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A path planning algorithm for human evacuations with an environment dependent motion

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Abstract— **This work proposes an algorithm for the optimal guidance of evacuees in indoor environments. The proposed work examines the path-dependent accumulation of hazardous substances inhaled, such as carbon monoxide, and provides an optimal path that ensures that evacuees can survive to emergency exits by guaranteeing the accumulated inhalation of carbon monoxide is below life-threatening levels. The spatiotemporal variation of the hazardous and toxic field, as described by either Poisson or advection-diffusion partial differential equation, is used to compute the accumulated amount of carbon monoxide inhaled. The accumulated amount is given in terms of the line integral of the hazardous field along the escape paths. The effects of carbon monoxide on evacuee speed are considered. To ensure a path with lower carbon monoxide inhalation levels as well as reduced flight times, level sets are used to generate the set of angles for each path. This is done using a coefficient that changes the direction of motion based on the instantaneous carbon monoxide concentration. This coefficient varies based on the level set with a specific critical level set declared such that it is never crossed. The optimization scheme provides the optimal path among all admissible paths having the smallest value below the tolerance levels of the substance. Simulation studies considering both spatially and spatiotemporally varying functions of carbon monoxide in an indoor environment representing a floor of an office building, are provided to further demonstrate evacuation policies in contaminated indoor environments.**

I. INTRODUCTION

In indoor evacuations of highly populated buildings, the number of fatalities can increase due to the chaotic nature of the escape procedures. One of the reasons for this, is the evacuees' ignorance about the location of the threat field and their inability to either locate one of the escape exits or travel a safe path towards one. Research has shown, see [1] and references therein, that evacuees have trouble locating the exit signs amidst the smoke and fire with just 8% of them seeing the signs. Furthermore people in danger are susceptible to herd behavior, often following others even though the leader does not know the safest path. Finally due to disorientation and the effects that smoke, and its various harmful constituents, can have on evacuees such as dizziness, additional injury dangers arise in the form of stampede. This is due to psychological factors and flight instincts, [2], [3].

Carbon monoxide is one of the leading causes of death in a fire. When inhaled, carbon monoxide molecules bind with hemoglobin forming carboxyhemoglobin which has a lower oxygen-carrying capacity, [4]. Being colorless and odorless makes this gas even more dangerous as victims do not realize they are being poisoned. Even at low concentrations such as 100ppm exposure to carbon monoxide can be deadly over an extended period of time. During a fire, however, carbon monoxide levels reach as high as 4000ppm which can lead to a loss of consciousness in less than five minutes, [5]. This time is dramatically decreased as evacuees breathe faster due to both physical activity and the enhanced presence of carbon dioxide (CO_2) from the fire, [6].

To model the ability of an evacuee to escape from the building several physiological aspects can be employed such as breathing capabilities, physical conditioning as well as the dynamics of human motion. The acceleration and top speed of a person depend on several aspects of their motion such energy and impulse applied by the person on the ground, [7], [8]. The higher the impulse and the peak force, the faster the person can accelerate and the higher the top speed achieved, [9], [10]. We do acknowledge that for a completely accurate model these factors must be considered. However for the purposes of our research here we can model human motion using the kinematic model instead. This is because the acceleration time is negligible compared to the evacuation time and the top speed is constant. For a multi agent evacuation model with interaction between evacuees other models can be considered such as either a collision particle model or a macroscopic model, [11], [12].

Further modelling requirements include that of carbon monoxide breathing. It is important to note that no research has been conducted on carbon monoxide breathing due to its poisonous nature. What is known is that carbon monoxide poisoning is more acute when a person breathes faster. Due to the panic situation as well as the physical effort to escape, *hyperventilation* can occur, [13]. Hyperventilation can cause the breathing rate of a person to rise as high as 40 times per minute. Hyperventilation occurs when the body detects high levels of carbon monoxide and not low levels of oxygen, [14]. Hyperventilation is by itself a dangerous condition that can cause *hypocapnia* and *respiratory alkalosis*. These can in turn cause dizziness, fainting and seizures, [15]. As mentioned above, during a fire evacuation, evacuee breathing rate increases due to the presence of higher CO_2 levels and this causes a faster CO poisoning.

Due to panic and fear, people are generally proven inefficient in evacuation scenarios. This is because they often run at a speed higher than the optimal evacuation speed creating a bottleneck effect, [16]. Injuries resulting from this bottleneck effect are attributed to either stampede or being crushed against a wall due to panic, [2]. This is despite the fact that exit options are available and well indicated as people will

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follow the herd. Several works of research on dynamic crowd emergency exit selection have been performed but they do not take into account physical models for the hazardous field not do they explore detailed path planning, [17], [18].

For path planning two methods are considered. Firstly the Rapidly-Exploring Random Trees (RRT) that uses a branch system to create a path from a known origin to a known goal, [19], [20]. This method uses a known origin and creates a branching path of known distance in a random direction. The process is repeated until the path reaches an obstacle or the goal set. Certain variations have been suggested and implemented such as speeding up the process by running multiple trees simultaneously with one starting from the origin and another starting from the goal, [21]. This method is better suited for a known environment due to the time limitations of an evacuation scenario. The second method is a level set based approach where the constraint is not the environment or any physical obstacles but rather the hazardous field so that the total carbon monoxide inhaled is not lethal. The path that will be generated will follow a specific level set of the carbon monoxide field. It is important for the path to remain as short as possible so as to not increase the evacuee's exposure to the carbon monoxide. Therefore the length of the level set path should be similar to that of the constant angle (straight line) path, [22], [23].

The goal of our work here is to ensure that evacuees are presented with an evacuation path that increases their survivability by reducing their exposure to the carbon monoxide and the rest of the harmful substances in smoke. With this, any decision making is not needed anymore and therefore the effect of panic is also reduced. Furthermore any effects due to the exposure to the carbon monoxide such as disorientation and dizziness will be less likely to affect the evacuees as the path generated will limit their exposure. To ensure the above, our research examines the effects on the evacuee for each available path due to the accumulated as well as the instantaneous CO concentration and limits them by generating a path with less accumulated concentration. To ensure that the instantaneous concentration does not exceed deadlier levels, a level set method will be applied. Both stationary and time-varying spatial fields of CO will be considered. The algorithm makes certain physiological modelling assumptions to calculate the CO inhaled and determine which path is the safest. At this stage a single evacuee is evaluated but the algorithm will be expanded to a multi-agent problem with collisions and interaction.

To physically achieve this, the authors envision a smart phone application to be created at a later stage where the user receives a notification and an up-to-date path generation that will help them escape the building. To gather information regarding the threat field, sensors are to be installed inside the building. Using this information the field can be reconstructed and the concentrations at each location be estimated.

Building on and extending the earlier work [24], the contributions of the paper are:

1) Model the CO-dependent velocity of an evacuee.

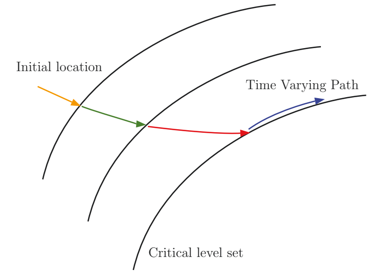

Fig. 1. Depiction of the proposed weighted level set-based escape path.

2) Express the accumulated amount inhaled up to current time in terms of the line integral of the CO concentration field along the escape path.
3) Propose a weighted level set-based path planning method that depends on the initial value of CO, as well as the instantaneous and the accumulated amounts inhaled by the evacuee as shown in Figure 1.

The remainder of the paper is as follows. The mathematical framework for the process describing the spatial distribution of carbon monoxide concentration along with the equations of motions of humans during evacuation which are based on kinematic models are summarized in Section II. The main results on evacuation policies that are dictated by the accumulated amount of carbon monoxide inhalation, posed as an optimization of environment-dependent path planning are presented in Section III. Extensive simulation studies over a typical industrial building floor, such as the one found in typical office and university settings, are discussed in Section IV. Conclusions along with discussions on future work by the authors are given in Section V.

II. Mathematical Framework and Modeling

While the spatial distribution of the CO concentration field is defined over a 3D domain, we will consider a 2D distribution. The distribution is modelled by Poisson-type elliptic partial differential equations (PDEs) or advection-diffusion PDEs defined over a 3D spatial domain [25]. Assuming a single species, the steady spatial distribution over a typical indoor environment representing the spatial domain $\Omega = [0,L_x]\times[0,L_y]\times[0,L_z]$, is given via the PDE

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)c = f, \tag{1}$$

where $c(x,y,z)$ denotes the species concentration at spatial coordinates $(x,y,z)\in\Omega$ and $f(x,y)$ is/are the source(s). Appropriate conditions at the boundary $\partial\Omega = \Gamma_D \cup \Gamma_N$ must be furnished to uniquely determine the solution $c(x,y,z)$; in this case, mixed boundary conditions can be assumed. For part of the boundary Γ_D , Dirichlet boundary conditions are imposed and for the other part of the boundary Γ_N, Neumann boundary conditions are imposed. The unsteady process is governed by the advection-diffusion PDE with reaction

$$\frac{\partial c}{\partial t} = \nabla\cdot(D\nabla c) - \nabla\cdot(\upsilon c) + f, \tag{2}$$

where now the scalar function $c(t,x,y,z)$ denotes the time-varying concentration. The parameter D denotes the diffusivity, υ denotes the velocity and $f(t,x,y,z)$ denotes the source. Inhomogeneous boundary conditions can be assumed in different parts of the boundary $\partial\Omega$ as in (1).

Within an indoor environment, the vertical variation (z-direction) of the concentration can be considered over a small interval bounded by a standard deviation from the height of a person's face. Denoting z_l the average height of a shorter person (one standard deviation below the average adult height) and z_h the average height of a taller person (one standard deviation above the average adult height), we consider the slice $[z_l, z_h]$ as the interval in the vertical direction. The effective domain is now $\Omega_e = [0, L_x] \times [0, L_y] \times [z_l, z_h]$. To justify axisymmetry, we further assume that the vertical variation of the concentration c within Ω_e is smaller than the variation in the x and y directions. Therefore the concentration is justifiably given by the 2D solution to either (1) or (2) and denoted by $c(x,y)$ or $c(t,x,y)$.

The solution to (1) or (2) will be assumed known; i.e. we have full state information. While this condition may be infeasible, we nonetheless make this assumption to develop the evacuation path planning. In a subsequent study we will consider an integrated real-time estimation of the field $c(t,x,y)$ and the real-time path planning. The former will be based on techniques developed by the first author [26], [27].

As the main thrust of this paper is on the generation of paths that ensure survivability of an evacuee to the cumulative effects of CO, we assume that there is a single evacuee in an indoor environment. The equations describing the motion within the horizontal plane of Ω_e of a single evacuee in an indoor environment are based on a simple kinematic model [28] with a CO-dependent speed

$$\begin{aligned} \dot{x}(t) &= v(t)\cos(\theta), \quad x(0) = x_0, \\ \dot{y}(t) &= v(t)\sin(\theta), \quad y(0) = y_0. \end{aligned} \tag{3}$$

Equation (3) does not consider interactions due to collisions with other evacuees or obstacle avoidance, [11], [12]. However, the CO concentration will act as an obstacle that depends on the accumulated amount of CO inhaled.

The angle θ in (3) is the angle between the direction of motion and the x-axis of the reference frame. Since no obstacles are assumed, then the path planning reduces to selecting the angle θ, [24]. However, a possibility exists that no constant-angle path can be found that ensures that the accumulated CO concentration inhaled is below the threshold. This then justifies the proposed work, which allows time-varying angles $\theta(t)$ to be used for planning a viable path to safety. The proposed modification ensures that an evacuee follows a level set boundary of the CO concentration to increase the chances of survivability. Only the angle $\theta(t)$ is assumed to be the control variable. The speed $v(t)$ of an evacuee is assumed to be initially constant and given by v_0. It is assumed that the maximum speed is attained in infinitesimal time. However, it becomes time-varying (reduces) as the accumulated amount of the hazardous substance inhaled increases.

As the accumulated amount of CO inhaled along a path is essential in determining the survivability of an evacuee, one must provide a model for computing this accumulated amount. For a prescribed path $\theta(t)$ and a fixed velocity v, the kinematic equations (3) provide the coordinates of an evacuee in terms of time. To estimate the amount of CO inhaled for a particular path $\theta(t)$, we calculate the line integral of the concentration $c(t,x,y)$ over the path defined by $\theta(t)$. However this line integral does not take into account the breathing rate of an evacuee. An evacuee will not breathe continuously and therefore the line integral must be scaled accordingly. We must use the fact that the breathing rate of a human under duress is around 20 breaths per minute with an equal length of inhaling and exhaling. Using the data from Table I we also obtain information on CO poisoning, which varies from person to person. This poisoning is also affected by a person's breathing rate and whether they are performing a certain activity that increases heart rate and blood flow. These will provide a safe limit in the total CO inhalation over a given path $\theta(t)$. Following [24], a safe limit in total CO inhalation would be $25,000$ ppm if the peak CO concentration is higher than $2,000$ ppm, and $64,000$ ppm if the peak CO concentration is less. This makes the level set of $2,000$ ppm a very important one in the survivability of the evacuee. This limit also includes a safety factor of 5 as it was desired to reduce the probability of a person experiencing dizziness or nausea. Using the above information for the total CO inhaled by an evacuee, the line integral of the CO concentration over the path $\theta(t)$ travelled via (3) can be used with a scaling. Using the fact that person would only inhale on an average half of the time, the line integral will in fact be scaled by 50% and therefore, the total CO inhaled is

CO Poisoning	
Concentration	Effects
3,200 ppm	Headache, dizziness and nausea in five to ten minutes. Death in 30 minutes.
6,400 ppm	Headache and dizziness in one to two minutes. Convulsions, respiratory arrest and death in less than 20 minutes.
12,800 ppm	Unconsciousness after 2-3 breaths. Death in less than 3 minutes.

TABLE I
QUANTITATIVE EFFECTS OF CO POISONING.

$$J(\theta) = \frac{1}{2}\int_{\theta(t)} c(\mathbf{r})\,\mathrm{d}s,$$

where $\mathbf{r}(t) = (x(t), y(t))$. Using the properties of line integrals [29], the expression above simplifies to

$$J(t; t_0, \theta(t)) = \frac{1}{2}\int_{t_0}^{t} v(\tau)c(\tau, x(\tau), y(\tau))\,\mathrm{d}\tau, \tag{4}$$

where $c(t, x(t), y(t))$ denotes the CO concentration at the coordinates $(x(t), y(t))$ provided by the solution to (3).

Finally to model the effects of carbon monoxide on the evacuee's speed we set the limit where the effects are first felt. From the data in Table I, this is estimated to be close to $20,000$ ppm. This assumes lightheadedness, dizziness and breathing complications. At this threshold the velocity of the evacuee becomes dependent on CO. The model takes into consideration both lethal limits of $25,000$ and $64,000$ ppm. If the peak instantaneous concentration at any point never reaches $2,000$ ppm the velocity of the evacuee is

$$v(t) = \begin{cases} \frac{v_0}{2}(1+\frac{20,000}{J(t)}) & J(t) \in [20,000\;\; 63,800] \\ 1+\frac{419}{638}v_0 - e^{\frac{(J(t)-63,800)}{64,000-63,800}} & J(t) > 63,800 \end{cases} \quad (5)$$

where $J(t)$ is the cumulative inhalation of CO at time t. For a path where the instantaneous peak concentration reaches $2,000$ ppm the expression for the velocity is

$$v(t) = \begin{cases} \frac{v_0}{2}(1+\frac{20,000}{J(t)}) & J(t) \in [20,000\;\; 24,800] \\ 1+\frac{28}{31}v_0 - e^{\frac{(J(t)-24,800)}{25,000-24,800}} & J(t) > 24,800 \end{cases} \quad (6)$$

III. Level set-based evacuation in presence of hazardous fields in indoor environments

The optimization problem, as stated in [24], for the case of constant angle paths, is to ensure that an evacuee with initial location $(x_i(0), y_i(0))$, $i = 1,\dots,n$, can follow a recommended escape path obeying the kinematic equations (3) and at the same time to ensure that the accumulated CO inhaled, as predicted by (4), is below the allowable limit of $25,000$ ppm for a peak concentration of $2,000$ ppm or, below $64,000$ ppm for a higher peak concentration; the critical amount is

$$J_{\text{crit}} = \begin{cases} 25{,}000 \text{ ppm if } c(t,x,y) > 2,000 \text{ ppm} \\ 64{,}000 \text{ ppm if } c(t,x,y) < 2,000 \text{ ppm} \end{cases} \quad (7)$$

These conditions ensure that each evacuee is still conscious when they reach any of the escape exits. We define Θ as the set of admissible paths θ that satisfy the motion constraints. Then the optimization problem can be stated as the selection of path $\theta(t) \in \Theta$ ensuring that the CO inhaled given by (4), is below the threshold J_{crit} in (7).

Problem statement: Design the path, via the time varying angle $\theta(t) \in \Theta$, so that the CO inhaled as given by (4) is less than $25,000$. This constrained optimization is stated below.

$$\underline{\text{optimization}:} \quad \begin{cases} \underset{\theta(t)\in\Theta}{\text{maximize}} \quad J_{\text{crit}} - J(\theta) \\ \text{subject to} \quad \begin{cases} \dot{x}(t) = v(t)\cos(\theta(t)) \\ \dot{y}(t) = v(t)\sin(\theta(t)) \\ v(t) \text{ given by (5)} \end{cases} \end{cases}$$

The above optimization assumes that there is at least one value of the constant angle θ for which $J(\theta) < J_{\text{crit}}$. The set of angles θ defining the candidate paths is the admissible set of the angles and essentially dictated by the number of emergency exit doors and windows.

For the i^{th} evacuee with initial position $(x_i(0), y_i(0))$, $i = 1,\dots,n$, Θ_i defines the set of constant-angle paths connecting $(x_i(0), y_i(0))$ to each exit with coordinates (X_j, Y_j)

$$\Theta_i = \left\{\theta_{ij} : \tan(\theta_{ij}) = \frac{Y_j - y_i(0)}{X_j - x_i(0)}, \forall j = 1,\dots,N\right\}. \quad (8)$$

A. Algorithm 1: constant angle paths

The optimization algorithm considered here, is designed for an empty floor i.e. no obstacles such as cubicles, divisions, furniture, incapacitated humans, etc. This means that the path to an exit is unobstructed and for now it is a straight line. As inputs we have the origin of each evacuee (i.e. coordinates $(x_i(0), y_i(0))$, $i = 1,\dots,n$), the exit coordinates (X_j, Y_j), $j = 1,\dots,N$, and the concentration of the carbon monoxide. Additionally, for the speed of a person, we use the nominal initial speed $v_0 = 7ms^{-1}$ which is the average top speed of a human. The optimization algorithm predicts the total accumulated carbon monoxide inhaled for each admissible path/exit and selects the smallest one that result in total accumulated inhalation that is below the allowable threshold necessary for survival. As the breathing rate of humans is on a timescale of seconds, the sampling rate corresponding to the amount of carbon monoxide inhaled is calculated every second.

Algorithm 1 Evacuation based on constant-angle paths

1: Use physiological data to estimate the maximum speed for each evacuee
2: Generate admissible path set, defined by (8) for each person with coordinates $(x_i(0), y_i(0))$, $i = 1,\dots,n$
3: For each i, $i = 1,\dots,n$, predict the accumulated CO inhaled, given by (4), over all admissible paths defined by the angles $\theta_{ij} \in \Theta_i^{adm}$
4: Generate the subset of Θ_i for which $J(\theta_{ij}) < 25,000$, i.e.

$$\Theta_i^{opt} = \{\theta_{ij} \in \Theta_i : J(\theta_{ij}) < 25,000\} \quad (9)$$

5: Optimal path θ_i^{opt} is chosen as the one in Θ_i^{opt} with the smallest level below $25,000$ ppm, i.e.

$$\theta_i^{opt} = \arg \min_{\theta\in\Theta_i^{opt}} J(\theta). \quad (10)$$

Algorithm 1 may be infeasible necessitating the time-varying angle escape path.

B. Algorithm 2: time-varying angle paths

In order to minimize the exposure to carbon monoxide and its effect on the evacuee's ability to escape the building we consider different level sets. At each one we find a motion change coefficient. These values are based on the initial instantaneous value of carbon monoxide. A higher number indicates a path that resembles a straight line from the current location to the destination point while a lower number indicates a path following the level set. Hence the level sets are not followed but rather affect the motion. The higher the value of the level set curve the lower the value of the coefficient. This means the path selected is such that both the escape time as well as the CO inhalation are minimized.

The level set equation is defined as

$$L_m = \{(x,y) \in \Omega : c(t,x,y) = m\} \quad (11)$$

where m is each separate level set. In order to set up the algorithm, we have to define the following parameters: K_m, ψ' and θ'. K_m is the weight coefficient for each level set L_m of interest. This is dependent on the initial instantaneous concentration of carbon monoxide; ψ' is the angle between the gradient of the field and the x-axis and finally θ' is the angle between the x-axis and the straight line from the current location to the exit. This is demonstrated in Figure 2.

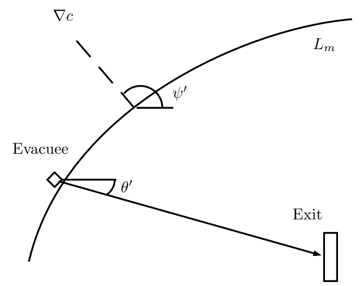

Fig. 2. Depiction of parameters used in Algorithm 1.

Algorithm 2 Evacuation based on a varying-angle paths

1. Calculate the initial concentration of CO
2. Generate the set of weight coefficients K_m
3. Find the gradient ∇c of the CO field at the current position and its angle of direction ψ'.
4. Find the new vector of motion in the x and y directions respectively as described by $\cos(\psi') + K_m \cos(\theta')$ and $\sin(\psi') + K_m \sin(\theta')$.
5. The angle of the new direction vector with respect to the x-axis is calculated and it is the new θ

IV. Numerical results

Two cases are investigated over the rectangular domain $\Omega = [0, 100] \times [0, 30]$m. We will look at a steady field and then an unsteady field. Firstly though we compare the time-varying angle algorithm to the constant angle path (straight line path) algorithm. In the case presented in Figure 3, the constant angle path (straight line path) fails to provide a viable option for evacuation, i.e. $\Theta^{opt} = \emptyset$. The evacuee will lose consciousness around the spatial coordinates $(60, 12)$ as indicated by the black cross. The dotted line depicts the would-be unimpeded path had the CO effects not taken into account. This highlights the necessity for an algorithm such as the one we describe in the previous section. An important factor to the problem is the threat field-dependent velocity, as given by (5), meaning that a constant angle path (straight line path) is more time consuming as it results in larger reductions in velocity.

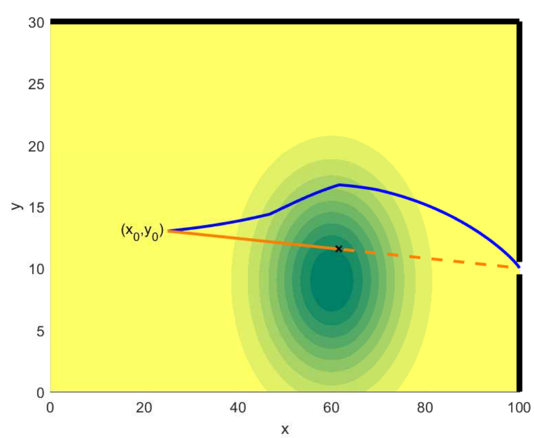

Fig. 3. Comparison between constant angle θ escape path and proposed time-varying angle $\theta(t)$ escape path algorithms; x, y distances in meters.

A. Case 1: Steady CO field

The CO field is the solution to (1) and is given by

$$c(x,y) = 4100 e^{-\left(\frac{(x-\mu_{x0})^2}{2\sigma_x^2} + \frac{(y-\mu_{y0})^2}{2\sigma_y^2}\right)}$$

where the mean is $(\mu_{x0}, \mu_{y0}) = (60, 9)$ and the standard deviations are $\sigma_x = 10$, $\sigma_y = \sqrt{30}$.

Three escape exits were selected at coordinates $(X_1, Y_1) = (100, 10)$, $(X_2, Y_2) = (100, 30)$ and $(X_3, Y_3) = (60, 30)$. For an evacuee initially at $(x_1(0), y_1(0)) = (30, 5)$, we use Algorithm 2 to generate the Θ set. Figure 4 depicts the results. The three level sets selected in this simulation are 1,050, 1,500 and 1,950 ppm therefore four different coefficients are selected, one for concentration lower than 1,050 ppm and one more for concentrations higher than each of the level set. During this simulation, the peak value of all paths never crossed 2,000 ppm meaning that the evacuee would not lose consciousness before 64,000 ppm. The highest total CO was inhaled on the first exit path for a total of 29,734 ppm. This means that all three exits are viable with a significant improvement over the straight line path which had the evacuee breathing a total of 43,511 ppm. Meanwhile the time taken for each path is 11.1, 10.5 and 5.54 seconds respectively making Exit #3 the optimal option.

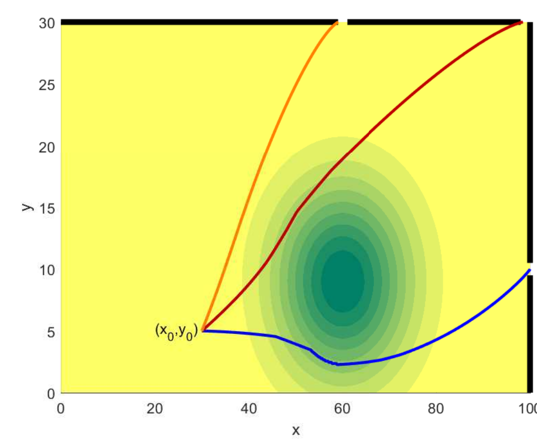

Fig. 4. Case1: time-varying angle $\theta(t)$ escape paths in a stationary concentration field $c(x,y)$; x, y distances in meters.

B. Case 2: Unsteady CO field

The CO field is time-varying with the time variation expressed in terms of time varying means and given

$$c(t,x,y) = 4100 e^{-\left(\frac{(x-\mu_x(t))^2}{2\sigma_x^2} + \frac{(y-\mu_y(t))^2}{2\sigma_y^2}\right)}$$

where $\mu_x(t) = 5\cos(9t - \frac{\pi}{2}) + \mu_{x0}$, $\mu_y(t) = 5\sin(9t - \frac{\pi}{2}) + \mu_{y0}$. This equation simulates the time varying center of the CO field. This can be due to a variety of reasons such as a change in the material burned in the fire. The exits are the same as Case 1. The CO source is initially centered at $(\mu_{x0}, \mu_{y0}) = (60, 9)$. The motion of the mean $(\mu_x(t), \mu_y(t))$ is depicted in Figure 5 by the black line.

The evacuee starts from the same initial coordinates as before. As seen in Figure 5 the algorithm (Algorithm 2) ensures the survivability of the evacuee for all three paths with none of the paths accumulating over 25,000 ppm, the highest being the path towards exit number 1 at 24,960 ppm. The time to evacuate is also improved over the constant angle path since the effect of the carbon monoxide on the evacuee's velocity is minimal in comparison dropping to 90% of the starting velocity v_0. The flight times to reach each escape exit are 10.5, 10.7 and 5.55 respectively, making Exit #3 the optimal escape exit once more.

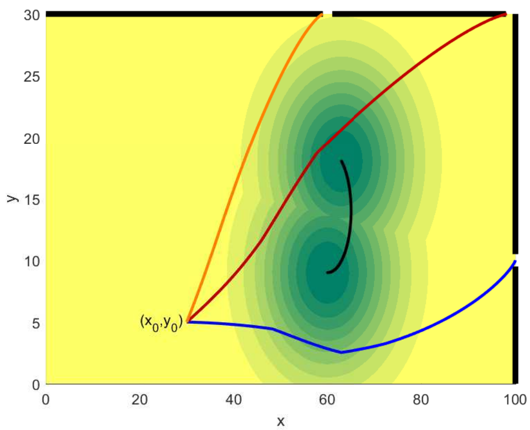

Fig. 5. Case 2: time-varying angle $\theta(t)$ escape paths in an unsteady concentration field $c(t,x,y)$; x,y distances in meters.

V. Conclusions

A model for CO-dependent evacuee velocity that takes into account both the instantaneous concentration as well as the accumulated inhaled amount, has been proposed. This showed that a shorter path might not always result to a faster evacuation. To better optimize the path planning algorithm, a weighted level set based approach was used where the angles of the path varied depending on the concentration levels. Usage of this algorithm prevented the evacuee's speed from decreasing significantly. The algorithm also ensured that the evacuee had a viable path towards the exit. Future work will involve an unknown field meaning limited access to the CO concentrations.

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References

[1] M. J. Ouellette, "Visibility of exit signs through smoke," *Progressive Architecture*, pp. 39–42, 7 1993.

[2] D. Helbing, I. Farkas, and T. Vicsek, "Simulating dynamical features of escape panic," *Nature*, vol. 407, pp. 487–490, 2000.

[3] M. Kobes, I. Helsloot, B. de Vries, and J. G. Post, "Building safety and human behaviour in fire: A literature review," *Fire Safety Journal*, vol. 45, no. 1, pp. 1 – 11, 2010.

[4] P. M. Winter and J. N. Miller, "Carbon monoxide poisoning," *JAMA*, vol. 236(13), pp. 1502–1504, 1976.

[5] R. J. Barnard and J. S. Weber, "Carbon monoxide: A hazard to fire fighters," *Archives of Environmental Health: An International Journal*, vol. 34(4), pp. 255–257, 1979.

[6] IAFF. Carbon monoxide and fire fighters. [Online]. Available: http://www.iaff.org/hs/PDF/CarbMnxde.pdf

[7] S. W. Lipfert, M. GÃijnther, D. Renjewski, S. Grimmer, and A. Seyfarth, "A model-experiment comparison of system dynamics for human walking and running," *J. of Theoretical Biology*, vol. 292, pp. 11 – 17, 2012.

[8] D. J. Farris and G. S. Sawicki, "The mechanics and energetics of human walking and running: a joint level perspective," *J. of The Royal Society Interface*, 2011.

[9] K. P. Clark, L. J. Ryan, and P. G. Weyand, "Foot speed, foot-strike and footwear: linking gait mechanics and running ground reaction forces," *J. of Experimental Biology*, 2014. [Online]. Available: http://jeb.biologists.org/content/early/2014/04/01/jeb.099523

[10] T. A. McMahon and G. C. Cheng, "The mechanics of running: How does stiffness couple with speed?" *J. of Biomechanics*, vol. 23, pp. 65 – 78, 1990.

[11] P. Kachroo, S. Al-nasur, S. Wadoo, and A. Shende, *Pedestrian Dynamics: Feedback Control of Crowd Evacuation*. Berlin-Heidelberg: Springer-Verlag, 2008.

[12] E. Cristiani, B. Piccoli, and A. Tosin, *Multiscale modeling of pedestrian dynamics*. Springer, Cham, 2014, vol. 12.

[13] M. Augustynek, D. Friedmannova, and M. Cmielova, "Measuring of dependency between heart rate, respiratory rate and the human movement," *IFAC Proc. Volumes*, vol. 46(28), pp. 292–297, 2013.

[14] R. D. Dripps and J. H. Comroe Jr., "The effect of the inhalation of high and low oxygen concentrations on respiration, pulse rate, ballistocardiogram and arterial oxygen saturation of normal individuals," *J. of Physiology*, vol. 149(2), pp. 277–291, 5 1947.

[15] R. P. Byrd. Respiratory alkalosis. [Online]. Available: https://emedicine.medscape.com/article/301680-overview

[16] V. Dossetti, S. Bouzat, and M. Kuperman, "Behavioral effects in room evacuation models," *Physica A: Statistical Mechanics and its Applications*, vol. 479, pp. 193 – 202, 2017.

[17] M. N. Akmal Khalid and U. K. Yusof, "Dynamic crowd evacuation approach for the emergency route planning problem: Application to case studies," *Safety Science*, vol. 102, pp. 263 – 274, 2018.

[18] M. M. Shuaib, "Incorporating intelligence for typical evacuation under the threat of fire spreading," *Safety Science*, vol. 106, pp. 1 – 9, 2018.

[19] S. M. LaValle, *Planning algorithms*. Cambridge University Press, Cambridge, 2006.

[20] J. Bruce and M. M. Veloso, "Real-time randomized path planning for robot navigation," in *RoboCup 2002: Robot Soccer World Cup VI*, G. A. Kaminka, P. U. Lima, and R. Rojas, Eds. Berlin, Heidelberg: Springer, 2003, pp. 288–295.

[21] A. Abbadi and R. Matousek, *Path planning implementation using Matlab*, 11 2014.

[22] T. Lolla, M. P. Ueckermann, K. YiÄ§it, P. J. Haley, and P. F. J. Lermusiaux, "Path planning in time dependent flow fields using level set methods," in *IEEE International Conference on Robotics and Automation*, May 2012, pp. 166–173.

[23] Y. A. Kapitanyuk, A. V. Proskurnikov, and M. Cao, "A guiding vector-field algorithm for path-following control of nonholonomic mobile robots," *IEEE Trans. on Control Systems Technology*, vol. 26(4), pp. 1372–1385, July 2018.

[24] M. A. Demetriou and M. Kontopyrgos, "A level-set based approach for the path planning of human evacuations in contaminated indoor environments," in *Proc. of the European Control Conference*, Naples, Italy, June 25-28 2019.

[25] R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, 2nd ed. New York, NY: John Wiley & Sons, Inc., 2010.

[26] M. Demetriou, "Guidance of mobile actuator-plus-sensor networks for improved control and estimation of distributed parameter systems," *IEEE Trans. on Automatic Control*, vol. 55(7), pp. 1570–1584, 2010.

[27] M. A. Demetriou, N. A. Gatsonis, and J. R. Court, "Coupled controls-computational fluids approach for the estimation of the concentration from a moving gaseous source in a 2-D domain with a Lyapunov-guided sensing aerial vehicle," *IEEE Trans. on Control Systems Technology*, vol. 22(3), pp. 853–867, May 2014.

[28] R. Siegwart, I. R. Nourbakhsh, and D. Scaramuzza, *Introduction to Autonomous Mobile Robots*, 2nd ed. The MIT Press, 2011.

[29] W. Kaplan, *Advanced Calculus*, 3rd ed. Addison-Wesley Publishing Company, Reading, MA, 1984.