

# A Fast Semi-Analytic Approach for Combined Electromigration and Thermomigration Analysis for General Multi-Segment Interconnects

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**Abstract**—Considering temperature gradient or thermomigration (TM) impacts on electromigration (EM) due to Joule heating was less studied in the past. In this paper, we propose a new semi-analytical stress transient analysis method to consider both EM and TM effects for general multi-segment interconnects. The new method is based on the separation of variables (SOV) approach to find the analytic solution of coupled EM-TM partial differential equation (PDE). The algorithm consists of several steps. We first develop analytic solutions to compute steady-state temperature distribution of multi-segment wires. Based on this, we derive closed-form solutions for steady-state hydrostatic stress distribution in the context of thermal gradients due to Joule heating for multi-segment interconnect wires. With the steady-state stress distribution, the coupled EM-TM PDE can be homogenized and solved by the SOV method. To deal with temperature/position-dependent diffusivity of metal migration process due to non-uniform temperature distribution, we utilize a piecewise linear technique to approximate the position-dependent diffusivity. Numerical results on multi-segment interconnects show that the proposed method has negligible error loss compared to commercial finite element analysis software COMSOL but is about an order of magnitude faster than COMSOL with 10x less memory footprint. The numerical results further show that temperature gradient due to Joule heating indeed has significant impacts on the EM failure process.

**Index Terms**—Analytical thermal model, electromigration (EM), Joule heating, separation of variables (SOV) method, temperature gradient effects.

## I. INTRODUCTION

**D**UE to the increasing current densities and decreasing dimensions of the interconnects, electromigration (EM) induced aging and failure effects remain the top reliability concerns for modern VLSI chips in 7-nm technology and below. Therefore, it is important to develop more accurate and less conservative EM sign-off and validation [1]. However, it is

well accepted that existing Black and Blech-based EM models are subject to growing criticism due to over conservativeness and they only work for a single wire segment [2], [3].

To mitigate the existing problem in EM models, a number of physics-based EM models and assessment techniques have been proposed [1], [4]–[16]. Those EM models primarily focus on solving the partial differential equation (PDE) (called Korhonen’s equation) of hydrostatic stress evolution in the confined multi-segment wires subject to blocking material boundary conditions.

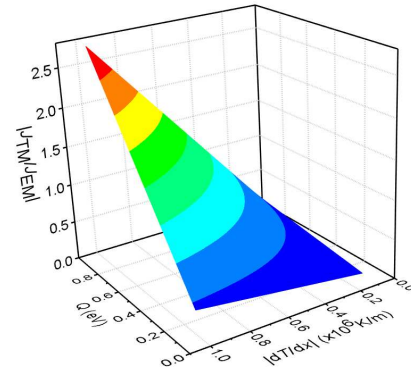


Fig. 1. The effect of the heat of transport  $Q$  and temperature gradient  $|dT/dx|$  on the ratio of TM flux to EM flux.

However, most of those EM models do not consider the temperature gradient impacts on the metal migration process. Instead, most of those approaches just assume constant wire temperature. However, recent study [16] shows that the temperature gradient effects can be quite significant (almost in the same magnitude as the current induced migration) due to the Joule heating effects. This situation will become even worse as technology advances to smaller features and 3D stacked integration, in which higher power density, larger Joule heating, and larger thermal resistances due to stacking will lead to even higher temperature and temperature gradients across chip [17]. To better illustrate importance of thermomigration (TM) effect, we show the ratio of TM flux over EM flux defined below under different conditions

$$\left| \frac{J_{TM}}{J_{EM}} \right| = \frac{Q/T \cdot \left| \frac{\partial T}{\partial x} \right|}{eZ\rho j}, \quad (1)$$

where  $Q$  is the specific heat of transport,  $T$  is the temperature,  $e$  is the elementary charge,  $Z$  is the effective charge number,  $\rho$  is the electrical resistivity,  $j$  is the current density, and  $J_{TM}$

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and  $J_{EM}$  are TM flux and EM flux, respectively. Fig. 1 shows the TM over EM ratio in (1) with different  $Q$  values and temperature gradients, where  $Q$  is ranged from 0.09 eV to 0.9 eV for different techniques [18]. As both  $Q$  values and temperature gradients (larger current densities) increase, the TM effect becomes more significant and can be even larger than the EM effect.

In this work, we propose a new semi-analytical solution to compute the transient hydrostatic stress evolution considering both EM and TM for the general multi-segment interconnect trees. Our new contributions are as followings:

- First, an accurate analytical thermal model is employed to capture the non-uniform temperature distribution of the interconnects due to Joule heating for a single wire. On top of this, we develop a closed-form expression for temperature distribution of multi-segment wires. With this result, we derive a more general and accurate analytical formula for steady-state stress distribution based on the computed non-uniform temperature distribution.
- Second, with the steady-state stress distribution, the coupled EM-TM PDE can be easily homogenized and solved by the separation of variables (SOV) method, in which Bessel function was derived as the eigenfunctions in the new SOV method, which is different from that in [19], [20]. To deal with temperature/position-dependent diffusivity of metal migration process due to non-uniform temperature distribution, we use a piecewise linear technique to approximate the position-dependent diffusivity, which is more suitable for the non-uniform diffusivity in contrast to stepwise uniform approximation [21].
- Third, compared with constant temperature, the effects of non-uniform temperature distribution on EM void nucleation phase for large multi-segment interconnect trees are studied, which shows that temperature gradient due to Joule heating indeed has significant impacts on the EM failure process. Numerical results on multi-segment interconnects show that the proposed method has negligible error loss compared to commercial finite element software COMSOL but is about an order of magnitude faster than COMSOL with 10x less memory footprint.

The rest of the article is organized as follows: Section II reviews the physics-based EM model considering TM effects. Section III gives a coupled EM-TM PDE describing hydrostatic stress. Section IV presents the analytical thermal model describing the temperature distribution of the interconnects and develops an approach to extract effective thermal length. Section V presents an accurate and analytical solution for steady-state EM-TM stress assessment. Section VI proposes a novel and efficient semi-analytical EM-TM method to estimate the hydrostatic stress evolution under non-uniform temperature distribution. Section VII shows numerical results for the proposed method and comparison against the commercial software COMSOL. Section VIII concludes this article.

## II. REVIEW OF RELEVANT WORK

Metal atoms migrate in the presence of thermal gradients. This effect is called *Soret Effect* or *thermomigration (TM)* [22]. The TM effects have been studied in the past and should be seriously considered in current and future interconnects

of ICs [23]–[26]. It has been experimentally observed that temperature gradient  $|dT/dx|$  can easily exceed  $0.1\text{K}/\mu\text{m}$  [16], [23], [24], [26], [27]. Li *et al.* show that temperature changes rapidly in the short and advanced interconnects using both measurement method and finite element based ANSYS simulation [28]. Temperature contour map obtained by scanning Joule expansion microscopy shows that Joule heating with current density  $3.13 \times 10^{10}\text{A}/\text{m}^2$  leads to temperature rise 30K on an interconnect of length  $9\mu\text{m}$  [29]. The large temperature gradients can cause significant TM, which can be in the same magnitude as EM [16], [24], [26]. Also, experimental data shows that thermal gradient has significant impacts on the EM-induced time to failure (TTF) for power electronics [25]. This work shows that thermal gradient of  $0.19\text{K}/\mu\text{m}$  can lead to 50% TTF reduction even with low oven temperature. In [24], the experiments and simulation show that when the direction of temperature gradient ( $25\text{K}/200\mu\text{m}$ ) changes from positive to negative, the mean time to failure (MTTF) improves by 213%. When the directions of temperature gradient and current are same, temperature gradient reduces MTTF by 57%. If they are in the opposite directions, temperature gradient improves MTTF by 33%. The effect of TM and EM on microelectronics solder joints is also studied with experiments and simulation in [30]. This paper shows that with  $2.4 \times 10^{10}\text{A}/\text{m}^2$  current density, 376K hot temperature and 366K cold temperature in the experiments, the test vehicle fails after 693h. It can be seen that crack in the solder joints which are stressed by TM+EM forces is the worst among the TM, TM-EM and TM+EM cases. As a result, TM effect has to be seriously considered due to significant Joule heating in modern ICs.

However, many existing EM methods only consider the transient/temporal thermal effects when solving the Korhonen's equation or derive analytic models to estimate transient EM stress under time-varying temperature [8], [10], [11], [31], [32]. Those works do not consider the spatial temperature or thermal gradient impacts on the multi-segments. After that, some research efforts are carried out recently to investigate EM considering the TM effects [16], [18], [33]–[36]. A coupled EM-TM equation based on Korhonen's equation is proposed and well accepted to estimate hydrostatic stress evolution due to EM, TM and stressmigration (SM).

To efficiently and accurately solve the Korhonen's equation, a number of numerical and analytic solutions are proposed recently, such as finite difference based methods [12], [37], [38], finite element based methods for post-voiding phase analysis [15], [39] and efficient analytical or semi-analytical solutions [6], [16], [19], [20], [40], [41]. Among them, SOV method [19], [20] is a promising semi-analytic method for general multi-segment interconnect trees.

However, applying SOV method for the coupled EM-TM equation becomes challenging as one has the location/temperature dependence of diffusivity of metal migration due to the presence of spatial thermal gradients. Recently Abbasinasab *et al.* propose analytical solutions for both transient and steady states of the EM-TM model [16], [18], [36]. However, their steady-state solution has more restricted assumption in which the temperature of surrounding dielectrics needs to be the average temperature of two end nodes in a segment. Furthermore, the analytical transient method only works for constant diffusivity of EM. For multi-branch interconnects structure, this method has to re-compute the coef-

ficients to satisfy the boundary conditions for all junctions in each time point, which requires very expensive computational cost. Therefore, more efficient and accurate transient coupled EM-TM analysis techniques considering spatial temperature-dependent diffusivity are highly desirable.

### III. ELECTROMIGRATION MODEL CONSIDERING THERMAL GRADIENT EFFECTS

EM is a diffusion phenomenon of the migration of metal atoms due to several driving forces. The momentum exchange from conducting electrons to metal atoms creates metal atoms tension at the cathode end and a corresponding metal compression at the anode ends of the metal wire. The temperature gradient is another important driving force to move atoms from hot to cool places. The lasting migration process increases the hydrostatic stress, which is a prime cause of forming void at cathode and hillock at anode of the interconnects. As the stress reaches the critical value, voids can be created. To make matters worse, the void causes open circuit and the hillock leads to short circuit in the interconnects, which is called EM-induced failure and a major reliability problem in 3D ICs.

The total atomic flux is the sum of the three fluxes due to EM, SM, and TM, expressed as [16], [18], [33]–[36]

$$J = J_{EM} + J_{SM} + J_{TM}$$

$$= -D_v \left( \frac{\Omega}{k_B T} C_v \frac{\partial \sigma}{\partial x} - C_v \frac{eZ\rho j}{k_B T} - \frac{Q}{k_B T^2} C_v \frac{\partial T}{\partial x} \right) \quad (2)$$

where  $D_v$  is the vacancy diffusivity,  $C_v$  is the vacancy concentration,  $k_B$  is Boltzmann's constant,  $\Omega$  is the atomic volume, and  $\sigma$  is the hydrostatic stress.

Then, Korhonen's equations considering temperature gradients can be rewritten as [33]

$$\frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x} \left[ \kappa(x) \left( \frac{\partial \sigma}{\partial x} - \frac{eZ\rho j}{\Omega} - \frac{Q}{\Omega T} \frac{\partial T}{\partial x} \right) \right] \quad (3)$$

where  $\kappa(x) = D_a(T(x))B\Omega/(k_B T(x))$  is a position-dependent diffusivity due to non-uniform temperature, and  $D_a = D_0 \exp(-E_a/(k_B T))$  is atomic diffusion coefficient.  $D_0$  is a constant and  $E_a$  is the EM activation energy. (3) can be viewed as coupled EM and TM equation for hydrostatic stress and its detail derivation is described in Appendix A.

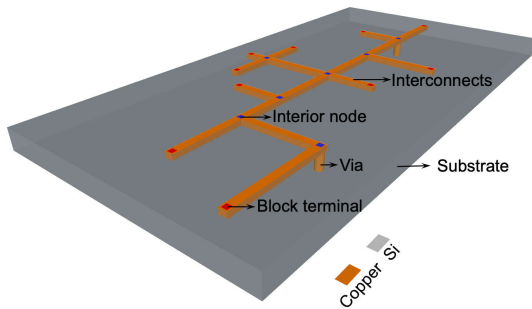


Fig. 2. 3D view of a general multi-segment Interconnects

Real on-chip power grid networks consist of many single branches, which are connected with each other by the junction, as shown in Fig. 2. Several researches show the hydrostatic stress can't be computed in the individual branch because each branch is not independent [1], [6], [19], [41].

To consider these connected effects, for a general interconnect wires with  $n$  nodes, including  $p$  interior junction nodes  $x_r \in \{x_{r1}, x_{r2}, \dots, x_{rp}\}$  and  $q$  block terminals  $x_b \in \{x_{b1}, x_{b2}, \dots, x_{bq}\}$ , as shown in Fig. 2 (blue nodes for  $x_r$  and red nodes for  $x_b$ ), the hydro-static stress distribution  $\sigma(x, t)$  along the wire is described by the following coupled EM-TM equation.

$$\frac{\partial \sigma_{ij}}{\partial t} = \frac{\partial}{\partial x} \left[ \kappa_{ij}(x) \left( \frac{\partial \sigma_{ij}}{\partial x} - S_{ij} - M_{ij} \right) \right], t > 0$$

$$BC : \sigma_{ij_1}(x_i, t) = \sigma_{ij_2}(x_i, t), t > 0$$

$$BC : \sum_{ij} \kappa_{ij}(x_r) \left( \frac{\partial \sigma_{ij}}{\partial x} \Big|_{x=x_r} - S_{ij} - M_{ij} \right) \cdot n_r = 0, t > 0$$

$$BC : \kappa_{ij}(x_b) \left( \frac{\partial \sigma_{ij}}{\partial x} \Big|_{x=x_b} - S_{ij} - M_{ij} \right) = 0, t > 0$$

$$IC : \sigma_{ij}(x, 0) = \sigma_{ij,T} \quad (4)$$

where  $ij$  denotes a branch connected to nodes  $i$  and  $j$ ,  $n_r$  represents the unit inward normal direction of the interior junction node  $r$  on branch  $ij$ , which is  $+1$  for right direction and  $-1$  for left direction of branch with assumption of  $x_i < x_j$ .  $S_{ij} = eZ\rho j/\Omega$  is EM flux and  $M_{ij} = Q/(\Omega T) \cdot \partial T/\partial x$  for TM flux for branch  $ij$ .  $\sigma_{ij,T}$  is the initial thermal-induced residual stress for segment or branch  $ij$ . Fig. 3(left) shows a cross interconnect tree, which is a two dimensions structure. To calculate the hydrostatic stress efficiently and conveniently in our work, we reduced 2D problem ( $x$ - $y$ ) to 1D problem ( $x$ ) which is described by (4). By performing such 2D to 1D mapping, we will end up with the same value of  $x$  but at different wire segments, as shown in Fig. 3 (right). The value of  $x$  only represents the relative position in each segment.

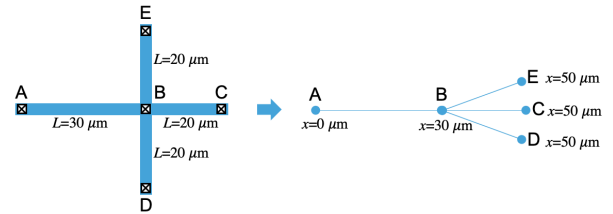


Fig. 3. The 2D problem to 1D problem mapping in the proposed analytic solution.

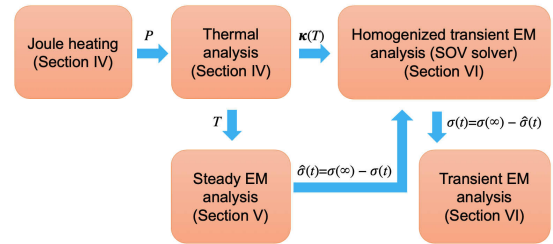


Fig. 4. Algorithmic flow of the proposed combined EM and TM analysis

Based on the physical EM-TM model, we can carry out a combined EM and TM analysis for general multi-segment interconnects, as shown in Fig. 4. At the beginning, the dissipated power calculated by Joule heating due to high current density is treated as heat source in thermal analysis.

Then, we develop an analytical temperature solution of the interconnects, as described in Section IV. With temperature distribution, steady-state solution of coupled EM-TM equation can be obtained, as shown in Section V. After that, considering non-uniform diffusivity  $\kappa(T)$ , we use steady-state solution to homogenize the EM-TM equation (4). To solve homogeneous EM-TM equation, an efficient separation of variables (SOV) method is proposed to do transient analysis. Finally, we can obtain the original transient response of hydrostatic stress by inverse transformation, as shown in Section VI.

#### IV. THERMAL ANALYSIS OF INTERCONNECTS DUE TO JOULE HEATING

To consider the non-uniform temperature effects, we have to capture the temperature profiles of the interconnects due to Joule heating. Generally, heat conduction equation with boundary conditions is utilized to model heat transfer and obtain temperature [42]. Many numerical methods, such as finite element method and finite difference method, have been proposed to solve the heat conduction equation. Numerical methods require expensive computation cost for very large scale integrated (VLSI) circuits. Because of uniform structure along the interconnects, we can reduce 3D structure to 1D problem and develop an analytical solution for heat conduction on the interconnects [16], [43]. However, this method only works for a single wire, not for general multi-segment interconnects. In addition, the expression for effective thermal length  $\Gamma$  is not accurate enough. Based on that, we develop a novel analytical approach to characterize thermal behavior of general multi-segment interconnects, which is efficient and accurate.

Specifically, the dissipated power density due to Joule heating is calculated by

$$P = \frac{I^2 R}{V} = \frac{(jA)^2 \rho \frac{L}{A}}{LA} = j^2 \rho \quad (5)$$

where  $I$  is the current,  $R$  is the resistance,  $V$  is the volume,  $A$  is the cross sectional area and  $L$  is the length of the interconnects.

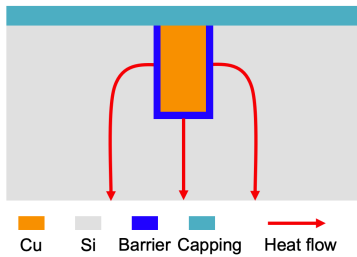


Fig. 5. Cross view of a interconnect

For interconnect wires, we can consider this problem as 1D problem. Then the governing heat equation along the Cu interconnects can be described by

$$\kappa_{Cu} \frac{\partial^2 T(x)}{\partial x^2} - h_{eff}(T - T_0) + j^2 \rho = 0 \quad (6)$$

where  $\kappa_{Cu}$  is thermal conductivity of Cu interconnects, and  $h_{eff}$  is the coefficient of convective heat transfer from Cu interconnect to surrounding substrate,  $T_0$  is temperature on the bottom side of Si substrate. The item  $h_{eff}(T - T_0)$  in (6) is

introduced to represent heat which flows from interconnects to surrounding dielectric, as shown in Fig. 5. With this item, we can reduce complex 3D thermal analysis of the interconnects to 1D problem.

With the boundary conditions  $T(-\frac{L}{2}) = T_1$  and  $T(\frac{L}{2}) = T_2$ , the solution of (6) is obtained by [18], [43]

$$T(x) = [\bar{T} - (T_0 + T_m)] \text{sech}\left(\frac{L}{2\Gamma}\right) \cosh\left(\frac{x}{\Gamma}\right) + T_n \text{csch}\left(\frac{L}{2\Gamma}\right) \sinh\left(\frac{x}{\Gamma}\right) + (T_0 + T_m) \quad (7)$$

where

$$\bar{T} = \frac{T_1 + T_2}{2}, T_n = \frac{T_2 - T_1}{2}, T_m = \frac{j^2 \rho \Gamma^2}{\kappa_{Cu}}, \quad (8)$$

where  $\Gamma$  can be viewed as effective thermal length, which can be expressed as [43]

$$\Gamma = \sqrt{\frac{\kappa_{Cu}}{h_{eff}}} \approx \sqrt{\frac{\kappa_{Cu} t_{Cu} t_{ILD}}{\kappa_{ILD}}} \quad (9)$$

where  $\kappa_{ILD}$  is the thermal conductivity of dielectric,  $t_{ILD}$  and  $t_{Cu}$  are the thickness of dielectric and copper, respectively. Approximated equation (9) illustrates  $\Gamma$  is determined by the dimensions and thermal conductivity of interconnect and surrounding dielectrics, as shown in Fig. 5. However, expression (9) only gives an approximated value, which may lead to inaccurate temperature results. To get accurate estimation of  $\Gamma$ , we propose to use finite element based COMSOL to extract the effective thermal length of the solution (7). Note for a given technology node, in general we can assume  $\Gamma$  is the same as first order approximation across the whole chip as the cross section of interconnects in a chip does not change significantly. If necessary, we can compute the  $\Gamma$  for each specific layer as well.

First, finite element analysis based on COMSOL is performed to calculate temperature  $T_i \in \mathbf{T} (i = 1, \dots, N)$  at corresponding position  $x_i \in \mathbf{x} (i = 1, \dots, N)$  along the Cu interconnects.  $\mathbf{T}$  and  $\mathbf{x}$  are vectors, which represent temperature and position, respectively.

Second, we can tune the value of  $\Gamma$  and fit the reference results  $\mathbf{T}$  and  $\mathbf{x}$  by using expression (7). To estimate  $\Gamma$  accurately, the destination is to minimize error  $\epsilon$  defined below

$$\begin{aligned} \epsilon &= \frac{1}{N} \sum_{i=1}^N (T_i - T(x_i, \Gamma))^2 \\ &= \frac{[\mathbf{T} - T(\mathbf{x}, \Gamma)]^T [\mathbf{T} - T(\mathbf{x}, \Gamma)]}{N} \end{aligned} \quad (10)$$

Instead of tuning the value of  $\Gamma$  manually, the accurate  $\Gamma$  is searched automatically by the Newton's iteration

$$\Gamma_{n+1} = \Gamma_n + \frac{[\mathbf{T} - T(\mathbf{x}, \Gamma_n)]^T [\mathbf{T} - T(\mathbf{x}, \Gamma_n)]}{2[\mathbf{T} - T(\mathbf{x}, \Gamma_n)]^T \frac{\partial T(\mathbf{x}, \Gamma)}{\partial \Gamma} |_{\Gamma=\Gamma_n}} \quad (11)$$

with the initial value from (9). Notice  $\Gamma$  can also be obtained by directly measurement or by computing the accurate coefficient of the effective convective heat transfer,  $h_{eff}$ , as shown in (9).

For general multi-segment wires, we assume that each wire segment  $ij$  has a direction defined for the sign of its heat flux  $\mathbf{q}_{ij}(x)$  at interior junction nodes  $x_r \in \{x_{r1}, x_{r2}, \dots, x_{rp}\}$ .

Then we observe that heat flux conservation is satisfied at location  $x_r$ , which is described by

$$\sum_{ij} \mathbf{q}_{ij}(x_r) = 0. \quad (12)$$

We note that this heat flux conservation ignores the heat loss to the surrounding dielectrics at  $x_r$ , which we assume is very small in general.

At the block terminals  $x_b \in \{x_{b1}, x_{b2}, \dots, x_{bq}\}$ , heat flux flows from the terminal to its surrounding dielectrics due to convective boundary condition, which is modeled by

$$-\mathbf{q}_{ij}(x_b) \cdot \mathbf{n}_b = h_b(T_{ij}(x_b) - T_0) \quad (13)$$

where  $h_b$  is the convective coefficient. The heat flux and temperature are related by Fourier's law, which is expressed as

$$\mathbf{q} = -\kappa_{Cu} \nabla T \quad (14)$$

Therefore, substituting (14) to boundary conditions (12) and (13), we have

$$\begin{aligned} BC : \sum_{ij} \frac{\partial T_{ij}(x)}{\partial x} \Big|_{x=x_r} \cdot n_r &= 0 \\ BC : \frac{\partial T_{ij}(x)}{\partial x} \Big|_{x=x_b} \cdot n_b &= \Gamma_b^{-2}(T_{ij}(x_b) - T_0) \end{aligned} \quad (15)$$

where terminal thermal length is given by

$$\Gamma_b = \sqrt{\frac{\kappa_{Cu}}{h_b}} \quad (16)$$

Finally, based on boundary conditions (15), temperature expression (7) and its derivative, we can form the linear equations

$$\mathbf{A} \cdot \boldsymbol{\beta} = \mathbf{B} \quad (17)$$

where  $\mathbf{A}$  is an  $(p+q) \times (p+q)$  matrix,  $\mathbf{B}$  is an  $(p+q) \times 1$  vector and the vector to be determined is represented by

$$\boldsymbol{\beta} = [T_{ij}(x_1), T_{ij}(x_2), \dots, T_{ij}(x_n)]^T \quad (18)$$

The elements of matrix  $\mathbf{A}$  are connected with a tree structure, which can be solved linearly and efficiently. Once the temperature of  $n$  nodes is obtained, temperature distribution along the interconnects can be determined by (7).

We notice that at terminal nodes (vias typically), the related elements of  $\boldsymbol{\beta}$  are known already, which are called  $\boldsymbol{\beta}_d$ . To solve this problem, we reorder and decompose equation (17), which is expressed as

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\beta}_u \\ \boldsymbol{\beta}_d \end{bmatrix} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} \quad (19)$$

where  $\boldsymbol{\beta}_u$  denotes unknown elements of  $\boldsymbol{\beta}$  and  $\boldsymbol{\beta}_d$  represents Dirichlet boundary elements of  $\boldsymbol{\beta}$ . Therefore, the unknown temperature can be calculated by

$$\boldsymbol{\beta}_u = \mathbf{A}_{11}^{-1}(\mathbf{B}_1 - \mathbf{A}_{12} \cdot \boldsymbol{\beta}_d) \quad (20)$$

We remark that we only need to consider the steady-state temperature distributions in the multi-segment wires in our problem. The reason is that transient behavior of the temperature is not important as temperature changing rates (time constant) is in the range of mili-second for VLSI systems [42],

while the time constant of EM is in the range of days or months.

We note that we also need to consider the temperature changing from active devices, which basically are computed by the traditional full-chip thermal analysis or coupled electrical-thermal analysis with proper power modeling for the devices at different levels [42] and many works have been proposed in the past [44]. We can treat the full-chip thermal analysis results as the ambient temperature inputs to the proposed EM analysis, as shown in (6). If the full-chip thermal analysis already includes the Joule-heating analysis, then we can directly use the computed temperature profile for both steady-state hydrostatic stress analysis and transient stress analysis with location dependent EM diffusivity, which will be discussed in following sections.

## V. STEADY-STATE ANALYTIC SOLUTION CONSIDERING BOTH THERMOMIGRATION AND RESIDUAL THERMO-MECHANICAL STRESS

In this section, we derive the steady-state solution of the coupled EM-TM equation in (4) considering the non-zero residual initial stress mainly from thermo-mechanical effects. Also, the steady analysis of the EM-TM model will be used to transfer the coupled EM-TM equation into homogeneous one in transient analysis as we show later.

Specifically, let  $\sigma_0$  is the non-zero initial residual stress of the wire, which may come from thermo-mechanical effects during the back end of line (BEOL) process. When the stress distribution comes to steady state, the atom flux no longer changes with time, and the constant atom flux in the wire becomes zero, which leads to

$$J_a = \kappa(x) \left( \frac{\partial \sigma(x, \infty)}{\partial x} - \frac{eZ\rho j}{\Omega} - \frac{Q}{\Omega T} \frac{\partial T}{\partial x} \right) = 0 \quad (21)$$

By integration of (21), we obtain the steady-state stress solution as

$$\sigma(x, \infty) = \frac{eZ\rho j}{\Omega} x + \frac{Q}{\Omega} \ln(T(x)) + C \quad (22)$$

where  $C$  is the constant to be determined.

We notice that instead of finding  $\sigma(x, \infty)$  for all possible  $x$ , we just need to find the  $\sigma(x, \infty)$  at the junction nodes  $x_r \in \{x_{r1}, x_{r2}, \dots, x_{rp}\}$  and block terminals  $x_b \in \{x_{b1}, x_{b2}, \dots, x_{bq}\}$ . The total number of nodes is  $p+q$  with  $p+q-1$  wire segments. Then, we need  $p+q$  equations to solve for the  $p+q$  unknowns.

The difference of the hydrostatic stress at two ends of a wire segment can be written as

$$\sigma(x_i) - \sigma(x_j) = \frac{eZ\rho j}{\Omega} L_{ij} + \frac{Q}{\Omega} \left[ \ln(T(x_i)) - \ln(T(x_j)) \right] \quad (23)$$

As a result, we have  $p+q-1$  equations already. Now we still need one more equation. By integration of (22), we have

$$\begin{aligned} \int_{x_i}^{x_j} \sigma(x) dx &= \frac{\sigma(x_i) + \sigma(x_j)}{2} L_{ij} + \frac{Q}{\Omega} \left[ \ln \left( \frac{T_0 + T_m}{\sqrt{T_i T_j}} \right) L_{ij} \right. \\ &\quad \left. + \frac{(T_i + T_j - 2(T_0 + T_m))\Gamma}{(T_0 + T_m)} \tanh \left( \frac{L_{ij}}{2\Gamma} \right) \right] \end{aligned} \quad (24)$$

For general multi-segment interconnect trees, with the atom conservation  $\int \sigma(x)dx = \int \sigma_0(x)dx$ , we obtain the last equation:

$$\sum_{ij} \left[ \frac{\sigma_i + \sigma_j}{2} L_{ij} + \frac{Q}{\Omega} \left( \ln \left( \frac{T_0 + T_{m,ij}}{\sqrt{T_i T_j}} \right) L_{ij} + \frac{(T_i + T_j - 2(T_0 + T_{m,ij}))\Gamma}{(T_0 + T_{m,ij})} \tanh\left(\frac{L_{ij}}{2\Gamma}\right) \right) \right] = \int \sigma_0(x)dx \quad (25)$$

With the  $p+q$  equations, we can obtain the steady-state stress  $\sigma(x, \infty)$  at all the nodes that we are interested in.

## VI. TRANSIENT STRESS SOLUTION

In this section, we develop a semi-analytic transient solution for the coupled EM-TM equation (4) using the SOV method. The SOV method has been applied to solve the Korhonen's equations without considering the thermal effects [19], [20]. However, it is difficult to use the SOV method to solve the EM-TM PDE due to the position-dependent diffusivity of copper wires.

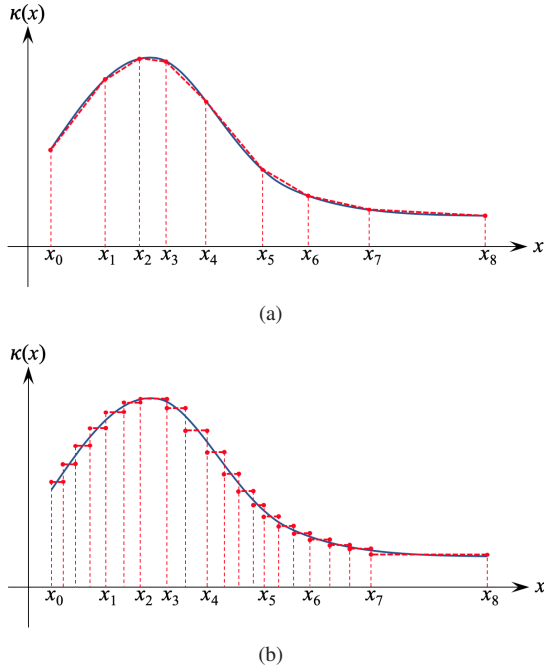


Fig. 6. (a) A piecewise linear approximation and (b) step-wise constant approximation of diffusion coefficient  $\kappa(x)$ .

As we can see from Fig. 6, piecewise linear and step-wise constant approximations are used to approximate the non-uniform diffusivity. It is evident that piecewise linear approximation shown in Fig. 6(a) requires less segments while maintaining high accuracy in contrast to stepwise constant approximation [45] shown in Fig. 6(b). Therefore, it is essential to develop piecewise linear approximation based SOV method to solve the transient EM-TM partial differential equation.

In this work, we employ a piecewise linear approximation to model temperature gradients across a wire. Specifically, as shown in Fig. 6(a), the diffusivity can be approximated by

$$\kappa(x) = \begin{cases} k_1 x + b_1 & x_0 \leq x < x_1 \\ k_2 x + b_2 & x_1 \leq x < x_2 \\ \dots & \dots \\ k_n x + b_n & x_{n-1} \leq x < x_n \end{cases} \quad (26)$$

Then, we can homogenize both the governing equation and boundary conditions with following transformation

$$\hat{\sigma}(x, t) = \sigma(x, \infty) - \sigma(x, t) \quad (27)$$

After that, the EM-TM PDE (4) can be transformed into homogeneous initial-boundary value problem (IBVP), which is described by

$$\frac{\partial \hat{\sigma}_{ij}}{\partial t} = \frac{\partial}{\partial x} \left[ \kappa_{ij}(x) \frac{\partial \hat{\sigma}_{ij}}{\partial x} \right], t > 0 \quad (28a)$$

$$BC : \hat{\sigma}_{ij_1}(x_i, t) = \hat{\sigma}_{ij_2}(x_i, t), t > 0 \quad (28b)$$

$$BC : \sum_{ij} \kappa_{ij}(x_r) \frac{\partial \hat{\sigma}_{ij}}{\partial x} \Big|_{x=x_r} \cdot n_r = 0, t > 0 \quad (28c)$$

$$BC : \kappa_{ij}(x_b) \frac{\partial \hat{\sigma}_{ij}}{\partial x} \Big|_{x=x_b} = 0, t > 0 \quad (28d)$$

$$IC : \hat{\sigma}_{ij}(x, 0) = \sigma(x, \infty) - \sigma_{ij,T} \quad (28e)$$

Applying separation of variables method  $\hat{\sigma}_{ij}(x, t) = \phi_{ij}(x)\theta(t)$ , the governing equation (28a) becomes

$$\frac{\theta'(t)}{\theta(t)} = \frac{\frac{\partial}{\partial x} \left[ \kappa_{ij}(x) \frac{\partial \phi_{ij}(x)}{\partial x} \right]}{\phi_{ij}(x)} = -\lambda^2 \quad (29)$$

where  $\lambda$  is the eigenvalue, and  $\phi_{ij}(x)$  and  $\theta(t)$  are temporal and spatial eigenfunctions, respectively. In general, there are a number of eigenvalues  $\lambda_m (m = 1, 2, 3, \dots)$  to satisfy the boundary conditions. For each eigenvalue, the solution of temporal function is

$$\theta_m(t) = C_m \cdot e^{-\lambda_m^2 t} \quad (30)$$

where  $C_m$  is a coefficient. The corresponding spatial distribution equation is given by

$$\frac{\partial}{\partial x} \left[ \kappa_{ij}(x) \frac{\partial \phi_{ij}(x)}{\partial x} \right] + \lambda_m^2 \phi_{ij}(x) = 0 \quad (31)$$

Replacing the position-dependent diffusivity with a linear function (26), (31) is simplified into

$$\left(x + \frac{b_{ij}}{k_{ij}}\right) \frac{\partial^2 \phi_{ij}(x)}{\partial x^2} + \frac{\partial \phi_{ij}(x)}{\partial x} + \frac{\lambda_m^2}{k_{ij}} \phi_{ij}(x) = 0 \quad (32)$$

which can be transformed to Bessel equation, and detail derivation is described in Appendix B [46]. The analytical solution of (32) is given by

$$\phi_{ij,m}(x) = A_{ij,m} J_0(\omega_{ij,m}(x)) + B_{ij,m} Y_0(\omega_{ij,m}(x)) \quad (33)$$

$$\begin{bmatrix} \left. \frac{\partial \phi_{ij,m}(x)}{\partial x} \right|_{x=x_i} \\ - \left. \frac{\partial \phi_{ij,m}(x)}{\partial x} \right|_{x=x_j} \end{bmatrix} = \begin{bmatrix} -\gamma_{ij,m}(x_i)J_1(\omega_{ij,m}(x_i)) & -\gamma_{ij,m}(x_i)Y_1(\omega_{ij,m}(x_i)) \\ \gamma_{ij,m}(x_j)J_1(\omega_{ij,m}(x_j)) & \gamma_{ij,m}(x_j)Y_1(\omega_{ij,m}(x_j)) \end{bmatrix} \cdot \begin{bmatrix} A_{ij,m} \\ B_{ij,m} \end{bmatrix} \quad (36)$$

$$\begin{bmatrix} \phi_{ij,m}(x_i) \\ \phi_{ij,m}(x_j) \end{bmatrix} = \begin{bmatrix} J_0(\omega_{ij,m}(x_i)) & Y_0(\omega_{ij,m}(x_i)) \\ J_0(\omega_{ij,m}(x_j)) & Y_0(\omega_{ij,m}(x_j)) \end{bmatrix} \cdot \begin{bmatrix} A_{ij,m} \\ B_{ij,m} \end{bmatrix} \quad (37)$$

$$J_1(\omega_{ij,m}(x))Y_0(\omega_{ij,m}(x)) - J_0(\omega_{ij,m}(x))Y_1(\omega_{ij,m}(x)) = \frac{2}{\pi\omega_{ij,m}(x)} \quad (39)$$

$$\begin{aligned} D_{11} &= \gamma_{ij,m}(x_i) \frac{J_0(\omega_{ij,m}(x_j))Y_1(\omega_{ij,m}(x_i)) - J_1(\omega_{ij,m}(x_i))Y_0(\omega_{ij,m}(x_j))}{J_0(\omega_{ij,m}(x_i))Y_0(\omega_{ij,m}(x_j)) - J_0(\omega_{ij,m}(x_j))Y_0(\omega_{ij,m}(x_i))} \\ D_{12} &= \gamma_{ij,m}(x_i) \frac{2/(\pi\omega_{ij,m}(x_i))}{J_0(\omega_{ij,m}(x_i))Y_0(\omega_{ij,m}(x_j)) - J_0(\omega_{ij,m}(x_i))Y_0(\omega_{ij,m}(x_i))} \\ D_{21} &= \gamma_{ij,m}(x_j) \frac{2/(\pi\omega_{ij,m}(x_j))}{J_0(\omega_{ij,m}(x_i))Y_0(\omega_{ij,m}(x_j)) - J_0(\omega_{ij,m}(x_j))Y_0(\omega_{ij,m}(x_i))} \\ D_{22} &= \gamma_{ij,m}(x_j) \frac{J_0(\omega_{ij,m}(x_i))Y_1(\omega_{ij,m}(x_j)) - J_1(\omega_{ij,m}(x_j))Y_0(\omega_{ij,m}(x_i))}{J_0(\omega_{ij,m}(x_i))Y_0(\omega_{ij,m}(x_j)) - J_0(\omega_{ij,m}(x_j))Y_0(\omega_{ij,m}(x_i))} \end{aligned} \quad (41)$$

where  $A_{ij,m}$  and  $B_{ij,m}$  are the coefficients to be determined,  $J_0(\omega_{ij,m}(x))$  and  $Y_0(\omega_{ij,m}(x))$  are the zero-order Bessel functions of the first and second kinds, respectively, and  $\omega_{ij,m}(x)$  is represented by

$$\omega_{ij,m}(x) = \frac{2\lambda_m}{\sqrt{k_{ij}}} \sqrt{x + \frac{b_{ij}}{k_{ij}}} \quad (34)$$

Based on the temporal function (30) and spatial function (33), the general solution for (28) is an infinity series, which is given by

$$\hat{\sigma}_{ij}(x, t) = \sum_{m=1}^{\infty} \theta_m(t) \cdot \phi_{ij,m}(x) \quad (35)$$

Once the general solution is obtained, we need to determine eigenvalues  $\lambda_m$  and eigenfunctions (33). In general multi-segment interconnect trees, branch  $ij$  with node  $i$  and  $j$  is the fundamental element, and connects with each other through the junction nodes. Therefore, boundary information at  $x_i$  and  $x_j$  is critical to determine the coefficients and eigenvalues in (33). Accordingly, the general solutions (33) and their derivatives at  $x_i$  and  $x_j$  are respectively formulated in (36) and (37), where  $\gamma_{ij,m}(x)$  is written as

$$\gamma_{ij,m}(x) = \frac{\lambda_m}{\sqrt{k_{ij}} \sqrt{x + \frac{b_{ij}}{k_{ij}}}} \quad (38)$$

In [46], Bessel functions also have the formula (39). Based on (36), (37) and (39), the inward normal component of atom flux on both ends of brach  $ij$  can be calculated by

$$\begin{bmatrix} \left. \frac{\partial \phi_{ij,m}(x)}{\partial x} \right|_{x=x_i} \cdot n_i \\ \left. \frac{\partial \phi_{ij,m}(x)}{\partial x} \right|_{x=x_j} \cdot n_j \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \cdot \begin{bmatrix} \phi_{ij,m}(x_i) \\ \phi_{ij,m}(x_j) \end{bmatrix} \quad (40)$$

where the elements  $D_{11}$ ,  $D_{12}$ ,  $D_{21}$ , and  $D_{22}$  are described in (41).

To satisfy the conservation of the atom flux at junction nodes and block boundaries, substituting (35) and (40) to the BCs (28c) and (28d), we obtain

$$\mathbf{K}(\lambda_m) \cdot \phi = \mathbf{0} \quad (42)$$

where  $\mathbf{K}$  is an  $(p+q) \times (p+q)$  matrix, and the vector is

$$\phi = [\phi_{ij,m}(x_1), \phi_{ij,m}(x_2), \dots, \phi_{ij,m}(x_n)]^T \quad (43)$$

In order to find the solution of the nonlinear transcendental equation  $|\mathbf{K}(\lambda_m)| = 0$  which is equivalent to (42), Wittrick-Williams (WW) algorithm [45], [47] is proposed to calculate all eigenvalues without missing anyone of them, which is also an extremely efficient method. The method is described as follows. We need to determine the number of eigenvalues between zero and value of  $\mu$ , which is calculated by

$$N(\mu) = N_{0n}(\mu) + s\{\mathbf{K}^\Delta(\mu)\} \quad (44)$$

where  $\mathbf{K}^\Delta(\mu)$  is the triangular form of  $\mathbf{K}(\mu)$  using Gaussian elimination process,  $s\{\cdot\}$  is a sign-count function which returns the number of negative main diagonal elements of the matrix  $\mathbf{K}^\Delta(\mu)$ , and  $N_{0n}(\mu)$  is the number of natural eigenvalues on decoupled branches with zero Dirichlet boundary conditions on both ends. Therefore, to decouple branches and find  $N_{0n}(\mu)$ , the vector  $\phi$  is set to  $\mathbf{0}$ , and (37) on brach  $ij$  becomes

$$f_0(\mu) = J_0(\omega_{ij,m}(x_i))Y_0(\omega_{ij,m}(x_j)) - J_0(\omega_{ij,m}(x_j))Y_0(\omega_{ij,m}(x_i)) = 0 \quad (45)$$

For positive and large  $x$ , Jacobi [46] obtained the approximate formulae for  $J_0(x)$  and  $Y_0(x)$ , which are respectively written as

$$J_0(x) \approx \sqrt{\frac{2}{\pi x}} \cos(x - \frac{\pi}{4}), Y_0(x) \approx \sqrt{\frac{2}{\pi x}} \sin(x - \frac{\pi}{4}) \quad (46)$$

Substituting (46) into (45), we have

$$\sin(\omega_{ij,m}(x_i) - \omega_{ij,m}(x_j)) = 0 \quad (47)$$

Therefore, the approximated eigenvalue of (45) is given by

$$\tilde{\lambda}_m = \frac{m\pi}{\left[ \frac{2}{\sqrt{\kappa_{ij}}} \left( \sqrt{x_i + \frac{b_{ij}}{\kappa_{ij}}} - \sqrt{x_j + \frac{b_{ij}}{\kappa_{ij}}} \right) \right]} \quad (48)$$

where  $\tilde{\lambda}_m$  denotes approximate value of  $\lambda_m$ . To correct errors of approximation (46), the accurate number of natural eigenvalues on branch  $ij$  is determined by

$$N_{0,ij}(\mu) = \tilde{N}_{0,ij}(\mu) \pm 1 \quad (49)$$

where  $\tilde{N}_{0,ij}(\mu)$  is the approximated value of  $N_{0,ij}(\mu)$ , which is written as

$$\tilde{N}_{0,ij}(\mu) = \left\lfloor \frac{|\omega_{ij,m}(x_i) - \omega_{ij,m}(x_j)|}{\pi} \right\rfloor \quad (50)$$

where  $\lfloor x \rfloor$  is the floor function which outputs the greatest integer less than or equal to  $x$ . In order to obtain the accurate value of  $N_{0,ij}(\mu)$ , it is necessary to correct  $\tilde{N}_{0,ij}(\mu)$  with the error  $\pm 1$ , which is the purpose of equation (49). As shown in Fig. 7, a number of numerical results have shown that all approximate eigenvalues represented by (48) are offset to the left or right with respect to accurate eigenvalues of (45). Fig. 7(a) and Fig. 7(c) show the approximate eigenvalues are less than the accurate eigenvalues. On the contrary, Fig. 7(b) and Fig. 7(d) show the opposite. Red number denotes  $N_{0,ij}(\mu)$  and green number represents  $\tilde{N}_{0,ij}(\mu)$ . Based on the relationship between eigenvalues and  $N_{0,ij}(\mu)$  shown in Fig. 7, there are four cases to correct errors of  $\tilde{N}_{0,ij}(\mu)$  and obtain the accurate  $N_{0,ij}(\mu)$ , which are listed in Table I. The total number of natural eigenvalues is the sum of  $N_{0,ij}(\mu)$

$$N_{0n}(\mu) = \sum_{ij} N_{0,ij}(\mu) \quad (51)$$

With the number of eigenvalue (44), we can use bisection method to separate and approach eigenvalues in an iterative way. Once the eigenvalues  $\lambda_m$  are determined, a linear equations formed by BCs (28b), (28c) and (28d) are solved for eigenfunctions. For the linear equations, we set some elements such as  $A_{12,m}(m = 1, 2, \dots)$  to 1 and the coefficients  $A_{ij,m}$  and  $B_{ij,m}$  can be obtained.

After that, using orthogonality of eigenfunctions and initial conditions (28e), the coefficients  $C_m$  can be determined by

$$C_m = \frac{\sum_{ij} \langle \phi_{ij,m}(x) \cdot \hat{\sigma}_{ij}(x, 0) \rangle}{\sum_{ij} \langle \phi_{ij,m}(x) \cdot \phi_{ij,m}(x) \rangle} \quad (52)$$

where the inner product is defined by

$$\langle f_{ij}(x) \cdot g_{ij}(x) \rangle = \int_{x_i}^{x_j} f_{ij}(x) g_{ij}(x) dx \quad (53)$$

Finally, the original transient hydrostatic stress  $\sigma(x, t)$  is obtained by inverse transformation

$$\sigma(x, t) = \sigma(x, \infty) - \hat{\sigma}(x, t) \quad (54)$$

## VII. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present some numerical results and validate the accuracy of the proposed analytical method with three typical structures in VLSI interconnects, including a single wire, straight line and multi-segment interconnects, as shown in Fig. 8, Fig. 11 and Fig. 14, respectively. All programs are

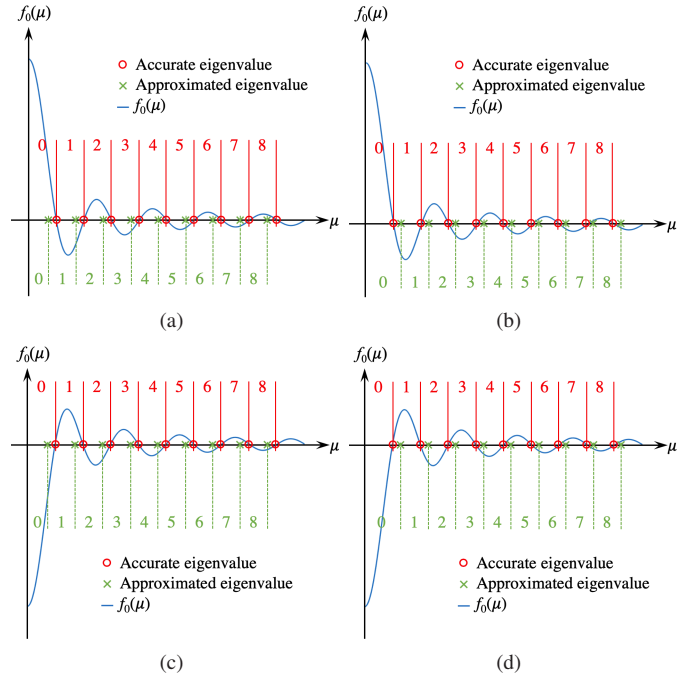


Fig. 7. Four types of relationship between the approximated and accurate eigenvalues to determine  $N_{0,ij}(\mu)$  and  $\tilde{N}_{0,ij}(\mu)$ .

TABLE I  
DETERMINATION OF NUMBER OF NATURAL EIGENVALUES

| Figure    | Case                                             | Subcase                                               | Correction                                  |
|-----------|--------------------------------------------------|-------------------------------------------------------|---------------------------------------------|
| Fig. 7(a) | $f_0(0) > 0$<br>and $f_0(\tilde{\lambda}_1) > 0$ | $\tilde{N}_{0,ij}(\mu)$ is odd<br>and $f_0(\mu) > 0$  | $N_{0,ij}(\mu) = \tilde{N}_{0,ij}(\mu) - 1$ |
|           |                                                  | $\tilde{N}_{0,ij}(\mu)$ is even<br>and $f_0(\mu) < 0$ |                                             |
| Fig. 7(c) | $f_0(0) < 0$<br>and $f_0(\tilde{\lambda}_1) < 0$ | $\tilde{N}_{0,ij}(\mu)$ is odd<br>and $f_0(\mu) < 0$  | $N_{0,ij}(\mu) = \tilde{N}_{0,ij}(\mu) - 1$ |
|           |                                                  | $\tilde{N}_{0,ij}(\mu)$ is even<br>and $f_0(\mu) > 0$ |                                             |
| Fig. 7(b) | $f_0(0) > 0$<br>and $f_0(\tilde{\lambda}_1) < 0$ | $\tilde{N}_{0,ij}(\mu)$ is odd<br>and $f_0(\mu) > 0$  | $N_{0,ij}(\mu) = \tilde{N}_{0,ij}(\mu) + 1$ |
|           |                                                  | $\tilde{N}_{0,ij}(\mu)$ is even<br>and $f_0(\mu) < 0$ |                                             |
| Fig. 7(d) | $f_0(0) < 0$<br>and $f_0(\tilde{\lambda}_1) > 0$ | $\tilde{N}_{0,ij}(\mu)$ is odd<br>and $f_0(\mu) < 0$  | $N_{0,ij}(\mu) = \tilde{N}_{0,ij}(\mu) + 1$ |
|           |                                                  | $\tilde{N}_{0,ij}(\mu)$ is even<br>and $f_0(\mu) > 0$ |                                             |

implemented in MATLAB and tested on a standard computer with a 2.7 GHz i5 CPU, 8 GB memory and macOS operating system.

### A. A single wire interconnect case

Fig. 8 shows the 3D geometry of a single wire structure with barrier and capping layer. The bottom side is set to a constant temperature  $T_0$ , two ends of interconnect are set to constant temperature  $T_1$  and  $T_2$  and other sides are adiabatic boundary. Current density is  $6.6 \times 10^{10}$  A/m<sup>2</sup>, which becomes heat source due to Joule heating. The initial stress  $\sigma_0$  is set to be 10 MPa.

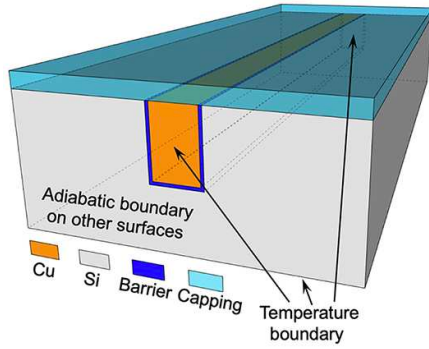


Fig. 8. A single wire structure with thermal boundaries

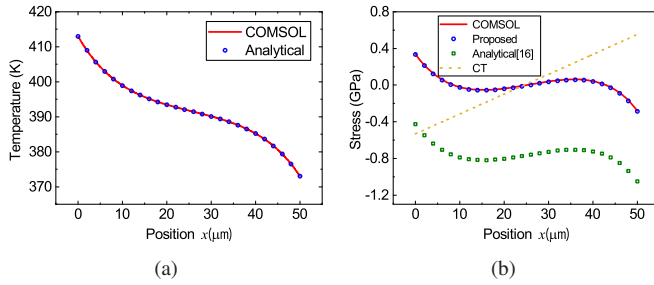


Fig. 9. (a) Temperature distribution and (b) steady stress distribution of a single wire.

The width of the wire is  $0.3 \mu\text{m}$  and thickness is  $0.8 \mu\text{m}$ . In order to obtain the accurate  $\Gamma$ , COMSOL is employed to perform thermal simulation. Based on the formula (11), the value of  $\Gamma$  is extracted by using the results from COMSOL simulation, which is  $9.718 \times 10^{-6}$  in this case. As shown from Fig. 9(a), the temperature profile computed by the proposed analytical solution (7) agrees very well with the COMSOL simulation.

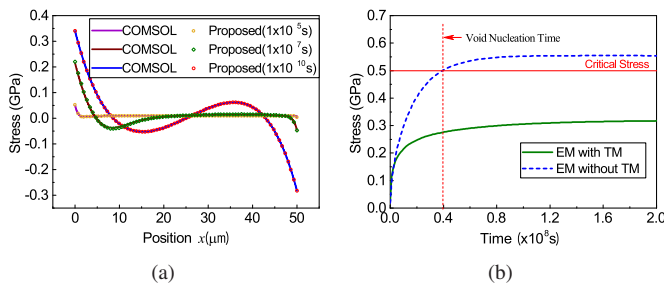


Fig. 10. (a) Transient stress distribution and (b) maximum stress over time of a single wire.

Once temperature distribution along the single wire is captured, we can obtain the steady state hydrostatic stress using equations (23) and (25). Fig. 9(b) shows the proposed method has a good agreement with COMSOL. However, due to using more restricted assumption on boundary conditions, the analytical solution in [16] leads to less accurate results. In the Fig. 9(b), the legend ‘CT’ denotes the constant temperature, which represents the case without considering TM effects. As we can see in Fig. 9(b), EM steady state stress considering TM effects is different from that of ‘CT’ case, and the maximum stress for EM-TM analysis is smaller than that of ‘CT’ case.

Then, we can perform EM-TM transient analysis. Fig. 10(a) shows that there exists a very good agreement between the proposed method and COMSOL results at different time points. To investigate the impact of TM on transient simulation, the maximum hydrostatic stresses over time of the single wire with and without TM effects are depicted in Fig. 10(b). In this paper, we set the critical stress to be 500 MPa [48]. As we can see, If we do not consider TM effects, void will be formed in cathode. In fact, the wire will never fail.

### B. A straight line interconnect case

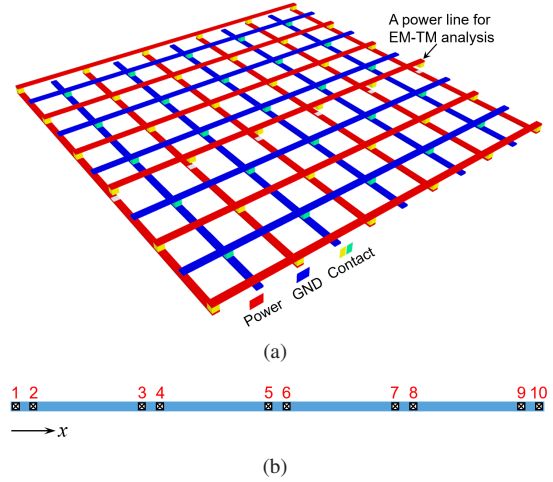


Fig. 11. (a) Structure of a typical power delivery network. We can evaluate EM stress separately for each power line (red) because diffusion barrier (Ta/TaN) prevents Cu atoms from diffusing into other metal layers. (b) A straight line structure extracted from IBMPG1 benchmark [49], which is drawn from real design. Positive value of current density denotes its direction is along  $x$  positive direction.

TABLE II  
PARAMETERS FOR THE STRAIGHT LINE INTERCONNECT WIRE

| Brch#     | $j$ (A/m <sup>2</sup> ) | $L$ (μm) | Brch#      | $j$ (A/m <sup>2</sup> ) | $L$ (μm) |
|-----------|-------------------------|----------|------------|-------------------------|----------|
| $l_{1,2}$ | $-1.06\text{E}10$       | 11       | $l_{6,7}$  | $-4.46\text{E}10$       | 119      |
| $l_{2,3}$ | $-5.09\text{E}9$        | 119      | $l_{7,8}$  | $-5.76\text{E}10$       | 11       |
| $l_{3,4}$ | $-4.72\text{E}10$       | 11       | $l_{8,9}$  | $2.22\text{E}10$        | 119      |
| $l_{4,5}$ | $-4.43\text{E}10$       | 119      | $l_{9,10}$ | $1.86\text{E}10$        | 11       |
| $l_{5,6}$ | $-4.41\text{E}10$       | 11       |            |                         |          |

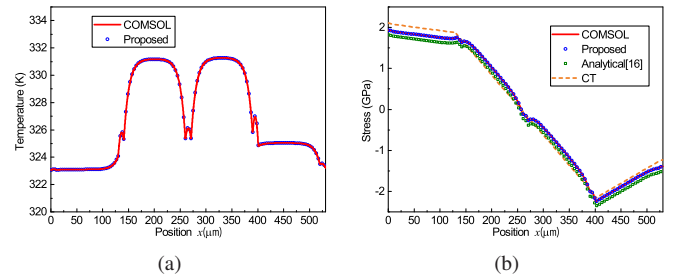


Fig. 12. (a) Temperature distribution and (b) steady stress distribution of a straight line.

An example of power delivery network (PDN) composed of two metal levels is described in Fig. 11(a). Power and ground lines are illustrated by red and blue lines, respectively.

Typical interconnect metals are covered with diffusion barrier (Ta/TaN) layer, which demonstrates strong resistance to EM from one metal layer to another metal layer. Therefore, EM analysis for each continuously connected interconnect can be analyzed separately. For a real application, a straight line is selected from realistic IBMPG1 benchmark [49], as shown in Fig. 11(b). Its size and current densities obtained by SPICE circuits simulation for IBMPG1 benchmark are illustrated in Table II. As we can see, there are global wires ( $119\mu\text{m}$ ) and local wires ( $11\mu\text{m}$ ) in the straight line. For the thermal simulation,  $\Gamma$  calculated by (11) is  $9.624 \times 10^{-6}$ .

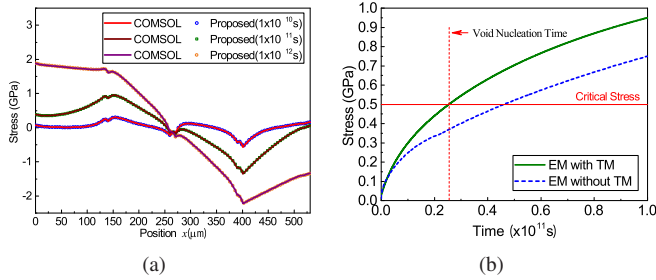


Fig. 13. (a) Transient stress distribution and (b) maximum stress over time of a straight line.

Fig. 12 and Fig. 13(a) show that there exists a very good agreement between the proposed method and COSMOL results for temperature distribution, steady stress and transient stress. It can be observed from Fig. 13(b) that TM effects lead to decreasing of void nucleation time.

### C. A multi-segment interconnect case

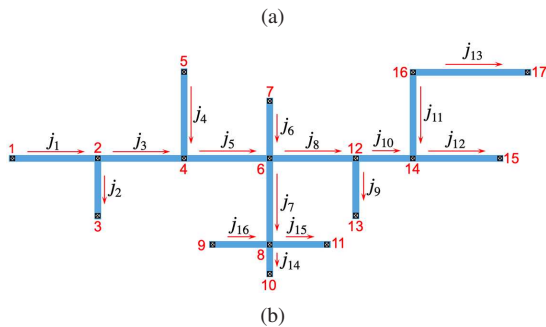
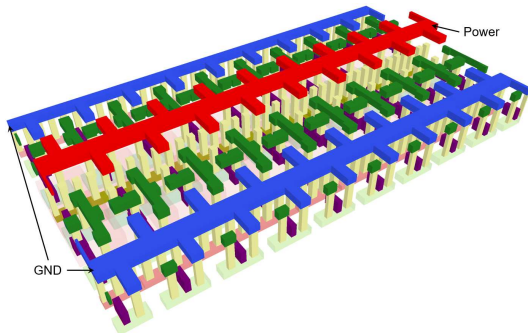


Fig. 14. (a) 3D schematic of standard cell for inverter circuits. Power line (red) is a tree structure in standard cell. We use (b) a multi-segment interconnect structure to describe complex power line tree.

To further demonstrate the efficiency and accuracy of the proposed semi-analytical transient solution for EM-TM

TABLE III  
PARAMETERS FOR THE MULTI-SEGMENT INTERCONNECT WIRE

| Brch# | $j$ (A/m <sup>2</sup> ) | $L$ (μm) | Brch# | $j$ (A/m <sup>2</sup> ) | $L$ (μm) |
|-------|-------------------------|----------|-------|-------------------------|----------|
| 1     | $5 \times 10^{10}$      | 30       | 9     | $4 \times 10^{10}$      | 20       |
| 2     | $1 \times 10^{10}$      | 20       | 10    | $2 \times 10^{10}$      | 20       |
| 3     | $2 \times 10^{10}$      | 30       | 11    | $2 \times 10^{10}$      | 30       |
| 4     | $4 \times 10^{10}$      | 30       | 12    | $1 \times 10^{10}$      | 30       |
| 5     | $2 \times 10^{10}$      | 30       | 13    | $3 \times 10^{10}$      | 40       |
| 6     | $3 \times 10^{10}$      | 20       | 14    | $1.5 \times 10^{10}$    | 10       |
| 7     | $2 \times 10^{10}$      | 30       | 15    | $1 \times 10^{10}$      | 20       |
| 8     | $5 \times 10^{10}$      | 30       | 16    | $2 \times 10^{10}$      | 20       |

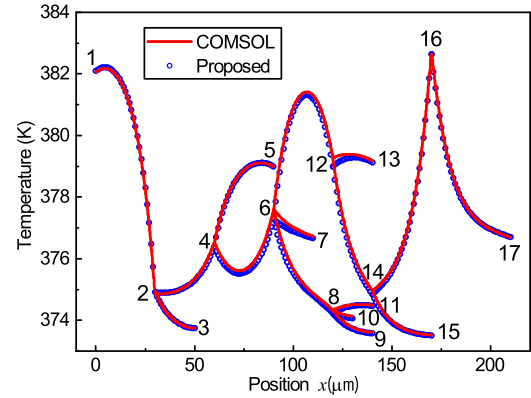


Fig. 15. Temperature distribution of a multi-segment interconnect.

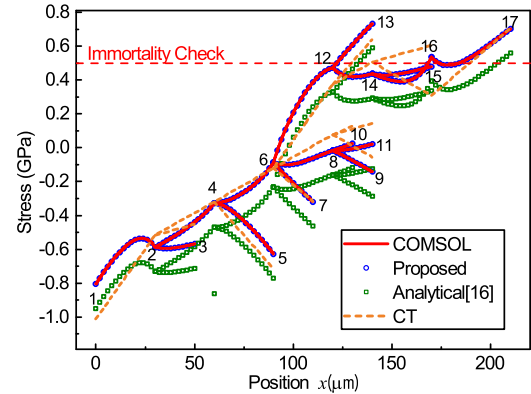


Fig. 16. Steady stress of a multi-segment interconnect.

analysis, we consider power interconnects of standard cell for designing ICs. Fig. 14(a) shows 3D view of a standard cell for inverters circuits. As we can see, the power line (red) is a tree structure and has multi-segments. Therefore, a more complicated and general multi-segment wire structure as shown in Fig. 14(b) is employed as a test case. The current densities and lengths of all the segments are given in Table III, where *Brch* is the branch index, *j* is the current density, and *L* is the length of the wire.

We use the proposed analytical method to capture thermal profile due to Joule heating. Accurate  $\Gamma$  and  $\Gamma_b$  are extracted by (11), which are respectively  $9.615 \times 10^{-6}$  and 0.0125 for this case. The effective thermal length  $\Gamma$  of the three cases are very similar due to their same dimensions and material in cross view. We set constant temperature at node 2 and 16 with 373K and 393K, respectively. Temperature at all nodes is

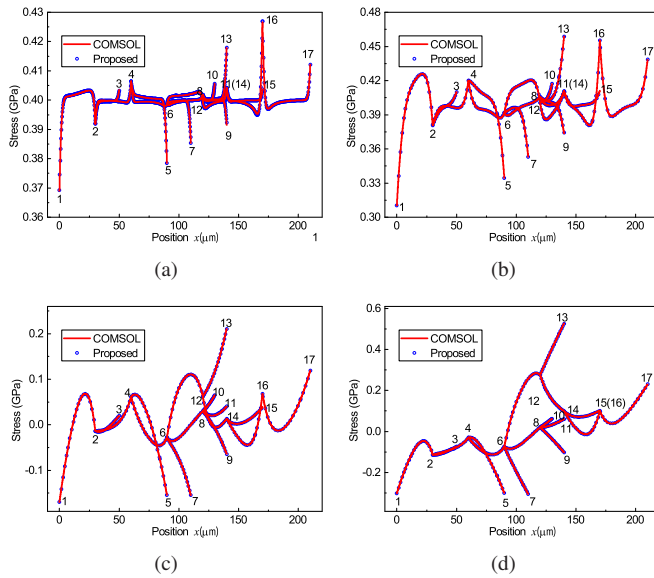


Fig. 17. Transient stress distribution of a multi-segment interconnect at (a)  $t = 1 \times 10^5$  s, (b)  $t = 1 \times 10^6$  s, (c)  $t = 1 \times 10^7$  s, and (d)  $t = 1 \times 10^8$  s.

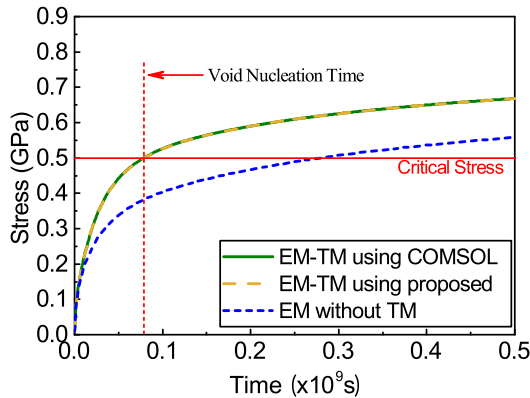


Fig. 18. Maximum stress over time of a multi-segment interconnect.

estimated by the thermal solver (20). As we can see that the analytical solution for temperature distribution is very accurate compared to the results of COMSOL as shown in Fig. 15.

Fig. 16 and Fig. 17 show that the results from the proposed method have a good agreement with that of COMSOL in both steady and transient analysis. As we can see in Fig. 16, analytical solution of steady state in [16] is again less accurate in the multi-segment interconnect case. In addition, the maximum stress of EM-TM analysis is larger than that of ‘CT’.

As shown in Fig. 18, the accuracy of our proposed EM-TM method is demonstrated by COMSOL. If we do not consider TM effects, void nucleation time is overestimated. Therefore, based on above mentioned three cases, *TM effects or temperature gradient impacts are significant for accurate prediction of hydrostatic stress evolution*. The TM impacts on EM stress evolution actually depend on the temperature gradient direction and current flow direction of the specific wire segment. The direction of TM driving force is from high temperature to low temperature, and the direction of EM driving force is opposed to that of the current. When the cathode node has lower temperature than the anode node, the TM

driving force can attenuate EM driving force as in the single wire case. When the cathode node has higher temperature than the anode node, the TM driving force can enhance EM driving force as shown in the *Brch 3* of the multi-segment interconnect case. Therefore, steady state  $\Delta\sigma_{2,4}$  with TM effect is greater than that without TM effect, as shown in Fig. 16. Experiments in [24], [30] also demonstrated the collective effects of EM and TM. However, the interaction between TM and EM for multi-segment interconnect is complex as both two situations exist. Overestimated void nucleation time is the result impacted by the superposition of each branch effect. Another important factor impact void nucleation time is non-uniform diffusivity  $\kappa(T)$  [31], which affects time to steady state but does not change the steady results, as shown in Fig. 19. Compared with TM effects, the impact of temperature-dependent diffusivity on void nucleation time is limited.

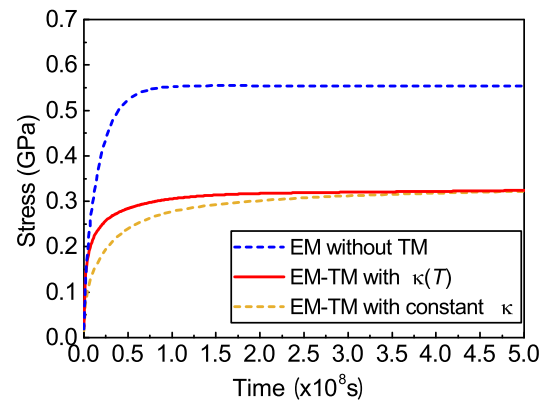


Fig. 19. Maximum stress over time of a single wire for different EM models.

Furthermore, to validate the efficiency of the proposed method, time step is set to  $\Delta t = 1 \times 10^6$  s and total simulation time lasts  $2 \times 10^9$  s. Table IV shows that the semi-analytical method reaches a  $9.1\times$  or about one order of magnitude speedup over COMSOL (with almost same accuracy) and reduces memory footprint by  $10.6\times$  compared with the FEM analysis.

TABLE IV  
COMPARISON OF PROPOSED METHOD AND COMSOL

|             | Proposed | COMSOL | Ratio |
|-------------|----------|--------|-------|
| Time (sec)  | 31       | 283    | 9.1   |
| Memory (MB) | 58       | 617    | 10.6  |

## VIII. CONCLUSION

In this paper, we have proposed a new semi-analytical transient solution for the electromigration (EM) hydrostatic stress evolution considering thermomigration (TM) effects on general multi-segment interconnects. The new method is based on the SOV approach to find the analytic solution of coupled EM and TM partial differential equations. We first develop analytic solutions to compute steady-state temperature distribution of multi-segment wires. Then, we derive a more general and accurate analytical formula for steady state EM-TM stress profile based on the analytical non-uniform temperature profile. Finally, we propose to use piecewise linear technique to approximate the position-dependent diffusivity of

metal migration. Numerical results on multi-segment interconnects show that the proposed method has negligible error loss compared to COMSOL but is about an order of magnitude faster than COMSOL with 10x less memory footprint. The numerical results further show that temperature gradient due to Joule heating indeed has significant impacts on the EM failure process.

#### APPENDIX A

There are three types of driving forces leading atomic transport along the interconnect line, such as EM, stressmigration (SM), thermomigration (TM). EM driving force caused by the momentum transfers between moving electrons and metal atoms increases with the increasing current density due to the applied electrical field. The atomic flux due to EM driving force is defined by

$$J_{EM} = D_v C_v \frac{eZ\rho j}{k_B T} \quad (55)$$

A vacancy concentration gradient decreasing from the cathode to the anode because of the migration of metal atoms forms the mechanical stress. The atomic flux caused by the mechanical force is given by

$$J_{SM} = -D_v \frac{\Omega}{k_B T} C_v \frac{\partial \sigma}{\partial x} \quad (56)$$

Temperature gradients caused by Joule heating can generate a driving force to the mass flow, namely thermomigration. The atomic flux generated by TM is formulated as

$$J_{TM} = D_v \frac{Q}{k_B T^2} C_v \frac{\partial T}{\partial x} \quad (57)$$

The total atomic flux is expressed as [16], [33]–[36]

$$J = -D_v \left( \frac{\Omega}{k_B T} C_v \frac{\partial \sigma}{\partial x} - C_v \frac{eZ\rho j}{k_B T} - \frac{Q}{k_B T^2} C_v \frac{\partial T}{\partial x} \right) \quad (58)$$

The vacancies accumulate or vanish at the divergence of the atomic flux. The vacancy continuity equation can be written as [13], [33]

$$\frac{\partial C_v}{\partial t} = -\frac{\partial J_v}{\partial x} + G \quad (59)$$

where  $G = \partial C_L / \partial t$  denotes a generation or annihilation term [50],  $C_L$  is the concentration of lattice sites.

Based on Hooke's law, the relationship of lattice concentration and hydrostatic stress is described by

$$\frac{dC_L}{C_L} = -\frac{d\sigma}{B} \quad (60)$$

where  $B$  is the Young's Modulus. By substituting (60) into (59), we have

$$-\frac{\partial J_v}{\partial x} = \frac{\partial C_v}{\partial t} + \frac{C_L}{B} \frac{\partial \sigma}{\partial t} \quad (61)$$

where the vacancy concentration is represented by [33]

$$C_v = C_{v0} \exp\left(\frac{\Omega \sigma}{k_B T}\right) \quad (62)$$

and its corresponding derivative with respect to time  $t$  is

$$\frac{\partial C_v}{\partial t} = C_v \frac{\Omega}{k_B T} \frac{\partial \sigma}{\partial t} \quad (63)$$

By substituting (63) and (58) into (61), we have

$$\left( \frac{BC_v \Omega}{C_L k_B T} + 1 \right) \frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x} \left[ \kappa \left( \frac{\partial \sigma}{\partial x} - \frac{eZ\rho j}{\Omega} - \frac{Q}{\Omega T} \frac{\partial T}{\partial x} \right) \right] \quad (64)$$

where  $\kappa = \frac{BD_v C_v \Omega}{C_L k_B T}$  is diffusivity of the stress. Based on the value of EM parameters, Korhonen *et al* found  $\frac{C_v \Omega}{k_B T} / \frac{C_L}{B} \ll 1$ , which can be neglected, and  $C_L = C_v = 1/\Omega$  [40]. Due to the above approximation, (64) can be rewritten as [33]

$$\frac{\partial \sigma}{\partial t} = \frac{\partial}{\partial x} \left[ \kappa(x) \left( \frac{\partial \sigma}{\partial x} - \frac{eZ\rho j}{\Omega} - \frac{Q}{\Omega T} \frac{\partial T}{\partial x} \right) \right] \quad (65)$$

#### APPENDIX B

As a spatial distribution equation, (32) can be transformed into

$$\begin{aligned} (x + \frac{b_{ij}}{k_{ij}})^2 \frac{\partial^2 \phi_{ij}(x)}{\partial x^2} + (x + \frac{b_{ij}}{k_{ij}}) \frac{\partial \phi_{ij}(x)}{\partial x} \\ + \frac{\lambda_m^2}{k_{ij}} (x + \frac{b_{ij}}{k_{ij}}) \phi_{ij}(x) = 0 \end{aligned} \quad (66)$$

It can be transformed into a general form as

$$u^2 \frac{\partial^2 y}{\partial u^2} + (1 - 2a)u \frac{\partial y}{\partial u} + [b^2 c^2 u^{2c} + (a^2 - c^2 p^2)]y = 0 \quad (67)$$

$p \geq 0, b > 0$

by setting the following transformation relations:

$$\begin{cases} u = x + \frac{b_{ij}}{k_{ij}} \\ a = 0, c = 1/2 \\ b = \frac{2\lambda}{\sqrt{k_{ij}}} \\ p = 0 \end{cases} \quad (68)$$

Then, replacing  $y(u)$  with  $u^a z$ , (67) can be written as

$$u^2 \frac{\partial^2 z}{\partial u^2} + u \frac{\partial z}{\partial u} + (b^2 c^2 u^{2c} - c^2 p^2)z = 0 \quad (69)$$

Furthermore, using  $t = u^c$ , we obtain the Bessel equation

$$t^2 \frac{\partial^2 z}{\partial t^2} + t \frac{\partial z}{\partial t} + (b^2 t^2 - p^2)z = 0 \quad (70)$$

Therefore, the analytical solution of (67) is given by [46]

$$y(u) = u^a [AJ_p(bu^c) + BY_p(bu^c)] \quad (71)$$

where  $A$  and  $B$  are the two coefficients,  $J_p(\cdot)$  and  $Y_p(\cdot)$  are the first and second kind Bessel functions, respectively.

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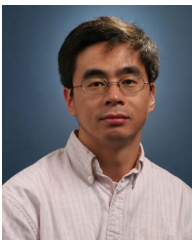
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