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Trajectory Generation on $\mathrm{SE}(3)$ for an Underactuated Vehicle with Pointing Direction Constraints

Mani H. Dhullipalla, Reza Hamrah, Rakesh R. Warier and Amit K. Sanyal

Abstract— **This paper addresses the problem of generating a position trajectory with pointing direction constraints at given waypoints for underactuated unmanned vehicles. The problem is initially posed on the configuration space $\mathbb{R}^3 \times \mathbb{S}^2$ and thereafter, upon suitable modifications, is re-posed as a problem on the Lie group $\mathrm{SE}(3)$. This is done by determining a vector orthogonal to the pointing direction and using it as the vehicle's thrust direction. This translates to converting reduced attitude constraints to full attitude constraints at the waypoints. For the position trajectory, in addition to position constraints, this modification adds acceleration constraints at the waypoints. For real-time implementation with low computational expenses, a linear-quadratic regulator (LQR) approach is adopted to determine the position trajectory with smoothness upto the fourth time derivative of position (snap). For the attitude trajectory, the thrust direction extracted from the position trajectory is used to first propagate the attitude to the subsequent waypoint and then correct it over time to achieve the desired attitude at this waypoint. Finally, numerical simulation results are presented to validate the trajectory generation scheme.**

I. INTRODUCTION

This paper investigates the problem of generating a trajectory for an underactuated vehicle to maneuver along a given set of waypoints while simultaneously pointing towards predetermined directions at those waypoints. The underactuated vehicle is modeled as a rigid body with three fully actuated rotational degrees of freedom (DOF) and one fully actuated translational DOF. This actuation model includes a wide range of unmanned vehicles like fixed-wing and rotorcraft unmanned aerial vehicles (UAVs), unmanned underwater vehicles (UUVs), and spacecraft. A rigid body with this type of actuation is controllable globally over its state space as shown in [1]–[3]. To the best of our knowledge, trajectory generation, tracking and pointing control with simultaneous pointing direction constraints at waypoints has not been done and reported for underactuated multi-rotor UAVs in prior literature.

Autonomous unmanned vehicles are of interest in applications that reduce human effort or where human piloting is infeasible. These applications include, but are not limited to, precision agriculture, structural inspection, surveillance, environmental explorations, and photography. The vehicle's operation across a diverse class of applications necessitates safe and reliable guidance, navigation and control schemes. This aspect is particularly important for operations beyond visual-line-of-sight (BVLOS). Particular to operations concerning applications like surveillance [4], agriculture [5], drone photography or explorations it might be necessary for the vehicle to point an imager at objects of interest while maneuvering itself around the environment, and going through regions of interest.

In this paper, the authors embed the problem of trajectory generation on $\mathbb{R}^3 \times \mathbb{S}^2$ to that on $\mathrm{SE}(3)$, the Lie group associated with rigid body motion. This is done by constructing a vector orthogonal to the pointing direction at the waypoint and using it as a thrust vector. Trajectories on $\mathrm{SE}(3)$ have been previously addressed in [6]–[8]. For the position trajectory, a nominal thrust magnitude is assumed to add an acceleration constraint to each waypoint. This step ties the position trajectory with the attitude trajectory through the thrust vector.

There are multiple ways of generating position trajectories that satisfy the constraints of position and acceleration at the waypoints. An integrated guidance and control scheme for finite-time position and pointing direction tracking of a class of underactuated vehicles is provided in [9]. However, the pointing direction trajectories are assumed to be given. Authors in [10] address problems involving constraints on thrust magnitude and direction using interior point methods of convex optimization. Authors in [11], [12] provide polynomial-based trajectory schemes in order to address these constraints at the waypoints. However, constrained trajectory generation techniques in [11] demonstrate low success rate and don't guarantee stability even though these constraints are met for a small number of waypoints. Any of these approaches can be used to generate trajectories that satisfy the position and acceleration constraints at the waypoints. In this work, we adopt linear-quadratic regulator (LQR) approach and treat the constraints at the waypoints to be soft. This leads to a relaxation on satisfying the waypoint constraints exactly but provides a stable trajectory generation technique that can handle large numbers of position waypoints to within desirable tolerance. Additionally, often the imagers (or sensors) have a field of view that can accommodate the errors in position and pointing direction at the waypoints. Once the position trajectory is determined, the vehicle's translational dynamics determines the time trajectory for the thrust vector. Using this thrust vector direction, the attitude trajectory is then constructed.

Trajectory generation on the Lie group of rotations, $\mathrm{SO}(3)$, for the attitude have been treated in [13]–[15]. These approaches use techniques of optimal control on the Lie group generating finite-time and infinite-time horizon trajectories. [16], [17] generate trajectories using splines on $\mathrm{SO}(3)$. In this work, the algorithm for attitude trajectory relies on the known

Dept. of Mechanical and Aerospace Engineering, Syracuse University, Syracuse, NY 13244, USA mdhullip,rhamrah,rwarier,aksanyal@syr.edu

trajectory of the thrust vector. Depending on how the position and acceleration constraints are treated, the desired attitude at the waypoints might have to be corrected. Using the thrust vector and its derivative, the attitude is propagated forward in time to the next waypoint. The difference between the desired and propagated attitudes is compensated over time by suitably rotating about the thrust vector. We adopt a particular choice for this compensation, and our numerical simulations validate this algorithm for attitude trajectory generation. It is important to emphasize that the attitude trajectory can be determined with the availability of time trajectory for thrust direction and does not depend on the kind of position trajectory adopted.

This paper is organized as follows: Section II provides a brief introduction to the terminology and establishes preliminary background necessary. The problem of trajectory generation is posed and recast in III. Further, this section presents an algorithm for position trajectory in III-A and utilizing quantities extracted from this, it details an algorithm for the attitude trajectory generation in III-B. Section IV presents numerical results to validate the algorithm discussed. Concluding remarks and future related research directions are provided in V.

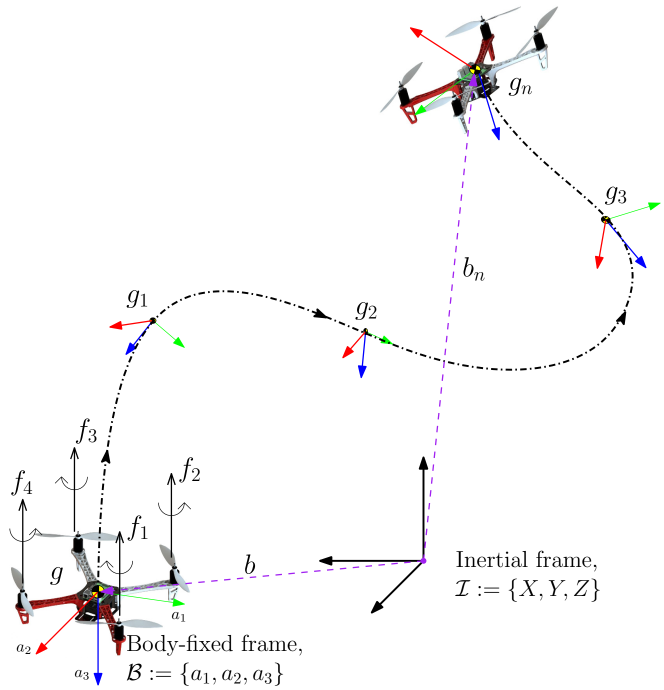

Fig. 1: Trajectory of a UAV between initial and final configurations on SE(3).

II. MATHEMATICAL PRELIMINARIES

The configuration of an unmanned vehicle modeled as a rigid body is given by its position and orientation, which are together referred to as its pose. To define pose of a vehicle, a coordinate frame $\mathcal{B}$ is attached to its body and another coordinate frame $\mathcal{I}$ is fixed in space and takes the role of an inertial coordinate frame. Let $b \in \mathbb{R}^3$ denote the position vector of the origin of frame $\mathcal{B}$ with respect to frame $\mathcal{I}$, expressed in frame $\mathcal{I}$. Let $R \in \mathrm{SO}(3)$ represent the orientation, defined as the rotation matrix from frame $\mathcal{B}$ to frame $\mathcal{I}$. $\mathrm{SO}(3)$, the special orthogonal group in three dimensions, is the group of rotations in Euclidean space $\mathbb{R}^3$. The pose of the vehicle can be represented in matrix form as follows:

$$g = \begin{bmatrix} R & b \\ 0 & 1 \end{bmatrix} \in \mathrm{SE}(3), \tag{1}$$

where $\mathrm{SE}(3) \in \mathbb{R}^{4\times 4}$, the special Euclidean group in three dimensions, is the six-dimensional Lie group of rigid body motions (translational and rotational) that is obtained as the semi-direct product of $\mathbb{R}^3$ with $\mathrm{SO}(3)$. A conceptual diagram depicting a trajectory on $\mathrm{SE}(3)$ is given in Fig. 1.

The vehicle satisfies the kinematics relation

$$\dot{b} = v, \tag{2}$$
$$\dot{R} = R\Omega^\times, \tag{3}$$

where v is the vehicle's translational velocity expressed in frame $\mathcal{I}$, Ω is the vehicle's angular velocity expressed in frame $\mathcal{B}$ and $(\cdot)^\times : \mathbb{R}^3 \to \mathfrak{so}(3) \subset \mathbb{R}^{3\times 3}$ is the skew-symmetric cross product operator that gives the vector space isomorphism between $\mathbb{R}^3$ and $\mathfrak{so}(3)$, the Lie algebra of the Lie group $\mathrm{SO}(3)$. The dynamics of the vehicle is given by:

$$m\dot{v} = fRe_3 - mge_3, \tag{4}$$
$$J\dot{\Omega} = J\Omega \times \Omega + \tau, \tag{5}$$

where m is the mass, $e_3 = [0\ 0\ 1]^{\mathrm{T}}$, Re_3 is the direction of thrust, f is the thrust magnitude, J is the moment of inertia and τ is the external torque applied to the vehicle [18].

III. TRAJECTORY GENERATION

Let a set of position waypoints in $\mathbb{R}^3$ be identified by an index set $W = \{1, 2, \ldots, N\}$. At each of these waypoints, position vector b_i and pointing direction s_i, expressed in frame $\mathcal{I}$, are provided i.e.

$$(b_i, s_i) \in \mathbb{R}^3 \times \mathbb{S}^2 \ \forall i \in W.$$

Let t_i be the time taken from the start to reach the i^{th} waypoint. Therefore the set of waypoints are arranged in the increasing order of time, i.e, if $i > j$ then $t_i > t_j \ \forall i, j \in W$. As stated here, the problem of trajectory generation is on $\mathbb{R}^3 \times \mathbb{S}^2$, however it can be posed as a problem of trajectory generation on $\mathrm{SE}(3)$ by defining and utilizing a vector orthogonal to s_i.

Assume that the sensor does not have to point vertically at any waypoint i.e. $s_i \neq \pm e_3 \ \forall i \in W$. Using this, define a unit vector q_i as

$$q_i = \frac{e_3 - (e_3^{\mathrm{T}} s_i) s_i}{\| e_3 - (e_3^{\mathrm{T}} s_i) s_i \|} \tag{6}$$

such that $q_i \perp s_i$. The two orthonormal vectors s_i and q_i at the i^{th} waypoint describe the desired attitude R_i

$$R_i = \begin{bmatrix} s_i \times q_i & s_i & q_i \end{bmatrix}. \tag{7}$$

The waypoints are now characterized by the desired pose g_i given by

$$g_i = \begin{bmatrix} R_i & b_i \\ 0 & 1 \end{bmatrix} \in \mathrm{SE}(3)\ \forall i \in W. \tag{8}$$

The objective of this paper is to provide a framework to generate a trajectory in SE(3) that addresses these pose constraints.

A. Position Trajectory

To generate a position trajectory, a linear-quadratic regulator (LQR) approach is presented where the constraints at the waypoints are considered to be soft. The system is described by the state $x(t)$ which is the concatenation of position, velocity, acceleration, jerk and snap of the vehicle at time t i.e.

$$x(t) = \begin{bmatrix} b(t) & \dot{b}(t) & \ddot{b}(t) & \dddot{b}(t) & \ddddot{b}(t) \end{bmatrix}^{\mathrm{T}}. \tag{9}$$

Using position and four of its derivatives in the state vector $x(t)$ ensures that the control torque evaluated using (5) is atleast C^4 continuous. Because the direction of thrust at the desired waypoints is pre-determined by the attitude R_i, the output $y(t)$ is constructed so that the desired output y_i at the waypoints is given by

$$y_i = \begin{bmatrix} b_i \\ a_i \end{bmatrix}.$$

Here a_i is the acceleration of the vehicle at the waypoint given by

$$a_i = \frac{f}{m} q_i - g e_3 \tag{10}$$

assuming a nominal thrust magnitude $f \in \mathbb{R}$ where q_i is given by (6).

The objective function for the position trajectory that is to be minimized is then given by

$$\begin{aligned} J = &\sum_{i=1}^{N} (y(t_i) - y_i)^{\mathrm{T}} S (y(t_i) - y_i) \\ &+ \int_{t_0}^{t_N} \frac{1}{2}\{x^{\mathrm{T}} Q x + u^{\mathrm{T}} R u\} dt, \end{aligned} \tag{11}$$

subject to the constraint equations

$$\dot{x} = Ax + Bu, \tag{12}$$
$$y = Cx, \tag{13}$$

where

$$A = \begin{bmatrix} 0_{12\times3} & I_{12\times12} \\ 0_{3\times3} & 0_{3\times12} \end{bmatrix},\ B = \begin{bmatrix} 0_{12\times3} \\ I_{3\times3} \end{bmatrix},$$

$$C = \begin{bmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times6} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} & 0_{3\times6} \end{bmatrix},$$

$I_{n\times n}$ is the identity matrix of dimension n, t_0 is the starting time and the control input $u = b^{(5)}(t)$ is the fifth derivative of position vector called crackle. The augmented objective function J' is

$$\begin{aligned} J' = &\sum_{i=1}^{N} (y(t_i) - y_i)^{\mathrm{T}} S (y(t_i) - y_i) \\ &+ \int_{t_0}^{t_N} \{\frac{1}{2} x^{\mathrm{T}} Q x + \frac{1}{2} u^{\mathrm{T}} R u + \lambda^{\mathrm{T}} (Ax + Bu - \dot{x})\} dt, \end{aligned} \tag{14}$$

where $\lambda \in \mathbb{R}^{15}$ is the Lagrange multiplier or co-state that incorporates the constraint (12) to the objective in (11). Let the Hamiltonian $\mathcal{H}$ be defined as

$$\mathcal{H} = \frac{1}{2} x^{\mathrm{T}} Q x + \frac{1}{2} u^{\mathrm{T}} R u + \lambda^{\mathrm{T}} (Ax + Bu),$$

then the conditions for optimality expressed using $\mathcal{H}$ are

$$\dot{x} = \frac{\partial \mathcal{H}}{\partial \lambda} = Ax + Bu, \tag{15}$$
$$\dot{\lambda} = -\frac{\partial \mathcal{H}}{\partial x} = -Qx - A^{\mathrm{T}} \lambda, \tag{16}$$
$$0 = \frac{\partial \mathcal{H}}{\partial u} = Ru + B^{\mathrm{T}} \lambda, \tag{17}$$
$$0 = \frac{1}{2} \frac{\partial (y(t_i) - y_i)^{\mathrm{T}} S (y(t_i) - y_i)}{\partial x(t_i)} + \lambda(t_i^+) - \lambda(t_i^-), \tag{18}$$

where t_i^+ is the time instant t_i when approached from times $t > t_i$ and t_i^- is the time instant t_i when approached from times $t < t_i$. These conditions can also be found in [19], [20]. Expressing the co-state as $\lambda(t) = P(t)x(t) + \eta(t)$, the governing equations for minimizing J' are

$$\dot{P} = -PA - A^{\mathrm{T}} P - Q + PBR^{-1} B^{\mathrm{T}} P, \tag{19}$$
$$\dot{\eta} = (-A + PBR^{-1} B^{\mathrm{T}}) \eta, \tag{20}$$
$$\dot{x} = (A - BR^{-1} B^{\mathrm{T}} P) x - BR^{-1} B^{\mathrm{T}} \eta, \tag{21}$$
$$u = -R^{-1} B^{\mathrm{T}} (Px + \eta), \tag{22}$$

with boundary conditions $\forall i \in W \setminus \{N\}$ being

$$\begin{aligned} P(t_i^-) &= P(t_i^+) + C^{\mathrm{T}} S C, \\ \eta(t_i^-) &= \eta(t_i^+) - C^{\mathrm{T}} S y_i. \end{aligned} \tag{23}$$

At $i = N$, $P(t_N) = C^{\mathrm{T}} S C$ and $\eta(t_N) = -C^{\mathrm{T}} S y_N$. The equations (19), (20) are solved backward in time starting at t_N and updating the boundary conditions at every t_i, $i < N$ as expressed in (23). Solving these equations determines the state vector $x(t)$ for all $t \in [t_0, t_N]$. The state vector provides acceleration $a(t)$ of the vehicle at every instant $t \in [t_0, t_N]$. This implies that the thrust direction $q(t)$ can be determined as follows:

$$q(t) = \frac{g e_3 + a(t)}{\| g e_3 + a(t) \|}. \tag{24}$$

In order to track the obtained position trajectory, the desired attitude trajectory has to be generated such that the direction of $R(t)e_3$, the third column vector of the rotation matrix $R(t)$, agrees with the thrust direction given in (24), and the direction of $R(t_i)e_2$ aligns with the desired pointing direction s_i. This is done in the following subsection.

B. Attitude Trajectory

The position trajectory obtained in III-A provides the thrust direction $q(t)$, in (24), at every instant t. Since the constraints at the waypoints are soft, the desired thrust direction q_i in (6) could be different from $q(t_i)\ \forall i \in W$. Therefore the desired attitudes R_i and pointing directions s_i at the waypoints stated in (7) have to be corrected.

1) Corrected desired attitude R_i^c: Let R_i^c be the corrected attitude such that

$$R_i^c = \begin{bmatrix} s_i^c \times q(t_i) & s_i^c & q(t_i) \end{bmatrix}, \tag{25}$$

where s_i^c is the corrected pointing direction. The corrected pointing direction $s_i^c \perp q(t_i)$ is then given by

$$s_i^c = \frac{s_i - (s_i^{\mathrm{T}} q(t_i))q(t_i)}{\|s_i - (s_i^{\mathrm{T}} q(t_i))q(t_i)\|}. \tag{26}$$

Note that as $q(t_i) \to q_i$, $R_i^c \to R_i$.

2) Attitude trajectory R(t): The direction of thrust is known from the position trajectory, so the attitude trajectory $R(t)$ has to be generated such that

$$R(t)e_3 = q(t)\ \forall t \in [t_0, t_N], \tag{27}$$

and $R(t_i) = R_i^c$. Let the angular velocity $\Omega(t)$ of the vehicle, expressed in the body frame $\mathcal{B}$, be of the form

$$\Omega(t) = R(t)^{\mathrm{T}}(q(t) \times \dot{q}(t) + \alpha(t)q(t)),$$

where $\alpha(t) \in \mathbb{R}$. This choice of $\Omega(t)$ is such that $\dot{q}(t) = R(t)\Omega(t) \times q(t)$. The derivative $\dot{q}(t)$ is arrived at by differentiating the expression for $q(t)$ in (24)

$$\dot{q}(t) = \frac{\dot{a}(t) - (\dot{a}(t)^{\mathrm{T}} q(t))q(t)}{\|ge_3 + a(t)\|}.$$

As a consequence, the thrust direction constraint expressed in (27) is satisfied $\forall t \in [t_0, t_N]$. It remains to determine an appropriate function $\alpha(t)$ such that $R(t_i) = R_i^c$. The choice for $\alpha(t)$ is addressed piecewise.

The algorithm between two waypoints at time instances t_{i-1} and t_i is as follows:

- Propagate the attitude $R(t_{i-1})$ forward in time to the instant t_i using the kinematics relation
$$\dot{R} = R\Omega^{\times},$$
where $\Omega(t)$ is such that $\alpha(t) = 0\ \forall t \in [t_{i-1}, t_i]$. Let this attitude be R_i^p.
- Determine the difference between the propagated attitude R_i^p and the corrected desired attitude R_i^c using the relation
$$\Theta_i = \log_{\mathrm{SO(3)}}(R_i^{c\mathrm{T}} R_i^p), \tag{28}$$
where Θ_i is the exponential coordinate characterized by principal angle and principal axis. The logarithmic map $\log_{\mathrm{SO(3)}}(\cdot)$ is defined in [21].
- Since $R_i^c e_3 = R_i^p e_3 = q(t_i)$ and the relation between R_i^c and R_i^p is rotation about the third body axis e_3, the exponential coordinate Θ_i is of the form
$$\Theta_i = (\Delta\theta_i)e_3, \tag{29}$$
where $\Delta\theta_i$ is the principal angle of rotation. In other words, the attitude R_i^p has to be rotated by an angle $\Delta\theta_i$ about its $z-$axis, $R_i^p e_3$.
- This implies that the rotation in plane over the time $(t_i - t_{i-1})$ should accumulate to $\Delta\theta_i$. Therefore the function $\alpha(t)$ is chosen such that
$$\Delta\theta_i = \int_{t_{i-1}}^{t_i} \alpha_i(t)dt. \tag{30}$$
- To avoid discontinuities in control torque that might arise while combining this scheme with attitude tracking controls used in [1], [18], [22], [23], it is desirable to ensure that $\dot{\Omega}(t_i^-) = \dot{\Omega}(t_i^+)$. Because $q(t)$ is continuous and differentiable, the choice for $\alpha(t)$ is such that
$$\frac{d}{dt}\alpha(t)|_{t_i^-} = \frac{d}{dt}\alpha(t)|_{t_i^+}. \tag{31}$$

Note: The algorithm described above provides steps to arrive at a function $\alpha(t)$ defined piecewise. It can also be extended to find $\alpha(t)$ globally i.e. $\forall t \in [t_0, t_N]$.

3) Choice for $\alpha(t)$: There are numerous choices for $\alpha(t)$ satisfying the conditions (30), (31). In this work, $\alpha(t)$ is chosen to be a fourth order polynomial

$$\alpha(t) = c_i(t - t_{i-1})^2(t - t_i)^2 \tag{32}$$

having double roots at the waypoints $t = t_{i-1}, t_i$. This ensures C^2 continuity in the angular velocity $\Omega(t)$ and minimizes the number of coefficients that need to be determined. The choice also makes it convenient to evaluate the integral

$$\Delta\theta_i = \int_{t_{i-1}}^{t_i} \alpha(t)dt = \frac{c_i}{30}(t_i - t_{i-1})^5, \tag{33}$$

and thus the coefficient c_i.

Using the attitude dynamics in (5), the torque τ can be evaluated.

Remark: It is necessary to note that the position and attitude trajectories, as described in III-A and III-B respectively, can be used in unison with trajectory tracking algorithms on SE(3) such as [9], [22]. For online implementation, the user generates, a priori, a trajectory through first few waypoints with pointing direction constraints. This trajectory can be tracked in real-time using [9], [22]. Simultaneously, if subsequent set of waypoints are available then the algorithm described in this work generates a C^4 continuous trajectory addressing pointing direction constraints for further tracking.

IV. NUMERICAL RESULTS

This section presents numerical simulation results for an unmanned aerial vehicle of mass $m = 4.34$ kg and moment of inertia J

$$J = \begin{bmatrix} 0.820 & 0 & 0 \\ 0 & 0.0845 & 0 \\ 0 & 0 & 0.1377 \end{bmatrix} \mathrm{kgm}^2.$$

The vehicle starts from rest at origin i.e. $b_0 = 0_{3\times 1}$ and its initial state is considered to be $x(t_0) = 0_{15\times 1}$. The vehicle's initial attitude, without loss of generality, aligns with the inertial frame $\mathcal{I}$, therefore the initial pose is

$$g(t_0) = \begin{bmatrix} 0_{3\times 1} & I_{3\times 3} \\ 0 & 1 \end{bmatrix},$$

where starting time $t_0 = 0$ s. Four waypoints are specified. The waypoints b_i are obtained from the expressions:

$$b_i = [4(\frac{i+2}{2})\cos\frac{\pi t_i}{20} \quad 6(\frac{i+2}{2})\sin\frac{\pi t_i}{20} \quad 0.6t_i] \tag{34}$$

where $t_i = 4i \; \forall i \in W = \{1, 2, 3, 4\}$. The pointing directions at these waypoints are

$$s_i = \frac{Sp - b_i}{\|Sp - b_i\|} \; \forall i \in W, \tag{35}$$

where a sphere, centered at $Sp = (-5, 0, 4)$ m as shown in Fig. 2(a), is considered to be the object of interest. Using (6) the thrust direction q_i at the waypoints is determined. Fig. 2(a) shows the trajectory maneuvering around the waypoints and pointing towards the object of interest. Pointing direction shown in Fig. 2(b) demonstrates the trajectory on $\mathbb{S}^2$ passing through the desired pointing directions at the waypoints that are displayed using markers. The thrust direction is shown in Fig. 2(c). The thrust and torque magnitudes based on the dynamics model given by equations (4)-(5), are depicted in Fig. 2(d), (e) respectively. These magnitudes are reasonable considering the vehicle's inertial parameters and typical actuator capabilities onboard such a vehicle.

V. CONCLUSIONS AND FUTURE WORK

A trajectory generation scheme with pointing direction constraints is formulated for autonomous operations of an underactuated vehicle, a rigid body with one actuated translational degree of freedom and three actuated rotational degrees of freedom, that can model UAVs, UUVs and spacecraft. The scheme embeds the trajectory generation problem from $\mathbb{R}^3 \times \mathbb{S}^2$ onto $\mathrm{SE}(3)$. This step ties the position trajectory with the attitude trajectory through the thrust vector that is orthogonal to the pointing direction. This modification adds acceleration constraints to the waypoints and the scheme uses an LQR approach to generate a position trajectory of appropriate smoothness. The attitude trajectory is generated using the time trajectory of thrust direction obtained as a result of the position trajectory. This is done by first propagating forward in time to the next waypoint in order to determine the difference between the propagated and desired attitudes. The difference amounts to planar rotation about the thrust vector. This difference is corrected over the path to attain the desired attitude. Numerical simulation results demonstrate the validity of the scheme. Future work would explore stable feedback tracking schemes on the Lie group $\mathrm{SO}(3)$ to generate and track the attitude trajectory.

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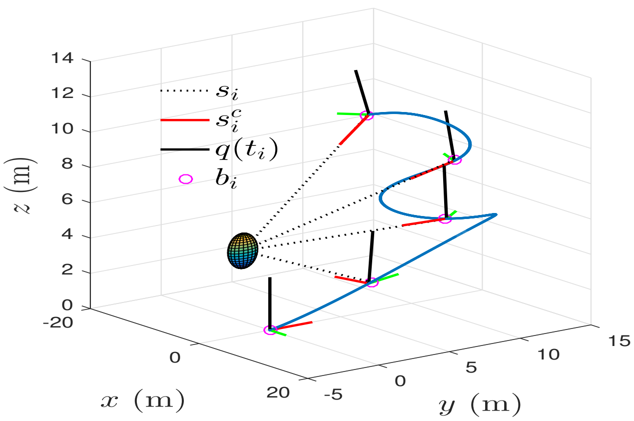
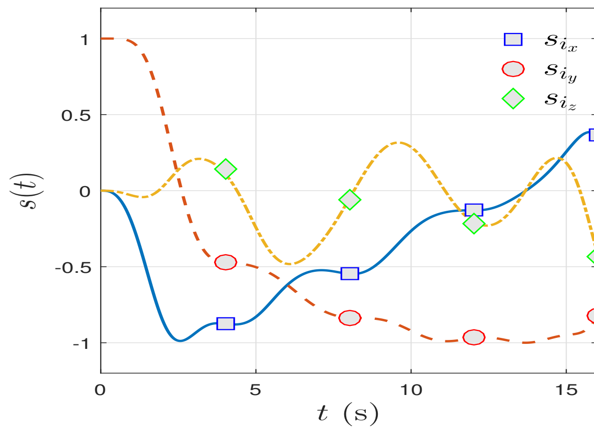
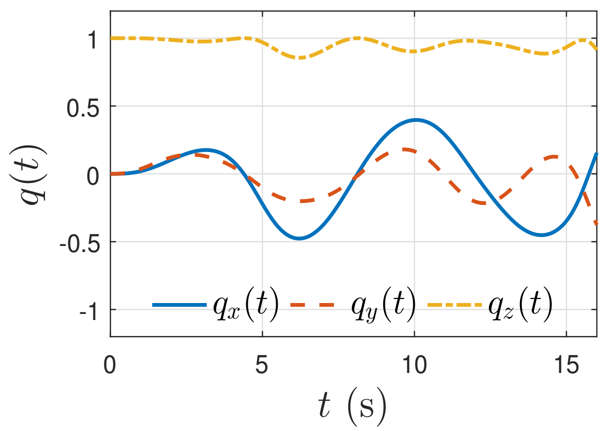


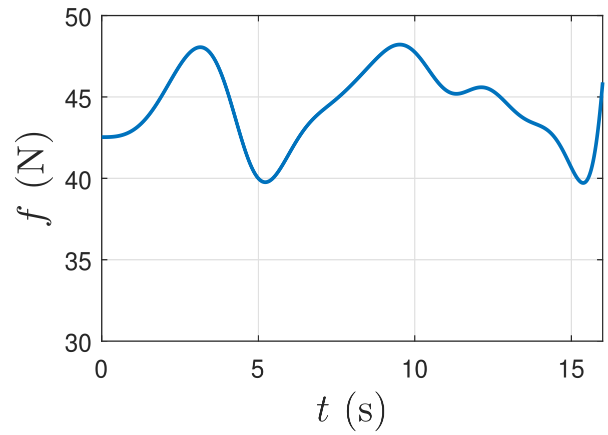
(a) Trajectory on SE(3)



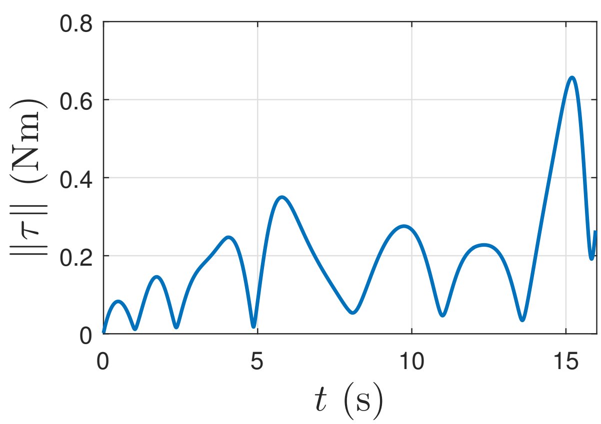
(b) Pointing direction $s(t) = R(t)e_2$



(c) Thrust direction



(d) Magnitude of thrust



(e) Magnitude of Torque

Fig. 2: Simulation results

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