t-1)\psi_i}\right]^T \quad (32)$$

where $\psi_i = -\pi + (2i-1)\frac{\pi}{M_t}, i = 1, \cdots, M_t$. The centers ψ_i of the beams are spaced equally in the $\psi \in [-\pi, \pi]$ domain so that the resulting beams are orthonormal i.e. $|\mathbf{a}_i^* \mathbf{a}_j| = 0$ when

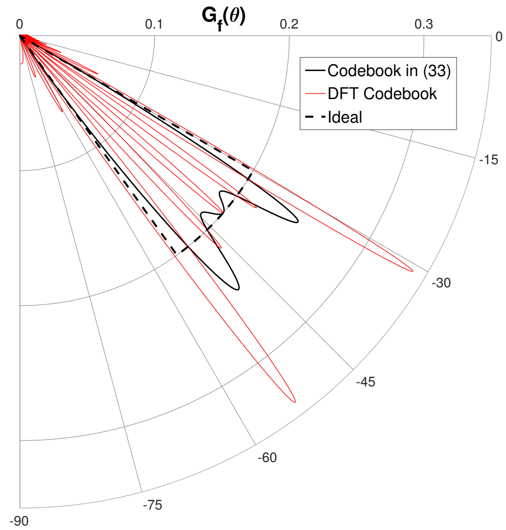

Figure 8. Consider $M_t = 32, \theta_{min} = -90°, \theta_{max} = 90°$. Spatial pattern $G_{\mathbf{f}}(\theta)$ of a beamformer designed to sound the region $S = \{4, 5, 6, 7, 8\}$ in (4) with DFT type beams vs the codebook in (32). Notice the maximum gain of an ideal beam is $\frac{1}{|S|} = 0.2$.

$i \neq j$. While similar in form to the beam steering vector, the additional phase shift term $e^{-j\frac{(M_t-1)}{2}\psi_i}$ applied to the beams ensures that when the individual harmonics are summed together in (4), the resulting beam patterns have nearly flat gains in the regions of interest and low side-lobes in others. In comparison, beams from a simple DFT-type codebook work poorly due to the nulls that occur on summing of harmonics. This is illustrated in Fig. 8. We also define idealized beams to simplify the detector design. Let $\tilde{\mathcal{C}} = \{\tilde{\mathbf{a}}_1, \cdots, \tilde{\mathbf{a}}_{M_t}\}$ be the set of ideal beams where beam $\tilde{\mathbf{a}}_i$ has a normalized spatial pattern given by

$$G_{\tilde{\mathbf{a}}_i}(\theta) = \begin{cases} 1 & \theta \in \mathcal{G}_i \\ 0 & \text{otherwise} \end{cases} \quad (33)$$

where $\mathcal{G}_i = \left\{\theta : \pi \sin(\theta) \in \left[-\pi + \frac{2\pi}{M_t}(i-1), -\pi + \frac{2\pi}{M_t} i\right]\right\}$.

A. Single Path Channel

Consider the channel with one dominant path $\mathbf{h} = \alpha \mathbf{a}(\theta_0)$, where $\theta_0 \sim \mathcal{U}(-\frac{\pi}{2}, \frac{\pi}{2})$ and $\alpha \sim \mathcal{CN}(0, 1)$. The optimal beam then achieves a normalized gain of $\max_i |\mathbf{a}(\theta_0)^* \mathbf{a}_i|^2$. On average, the best achievable beamforming gain for the codebook in (32) is equal to

$$\begin{aligned} A_{max} &= \mathbb{E}_{\theta_0}\left[\max_i |\mathbf{a}(\theta_0)^H \mathbf{a}_i|^2\right] \\ &= \mathbb{E}_{\theta_0}\left[\max_i \frac{\sin^2(\frac{M_t(\psi_i - \pi\sin(\theta))}{2})}{\sin^2(\frac{(\psi_i - \pi\sin(\theta))}{2})}\right]. \end{aligned} \quad (34)$$

For $M_t = 32$ antennas for example, the performance limit is numerically evaluated to be $A_{max} \approx 75.4\%$.

Since the detector outputs either an ACK or a NACK, we formulate the detector design problem as a hypothesis test assuming idealized beams. For practicality of our schemes, we consider a simple threshold detector of the form $r_\ell = \mathbf{I}(|y_\ell| >$

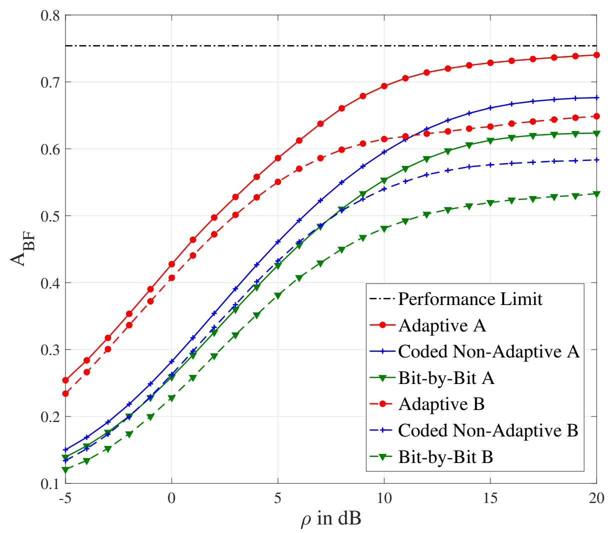

Figure 9. Expected channel-normalized beamforming gain for single path millimeter wave channel as a function of sounding SNR ρ. The parameters are fixed at $M_t = 32, N = 16, K = 5$ and full 180° beamforming is considered. The feedback link is assumed to be perfectly noiseless.

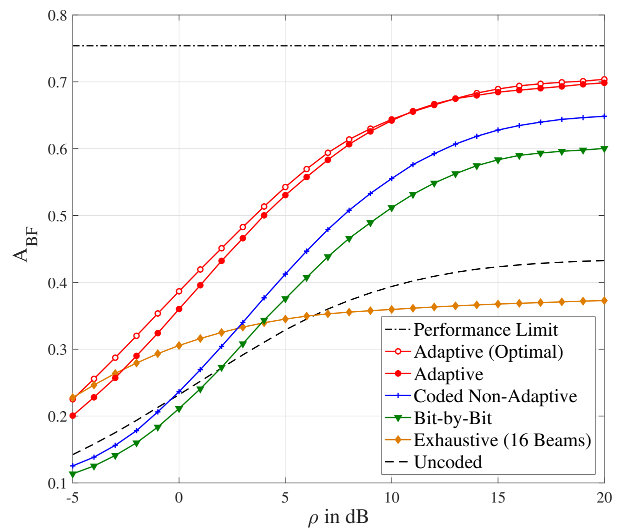

Figure 10. Performance when the feedback link is a binary symmetric channel with error probability 5%. Detection threshold was designed based on (35).

γ) where the decision threshold γ is chosen independent of the beamformer $\mathbf{f}_\ell$ selected and sounding iteration ℓ. On receiving the complex symbol y_ℓ, the beam detection problem from (1) then reduces to the hypothesis test

$$\begin{cases} \mathcal{H}_0 : |y_\ell| \sim Rician\left(|\alpha|\sqrt{2}, \frac{1}{2\rho}\right) \\ \mathcal{H}_1 : |y_\ell| \sim Rayleigh\left(\frac{1}{2\rho}\right) \end{cases} \tag{35}$$

where we have assumed that the transmitter always sounds half the number of bins. In other words, in (4) we always have that $|S| = \frac{T}{2}$. Threshold rule γ is then selected based on the ROC curves. If an estimate of the fading gain $|\alpha|$ is not available at the receiver, the hypothesis test is formulated as

$$\begin{cases} \mathcal{H}_0 : |y_\ell| \sim Rayleigh\left(1 + \frac{1}{2\rho}\right) \\ \mathcal{H}_1 : |y_\ell| \sim Rayleigh\left(\frac{1}{2\rho}\right) \end{cases} . \tag{36}$$

In Fig. 9, the transmitter is equipped with $M_t = 32$ antennas and the given sounding time budget is $N = 16$. The feedback link is assumed to be perfectly noiseless and the errors are only due to beam detection errors. The scheme labelled 'Bit-by-Bit' is a simple non-adaptive scheme chosen as the baseline where the first $N = 16$ columns of the matrix $[\mathbf{I}_5|\mathbf{I}_5|\cdots\mathbf{I}_5]$ are set as the generator. This yields a $(16, 5, 3)$ code and corresponds to asking questions per bit, and cycling through them repeatedly. The coded non-adaptive scheme is based on the best known open-loop code for given K and N which is the $(16, 5, 8)$ code constructed using MAGMA. A minimum distance of 8 ensures that up to 3 errors can always be corrected. The adaptive sounding scheme is implemented based on Algorithm 1 and we set the maximum number of lies parameter at $L = 3$. Thus any region receiving more than 3 negative votes is discarded during the sounding process. The natural performance metric is the channel normalized beamforming gain defined to be

$$A_{BF} = \frac{|\mathbf{h}^* \mathbf{f}_{sel}|^2}{\|\mathbf{h}\|^2} \tag{37}$$

where $\mathbf{f}_{sel}$ is the beam selected by the transmitter at the end of the sounding process. Two sets of curves are shown for comparison, 'A' is where the detection threshold is selected according to (35) and 'B' where it is chosen according to (36).

We repeat the same set of simulations for the case where the feedback link is noisy, specifically a binary symmetric channel with probability of error $p = 0.05$. The results are indicated in Fig. 10. We also compare our proposed techniques to two other schemes. One is where the transmitter exhaustively sweeps over a codebook containing $T = 16$ orthonormal beams that approximately span the horizon $[-90^\circ, 90^\circ]$. The other is uncoded beam sounding wherein the transmitter sounds $K = 5$ beams, one for each bit, with their powers adjusted so that the total power budget across all schemes being compared is the same. We can draw the following conclusions from these simulation studies:

1) For adaptive sounding, optimal beam selection by solving the integer program in (23) each time performs only slightly better than greedy selection outlined in Algorithm 1. The corresponding performance curve is labelled as 'Adaptive (Optimal)' in Fig. 10.
2) Adaptive sounding outperforms the best non-adaptive coded strategy by 3-4 dB and the bit-by-bit scheme by up to 5 dB.
3) The proposed coded sounding schemes generally maintain good performance even when the feedback link is noisy as seen in Fig. 10.

Next, we study the performance of our algorithms as a function of the sounding time budget N. Fig. 12 shows two

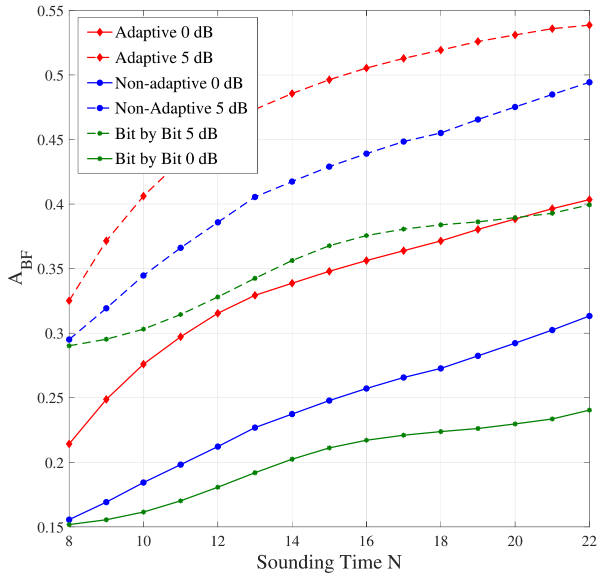

Figure 11. Expected channel-normalized beamforming gain for single path millimeter wave channel as a function of sounding time N. The parameters are fixed at $M_t = 32, K = 5$ and full 180° beamforming is considered. Two sets of curves, one at sounding SNR $\rho_1 = 0$ dB and the other at $\rho_2 = 5$ dB are shown.

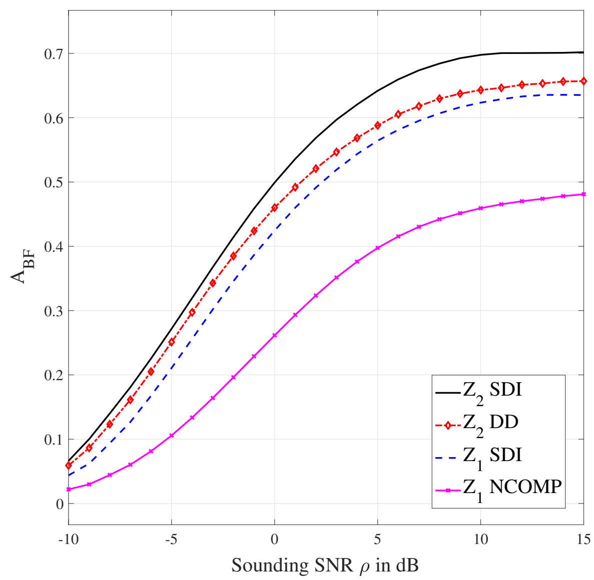

Figure 12. Expected channel-normalized beamforming gain for a 2-path millimeter wave channel. The parameters are fixed at $M_t = 512, N = 63$ and full 180° beamforming is considered. $\mathbf{Z}_1$ is a Bernoulli i.i.d. design while $\mathbf{Z}_2$ is a deterministic 2^3-disjunct matrix.

sets of performance curves for sounding SNR's fixed at $\rho_1 = 0$ dB and $\rho_2 = 5$ dB. For each N, the non-adaptive scheme was implemented by choosing the known length N code with the largest minimum distance. The adaptive scheme is based on Algorithm 1 with the threshold for all curves chosen based on (36). For the case of $\rho_1 = 0$ dB for example, adaptive channel sounding can achieve the same expected beamforming gain as non-adaptive schemes with just half the sounding time budget.

B. Multi-Path Channel

In this section, we consider a channel model with two paths $\mathbf{h} = (\alpha_1 \mathbf{a}(\theta_1) + \alpha_2 \mathbf{a}(\theta_2))$, where $\theta_1, \theta_2 \sim \mathcal{U}(-\frac{\pi}{2}, \frac{\pi}{2})$ and $\alpha_1, \alpha_2 \sim \mathcal{CN}(0, 1)$. We assume that the transmitter is equipped with $M_t = 512$ antennas and the sounding time is fixed at $N = 63$. The region $[-90^\circ, 90^\circ]$ is thus split into 512 bins, and the sounding signals for the multi-path scenario correspond to a suitably chosen group testing matrix. Similar to the single-path scenario, the detection threshold at the receiver is held constant throughout the sounding process for a given sounding SNR. At the end of channel sounding, the transmitter comes up with an estimate B of bin indices corresponding to the dominant path directions. It then performs training on beams specified by B to choose a beamformer as discussed in Section V-E.

We consider two different non-adaptive designs of the sounding matrix $\mathbf{Z}$. The first design $\mathbf{Z}_1$ is a randomly generated 0-1 matrix whose each entry is i.i.d. Ber(p), where $p = 1 - (2^{-\frac{1}{2}})$ is selected based on the analysis in [44]. The second test matrix $\mathbf{Z}_2$ is a carefully constructed deterministic 2^3-disjunct design based on "matrix-containment" construction techniques using the GAP software package [45]. In the notation of [46], the 63×512 submatrix of $M_2(6, 4, 1)$ is set as $\mathbf{Z}_2$.

Suppose that the received bit vector is **r'** in (30). We consider the following practical decoding strategies. The first two have a combinatorial flavour while the third is based on assuming a noise model and probabilistic.

- Noisy Combinatorial Orthogonal Matching Pursuit (NCOMP) algorithm [47]: For each column $\mathbf{z}_i$ of $\mathbf{Z}$, we define the metric $m_i \triangleq \frac{\left|\left\{j \mid \mathbf{z}_i(j)=1 \text{ and } r_j^{'}=1\right\}\right|}{\mathcal{H}(\mathbf{z}_i, \mathbf{0})}$. The indices corresponding to the two largest values of this metric are returned where ties are broken arbitrarily.
- Direct Decoding (DD) : We use Algorithm 2 for decoding for increasing values of e until the algorithm returns two indices. This is essentially picking out the two columns with the lowest values of $C(\mathbf{z}_i, \mathbf{r}^{'})$.
- (Modified) Separate Decoding of Items (SDI) algorithm: This type of decoder was first described in the Russian literature [44] and recently studied extensively in [48]. We assume that a bit $\mathbf{r}_i$ in (28) is flipped with some error probability q which we estimate via Monte-Carlo simulations. This is the so-called symmetric additive noise model. Following [48], the decoder involves computing for each column $\mathbf{z}_i$

$$\phi_i = \sum_{i=1}^{n} \ln \frac{f_1(\mathbf{r}_i^{'}, \mathbf{z}_i(j))}{f_2(\mathbf{r}_i^{'})} \tag{38}$$

where $f_1(0,0) = pq + (1-p)(1-q), f_1(1,0) = 1 - f_1(0,0), f_1(0,1) = q, f_1(1,1) = 1 - f_1(0,1), f_2(0) = (1-q)(2p-p^2) + q(1-p)^2$ and $f_2(1) = 1 - f_2(0)$. The bin indices corresponding to the two largest values of ϕ_i are then returned by the algorithm. We caution the reader that due to the final sorting step, the algorithm used here is not strictly separate decoding as defined in [48].

Fig. 11 shows the performance of the proposed techniques as a function of transmit SNR ρ. As one would expect, the carefully constructed deterministic design beats a random i.i.d. design for any decoding algorithm. In case of a random Bernoulli design, decoding with SDI beats NCOMP significantly which is in agreement with the simulations reported in [48]. For the case of a deterministic design, SDI provides a slight improvement over DD as it captures the probability of feedback bits being in error to make better decoding decisions. A simple Bernoulli i.i.d design is only worse by about 1.5 dB than the hard to construct deterministic design when SDI is used.

VII. Conclusions

In this work, we studied the problem of one-bit feedback-assisted beam alignment in millimeter wave networks. By interpreting the beamforming problem as one of searching in a finite set, we investigated adaptive and non-adaptive channel sounding strategies that were designed to be robust to noisy quantization. The open-loop technique is based on standard block codes while the closed-loop technique corresponds to playing Ulam's game against a liar. We showed that it is also possible to identify multi-paths by leveraging tools from group testing.

New beam adaption techniques can potentially be formulated by exploring other error models studied in the literature such as one where the ACK/NACK is erroneous with a certain error probability. Future work includes extending the proposed techniques to the case where there are restrictions on the beams that can be sounded. Questioning strategies studied traditionally in computer science may prove to be useful for other feedback based problems in communications and signal processing.

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