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Decision Support

The value of information for price dependent demand

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ABSTRACT

Predicting demand and determining optimal pricing are essential components of operations management. It is often useful to think in terms of the price elasticity of demand when reasoning about the demand curve. Firms wishing to invest in demand prediction and information gathering should reason about the relationship between the expected value of perfect information (EVPI) on demand and demand elasticity. Should firms pay more/less for information on demand if elasticity is high/low? Furthermore, when considering different product prices, correlation may exist between demand at different prices. Should firms pay more/less for information if the correlation between demand at different prices is high or low? This paper derives analytic and numeric results to answer these questions. We start with the assumption that demand is uncertain and follows a uniformly distributed band around a deterministic demand curve where the upper and lower bounds of the demand distribution vary with price. This formulation enables a closed form expression for EVPI that provides a useful benchmark. We find nuanced behavior of EVPI that depends on both the elasticity and the initial price preference. The EVPI approaches zero as elasticity increases (decreases) for a firm that initially prefers the low (high) price. Numerical results using the truncated normal and beta distributions relax assumptions about the uniform distribution and show EVPI is similar when the distribution variances are similar. Finally, we relax the assumption of perfect information and show the expected value of imperfect information (EVOI) follows similar patterns as EVPI with respect to demand elasticity.

1. Introduction

Predicting demand and determining an optimal price are essential components of operations management (Azadian & Murat, 2017; Hsieh, Liu & Wang, 2010; Özer & Phillips, 2012; Sun, Hupman, Ritchey & Abbas, 2016). Although there is a rich literature on pricing, challenges remain in practice. McKinsey & Company estimates that thirty percent of pricing decisions made by companies are suboptimal (Baker, Kiewell & Winkler, 2014). Firms spend resources gathering and analyzing data to better predict demand, but it is often unclear how much a firm should invest in this task, particularly as new data sources and analytical methods become available. In the presence of price-dependent demand, it is also unclear how properties of elasticity of demand relate to the value of information on demand, and consequently, how much companies should invest in information gathering. The expected value of perfect information (EVPI) is a decision analytic construct that can answer these questions. It places an upper bound on how much a firm should pay for any information gathering activity about an uncertainty of interest (Howard, 1966; Howard & Abbas, 2016). The EVPI differs from information theoretic measures (Shannon, 1948) that consider the entropy (or the quantity of information) separately from the consequences of a decision. In this paper, we examine EVPI in pricing decisions with price-dependent demand to answer these questions.

For a risk-neutral decision maker or one with an exponential utility function, the EVPI is found by comparing the value of a decision situation with and without the information (Howard, 1966; Howard & Abbas, 2016). The EVPI does not necessarily increase with the level of risk (Gould, 1974; Laffont, 1976) or with the risk aversion (Abbas, Bakir, Klutke & Sun, 2013; Freixas & Kihlstrom, 1984; Hilton, 1981), but in the presence of certain additional information about preferences, the EVPI does relate to risk aversion (Sun & Abbas, 2014). Additionally, when information is available on two uncertainties, the VOI to resolve both uncertainties can be greater than the sum of the VOI for each uncertainty individually (Howard & Abbas, 2016; Keisler, 2005).

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The value of information sharing among supply chain partners has garnered much attention in the operations literature. Much of the work examines sharing information in the context of inventory problems (Cachon & Fisher, 2000; Chen, 2011; Raghunathan, 2001). The information facilitates enhanced management of uncertainty including the bullwhip effect (Bray & Mendelson, 2012; Chen & Lee, 2009; Syntetos, Babai, Boylan, Kolassa & Nikolopoulos, 2016) through strategies that include coordination with partners (Aviv, 2001; Zhou, Dan, Songxuan & Xumei, 2017). Conditions that moderate the value of shared information have also been examined (Babai, Boylan, Syntetos & Ali, 2016; Ciancimino, Cannella, Bruccoleri & Framinan, 2012; Lee, So & Tang, 2000; Teunter, Babai, Bokhorst & Syntetos, 2018), and the benefits of additional information have been shown (Asgari, Nikbakhsh, Hill & Zanjirani Farahani, 2016; Chatfield, Kim, Harrison & Hayya, 2004). Alternatively, Perakis and Roels (2008) consider the value of information on assumptions about an unknown demand distribution in an inventory problem. However, relatively little work exists relating the value of information to pricing decisions, with a few notable exceptions. Iyer and Ye (2000) study promotional pricing and show that the presence or absence of information can affect the profitability of promotions. Petruzzi and Dada (1999) consider a newsvendor problem with pricing for additive and multiplicative demand and derive extensions that relate the value of information for a decision maker with constant absolute risk aversion, where the base demand is a linear function of price and an additive error is introduced to model uncertainty.

Our formulation differs from other analyses in the operations management literature in that we consider a pricing decision and that we consider the relation between the value of perfect demand information and price elasticity. This approach provides a useful benchmark for firms early in the process of determining an optimal price. This work relates to some prior results that show the EVPI is at a maximum at the demand elasticity that makes a firm indifferent between two possible selling prices (Zellner & Abbas, 2018), a finding that corresponds with other results showing the EVPI is at a maximum when there is indifference between two alternatives (Delequié, 2008; Mehrez & Stulman, 1982). Our work is distinct from these prior results in that we more broadly characterize the behavior of the EVPI, considering different levels of elasticity and correlation between demand at different prices, different distributions of demand, and the value of imperfect information. The analytic results also contribute to a better understanding of the drivers of the value of information in pricing decisions, a helpful result given the conflicting findings of many value of information studies (e.g. Ketzenberg, Rosenzweig, Marucheck & Metters, 2007).

We explicitly consider the relationship between the EVPI and price elasticity of demand given the importance of demand elasticity in the pricing decision. Price elasticity measures how changes in price affect changes in demand, and these price changes can have large impacts on profitability (Mercer, 1993; Pauler & Dick, 2006). The importance of price elasticity has motivated a large literature. For example, Tellies (1988) conducts a meta-analysis of econometric models in the estimation of the price elasticity of demand and shows the distribution of estimated price elasticities. George, Mercer and Wilson (1996) shows the price elasticities of market share for some competing items may not be constant on a relatively large scale. Casado and Ferrer (2013) model a price elasticity of demand, which is constant but different in three intervals along with two thresholds, based on the heterogeneous consumers' utilities.

Previous work has shown that the value of information on demand can be higher when demand is correlated over time (Lee et al., 2000). Given this result, our analysis also considers the possibility of correlation between the realized demand at two different prices, i.e. demand at one price is correlated with demand at another price. We use probability copulas to represent the relationship between the different demands and numerically identify the copula parameters that correspond to particular values of the Pearson correlation coefficient. This approach allows us to report the findings in terms of correlation between the demand at different prices.

The analysis also considers how the EVPI is affected by different distributions of demand. We derive analytic results for the maximum entropy case of uniform demand. This distribution is appropriate for the earliest stages of analysis, for example, as a prior before information is gathered, and is consistent with work on Bayesian approaches to demand estimation (Hill, 1997). However, in practice demand is often modeled as following a normal (Axsäter, 2013; Strijbosch & Moors, 2005) or a beta distribution (Berk, Gürler & Levine, 2007, Siblermayr et al. 2017). We therefore conduct numeric analyses using these distributions. We quantify the deviations between the analytic and numeric results and show how the analytic results can be adjusted based on the demand distribution to provide readily available estimates of EVPI early in the decision making process. Finally, we consider the case of the expected value of imperfect information (EVI) by adding error terms to the realized demand and shows that it follows similar behavior as EVPI with respect to demand elasticity.

We illustrate the applicability of the results with a motivating example of a firm purchasing information on uncertain demand. This purchase could be in various forms of data collection and/or analysis, such as investment in information sharing between supply chain partners.

1.1. Illustrative example

A major challenge for retailers is setting the price for a new product (Ferreira, Lee & Simchi-Levi, 2016). This challenge forms the motivating example for this paper and illustrates the applicability of the results. Consider a retailer that has a fully designed product and the ability manufacture sufficient product on demand, but it must determine the price to charge that will optimize value to the retailer. Demand is stochastic and depends on the selling price with higher prices resulting in lower average demand and vice versa. The retailer may also purchase information about the uncertain demand through means such as additional marketing analysis or hiring a consultant, but it is uncertain how much it should invest in this information, if anything at all. This decision motivates the need to calculate the EVPI to ensure it is nonzero and to provide an upper bound on the investment.

The remainder of this paper is organized as follows. Section 2 describes the problem formulation. Section 3 presents the results when demand at one price point is independent of demand at another price point. Section 4 presents analytic results for demand that is perfectly correlated at different prices, while Section 5 presents general results of correlation through a numeric example. Section 6 examines the effect of different demand distributions. Section 7 illustrates how numeric calculations can relax the assumption of perfect information. Section 8 discusses implications of the results, and Section 9 provides concluding remarks.

2. Problem formulation

We are interested in the value of information on uncertain demand for a decision maker who must determine what price to charge for a new product. The new product has uncertain, stochastic demand that is affected by the price. To model this situation, the following assumptions are made:

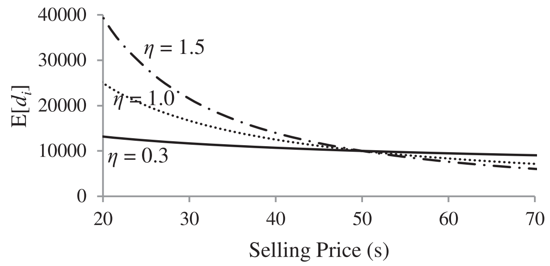

Fig. 1. The effect of constant elasticity η on mean demand when a price of \$50 is believed to result in E[d_i]=10,000 units.

i. The retailer considers one of the two given prices as its selling price, and only considers prices greater than the marginal production cost.
ii. The product cost is independent of the selling price and is deterministic.
iii. The price elasticity of the mean demand is constant and deterministic.
iv. The mean demand for one price (assume the low price without loss of generality) is given.
v The firm's belief of the prior distribution is a uniform band around the baseline demand curve.
vi. The widths of the demand distributions are a constant multiple of their means.
vii. The decision maker is risk neutral.

The assumption of constant elasticity of demand is commonly used (e.g. McAfee & te Velde, 2008). The remaining assumptions are consistent with a firm that has little information on the demand of a new product but operates in a market with well-known properties of demand elasticity. If the firm believes demand elasticity is constant but has difficulty specifying its value, then the firm may specify the mean demand at the second price and calculate the corresponding elasticity that would result in the specified mean demand. This approach results in specifications that are equivalent to these assumptions. We also note that this formulation implies no economies of scale and an ability to meet demand at any level.

With these assumptions in place, we can calculate the value of free perfect information, or clairvoyance, to the decision maker and analyze its sensitivity to the price elasticity of demand. We use indices $i = 1, 2$ to differentiate the two selling prices under consideration. The selling prices are denoted s_i, with $s_1 < s_2$, and the associated mean demand is denoted $\overline{d_i}$. Demand elasticity is represented by the parameter η. Using the elasticity of substitution and assuming elasticity is constant, we can write

$$\overline{d_i} s_i^{\eta} = \text{constant}. \tag{1}$$

The law of demand specifies a negative elasticity, meaning that increases in price have a negative relationship with demand. With (1), negative elasticity corresponds to $\eta > 0$.

To better illustrate this representation of price elasticity of demand, consider the case of a decision maker who believes the mean demand for a selling price of \$50 is 10,000 units. If $\eta = 0.3$, then demand is relatively inelastic with fluctuations in the price resulting in relatively small changes in the demand. As η becomes larger, elasticity increases, and changes in the price result in larger variations in the demand as shown in Fig. 1, the results of which are found by relating $\overline{d_i} s_i^{\eta}$ in (1) to values at different prices. For example, if we consider $\overline{d_1} s_1^{\eta}$ and wish to find the resulting mean demand at other prices, we rearrange (1) to solve for d_i, i.e. $\overline{d_i} = \overline{d_1} s_1^{\eta} / s_i^{\eta}$.

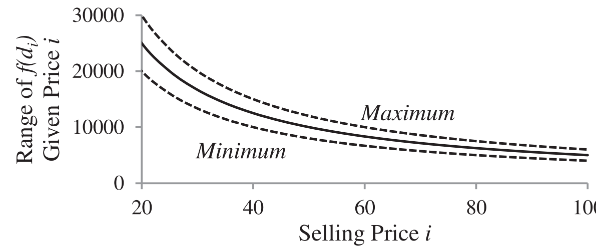

Fig. 2. The minimum, maximum, and midpoint of uniform distributions for the numeric example with η=1.0 and b=0.2.

The profit to the retailer is exclusively determined by the selling price and demand if the cost is assumed deterministic and independent of price. This cost is denoted c. Thus, the profit is

$$\text{Profit}_i = d_i (s_i - c). \tag{2}$$

The demand distribution is assumed uniform with mean $\bar{d_i}$. Following assumption vi, we formulate the uniform distributions by defining a parameter b_i, $0 < b_i < 1$, to represent the distribution as a function of $\bar{d_i}$, the mean. To construct the interval of the uniform distribution, b_i times of the mean, $\bar{d_i}$, is added to and subtracted from $\bar{d_i}$. With $b_1 = b_2 = b$, the intervals of the distributions become $[\bar{d_i}(1-b), \bar{d_i}(1+b)]$. Then the range of each distribution is $2b\bar{d_i}$, making the marginal density functions

$$f(d_i) = \begin{cases} \frac{1}{2b\bar{d_i}}, if \bar{d_i}(1-b) \le d_i \le \bar{d_i}(1+b) \\ 0, \text{otherwise} \end{cases}. \tag{3}$$

This formulation of the demand distributions is illustrated in Fig. 2 using the same numeric example from Fig. 1 with $\eta = 1.0$ and $b = 0.2$ Because the variance of the distribution decreases as the range decreases, an implication of (1) and (3) is that the variance decreases as the price increases. Stated alternatively, larger values of error are associated with larger quantities demanded, consistent with modeling assumptions in the literature (Leland, 1972).

For a risk neutral decision maker, the value of free perfect information in this pricing decision is the difference in the expected value of the decision with perfect information and the expected value without perfect information. Given this problem formulation, the next two sections examine the EVPI for a risk neutral decision maker when the demand at one price is independent of the demand at a second price and for the case when these two demands are correlated.

3. EVPI when demand is independent at two prices

We begin by examining the case when the uncertain demand d_1 at price s_1 is independent of the uncertain demand d_2 at price s_2. This case of independence is appropriate, for example, when the firm is introducing a new product and is unable to specify a belief about a dependence structure for observed demand at different prices. In this case, given the uniform marginal distributions in (3) for the two prices, the joint density function of d_1 and d_2 is

$$f(d_1, d_2) = \begin{cases} \frac{1}{4\bar{d_1}\bar{d_2}b^2}, & \forall [d_1, d_2] \in \left[\bar{d_1}(1-b), \bar{d_1}(1+b)\right] \\ \times \left[\bar{d_2}(1-b), \bar{d_2}(1+b)\right]; \\ 0, \; otherwise. \end{cases} \tag{4}$$

The bounds of the distribution give the maximal and minimal demands, which are $\bar{d_i}(1+b)$ and $\bar{d_i}(1-b)$, respectively. From (2), the maximal and minimal profit at selling price s_i are

$$M_i = (1+b)\bar{d_i}(s_i - c), \tag{5}$$

$$m_i = (1-b)\bar{d_i}(s_i - c), \tag{6}$$

where M_i denotes maximal profit and m_i denotes minimal profit. The relationship between M_i and m_i depends on the size of s_1 relative to s_2. Note that $M_1 \geq M_2$ if and only if $\bar{d}_1(s_1 - c) \geq \bar{d}_2(s_2 - c)$, i.e. $\eta \geq \log_{s_2/s_1} \frac{s_2-c}{s_1-c}$, due to (1), and we can also state $m_1 \geq m_2$ in this case. Similarly, $M_1 \geq m_2$ if and only if $\eta \geq \log_{s_2/s_1} \frac{(s_2-c)(1+b)}{(s_1-c)(1-b)}$, and $M_2 \geq m_1$ if and only if $\eta \geq \log_{s_2/s_1} \frac{(s_2-c)(1-b)}{(s_1-c)(1+b)}$. We summarize all possible orderings of m_1, M_1, m_2, and M_2 with respect to the elasticity η as follows:

(i) If $\eta \geq \log_{s_2/s_1} \frac{(s_2-c)(1+b)}{(s_1-c)(1-b)}$, then $m_2 < M_2 \leq m_1 < M_1$;
(ii) If $\log_{s_2/s_1} \frac{s_2-c}{s_1-c} < \eta < \log_{s_2/s_1} \frac{(s_2-c)(1+b)}{(s_1-c)(1-b)}$, then $m_2 < m_1 < M_2 < M_1$;
(iii) If $\log_{s_2/s_1} \frac{(s_2-c)(1-b)}{(s_1-c)(1+b)} < \eta \leq \log_{s_2/s_1} \frac{s_2-c}{s_1-c}$, then $m_1 \leq m_2 < M_1 \leq M_2$
(iv) If $\eta \leq \log_{s_2/s_1} \frac{(s_2-c)(1+b)}{(s_1-c)(1-b)}$, then $m_1 < M_1 \leq m_2 < M_2$.

Note that the expected value of the profit at s_i is $E[d_i(s_i - c)] = 0.5M_i + 0.5m_i$, $i = 1, 2$. A risk-neutral decision maker will ask for the low price s_1 in cases (i) and (ii) and the high price s_2 in cases (iii) and (iv) without the additional information. Next we consider the EVPI on the uncertain demands d_1 and d_2. In particular, we assume that the decision maker can observe the uncertain demands d_1 and d_2 before determining whether to set the selling price as s_1 or s_2. Then he/she will choose the low price s_1 if the profit at the low price is higher than the profit at the high price, i.e. $d_1(s_1 - c) > d_2(s_2 - c)$. He/she will choose the high price s_2 if the profit at the high price is higher than the profit at the low price, i.e. $d_2(s_2 - c) > d_1(s_1 - c)$. Therefore, the profit with the perfect information on the uncertain demands d_1 and d_2 is $\max\{d_1(s_1 - c), d_2(s_2 - c)\}$ and the expected information is

$$\begin{aligned} EVPI = {} & E[\max\{d_1(s_1 - c), d_2(s_2 - c)\}] \\ & - \max\{E[d_1(s_1 - c)], E[d_2(s_2 - c)]\}. \end{aligned} \tag{7}$$

Proposition 1. *The expected value of perfect information on the uncertain demands d_1 and d_2 is*

$$\begin{aligned} EVPI = {} & \frac{1}{6(M_1 - m_1)(M_2 - m_2)} \\ & \times \max\left\{0, \min\left\{(M_1 - m_2)^3, (M_2 - m_1)^3\right\}\right\}. \end{aligned} \tag{8}$$

The proof to Proposition 1 is in the Appendix.

This result enables firms to calculate the EVPI between two selling prices by specifying only η and the minimal and maximal demand at one price, simplifying the tasks of specifying demand distributions and of calculating the value of information. Using this approach, the firm can use the EVPI to determine whether the investment of time and resources on additional analysis is value-adding to the firm. This result also facilitates further analyses such as examining the sensitivity of the EVPI to the specified parameters.

Proposition 2. *If a risk neutral decision maker asks for the low price without the perfect information on the uncertain demands, then the expected value of the perfect information on the uncertain demands is strictly decreasing to zero as the demand elasticity increases.*

Heuristically speaking, if the firm prefers the low price without the information on the uncertain demands, then the information is valuable if and only if it indicates the high price will lead to more profit than the low price, inducing a change in the pricing decision. However, the mean demand of the high price decreases as the demand elasticity increases. Therefore, the EVPI also decreases as the demand elasticity increases. In particular, recall that if $\eta \geq \log_{s_2/s_1} \frac{(s_2-c)(1+b)}{(s_1-c)(1-b)}$, then $m_2 < M_2 \leq m_1 < M_1$. Hence, profit from the high price is always smaller than the profit from the low price, and the firm will not switch to the high price even if the uncertain demand at the high price achieves its maximum and the uncertain demand at the low price achieves its minimum simultaneously.

Proposition 3. *If a risk neutral decision maker asks for the high price without the perfect information on the uncertain demands, then the expected value of the perfect information on the uncertain demands is strictly decreasing to zero as the demand elasticity decreases.*

Again consider the heuristic logic. If the firm prefers the high price without the information on the uncertain demands, then the information is valuable if and only if it indicates the low price will lead to more profit than the high price, inducing a change in the pricing decision. However, the mean demand of the high price increases as the demand elasticity decreases. Therefore, the EVPI also decreases as the demand elasticity decreases. In particular, recall that if $\eta \leq \log_{1+h} \frac{(s_2-c)(1+b)}{(s_1-c)(1-b)}$, then $m_1 < M_1 \leq m_2 < M_2$. Hence, the profit from the high price is always larger than the profit from the low price, and the firm will not switch to the low price even if the uncertain demand at the low price achieves its maximum and the uncertain demand at the high price achieves its minimum simultaneously.

Proofs of Propositions 2 and 3 are straightforward by checking the sign of the partial derivative of EVPI with respect to $\bar{d}_2$ and are included in the Appendix.

Propositions 2 and 3 describe conditions in which a firm should not risk losing value by investing in information about the demand uncertainty. If the lower price is preferred and the product has highly elastic demand, or if the higher price is preferred and the product has highly inelastic demand, then investing additional resources to resolve the demand uncertainty risks losing value for the firm. As EVPI decreases to zero, investment in information on demand will become increasingly detrimental to the firm. These conditions also underscore the importance that a firm understands the market in which it operates. If the firm is unaware of whether the product has elastic or inelastic demand, then the firm cannot intelligently invest in, or abstain from investing in, obtaining additional information.

Proposition 4. *A risk neutral decision maker values the information on the uncertain demands highest when he/she is indifferent between the high selling price and the low selling price without the information among all possible demand elasticities.*

Proposition 4 is a corollary of Propositions 2 and 3.

Proposition 4 is complementary to Propositions 2 and 3. While the prior results showed the cases in which EVPI decreases to zero, Proposition 4 highlights when EVPI is highest. If the firm has no strict preference over the two possible selling prices without the information on the uncertain demands, then the information is most valuable because it will almost always change the firm's preference from indifference to the price with the higher profit. Propositions 2, 3, and 4 show the sensitivity of the EVPI to the demand elasticity for a risk neutral firm. The following example illustrates the sensitivity analysis on the value of information with respect to the demand elasticity.

We consider a numeric example of a firm that is considering a low price of \$50 versus a high price of \$100. The fixed marginal product cost is \$40. At the low price, the firm estimates the mean demand is 10,000 units, with a range from 8000 to 12,000 units (i.e.$b = 0.2$). Fig. 3 shows the sensitivity analysis of the EVPI to the demand elasticity. If the demand elasticity is low, i.e. $\eta < \log_{s_2/s_1} \frac{s_2-c}{s_1-c} = 2.585$, then the firm will prefer the high price without the information, and the EVPI will be increasing with demand

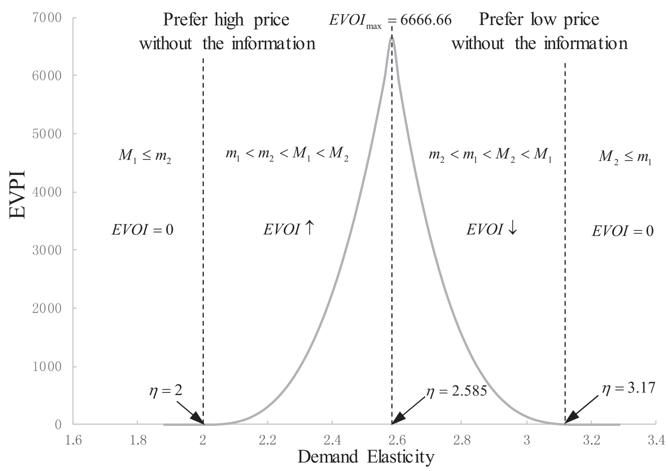

Fig. 3. The sensitivity of the EVPI to changes in elasticity for numeric example with $b = 0.2$, $s_1 = 50$, $s_2 = 100$, $c = 40$, and $\bar{d_1} = 10,000$..

elasticity following Proposition 3. Conversely, if the demand elasticity is high, i.e. $\eta = 2.585$, then the firm will prefer the low price without the information, and the EVPI will be decreasing with demand elasticity following Proposition 2.

Therefore, the firm will be indifferent between the two prices without the information when $\eta = 2.585$, and the EVPI achieves its maximum, in this case. Moreover, if the demand elasticity is very low, i.e. $\eta \le \log_{s_2/s_1} \frac{(s_2-c)(1+b)}{(s_1-c)(1-b)} = 2$, then $M_1 \le m_2$ and the information is valueless. Or if the demand elasticity is very high, i.e. $\eta \ge \log_{s_2/s_1} \frac{(s_2-c)(1-b)}{(s_1-c)(1+b)} = 3.17$, then $M_2 \le m_1$ and the information is also valueless.

4. EVPI with perfect correlation between demand at two prices

While the independence of demand at different prices is appropriate in some cases, there are also many cases in which relevance may exist. For example, it is reasonable that demand for a product is related to the success of the product design. If demand is high for the product at price s_1, then it is more likely the product design is favorable, and demand is more likely to be high at price s_2 as well. The existence of correlation can also be conceptualized in terms of epistemic and aleatory uncertainty. Given the price elasticity, observing demand at one price reduces epistemic uncertainty about the product and reduces, but does not eliminate, uncertainty about demand at another price, implying correlation between the demand distributions.

In this section, we examine EVPI when the demand for the product at price s_1 is relevant to the demand for the product at price s_2. We begin with the special case of perfect correlation and derive five propositions. The analysis of EVPI with perfect correlation between demand at two different prices provides useful insight to the behavior of EVPI as the correlation increases under various conditions. We begin with perfect positive correlation. If two uncertain demands, d_1 and d_2, at prices s_1 and s_2, respectively, are perfectly positively correlated, i.e.$\rho = 1$, then $d_2 = (s_1/s_2)^{\eta} d_1$.

Proposition 5. *If $d_2 = (s_1/s_2)^{\eta} d_1$, then the value of information on the uncertain demand is zero for any decision maker.*

For positive correlation, the EVPI will approach zero as the correlation increases. This result indicates when the correlation between demand at different prices is both extremely large and positive, the firm may not wish to invest in additional information on the demand. This result follows because with perfect positive correlation, additional information about the uncertain demand does not result in a change in the pricing decision. If demand is high at one price, it will also be high at the other price, and vice versa. Conceptualized another way, perfect positive correlation means there is a positive linear relationship between the demands. Any change in one demand will proportionally change the profits at both prices in the same way such that there is no change in the rank of the profits at the prices.

Next, we consider perfect negative correlation. If two uncertain demands, d_1 and d_2, at prices s_1 and s_2, respectively, are perfectly negatively correlated, i.e. $\rho = -1$, then $d_2 = 2\bar{d_2} - (s_1/s_2)^{\eta} d_1$.

Proposition 6. *If $d_2 = (s_1/s_2)^{\eta}(2\bar{d_1} - d_1)$, then the expected value of information on the uncertain demand is*

$$EVPI = \frac{1}{2b(M_1 + m_1 + M_2 + m_2)} \times \max\left\{0, \min\left\{(M_1 - m_2)^2, (M_2 - m_1)^2\right\}\right\}. \quad (9)$$

With perfect negative correlation, the EVPI is nonzero if $M_1 > m_2$ or if $M_2 > m_1$, i.e. the profit at one price is not dominant over the profit at the other price. The EVPI can be positive in this case because the firm's decision on the pricing may change based on the information obtained. For example, the firm may choose a price, and the information may indicate the demand at this price is very lower. For perfect negative correlation, this indication means that the demand at another price is so high that its profits will be higher than profit at the chosen price. Thus, the firm will change its decision and realize more profit due to the information.

Further investigation of negatively correlated demand results in the following propositions.

Proposition 7. *Assume that the uncertain demands at two selling prices are perfectly negatively correlated. If a risk neutral decision maker asks for the high selling price without the perfect information on the uncertain demands, then the expected value of the perfect information on the uncertain demands is strictly decreasing to zero as the demand elasticity decreases.*

Proposition 8. *Assume that the uncertain demands at two selling prices are perfectly negatively correlated. If a risk neutral decision maker asks for the low selling price without the perfect information on the uncertain demands, then the expected value of the perfect information on the uncertain demands is strictly decreasing to zero as the demand elasticity increases.*

Propositions 7 and 8 show two different sets of conditions under which the EVPI decreases to zero. These results again underscore the importance of a firm understanding the market in which it operates and the characteristics of demand for its product. Even perfect negative correlation that results in the largest change in relative value of alternatives cannot guarantee a positive EVPI. These results also highlight the complexities of working with the EVPI. Its behavior in this case is dependent on the interaction of two parameters; it cannot be summarized with a simple relationship with a single parameter.

Proposition 9. *Assume that the uncertain demands at two selling prices are perfectly negatively correlated. A risk neutral decision maker values the information on the uncertain demands highest when he/she is indifferent between the high selling price and the low selling price without the information among all possible demand elasticities.*

Proposition 9 complements the previous propositions which require some knowledge of the demand elasticity to be applied. If this preliminary information is unknown, it is reasonable that additional information on the demand would be beneficial, a line of reasoning supported by Proposition 9.

Finally, we consider a comparison of the EVPI in the cases of perfect positive correlation ($EVPI_1$), independence ($EVPI_0$), and perfect negative correlation ($EVPI_{-1}$).

Proposition 10. $0 = EVPI_1 \leq EVPI_0 \leq (2/3)EVPI_{-1}$*, where the strict inequality holds if and only if* $EVPI_0 > 0$.

Proposition 10 shows how the EVPI increases as the correlation between demand at two price points decreases from +1 to 0 to −1, indicating an inverse relationship between correlation and the EVPI.

5. EVPI with general correlation between demand at two prices

Next we consider the general case where the correlation coefficient is any value between −1 and 1. We use probability copulas to represent the joint distributions with correlation, specifically relying on the Gaussian and the Frank copulas. A probability copula provides a general structure to represent the joint density of two variables while maintaining the marginal distributions of each variable (Nelson, 1998; Sklar, 1959), enabling analysis with uniform marginal distributions consistent with the prior analysis. The parameters of the copulas govern the dependence structure, and the parameter that corresponds to a particular level of correlation may be found numerically. More precisely, copula structure requires the specification of the marginal distribution of the uncertain demand with the selling price as $s_i, i = 1, 2$,

$$P_i(x) = \frac{\min\left\{\max\left\{x - \bar{d}_i(1-b), 0\right\}, 1\right\}}{2b\bar{d}_i}, \quad i = 1, 2. \tag{10}$$

The Gaussian copula with parameter ρ is then formed from the multivariate Gaussian distributions as

$$G_\rho(P_1, P_2) = \Phi_\rho(\Phi^{-1}(P_1), \Phi^{-1}(P_2)), \tag{11}$$

where Φ_ρ is the joint cumulative distribution function of the two-dimensional normal distribution with standard normal distributions as its marginal distributions, with the correlation coefficient ρ, and with Φ^{-1} as the inverse function of the standard normal distribution. Therefore, the joint cumulative distribution function of the two uncertain demands is

$$F(x,y) = G_\rho(P_1(x), P_2(y)) = \Phi_\rho\left(\begin{matrix}\Phi^{-1}\left(\frac{\min\left\{\max\left\{x-\bar{d}_1(1-b),0\right\},1\right\}}{2b\bar{d}_1}\right), \\ \Phi^{-1}\left(\frac{\min\left\{\max\left\{y-\bar{d}_2(1-b),0\right\},1\right\}}{2b\bar{d}_2}\right)\end{matrix}\right). \tag{12}$$

In particular, if $\rho = -1$, then the two uncertain demands d_1 and d_2 are perfectly negatively correlated. If $\rho = 0$, then the two uncertain demands d_1 and d_2 are independent of each other. If $\rho = 1$, then the two uncertain demands d_1 and d_2 are perfectly positively correlated. Generally, we can numerically estimate the corresponding coefficient $\rho$for a given Pearson correlation coefficient such as −0.5 or 0.5.

The Frank copula is formed from a closed form generating function and has a single parameter δ that governs the strength of preference (Nelson, 1998) as

$$F_\delta(P_1, P_2) = \begin{cases} -\frac{1}{\delta}\ln\left(1 - \frac{(1-e^{-\delta P_1})(1-e^{-\delta P_2})}{1-e^{-\delta}}\right), & \delta \neq 0, \\ P_1 P_2, & \delta = 0. \end{cases} \tag{13}$$

Therefore, the joint cumulative distribution function of the two uncertain demands is

$$F(x,y) = F_\delta(P_1(x), P_2(y)) = \begin{cases} -\frac{1}{\delta}\ln\left(1 - \left(1 - e^{-\frac{\delta\min\left\{\max\left\{x-\bar{d}_1(1-b),0\right\},1\right\}}{2b\bar{d}_1}}\right)\left(1 - e^{-\frac{\delta\min\left\{\max\left\{x-\bar{d}_2(1-b),0\right\},1\right\}}{2b\bar{d}_2}}\right)/\left(1-e^{-\delta}\right)\right), \delta \neq 0, \\ \frac{\min\left\{\max\left\{x-\bar{d}_1(1-b),0\right\},1\right\}\min\left\{\max\left\{x-\bar{d}_2(1-b),0\right\},1\right\}}{4b^2\bar{d}_1\bar{d}_2}, \quad \delta = 0. \end{cases} \tag{14}$$

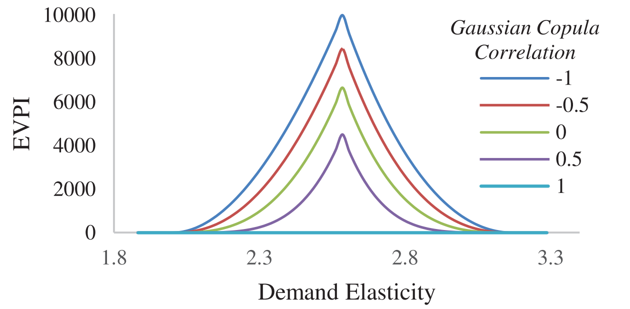

Fig. 4. EVPI for the numeric example with a Gaussian probability copula to represent correlation.

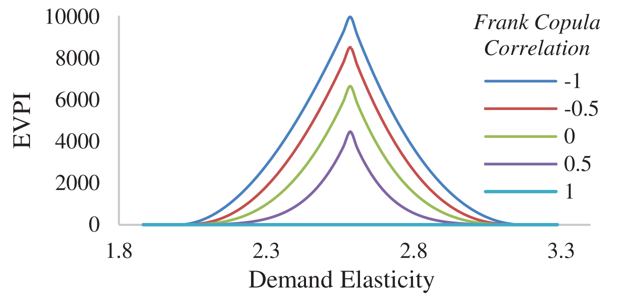

Fig. 5. EVPI for the numeric example with a Frank probability copula to represent correlation.

In particular, if $\delta \to -\infty$, then the two uncertain demands d_1 and d_2 are perfectly negatively correlated. If $\delta = 0$, then the two uncertain demands d_1 and d_2 are independent of each other. If $\delta \to +\infty$, then the two uncertain demands d_1 and d_2 are perfectly positively correlated. Generally, we can numerically estimate the corresponding coefficient δ for a given Pearson correlation coefficient such as −0.5 or 0.5.

In our example, we specify the Pearson correlation coefficient as −1, −0.5, 0, 0.5 and 1, respectively, and use the corresponding parameter for the copula.

We revisit the numeric example of a firm that is considering a low price of $50 versus a high price of $100. Recall, the fixed marginal product cost is $40. At the low price of $50, the firm estimates the average demand is 10,000 units, with a range from 8000 to 12,000 units (i.e. $b = 0.2$). We examine how the EVPI changes with different demand elasticities, different values of correlation, and different probability copulas to represent the demand distribution. The results are shown in Figs. 4 and 5 for the Gaussian copula and the Frank copula, respectively.

The effect of demand elasticity on the EVPI is not monotonic. For this example, the maximum EVPI occurs when $\eta = 2.585$ for all values of correlation and for both copulas. EVPI then decreases to zero as the elasticity decreases and increases away from 2.585. These results follow from the effect of elasticity on the relative attractiveness of the alternatives. As elasticity increases, average demand decreases more quickly with increases in price, until there is no overlap between $[m_1, M_1]$ and $[m_2, M_2]$, and the dominance of one alternative reduces the EVPI to zero. On the other hand,

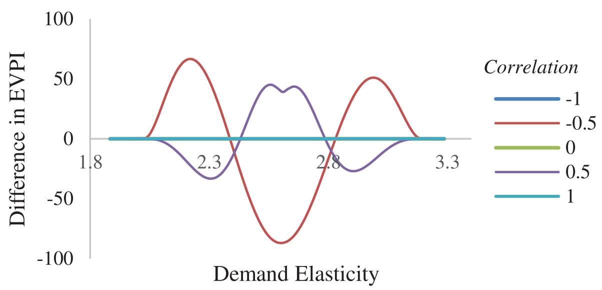

Fig. 6. The difference in EVPI between the Gaussian and Frank copulas for the numeric example.

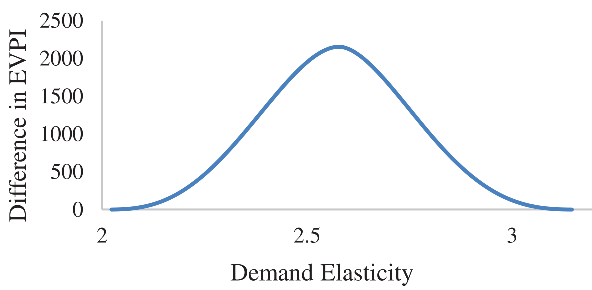

Fig. 7. The difference in EVPI between $\rho = 0$ and $\rho = 0.5$ for the Gaussian copula.

as elasticity decreases, the average demand becomes insensitive to price changes, again reducing the EVPI to zero.

Next we compare the results obtained with the Gaussian copula to those obtained with the Frank copula for different values of correlation. The results are shown in Fig. 6. For the cases of perfect negative correlation, independence, and perfect positive correlation, the difference is zero because these are all limiting cases that eliminate the effect of correlation. When the correlation is nonzero and is not perfect, we observe nonlinear behavior in the EVPI. Neither copula guarantees a higher or lower EVPI. Importantly, we find the magnitude of the differences in EVPI is small relative to the absolute value of EVPI for either copula. For $\rho = -0.5$, the maximum absolute difference is \$86.94 and occurs at $\eta = 2.61$, resulting in an error of only 1.2% for specifying the incorrect copula. For $\rho = 0.5$, the maximum absolute difference is \$45.08 and occurs at $\eta = 2.56$, resulting in a maximum error of 1.2% for specifying the incorrect copula.

Finally, we examine the effect of assuming independence between demand at different selling prices when there is actually positive correlation by comparing the EVPI results from the Frank copula with $\rho = 0$ and $\rho = 0.5$. The differences are shown in Fig. 7. The maximum difference is 2151.22 and occurs at $\eta = 2.585$, the same elasticity as the maximum EVPI. Thus, assuming independence when $\rho = 0.5$ results in an overestimation of EVPI by 32.27%.

6. EVPI for non-uniform demand distributions

Though uniform distributed uncertain demand shows interesting properties on value of information, it may not be always valid in practice. Now we relax Assumption V and consider the uncertain demand which is not uniformly distributed. Specifically, we consider the truncated normal distribution and the beta distribution, which are commonly used to represent uncertain demand. Although these distributions do not enable elegant analytic results on the EVPI, we can show similar propositions on the EVPI with numeric results. Recall, we examine a firm that is considering a low price of \$50 versus a high price of \$100 for a product. The fixed marginal product cost is \$40. At the low price of \$50, the

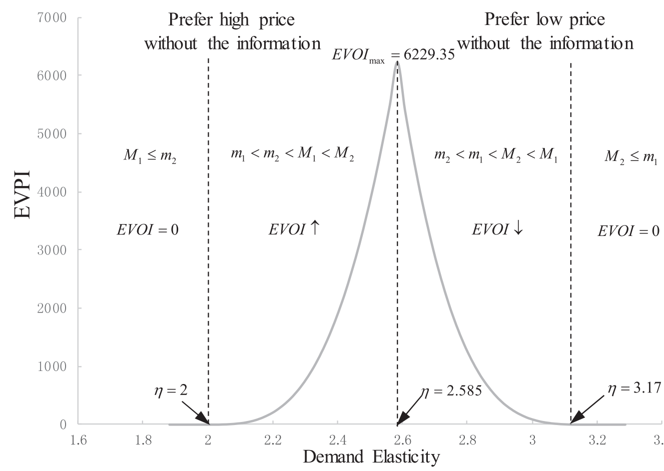

Fig. 8. The sensitivity of the EVPI to changes in elasticity for the truncated normal distribution with bounds of 8000 and 12,000 and with mean and standard deviation matching the uniform distribution of the same bounds.

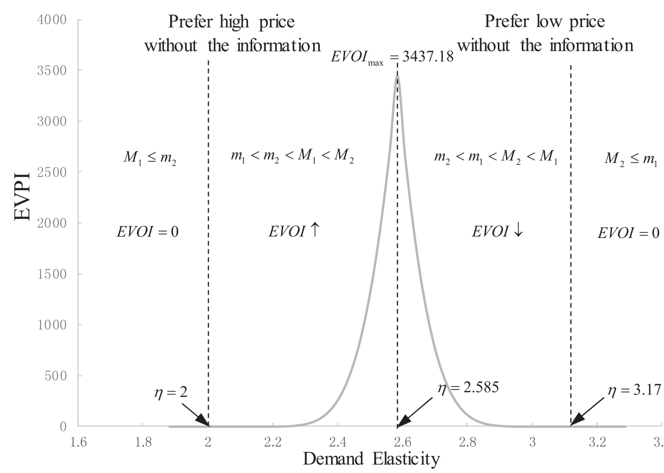

Fig. 9. The sensitivity of the EVPI to changes in elasticity for beta distributed demands with $k = r = 5$.

firm estimates the average demand is 10,000 units, with a range from 8000 to 12,000 units, i.e. $b = 0.2$, using the notation of (3).

First, we examine how EVPI changes with different demand elasticities when the uncertain demand d_i follows a truncated normal distribution. The bounds of the distribution are the same as those previously specified. The mean is 10,000, and the standard deviation is $\sigma_i = bd_i, i = 1, 2,$for the low and high price. The demand at the low price is independent of demand at the high price. The results of the EVPI are similar to those obtained with the uniform distribution and are shown in Fig. 8. The maximum EVPI is \$6229.35, representing a 6.55% decrease from the EVPI with a uniform distribution.

Next, we consider the case when demand follows a beta distribution. Fig. 9 plots the EVPI of uncertain demand when demand follows a beta distribution with parameters $k = r = 5$ for the low and high price. In this case, the maximum EVPI shows much more deviation from the case with uniform demand; the EVPI decreases by 48.44%. Note, however that the deviation in EVPI from the uniform distribution to the Beta distribution depends on the change in the variance of the demand distributions. If the beta parameters are $k = r = 1$, then the beta distribution becomes a uniform distribution, and there is no difference in EVPI. As the parameters k and r increase, the variance of the beta distribution decreases, causing the EVPI to decrease as well. We also examine the effect of correlation between demand with a beta distribution and find the same patterns of behavior as previously described.

7. The expected value of imperfect information

Finally, we consider the scenario where perfect information is not available but imperfect information can be obtained in which some uniformly distributed error is present. More precisely, the decision maker can obtain the extra information on the demands as two random variables q_1 and q_2, which are the exact demands plus two error terms e_1 and e_2, respectively. i.e.

$$q_i = d_i + e_i, i = 1, 2,$$

where e_1 and e_2 are independent and uniformly distributed in $[-a, a]$. Therefore, the conditional probability density of q_i given the demands d_1 and d_2 is

$$f_i(q_i|d_1, d_2) = \begin{cases} \frac{1}{2a}, d_i - a \le q_i \le d_i + a \\ 0, otherwise. \end{cases}$$

We revisit the independent case in Section 3 but with the imperfect information q_1 and q_2. The expected value of imperfect information (EVI) from the observations of q_1 and q_2 is the surplus in the profit as $EVOI = E[\max\{E[(s_1 - c)d_1|q_1, q_2], E[(s_1 - c)d_1|q_1, q_2]\}] - \max\{(s_1 - c)\bar{d}_1, (s_2 - c)\bar{d}_2\}$.

The joint density function of d_1, d_2, q_1 and q_2 is

$$f(d_1, d_2, q_1, q_2) = \begin{cases} \frac{1}{16a^2b^2\bar{d}_1\bar{d}_2}, & d_i - a \le q_i \le d_i + a, \bar{d}_i(1-b) \\ & \le d_i \le \bar{d}_i(1+b), i = 1, 2; \\ 0, \ otherwise. \end{cases}$$

Therefore, the joint density function of q_1 and q_2 is

$$f(q_1, q_2) = \begin{cases} \frac{(\alpha_1-\beta_1)(\alpha_2-\beta_2)}{16a^2b^2\bar{d}_1\bar{d}_2}, & \forall[q_1, q_2] \in [\bar{d}_1(1-b) - a, \bar{d}_1(1+b) \\ & +a] \times [\bar{d}_2(1-b) - a, \bar{d}_2(1+b) + a]; \\ 0, \ otherwise, \end{cases}$$

where $\alpha_i = \min\{\bar{d}_i(1+b), q_i + a\}$ and $\beta_i = \max\{\bar{d}_i(1-b), q_i - a\}$, $i = 1, 2$. The expected demands given the observations of q_1 and q_2 are $E[d_i|q_1, q_2] = \frac{\alpha_i+\beta_i}{2}, i = 1, 2$. Therefore, the expected profit after knowing q_1 and q_2 is

$$\begin{aligned} &E[\max\{E[(s_1 - c)d_1|q_1, q_2], E[(s_1 - c)d_1|q_1, q_2]\}] \\ &= \int_{\bar{d}_2(1-b)-a}^{\bar{d}_2(1+b)+a} \int_{\bar{d}_1(1-b)-a}^{\bar{d}_1(1+b)+a} \frac{(\alpha_1 - \beta_1)(\alpha_2 - \beta_2)}{32a^2b^2\bar{d}_1\bar{d}_2} \\ &\max\{(s_1 - c)(\alpha_1 + \beta_1), (s_2 - c)(\alpha_2 + \beta_2)\}dq_1dq_2 \end{aligned} \quad (15)$$

The integral in (15) does not have a simple analytic functional form as it did in the case of perfect information. However, we can calculate it numerically through software such as Matlab. We revisit the numeric example of a firm that is considering a low price of \$50 versus a high price of \$100 that was first presented in Section 3. We compare the EVPI and the EVI when the imperfect information has different accuracies as $a = 100$ and 500. The results are shown in Fig. 10. The EVI follows the same monotonicity as the EVPI. The value of the information increases becomes as a decreases to 0, where the EVPI is the upper bound of the value when $a = 0$. Notably, when a is as large as 100, value of information is nearly indistinguishable from EVPI in Fig. 10.

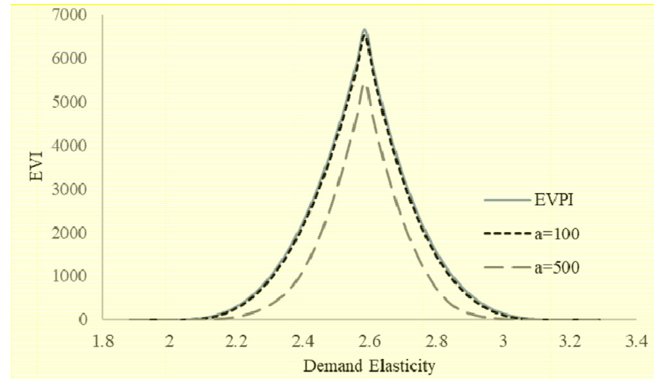

Fig. 10. The expected value of imperfect information (EVI) for the numeric example.

8. Discussion and application

This work is motivated by the need to provide an estimate of the EVPI on uncertain demand when little prior information is available. By providing an upper bound for how much a firm should invest in activities related to estimating demand, the results of this work provide a useful benchmark early in the process of determining product price and other decision making processes. In the case of a firm that is considering two prices for a new product and can specify its prior belief of the demand distribution as either (i) a uniform distribution over one price and the level of demand elasticity, or (ii) a uniform distribution for one price and the mean demand for a second price, information from which demand elasticity can be calculated, an exact expression for EVPI can be found. The firm should not invest more time or resources in clarifying the demand distribution than the EVPI. In the case of a firm that believes the demand distribution is more likely to follow a normal distribution, the firm may identify the uniform distribution that has the same mean and variance as the normal distribution. It may then use the EVPI to calculate an upper bound on information gathering activities, keeping in mind that the use of the uniform distribution overestimates the upper bound by a margin of approximately 6% in the numeric example studied.

Perhaps most usefully, the EVPI expression allows firms to determine situations in which EVPI is zero, emphasizing that firms can lose value by seeking information in the wrong situation. In the numeric example, the firm specifies an initial estimate of the demand distribution as uniform from 8000 to 12,000 units at a price of \$50. If the firm is also considering a price of \$100 and estimates the mean demand at \$100 to be any value less than 2500 units, then the EVPI is zero, and the firm should proceed with its preferred price. The firm may also conduct sensitivity analyses to the specification of the initial demand distribution and the mean demand at the second price to determine how much the initial estimates would need to change in order for the EVPI to be nonzero. Given the similarity of EVPI for the uniform and normal distribution, the ability to identify scenarios when the firm should proceed with the pricing decision extend to normally distributed demand as well.

Additionally, the results elucidate the nuanced relationships between price elasticity of demand, initial price preferences, and the correlation between demand at different prices. These patterns provide additional insight for firms on the basis of these parameters. For firms that initially prefer the low (high) price, the EVPI approaches zero as elasticity increases (decreases) for the cases of independent demand and perfectly negatively correlated. In the special case of perfect positive correlation, the EVPI is zero. This finding suggests that when a firm believes there is strong correlation for demand at the considered prices, it should carefully examine whether EVPI is zero. On the other hand, if the firm believes negative correlation may exist, then EVPI may be higher since EVPI for a particular initial price preference and elasticity is at a maximum in the case of perfect negative correlation. When EVPI is nonzero, the results highlight the importance of a firm's treatment of correlation. Assuming demand is independent when a positive correlation exists causes much more error in the calculation of the EVPI than it does to assume an incorrect correlational structure, i.e. assume the copula when the Gaussian copula is correct or vice versa. This result underscores the danger of assuming independence for the purpose of simplifying the problem.

Finally, we show that the EVI follows the same monotonic pattern with respect to demand elasticity, illustrating how numeric results may be used for further study of information gathering activities in the case the EVPI, the upper bound on the value of these activities, is nonzero.

9. Conclusion

This paper examines the EVPI on demand taking into account the price elasticity of demand and potential correlation between demand at two different prices. This work provides useful analytic results for EVPI that inform pricing decisions before a product is placed on the market, providing a useful upper bound on the value of investment in data acquisition and analysis early in decision making processes when minimal information is available to specify a prior demand distribution. Patterns of correlation and EVPI are examined, and numeric analysis of EVPI in the case of non-uniform demand distributions shows close alignment between the EVPI for demand that follows a uniform distribution or that follows a truncated normal distribution with the same first and second moments. Numeric results also show how the assumption of perfect information can be relaxed to quantify how the value of information decreases with decreases in the accuracy of the information that can be obtained. Firms, however, must remain cognizant of the limitations of this work which include the assumption that elasticity is constant and the assumption that the cost of producing the product is deterministic and known. Future research is needed to consider how relaxing these assumptions of the problem formulation might affect EVPI and how other factors may affect the value of information in pricing decisions.

Overall, the results underscore the nuanced behavior of EVPI. It is not monotonic with demand elasticity and can have special properties as elasticity increases or decreases, making the application of these results quite useful to fields with highly elastic or highly inelastic demand. The results provide insight on how demand elasticity and the correlation of demand for a product at different prices affect the EVPI and underscore the importance of including the effects of correlation in the calculation of the EVPI.

Acknowledgement

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Appendix

Proof of Proposition 1: We shall prove it by considering four different cases on M_i and m_i, $i = 1, 2$:

(i) $m_2 < M_2 < m_1 < M_1$: The risk-neutral decision maker will always ask for the low selling price s_1 with or without the information since the profit at s_1 is always higher than the profit at s_2. Therefore, $EVPI = 0$, which satisfies (8) since $(M_2 - m_1)^3 < 0$ in this case.

(ii) $m_2 < m_1 \le M_2 < M_1$: Recall that the risk-neutral decision maker will ask for the low selling price s_1 without the information. Therefore, the expected value of information

$$
\begin{aligned}
EVPI &= \int_{\bar{d}_1(1-b)}^{\bar{d}_1(1+b)} \int_{\bar{d}_2(1-b)}^{\bar{d}_2(1+b)} f(d_1, d_2) \\
&\quad (\max\{d_2(s_2-c), d_1(s_1-c)\} - d_1(s_1-c)) dd_2 dd_1 \\
&= \int_{\bar{d}_1(1-b)}^{\frac{\bar{d}_2(1+b)(s_2-c)}{s_1-c}} \int_{\frac{d_1(s_1-c)}{s_2-c}}^{\bar{d}_2(1+b)} f(d_1, d_2)(d_2(s_2-c) \\
&\quad - d_1(s_1-c)) dd_2 dd_1 \\
&= \frac{1}{4\bar{d}_1\bar{d}_2 b^2} \int_{\bar{d}_1(1-b)}^{\frac{\bar{d}_2(1+b)(s_2-c)}{s_1-c}} \int_{\frac{d_1(s_1-c)}{s_2-c}}^{\bar{d}_2(1+b)} (d_2(s_2-c) \\
&\quad - d_1(s_1-c)) dd_2 dd_1.
\end{aligned}
$$

We use the transformation $u = d_2(s_2-c) - d_1(s_1-c)$ and $v = d_2(s_2-c) + d_1(s_1-c)$. Therefore, $d_1 = \frac{v-u}{2(s_1-c)}$ and $d_2 = \frac{u+v}{2(s_2-c)}$, and the determinant of their Jacobian matrix is

$$
J = \begin{vmatrix} \frac{\partial d_1}{\partial u} & \frac{\partial d_1}{\partial v} \\ \frac{\partial d_2}{\partial u} & \frac{\partial d_2}{\partial v} \end{vmatrix} = \begin{vmatrix} -\frac{1}{2(s_2-c)} & \frac{1}{2(s_1-c)} \\ \frac{1}{2(s_2-c)} & \frac{1}{2(s_1-c)} \end{vmatrix} = -\frac{1}{2(s_1-c)(s_2-c)}.
$$

Hence,

$$
\begin{aligned}
EVPI &= \frac{1}{4\bar{d}_1\bar{d}_2 b^2} \int_0^{\bar{d}_2(1+b)(s_2-c)-\bar{d}_1(1-b)(s_1-c)} \int_{2\bar{d}_1(1-b)(s_1-c)+u}^{2\bar{d}_2(1+b)(s_1-c)-u} \\
|J| udvdu &= \frac{1}{8\bar{d}_1\bar{d}_2 b^2 (s_1-c)(s_2-c)} \int_0^{M_2-m_1} \int_{2m_1+u}^{2M_2-u} udvdu \\
&= \frac{1}{4\bar{d}_1\bar{d}_2 b^2 (s_1-c)(s_2-c)} \int_0^{M_2-m_1} [M_2 - m_1 - u] udv \\
&= \frac{1}{(M_1-m_1)(M_2-m_2)} \left(\left[\frac{M_2-m_1}{2} u^2 - \frac{u^3}{3} \right] \Bigg|_0^{M_2-m_1} \right) \\
&= \frac{(M_2-m_1)^3}{6(M_1-m_1)(M_2-m_2)}.
\end{aligned}
$$

Note that $M_1 - m_2 > M_2 - m_1 \ge 0$, since $m_2 < m_1 < M_2 < M_1$. Hence, $EVPI = \frac{(M_2-m_1)^3}{6(M_1-m_1)(M_2-m_2)}$ satisfies (8).

(iii) $m_1 \le m_2 < M_1 \le M_2$: Recall that the risk-neutral decision maker will ask for the high selling price s_2 without the information. Therefore, the EVPI becomes

$$
\begin{aligned}
EVPI &= \int_{\bar{d}_2(1-b)}^{\bar{d}_2(1+b)} \int_{\bar{d}_1(1-b)}^{\bar{d}_1(1+b)} f(d_1, d_2) \\
&\quad \times (\max\{d_2(s_2-c), d_1(s_1-c)\} - d_2(s_2-c)) dd_1 dd_2 \\
&= \int_{\bar{d}_2(1-b)}^{\frac{\bar{d}_1(1+b)(s_1-c)}{s_2-c}} \int_{\frac{d_2(s_2-c)}{s_1-c}}^{\bar{d}_1(1+b)} \\
&\quad \times f(d_1, d_2)(d_1(s_1-c) - d_2(s_2-c)) dd_2 dd_1 \\
&= \frac{1}{4\bar{d}_1\bar{d}_2 b^2} \int_{\bar{d}_2(1-b)}^{\frac{\bar{d}_1(1+b)(s_1-c)}{s_2-c}} \int_{\frac{d_2(s_2-c)}{s_1-c}}^{\bar{d}_1(1+b)} \\
&\quad \times (d_1(s_1-c) - d_2(s_2-c)) dd_2 dd_1.
\end{aligned}
$$

Using the same transformation in case (ii), we have

$$
\begin{aligned}
EVPI &= -\frac{1}{4\bar{d}_1\bar{d}_2 b^2} \int_{\bar{d}_2(1-b)(s_2-c)-\bar{d}_1(1+b)(s_1-c)}^{0} \int_{2\bar{d}_2(1-b)(s_2-c)-u}^{2\bar{d}_1(1+b)(s_1-c)+u} |J| udvdu \\
&= -\frac{1}{8\bar{d}_1\bar{d}_2 b^2 (s_1-c)(s_2-c)} \int_{m_2-M_1}^{0} \int_{2m_2-u}^{2M_1+u} udvdu \\
&= -\frac{1}{4\bar{d}_1\bar{d}_2 b^2 (s_1-c)(s_2-c)} \int_{m_2-M_1}^{0} [M_1 - m_2 + u] udv \\
&= \frac{1}{(M_1-m_1)(M_2-m_2)} \left(\left[-\frac{M_1-m_2}{2} u^2 - \frac{u^3}{3} \right] \Bigg|_{m_2-M_1}^{0} \right) \\
&= \frac{(M_1-m_2)^3}{6(M_1-m_1)(M_2-m_2)}.
\end{aligned}
$$

Note, $M_2 - m_1 \geq M_1 - m_2 > 0$, since $m_1 \leq m_2 < M_1 \leq M_2$. Hence, $EVPI = \frac{(M_1-m_2)^3}{6(M_1-m_1)(M_2-m_2)}$ satisfies (8).

(iv) $m_1 < M_1 \leq m_2 < M_2$: The risk-neutral decision maker will always ask for the high selling price s_2 with or without the information since the profit at s_2 is always higher than the profit at s_1. Therefore, $EVPI = 0$, which satisfies (8) since $(M_1 - m_2)^3 \leq 0$ in this case.

Proof of Propositions 2: Note that the expectation of the uncertain demand at the high selling price will increase as the demand elasticity decreases when the expectation of the uncertain demand at the low selling price is given. So does the expected profit at the high selling price. Therefore, if a risk neutral decision maker asks for the high selling price without the information on the uncertain demands at the two prices, then he/she will keep asking for the high selling price without the information as the demand elasticity decreases. Hence,

$$EVPI = \frac{(M_1 - m_2)^3}{6(M_1 - m_1)(M_2 - m_2)} \tag{16}$$

Recall that $M_2 = \overline{d_2}(1+b)(s_2-c)$, we know that $\frac{\partial M_2}{\partial \bar{d_2}} = (1+b)(s_2-c) = \frac{M_2}{\bar{d_2}}$. Similarly, $\frac{\partial m_2}{\partial \bar{d_2}} = \frac{m_2}{\bar{d_2}}$. Therefore,

$$\frac{\partial EVPI}{\partial \bar{d_2}} = \frac{\frac{\partial (M_1-m_2)^3}{\partial \bar{d_2}}(M_2-m_2) - \frac{\partial (M_2-m_2)}{\partial \bar{d_2}}(M_1-m_2)^3}{6(M_1-m_1)(M_2-m_2)^2}$$

$$= -\frac{3m_2(M_1-m_2)^2(M_2-m_2) + (M_2-m_2)(M_1-m_2)^3}{6\bar{d_2}(M_1-m_1)(M_2-m_2)^2} < 0.$$

Hence, the EVPI is strictly decreasing with the mean demand at the high selling price in this case. Note that the mean demand at the high selling price is strictly increasing as the demand elasticity decreases. Therefore, the EVPI is strictly decreasing as the demand elasticity decreases.

Proof of Propositions 3: Note that the expectation of the uncertain demand at the high selling price will decrease as the demand elasticity increases when the expectation of the uncertain demand at the low selling price is given. So does the expected profit at the high selling price. Therefore, if a risk neutral decision maker asks for the low selling price without the information on the uncertain demands at the two prices, then he/she will keep asking for the low selling price without the information as the demand elasticity increases. Hence the EVPI in this case is

$$EVPI = \frac{(M_2 - m_1)^3}{6(M_1 - m_1)(M_2 - m_2)} \tag{17}$$

Recall that $M_2 = \overline{d_2}(1+b)(s_2-c)$, we know that $\frac{\partial M_2}{\partial \bar{d_2}} = (1+b)(s_2-c) = \frac{M_2}{\bar{d_2}}$. Similarly, $\frac{\partial m_2}{\partial \bar{d_2}} = \frac{m_2}{\bar{d_2}}$. Therefore,

$$\frac{\partial EVPI}{\partial \bar{d_2}} = \frac{\frac{\partial (M_2-m_1)^3}{\partial \bar{d_2}}(M_2-m_2) - \frac{\partial (M_2-m_2)}{\partial \bar{d_2}}(M_2-m_1)^3}{6(M_1-m_1)(M_2-m_2)^2}$$

$$= \frac{3(M_2-m_1)^2(2M_2+m_1)}{6\bar{d_2}(M_1-m_1)(M_2-m_2)} > 0$$

Hence, the EVPI is strictly increasing with the mean demand at the high selling price in this case. Note that the mean demand at the high selling price is strictly decreasing as the demand elasticity increases. Therefore, the EVPI is strictly decreasing as the demand elasticity increases.

Proof of Proposition 4: If risk neutral decision maker values the information on the uncertain demands highest when he/she is indifferent between the high selling price and the low selling price without the information, then he/she will ask for the high selling price as the demand elasticity decreases and ask for the low selling price as the demand elasticity increases without the information. Therefore, his/her value of information will decrease whenever the demand elasticity decreases or increases per to Propositions 2 and 3. Hence, he/she will value the information highest among all possible demand elasticities in this case.

Proof of Proposition 5: If $d_2 = (\frac{s_1}{s_2})^\eta d_1$, then $\frac{d_1}{d_2} = (\frac{s_2}{s_1})^\eta$, which is a constant. Hence, $(s_1-c)d_1 < (s_2-c)d_2$ when $(\frac{s_2}{s_1})^\eta < \frac{s_2-c}{s_1-c}$ and $(s_1-c)d_1 \geq (s_2-c)d_2$, otherwise. Recall that the profits for the high selling price and the low selling price are $(s_1-c)d_1$ and $(s_2-c)d_2$, respectively. Therefore, any type of decision maker will ask for the high selling price if $(\frac{s_2}{s_1})^\eta < \frac{s_2-c}{s_1-c}$ and ask for the low selling price otherwise with or without the information on the uncertain demands at these two prices. Hence, any decision maker will pay nothing for the information on the uncertain demands.

Proof of Proposition 6: To simplify the notation, we denote the expected profit at the two prices s_1, s_2 as $p_i = (s_i - c)\bar{d_i} = 0.5M_i + 0.5m_i$, $i = 1, 2$. We shall prove it by considering four different cases on M_i andm_i, $i = 1, 2$:

(i) $m_2 < M_2 < m_1 < M_1$: The risk-neutral decision maker will always ask for the low selling price s_1 with or without the information since the profit at s_1 is always higher than the profit at s_2. Therefore, $EVPI = 0$, which satisfies (9) since $(M_2 - m_1)^2 < 0$.

(ii) $m_2 < m_1 \leq M_2 < M_1$: Recall that the risk-neutral decision maker will ask for the low selling price s_1 without the information. Therefore,

$$\begin{aligned}
EVPI = &\int_{\bar{d_1}(1-b)}^{\bar{d_1}(1+b)} f(d_1) \\
&\times \Big[\max\Big\{\Big(\frac{s_1}{s_2}\Big)^\eta \big(2\bar{d_1} - d_1\big)(s_2-c), d_1(s_1-c)\Big\} \\
&- d_1(s_1-c)\Big] dd_1 \\
= &\int_{\bar{d_1}(1-b)}^{\frac{2(M_2+m_2)\bar{d_1}}{M_1+M_2+m_1+m_2}} f(d_1) \\
&\times \Big[\Big(\frac{s_1}{s_2}\Big)^\eta \big(2\bar{d_1} - d_1\big)(s_2-c) - d_1(s_1-c)\Big] dd_1 \\
= &\int_{\bar{d_1}(1-b)}^{\frac{2(M_2+m_2)\bar{d_1}}{M_1+M_2+m_1+m_2}} \frac{1}{2b\bar{d_1}} \\
&\times \Big[2(M_2+m_2) - \frac{d_1}{\bar{d_1}}(M_1+M_2+m_1+m_2)\Big] dd_1 \\
= &\int_0^{M_2-m_1} \frac{t}{2b(M_1+M_2+m_1+m_2)} dt \\
= &\frac{(M_2-m_1)^2}{2b(M_1+M_2+m_1+m_2)},
\end{aligned}$$

Where $t = 2(M_2+m_2) - \frac{d_1}{\bar{d_1}}(M_1+M_2+m_1+m_2)$. Note that $M_1 - m_2 > M_2 - m_1 \geq 0$,since $m_2 < m_1 < M_2 < M_1$. Hence, $EVPI = \frac{(M_2-m_1)^2}{2b(M_1+M_2+m_1+m_2)}$, which satisfies (9).

(iii) $m_1 \leq m_2 < M_1 \leq M_2$: Recall that the risk-neutral decision maker will ask for the high selling price s_2 without the information. Therefore,

$$\begin{aligned}
EVPI = &\int_{\bar{d_1}(1-b)}^{\bar{d_1}(1+b)} f(d_1) \\
&\times \left[\begin{matrix}\max\Big\{\big(\frac{s_1}{s_2}\big)^\eta \big(2\bar{d_1} - d_1\big)(s_2-c), d_1(s_1-c)\Big\} \\ -\big(\frac{s_1}{s_2}\big)^\eta\big(2\bar{d_1} - d_1\big)(s_2-c)\end{matrix}\right] dd_1
\end{aligned}$$

$$= \int_{\frac{2(M_2+m_2)\bar{d}_1}{M_1+M_2+m_1+m_2}}^{\bar{d}_1(1+b)} \frac{1}{2b\bar{d}_1}\left[d_1(s_1-c)-\left(\frac{s_1}{s_2}\right)^{\eta}\left(2\bar{d}_1-d_1\right)(s_2-c)\right]dd_1$$

$$= \int_{\frac{2(M_2+m_2)\bar{d}_1}{M_1+M_2+m_1+m_2}}^{\bar{d}_1(1+b)} \frac{1}{2b\bar{d}_1}\left[\frac{d_1}{\bar{d}_1}(M_1+M_2+m_1+m_2)-2(M_2+m_2)\right]dd_1$$

$$= \int_0^{M_1-m_2} \frac{s}{2b(M_1+M_2+m_1+m_2)}ds = \frac{(M_1-m_2)^2}{2b(M_1+M_2+m_1+m_2)},$$

where $t = 2(M_2+m_2) - \frac{d_1}{\bar{d}_1}(M_1+M_2+m_1+m_2)$. Note that $M_2 - m_1 > M_1 - m_2 \geq 0$, since $m_1 \leq m_2 < M_1 \leq M_2$. Hence, $EVPI = \frac{(M_1-m_2)^2}{2b(M_1+M_2+m_1+m_2)}$ satisfies (9).

(iv) $m_1 < M_1 \leq m_2 < M_2$: The risk-neutral decision maker will always ask for the high selling price s_2 with or without the information since the profit at s_2 is always higher than the profit at s_1. Therefore, $EVPI = 0$, which satisfies (9) since $(M_1 - m_2)^2 < 0$.

Proof of Propositions 7: Note that the expectation of the uncertain demand at the high selling price will increase as the demand elasticity decreases when the expectation of the uncertain demand at the low selling price is given. So does the expected profit at the high selling price. Therefore, if a risk neutral decision maker asks for the high selling price without the information on the uncertain demands at the two prices, then he/she will keep asking for the high selling price without the information as the demand elasticity decreases. Hence,

$$EVPI = \frac{(M_1-m_2)^2}{2b(M_1+M_2+m_1+m_2)}. \tag{18}$$

Recall that $M_2 = \overline{d_2}(1+b)(s_2-c)$, we know that $\frac{\partial M_2}{\partial \bar{d}_2} = (1+b)(s_2-c) = \frac{M_2}{\bar{d}_2}$. Similarly, $\frac{\partial m_2}{\partial \bar{d}_2} = \frac{m_2}{\bar{d}_2}$. Therefore,

$$\frac{\partial EVPI}{\partial \bar{d}_2} = \frac{\frac{\partial (M_1-m_2)^2}{\partial \bar{d}_2}(M_1+M_2+m_1+m_2) - \frac{\partial (M_1+M_2+m_1+m_2)}{\partial \bar{d}_2}(M_1-m_2)^2}{2b(M_1+M_2+m_1+m_2)^2}$$

$$= \frac{-2(M_1-m_2)m_2(M_1+M_2+m_1+m_2)-(M_2+m_2)(M_1-m_2)^2}{2b\bar{d}_2(M_1+M_2+m_1+m_2)^2}$$

$$= -\frac{(M_2-m_1)[2m_2(M_1+M_2+m_1+m_2)+(M_2+m_2)(M_1-m_2)]}{2b\bar{d}_2(M_1+M_2+m_1+m_2)^2} < 0.$$

Hence, EVPI is strictly decreasing with the mean demand at the high selling price in this case. Note that the mean demand at the high selling price is strictly increasing as the demand elasticity decreases. Therefore, EVPI is strictly decreasing as the demand elasticity decreases. Also, if $M_2 \leq m_1$, then the risk-neutral decision maker will always ask for the low selling price s_1 with or without the information since the profit at s_1 is always higher than the profit at s_2. Therefore, EVPI=0, which is trivial in Propositions 7. This will be true in Proposition 8 as well. Therefore, we can assume $M_2 > m_1$ in these proofs.

Proof of Propositions 8: Note that the expectation of the uncertain demand at the high selling price will decrease as the demand elasticity increases when the expectation of the uncertain demand at the low selling price is given. So does the expected profit at the high selling price. Therefore, if a risk neutral decision maker asks for the low selling price without the information on the uncertain demands at the two prices, he/she will keep asking for the low selling price without the information as the demand elasticity increases. Hence the EVPI in this case is

$$EVPI = \frac{(M_2-m_1)^2}{2b(M_1+M_2+m_1+m_2)} \tag{19}$$

Recall $M_2 = \overline{d_2}(1+b)(s_2-c)$, and $\frac{\partial M_2}{\partial \bar{d}_2} = (1+b)(s_2-c) = \frac{M_2}{\bar{d}_2}$. Similarly, $\frac{\partial m_2}{\partial \bar{d}_2} = \frac{m_2}{\bar{d}_2}$. Therefore,

$$\frac{\partial EVPI}{\partial \bar{d}_2} = \frac{\frac{\partial (M_2-m_1)^2}{\partial \bar{d}_2}(M_1+M_2+m_1+m_2) - \frac{\partial (M_1+M_2+m_1+m_2)}{\partial \bar{d}_2}(M_2-m_1)^2}{2b(M_1+M_2+m_1+m_2)^2}$$

$$= \frac{2(M_2-m_1)M_2(M_1+M_2+m_1+m_2)-(M_2+m_2)(M_2-m_1)^2}{2b\bar{d}_2(M_1+M_2+m_1+m_2)^2}$$

$$= \frac{(M_2-m_1)[2M_2(M_1+M_2+m_1+m_2)-(M_2+m_2)(M_2-m_1)]}{2b\bar{d}_2(M_1+M_2+m_1+m_2)^2}$$

$$> \frac{(M_2-m_1)(M_2+m_2)[(M_1+M_2+m_1+m_2)-(M_2-m_1)]}{2b\bar{d}_2(M_1+M_2+m_1+m_2)^2}$$

$$= \frac{(M_2-m_1)(M_2+m_2)(M_1+2m_1+m_2)}{2b\bar{d}_2(M_1+M_2+m_1+m_2)^2} > 0$$

Hence, the EVPI is strictly increasing with the mean demand at the high selling price in this case. Note that the mean demand at the high selling price is strictly decreasing as the demand elasticity increases. Therefore, the EVPI is strictly decreasing as the demand elasticity increases.

Proof of Proposition 9: If risk neutral decision maker values the information on the uncertain demands highest when he/she is indifferent between the high selling price and the low selling price without the information, then he/she will ask for the high selling price as the demand elasticity decreases and ask for the low selling price as the demand elasticity increases without the information. Therefore, his/her EVPI will decrease whenever the demand elasticity decreases or increases per to Propositions 7 and 8. Hence, he/she will value the information highest among all possible demand elasticities in this case.

Proof of Proposition 10: Denote $p_i = (s_i - c)\bar{d}_i$ as the expected profit as selling price s_i, $i = 1, 2$, respectively. Then $M_i = p_i(1+b)$ and $m_i = p_i(1-b)$, $i = 1, 2$, respectively. If $M_1 \leq m_2$ or $M_2 \leq m_1$, then $EVPI_{-1} = EVPI_0 = 0$. Otherwise, if $m_2 < M_1 < M_2$, then $p_1 < p_2$ and $0 < M_1 - m_2 < M_2 - m_1$. Therefore,

$$\begin{aligned}\frac{EVPI_0}{EVPI_{-1}} &= \frac{(M_1-m_2)^3}{6(M_1-m_1)(M_2-m_2)} \cdot \frac{2b(M_1+M_2+m_1+m_2)}{(M_2-m_1)^2} \\ &= \frac{b(M_1+M_2+m_1+m_2)(M_1-m_2)}{3(M_1-m_1)(M_2-m_2)} \\ &= \frac{2b(p_1+p_2)(p_1(1+b)-p_2(1-b))}{12b^2p_1p_2} \\ &= \frac{(p_1+p_2)(p_1(1+b)-p_2(1-b))}{6bp_1p_2}\end{aligned} \tag{20}$$

Note that

$$\begin{aligned}&(p_1+p_2)(p_1(1+b)-p_2(1-b)) \\ &= (1-b)(p_1+p_2)(p_1-p_2)+2bp_1(p_1+p_2) \\ &< 2bp_1(p_1+p_2) < 4bp_1p_2.\end{aligned} \tag{21}$$

Substituting (16) into (14) gives $\frac{EVPI_0}{EVPI_{-1}} < \frac{2}{3}$. Therefore, $0 < EVPI_0 < \frac{2}{3}EVPI_{-1}$.

Similarly, we can prove that if $m_1 < M_2 < M_1$, then $0 < EVPI_0 < \frac{2}{3}EVPI_{-1}$.

References

Abbas, A. E., Bakir, N. O., Klutke, G. A., & Sun, Z. (2013). Effects of risk aversion on the value of information in two-action decision problems. *Decision Analysis, 10*(3), 257–275.

Asgari, N., Nikbakhsh, E., Hill, A., & Zanjirani Farahani, R. (2016). Supply chain management 1982-2015: A review. *IMA Journal of Management Mathematics, 27*, 353–379.

Aviv, Y. (2001). The effect of collaborative forecasting on supply chain performance. *Management Science, 47*(10), 1326–1343.

Axsäter, S. (2013). When is it feasible to model low demand by a normal distribution. *OR Spectrum, 35*(1), 153–162.

Azadian, F., & Murat, A. (2017). Service location grouping and pricing in transportation: Application in air cargo. *European Journal of Operational Research, 267*(3), 933–943.

Babai, M. Z., Boylan, J. E., Syntetos, A. A., & Ali, M. M. (2016). Reduction of the value of information sharing as demand becomes strongly auto-correlated. *International Journal of Production Economics, 143*, 463–471.

Baker, W., Kiewell, D., & Winkler, G. (2014). *Using big data to make better pricing decisions. McKinsey & Company: Marketing & Sales Insights*. Accessed March 17, 2019. <https://www.mckinsey.com/business-functions/marketing-and-sales/our-insights/using-big-data-to-make-better-pricing-decisions>.

Berk, E., Gürler, Ü., & Levine, R. A. (2007). Bayesian demand updating in the lost sales newsvendor problem: A two-moment approximation. *European Journal of Operational Research, 182*(1), 256–281.

Bray, R., & Mendelson, H. (2012). Information transmission and the bullwhip effect: An empirical investigation. *Management Science, 58*(5), 860–875.

Cachon, G. P., & Fisher, M. (2000). Supply chain inventory management and the value of shared information. *Management Science, 46*(8), 1032–1048.

Casado, E., & Ferrer, J. (2013). Consumer price sensitivity in the retail industry: Latitude of acceptance with heterogeneous demand. *European Journal of Operational Research, 228*(2), 418–426.

Chatfield, D. C., Kim, J. G., Harrison, T. P., & Hayya, J. C. (2004). The bullwhip effect-impact of stochastic lead time, information quality, and information sharing: A simulation study. *Production and Operations Management, 13*(4), 340–353.

Chen, J. (2011). The impact of sharing customer returns information in a supply chain with and without a buyback policy. *European Journal of Operational Research, 213*(3), 478–488.

Chen, L., & Lee, H. L. (2009). Information sharing and order variability control under a generalized demand model. *Management Science, 55*(5), 781–797.

Ciancimino, E., Cannella, S., Bruccoleri, M., & Framinan, J. M. (2012). On the bullwhip avoidance phase: The synchronized supply chain. *European Journal of Operational Research, 221*, 49–63.

Delequié, P. (2008). The value of information and intensity of preference. *Decision Analysis, 4*(3), 129–139.

Ferreira, K. J., Lee, B. H. A., & Simchi-Levi, D. (2016). Analytics for an online retailer: Demand forecasting and optimization. *Manufacturing & Service Operations Management, 18*(1), 69–88.

Freixas, X., & Kihlstrom, R. (1984). Risk aversion and information demand. In M. Boyer, & R. Kihlstrom (Eds.), *Bayesian models in economic theory* (pp. 93–104). Amsterdam: North Holland Publishing.

George, J., Mercer, A., & Wilson, H. (1996). Variations in price elasticities. *European Journal of Operational Research, 228*(2), 418–426.

Gould, J. P. (1974). Risk, stochastic preference, and the value of information. *Journal of Economic Theory, 8*, 64–84.

Hill, R. M. (1997). Applying Bayesian methodology with a uniform prior to the single period inventory model. *European Journal of Operational Research, 98*(3), 555–562.

Hilton, R. (1981). Determinants of information value. *Management Science, 27*(1), 57–64.

Howard, R. A. (1966). Information value theory. *IEEE Transactions on Systems Science and Cybernetics, 2*(1), 22–26.

Howard, R. A., & Abbas, A. E. (2016). *Foundations of decision analysis*. Boston, MA: Pearson Education Limited.

Hsieh, C. C., Liu, Y. T., & Wang, W. M. (2010). Coordinating ordering and pricing decisions in a two-stage distribution system with price-sensitive demand through short-term discounting. *European Journal of Operational Research, 207*(1), 142–151.

Iyer, A., & Ye, J. (2000). Assessing the value of information sharing in a promotional retail environment. *Manufacturing and Service Operations Management, 2*(2), 128–143.

Keisler, J. (2005). Additivity of information value in two-act linear loss decisions with normal priors. *Risk Analysis, 25*(2), 351–359.

Ketzenberg, E. K., Rosenzweig, E. D., Marucheck, A. D., & Metters, R. D. (2007). A framework for the value of information in inventory replenishment. *European Journal of Operational Research, 182*, 1230–1250.

Laffont, J. J. (1976). Risk, stochastic preference, and the value of information: A comment. *Journal of Economic Theory, 12*, 483–487.

Lee, H. L., So, K. C., & Tang, C. S. (2000). The value of information sharing in a two-level supply chain. *Management Science, 46*(5), 626–643.

Leland, H. E. (1972). Theory of the Firm Facing Uncertain Demand. *The American Economic Review, 62*(3), 278–291.

McAfee, R. P., & te Velde, V. (2008). Dynamic pricing with constant demand elasticity. *Production and Operations Management, 17*(4), 432–438.

Mehrez, A., & Stulman, A. (1982). Some aspects of the distributional properties of the expected value of perfect information (EVPI). *Journal of the Operational Research Society, 33*(9), 827–836.

Mercer, A. (1993). Developments in implementable retailing research. *European Journal of Operational Research, 68*(1), 1–8.

Nelson, R. B. (1998). *An introduction to copulas*. New York, NY: Springer-Verlag.

Özer, Ö., & Phillips, R. (Eds.). (2012). *The oxford handbook of pricing management*. Oxford: Oxford University Press.

Pauler, G., & Dick, A. (2006). Maximizing profit of a food retailing chain by targeting and promoting valuable customers using loyalty card and scanner data. *European Journal of Operational Research, 174*(1), 1260–1280.

Perakis, G., & Roels, G. (2008). Regret in the newsvendor model with partial information. *Operations Research, 56*(1), 188–203.

Petruzzi, N., & Dada, M. (1999). Pricing and the newsvendor problem: A review with extensions. *Operations Research, 47*(2), 183–194.

Raghunathan, S. (2001). Information-sharing in a supply chain. *Management Science, 30*(4), 1053–1071.

Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal, 27* 379-423 & 623-656.

Sklar, A. (1959). Fonctions de répartition à *n* dimensions et leurs marges. *Publications de l'Institut de Statistique de l'Université de Paris, 8*, 229–231.

Strijbosch, L. W. G., & Moors, J. J. A. (2005). The impact of unknown demand parameters on (r,s)-inventory control performance. *European Journal of Operational Research, 172*(1), 201–212.

Sun, Z., & Abbas, A. E. (2014). On the sensitivity of the value of information to risk aversion in two-action decision problems. *Environment Systems and Decisions, 1*(34), 24–37.

Sun, Z., Hupman, A. C., Ritchey, H. I., & Abbas, A. E. (2016). Bayesian updating of the price elasticity of uncertain demand. *IEEE Systems Journal, 10*(1), 136–146.

Syntetos, A., Babai, Z., Boylan, J. E., Kolassa, S., & Nikolopoulos, K. (2016). Supply chain forecasting: Theory, practice, their gap and the future. *European Journal of Operational Research, 252*, 1–26.

Tellies, G. (1988). The price elasticity of selective demand: A meta-analysis of econometric models of sales. *Journal of Marketing Research, 25*(4), 331–340.

Teunter, R. H., Babai, M. Z., Bokhorst, J. A. C., & Syntetos, A. A. (2018). Revisiting the value of information sharing in two-stage supply chains. *European Journal of Operational Research, 270*, 1044–1052.

Zellner, M., & Abbas, A. E. (2018). The effects of demand elasticity and selling price decisions on the value of information. In *Proceedings of the IEEE International Systems Conference (SysCon), Vancouver, BC, Canada* (pp. 1–8).

Zhou, M., Dan, B., Songxuan, M., & Xumei, Z. (2017). Supply chain coordination with information sharing: The informational advantage of GPOs. *European Journal of Operational Research, 256*(3), 785–802.