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# Costs of energy efficiency mandates can reverse the sign of rebound☆



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#### ABSTRACT

Improvements in energy efficiency reduce the cost of consuming services from household cars and appliances and can result in a positive rebound effect that offsets part of the direct energy savings. We use a general equilibrium model to derive analytical expressions that allow us to compare rebound effects from a costless technology shock (CTS) to those from a costly energy efficiency standard (EES). We decompose each total effect on the use of energy into a direct efficiency effect, direct rebound effect, and indirect rebound effect. We show which factors determine the sign and magnitude of each. Rebound from a CTS is generally positive, as in prior literature, but we also show how a pre-existing EES can negate the direct energy savings from the CTS – leaving only the positive rebound effect on energy use. Then we analyze increased stringency of an EES, and we show exactly when the increased costs reverse the sign of rebound. Using plausible parameter values in this model, we find that indirect effects can outweigh the direct effects captured in partial equilibrium models, and that the total rebound from a costly EES can be negative.

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New energy efficiency technology can reduce electricity or fuel use needed to get the same services such as cooling from an air conditioner or refrigerator, heat from a furnace, or miles driven in a car. It reduces the marginal cost of such services, so it encourages consumers to make more use of those appliances – and thus causes a "rebound" effect that offsets at least part of the energy savings. Many papers demonstrate this positive rebound effect in empirical partial equilibrium (PE) models and analytical or numerical general equilibrium (GE) models. They analyze effects of a costless technology shock and show important economic effects of technology. Many discuss policy implications.

Here, we demonstrate problems interpreting those estimates and their implications for energy policy. Our point is that policy cannot require a costless technology shock (CTS). It can require that firms provide more energy efficiency in appliances they sell, which likely adds costs. Extra costs mean that economic effects of such requirements cannot be inferred from studies of a CTS. A few papers consider the dollar costs of particular additions to energy efficiency, but none explicitly consider policy compliance costs: as an energy efficiency standard (EES) becomes more stringent, it has welfare costs that rise disproportionately, just as deadweight loss of a tax rises with the tax rate. Our GE model provides the first analysis of rebound effects from increased

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stringency of an EES mandate, and we also derive welfare effects. Ours is the first direct comparison of an EES mandate to a CTS.

Our simple GE model has many identical, rational, and fully informed consumers or households who maximize utility from vehicle and appliance services and from a composite of other goods. We analyze a costly and binding mandate that requires more energy efficiency than consumers would buy if unconstrained. Households own a single factor of production that they sell for income to buy appliances, energy, and the other good. We model general production functions, economywide resource constraints, and zero profits of competitive firms. All equations are differentiated to linearize and solve the model, and we analvze three exogenous shocks. First, like prior literature, we consider a CTS with no pre-existing EES. That shock raises welfare and reduces direct energy use - with positive rebound. Second, we consider that CTS with an existing costly EES. Third is the essential policy question: what is the effect of increasing the energy efficiency requirement of a pre-existing mandate? It depends on the curvature of the cost of acquiring additional energy efficiency (given existing technology). If that extra efficiency is cheap, then overall welfare costs are low. But we then consider the likely case that the marginal cost of achieving additional energy efficiency is not only positive but rising - as policymakers attempt to achieve greater energy efficiency.

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<sup>&</sup>lt;sup>1</sup> In a behavioral model where consumers fail to reduce their own costs by choosing more energy efficiency, a mandate could raise welfare – with effects very similar to the costless technology shock. Here, we use a simple rational consumer model to show how increased stringency can raise costs and thus have negative rebound.

Our model is solved analytically to decompose rebound into direct effects and indirect effects, each of which is explained by income and substitution effects. Closed-form solutions show exactly how each effect depends on parameters. Then we also calibrate the model to provide numerical illustrations. We show that this view of the policy experiment can indeed improve energy efficiency and has the same kind of direct efficiency effect, reducing energy use, but the sign of overall rebound can switch from positive to negative. The sign is formally ambiguous, and so our results show exactly the conditions under which rebound is negative. And plausible cases we consider in our numerical illustrations yield negative rebound.

The intuition is clear: the CTS is an exogenous improvement in technology that makes more possible with less, expanding the production possibility frontier (PPF). It raises real income, which induces consumers to purchase more heating or cooling services. In contrast, the EES can have costs that move us to a point inside the PPF. With no pre-existing mandate, introducing a small EES has small costs. But, most developed economies have substantial energy efficiency requirements. If technology is given, and consumers are constrained to buy more-expensive appliances with more energy efficiency than they would if unconstrained, then an increase in required stringency can raise marginal costs and reduce real income. Those extra costs could be justified by benefits of reduced negative externalities from energy use, but costs of complying with the mandate still reduce household incomes and thus have negative income effects on the purchases of all goods. We show how the negative income effect offsets part of the positive direct effect on energy used for energy services, and it creates a negative indirect rebound effect on energy used in production of other goods.

We also show how rebound relates to welfare and how both depend on parameters. The mandate can cause positive or negative rebound, either of which could be associated with a net welfare gain or loss. We do not analyze a Pigovian tax on energy, as it is well studied already. We know it causes no rebound and minimizes the welfare cost for any given energy reduction. Thus, as we confirm below, the mandate is a relatively costly way to reduce energy use.

Scores of economics papers provide theoretical and empirical analyses of rebound from energy efficiency improvements, but the size of rebound is still debated. Prior papers include both PE and GE models, but most study a costless technology improvement and find positive rebound. Many of them discuss policy implications.<sup>2</sup> Of course, PE studies do not incorporate economy-wide resource constraints. Thus, they can show positive rebound effects on energy use in the sector with the efficiency improvement, but they miss the fact that a costly EES mandate could have negative income effects on energy use in other sectors (a source of negative rebound we find below). Some PE studies consider costs voluntarily incurred to obtain more energy efficiency, and they find smaller rebound than in the case of a costless technology shock.<sup>3</sup> Some find negative rebound (for reasons other than policy costs).<sup>4</sup> If consumers voluntarily pay extra for appliances with more energy

efficiency, then this choice can be presumed to raise consumer welfare, with positive effects on real income (and thus on energy use). Here, in contrast, we model a mandate that forces the purchase of more energy efficiency than desired, which can have negative income effects on all energy use.

Rebound has also been studied numerically, either in multi-sectoral GE models or dynamic growth models.<sup>5</sup> These models can include considerable detail about the economy, policies, and important forms of heterogeneity. They assume specific functional forms, so they can solve for effects of large changes in energy efficiency. They vary parameters numerically, but they do not provide closed-form analytical expressions that show explicitly how the sign and size of rebound depend on parameters. 6 Some of those numerical GE models include voluntary costs, as mentioned, and some insert an assumed cost of a particular policy (which reduces total rebound). But none model how those policy costs depend on policy stringency, to show how the parameters of the cost function enter rebound. In contrast, our simple analytical GE model is not intended for numerical accuracy but instead to provide greater intuition and clarity about what factors are driving the results. It is a conceptual model, so our calibration below can only illustrate plausible magnitudes related to those concepts.

Thus, relative to pre-existing rebound literature, our paper makes four contributions. First, we provide the first analytical GE model of rebound to employ general functional forms and linearization methods, which allow us to derive closed-form expressions for the effects on all price and quantity outcomes from a small change in energy efficiency, expressions that show exactly how those effects depend on each parameter. Second, and most important, we provide the first explicit model of an energy efficiency mandate that imposes compliance costs larger than the voluntary cost of buying more energy efficiency. These extra costs reduce "direct rebound" in the energy services sector, but in our GE model they also reduce energy use in the other sector. Our general cost function allows us to show exactly how total rebound can then turn from positive to negative with consideration of these policy costs. Third, our paper is the first to use the same model for direct comparisons of a CTS and EES, showing exactly what parameter combinations can generate the result that rebound from a CTS is positive while rebound from an EES is negative. Fourth, our paper is the first to show the effects of a CTS with a pre-existing, binding, costly EES mandate.8

This fourth contribution follows naturally from the first three, and the result may seem so intuitive to be obvious. But it does not appear in prior literature. If an existing EES mandate is costly and binding, then consumers pay more for energy efficiency than if unconstrained. If so, then any small CTS reduces the cost of achieving the mandated level of energy efficiency. It therefore saves money for consumers but does not change chosen energy efficiency. The technology improvement has no direct effect that reduces energy use, but only the indirect effect of a positive income effect that increases energy use. This positive rebound effect is called "backfire" because it exceeds the (zero) direct effect on energy use.

<sup>&</sup>lt;sup>2</sup> Reviews are in Greening et al. (2000), Sorrell and Dimitropoulos (2008), Sorrell et al. (2009), and Gillingham et al. (2016). PE models are in Thomas and Azevedo (2013) and Chan and Gillingham (2015). GE models are in Grepperud and Rasmussen (2004), Wei (2007, 2010), Abdessalem and Labidi (2016), and Lemoine (2018, 2020). Most papers back to Khazzoom (1980) use costless variation in energy efficiency to estimate rebound, and many conclude that policy mandates cause positive rebound. Frondel et al. (2012) estimate "fuel price elasticities from which rebound effects can be derived" (p. 461), meaning that they do not account for capital costs. Then they conclude: "From a policy perspective, the fact that the estimated rebound is relatively high ... calls into question the effectiveness of efficiency standards as a pollution control instrument" (p. 466).

<sup>&</sup>lt;sup>3</sup> See Mizobuchi (2008) and Nässén and Holmberg (2009).

<sup>&</sup>lt;sup>4</sup> We recommend Borenstein (2015), who starts with a price of electricity that exceeds social marginal cost, a pre-existing distortion that encourages energy efficiency. He then considers the choice to spend more for additional energy efficiency, incurring a cost that further reduces income and cause negative rebound. Yet his paper does not consider the plausibly greater cost of complying with a binding efficiency mandate.

<sup>&</sup>lt;sup>5</sup> See Allan et al. (2007), Barker et al. (2007), Turner (2009), Broberg et al. (2015), and Chang et al. (2018).

<sup>&</sup>lt;sup>6</sup> Specific functional forms such as constant elasticity of substitution production functions allow analysis of large changes, but here we use general functional forms and differentiate to linearize the model. It limits our study to small changes, but we do not attempt to analyze specific large policies. We only study the size and direction of changes associated with small shocks, but most policy changes are incremental in any case.

<sup>&</sup>lt;sup>7</sup> The only analytical GE model on rebound prior to ours is in a working paper that considers only a costless technology shock (Lemoine, 2018). Then the author was motivated by a 2019 version of our current paper to add explicit costs of energy efficiency (see Lemoine, 2020, page 4, footnote 7).

<sup>&</sup>lt;sup>8</sup> The review by Gillingham et al. (2016) points out the distinction between rebound from a costless technology shock and a costly policy shock, and it describes how empirical estimates mix the two. Interestingly, however, none of the papers in this large rebound literature has analyzed an energy efficiency improvement – either costless or costly – as we do here in a world with a pre-existing policy that is both costly and binding.

Following sections describe background (Section 1), the model (Section 2), linearization (Section 3), theoretical results for the three shocks just described (Sections 4, 5, and 6), our calibration (Section 7), numerical results (Section 8), sensitivity (Section 9), welfare effects (Section 10), extensions (Section 11), and conclusions (Section 12). All appendices are below, but later will be online only. Our section on extensions discusses rebound with: (a) negative effects of energy use on productivity, (b) upward-sloping energy supply, (c) energy efficiency standards in the other sector, and (d) disaggregation.

# 1. Policy background

First, we note that a CTS is certainly possible, and worthy of analysis. Here is one example among many. While working on an eco-friendly oven for a high school research project, a 19-year-old Filipina student in 2016 accidentally discovered an air conditioner technology that uses a disc-shaped compressor, does not need a chemical refrigerant, and uses about a quarter of the power needed by other air conditioners for the same cooling. 9 If that discovery provides cheaper cooling, consumers can save money and spend it on other energy-using goods. But that positive rebound does not apply to analysis of policies. Even if free or subsidized energy-saving investments are provided by government, then recipients may have positive income effects that increase their energy use, but someone else must be taxed to pay for it. In this case, the policy costs of interest include the disproportionate increase in the deadweight loss of taxation from raising somebody's marginal tax rate to pay for the subsidy. That policy cost has negative income effects on total rebound of energy use.

For example, a U.K. residential retrofit program offers subsidized insulation.<sup>10</sup> Their analysis uses existing empirical estimates of rebound to assume energy savings are reduced by 15% positive rebound. Yet, those empirical estimates do not account for policy costs.

In the U.S., corporate average fuel economy (CAFE) standards have periodically and incrementally raised required miles per gallon (mpg) for new cars from 18 mpg in 1978 to 27.5 mpg in 1985 (NHTSA, 2011). Rules set in 2012 require an average of 54.5 mpg by 2025. In 2018, a U.S. report chooses to use 20% rebound to calculate effects of CAFE (pp. 43099–43105, DOT/EPA, 2018), based on many studies that estimate miles driven in response to changes in the cost per mile while not accounting for income effects of higher vehicle cost and other policy costs. Our Section 7 uses engineering studies of the cost of increasing existing energy efficiency requirement for seven examples. One example is "midsize cars with spark-ignition engines", from NRC (2015). Under their "low cost" scenario, starting at 36.5 mpg, the marginal cost of raising mpg is \$68 per mpg. At 52.1 mpg, that marginal cost is \$153 per mpg. The curvature parameter we need below is that change in marginal cost (124%). In the content of the cost of raising mpg is \$68 per mpg. At 52.1 mpg, that marginal cost is \$153 per mpg. The curvature parameter we need below is that change in marginal cost (124%).

Using our analytical model below, an illustration with plausible parameters finds that rebound from a *costless* technology shock is +12% of direct energy savings, a result roughly consistent with either of those rebound assumptions (15% in the U.K. example or 20% in the U.S. example). Then, when we use the same plausible parameters in our model to analyze increased stringency of an energy efficiency standard, we find total rebound is zero or slightly negative. This latter calculation reflects the appropriate policy experiment, such as raising CAFE

standards to 54.5 mpg, indicating that DOT/EPA (2018) grossly overstates rebound.

In addition, the U.S. Department of Energy (DOE) enforces energy efficiency rules for more than 60 categories of consumer products, but we describe only one more example here from the seven calibrations. For a refrigerator-freezer, the DOE sets a maximum energy use in kilowatthours per year (kWh/yr). Using engineering estimates from DOE (2011), we calculate the marginal cost of adding energy efficiency starting at 410 kWh/yr. Then, at more stringency (341 kWh/yr), that marginal cost is almost twice as high.

Using PE models, Borenstein (2015) or Chan and Gillingham (2015) can calculate rebound for particular vehicles or appliances. For a broader perspective, our GE model is most useful for a large sector. Therefore, we analyze energy efficiency in a general way for all appliances, an aggregation of everything that uses energy: washer, dryer, refrigerator, lights, furnace, and electric space heaters. Since vehicles also face fuel standards, they are included. <sup>13</sup> Then energy for appliance services is an aggregation of all household purchase of electricity, natural gas, heating oil, propane, gasoline, and other motor fuels.

Our last section below discusses how our results can help guide policy analysis.

#### 2. The analytical general equilibrium model

For simplicity, we assume a static, one-period, closed economy with competitive firms and a large number n of identical consumers (or households). They each own and supply a single primary factor (K), which can be labor, capital, or a composite of both. As specified below, K is used in production of: energy (E), the energy efficiency of appliances (A), and a composite of all other goods such as clothing, food, and shelter (X). With this aggregation for tractability, we define A to include the energy efficiency of air conditioners, furnaces, and all other consumer durables that use energy – including vehicles. Then a costless technological discovery might increase a refrigerator's cooling per kilowatt hour of electricity, or a car's miles per gallon. Or, many nations have standards and might increase their stringency.

In a form of home production, households produce services S such as refrigeration, cooling in summer, heat in winter, and miles driven. Their inputs are their purchases of energy efficiency, A, and energy used for services,  $E_S$ . They get utility from these services, and from the other composite good, and they get disutility from the economy-wide aggregate use of energy. Thus, each consumer's utility function takes the following form:

$$U = U(S, X; nE) \tag{2.1}$$

where U is twice continuously differentiable, quasi-concave, and homothetic. It is increasing in the first two arguments and decreasing in nE, aggregate energy use.<sup>14</sup> So that changes in nE do not affect consumer choices of S or X, we assume this public bad is separable in utility.<sup>15</sup>

<sup>&</sup>lt;sup>9</sup> Media coverage can be found at https://gulfnews.com/world/asia/philippines/air-conditioner-without-refrigerant-this-filipina-19-could-revolutionise-your-ac-1.2263116.
<sup>10</sup> Their technical guidance manual is at https://www.ofgem.gov.uk/ofgem-publications/58721/tm-guidance.pdf.

<sup>&</sup>lt;sup>11</sup> Vehicles with footprint over 55 square feet need to meet a standard of 46 mpg, while those 41 square feet or smaller must achieve 60 mpg. The final CAFE standard for light-duty vehicles for model years 2017 and beyond (DOT/EPA, 2012) can be accessed at https://www.govinfo.gov/content/pkg/FR-2012-10-15/pdf/2012-21972.pdf.

<sup>&</sup>lt;sup>12</sup> Our calibration section 7 below refers to online Appendix G for more detail. This calculation for midsize cars is in Appendix Table G1, and the calculation for refrigerator-freezers mentioned shortly is in Table G5.

<sup>&</sup>lt;sup>13</sup> Stringency differs across appliances. A disaggregated computational GE model can include each separately. For the same parameters, an individual appliance standard would have the same relative rebound effects as found for our aggregate, but smaller absolute sizes. Even for one appliance, an increasingly stringent EES still implies negative income effects and negative rebound. In a later extension, we analyze cars and appliances separately.
<sup>14</sup> If n is large enough, an individual household disregards its own contribution to aggre-

 $<sup>^{1\</sup>dot{4}}$  If n is large enough, an individual household disregards its own contribution to aggregate energy use and takes as fixed the third argument in utility (nE). Except for an extension in Section 11, we do not make use of this third argument in our analysis, but include it primarily as a reminder of the reason to have an energy efficiency policy.

<sup>&</sup>lt;sup>15</sup> Thus, changes in pollution *nE* have no effects on purchases of cooling services (not even income effects). We look at effects of policy *costs* on welfare and rebound, but not effects of policy benefits on welfare or rebound. This assumption allows for tractability, but it abstracts from major possible effects of policy. An extension below considers how reduced emissions can increase income, some of which might be spent on energy. For other examples with non-separable effects of pollution on private purchases, see Carbone and Smith (2008).

Our simple model cannot consider the discrete choice to buy an appliance or vehicle. Here, we ignore the possibility that the household forgos a car or furnace, and instead consider the continuous policymaker's choice of required energy efficiency A, and the household's choice of other features not related to energy efficiency. Leather seats are best considered to be part of the other good, X. Similarly, a home's double-pane insulated glass window is part of A, but the stained-wood window frame is part of X. Thus, we use A not to represent total cars and appliances, but only the portion devoted to energy efficiency.  $^{16}$ 

With those clarifications, the home-production function is:

$$S = \epsilon A E_S, \tag{2.2}$$

where overall energy efficiency  $\epsilon A$  is determined by a costless technology scalar  $\epsilon$  times purchased energy efficiency A. If  $\epsilon A$  is in miles per gallon, for example, then multiplication by  $E_S$  in gallons of gasoline yields S in miles. Or, if  $\epsilon A$  is in cooling per kwh, and  $E_S$  is kwh of electricity, then S is measured in degrees of cooling. With only one type of A, consumers who want more energy efficiency must buy more A. Then, with this model, we can study a small costless exogenous increase in the technology scalar,  $\epsilon$ , to solve for rebound effects from a CTS, and we can study a small increase in required A to analyze an EES.

Since energy efficiency A can be purchased, it must be produced by firms using extra resources,  $K_A$ , in a production function such as  $A = A(K_A)$ . The inverse of that function is the resource cost of making more energy efficiency, which we express as:

$$K_A = B(A - A_0)^{\beta}.$$
 (2.3)

The scale parameter B converts units (from A to K), so B > 0. The exponent  $\beta$  represents cost curvature. We assume  $\beta > 1$ , to capture the likely case that additional efficiency requires using successively more expensive technologies or materials. The parameter  $A_0$  provides flexibility to shift the intercept of the cost function; this shifted function is used in most engineering studies reviewed below and in our calibration appendix. The reasons described below, we assume  $0 \le A_0 < A$ . The general idea is that any car generates some positive miles per gallon (mpg), even when no costs are incurred trying to raise mpg. Some minimal energy efficiency ( $A_0$ ) comes with any car, while costs of additional efficiency could be quadratic (e.g.,  $\beta$ =2).

The model abstracts from various taxes on inputs or outputs, just as it abstracts from government expenditures. <sup>18</sup> The essential function of government modeled here includes only a required target total energy efficiency of household appliances, a target that can be stated in miles/gallon or degrees of cooling/kwh. That standard can be represented as  $\epsilon A \geq \eta$ , for a policy scalar  $\eta$ . We assume the existing policy is binding, so fuel efficiency matches the scalar:  $\epsilon A = \eta$ . For a given value of the technology parameter, government has essentially set A, which costs

 $K_A$ . Thus, a new regulation that raises required fuel efficiency must raise costs in this model, where the cost of A rises at an increasing rate (increasing marginal cost).<sup>19</sup>

When the initial equilibrium has a pre-existing efficiency standard that is both costly and binding, we have no need to describe how a consumer facing no standard would maximize  $U = U(\epsilon AE_S, X; nE)$  by their choices of  $A, E_S$ , and X. Instead, we assume that the government "distorts" those choices by requiring more A than unconstrained consumers would choose – presumably to reduce the negative effect on U from total energy use nE. Our linearization below can analyze small changes, so we also assume that a small increase in technology  $\epsilon$  would not be enough to make the policy non-binding (discussed more below).

In Proposition 2 below, we solve for a condition on parameters that must hold for our assumption that rational and fully-informed consumers are being forced to purchase more *A* than they would desire if unconstrained. When this condition holds, then any increase in the stringency of the energy efficiency requirement must be costly and therefore must reduce real income (ignoring the benefits of reduced external damages).<sup>20</sup>

Since  $\epsilon$  and A are given, then  $K_A$  is a fixed cost. Households choose only energy use  $E_S$  (at price  $P_E$ ) and other goods X (at price  $P_X$ ). Their only income is from a fixed total factor endowment,  $\overline{K}$ , which earns the rate of return  $P_K$ . Thus, the budget constraint is:

$$I = P_K \overline{K} = P_X X + P_K K_A + P_E E_S$$

Then, because  $S = \epsilon A E_S$ , the maximization of U(S,X) subject to this budget yields first order conditions that can be solved for the

marginal rate of substitution: 
$$\frac{\partial U}{\partial X}/\frac{\partial U}{\partial S} = P_X/(P_E/\epsilon A)$$
.

In other words, given the fixed cost  $P_KK_A$ , the *effective marginal price of appliance services* is  $P_E/\epsilon A$ . For example, if  $P_E$  is the price of gasoline in \$/gallon, and  $\epsilon A$  is fuel efficiency in miles/gallon, then the cost of an additional mile  $(P_E/\epsilon A)$  is in dollars per mile.

This model abstracts from the fact that the choice of car or furnace takes place before the choice of miles or heating. It collapses the dynamic problem to a year in which the economy is in long-run equilibrium, so the household pays the annualized cost of the car plus annual cost of miles. <sup>21</sup> Nonetheless, this model captures exactly the aspect of the problem that gives rise to rebound: once the car or appliance is purchased with a particular energy efficiency, the only marginal cost is energy use. Energy efficiency may rise through a costless shock to  $\epsilon$ , or from a requirement that raises the fixed cost of A, but either such shock *reduces* marginal cost ( $P_E/\epsilon A$ ) per mile driven in a car, or per degree of heating from the furnace.

<sup>&</sup>lt;sup>16</sup> With no data to identify the portion of a car spent on fuel efficiency, see our calibration below for alternative assumptions. Also, our static model abstracts from the timing of durable purchases, vintages that affect energy use, and the distinction between short run and long run effects. We compare post-adjustment equilibria. But, we capture the key fact that either a CTS or EES would reduce the price of services and thus give rise to rebound.

<sup>&</sup>lt;sup>17</sup> Those studies include NRC (2002, 2015), DOE (2011, 2016a, 2016b), and Lutsey et al. (2017). Our cost function allows  $A_0$  to be zero, and we later determine whether a positive  $A_0$  is necessary to fit the data.

 $A_0$  is necessary to fit the data. <sup>18</sup> Our model could include other taxes and second-best effects, but the papers in Goulder (2002) already make clear how other distortions could matter. If a car or appliance has a price above marginal cost because of taxes or market power, then that distortion is exacerbated by an increase in mandate stringency that raises product price further (reducing real income and potentially making rebound more negative). Moreover, this addition to consumer price effectively reduces the real net wage and thus could reduce labor supply (which also reduces cash income, with further negative effects on rebound). Second-best effects are important, but studied. They would add more terms to our equation for rebound but would not remove the key new terms we discuss. We prefer the simplest model necessary to demonstrate our new results (as summarized above and in propositions below).

<sup>&</sup>lt;sup>19</sup> These costs follow from our assumption that consumers are well-informed optimizers facing a costly mandate. In a behavioral model where consumers miss opportunities to reduce their own overall costs by choosing more energy efficiency, a mandate could raise welfare – with effects very similar to the CTS analyzed below. Here we use the simplest model to show how increased stringency can raise costs and thus have required to the condition is not attified these profiles in the condition is not attified.

<sup>&</sup>lt;sup>20</sup> Or, if that condition is not satisfied, then a policy to increase energy efficiency can raise real income. That case would require a behavioral model of how consumers choose energy efficiency (A) that does not maximize utility.

<sup>&</sup>lt;sup>21</sup> Costs of energy efficiency may include: (1) fixed costs of R&D required to meet the mandated efficiency, and (2) greater marginal cost to produce and sell appliances. Both are dollar costs passed back onto investors or forward onto consumers. Our model has 'many identical households' who earn all labor and capital income and buy all final goods, so they bear all these costs. With heterogeneity, effects on energy could depend on different energy spending by investors, workers, and consumers. Also, (3) an EES can induce trade-offs between fuel-economy vs. performance (Knittel, 2012), or fuel-economy vs. vehicle attributes that make driving less attractive (West et al., 2017). This cost is more difficult to capture in our simple model with only two goods in utility, first because some of those behaviors are substitution rather than income effects, and also because anything other than services *S* (e.g., miles driven) in our model must be part of the composite good *X* (leather seats or performance). The cost may be lost consumer surplus, rather than through changes to cash income or cash expenditures.

On the producer side, competitive firms are price takers in all markets, with constant returns to scale (CRTS) production. Firms in sector X use  $K_X$  and energy  $E_X$  as inputs. Energy E is generated from input of factor  $K_E$ . We choose units of measurement such that one unit of the primary factor  $K_E$  can produce one unit of E. Thus, the production functions are:

$$X = X(K_X, E_X) \tag{2.4}$$

$$E = K_E \tag{2.5}$$

Perfect competition and CRTS imply zero-profit conditions stating that the value of each sector's output produced and sold must equal the sum of amounts spent on inputs:

$$P_X X = P_K K_X + P_E E_X (2.6)$$

$$P_E E = P_K K_E \tag{2.7}$$

All markets must clear in equilibrium. That is, energy supply must equal the sum of all demands, and the factor endowment must equal the sum of all factor uses:

$$E = E_X + E_S, (2.8)$$

$$\overline{K} = K_X + K_A + K_E. \tag{2.9}$$

#### 3. Linearization

Many analytical GE models employ particular functional forms in order to be able to solve for outcomes in levels – both with and without a large policy or other large shock. <sup>22</sup> Instead, we differentiate general equations above to linearize the model and then solve the N linear equations for N unknown outcomes (prices, quantities, and welfare). This approach is only valid for small changes, but it has other advantages. First, it allows us to avoid restrictive assumptions about functional form and use general forms such as  $X = X(K_X, E_X)$ . Second, we have no need to compare a world with no mandate to one with a stringent mandate. Large new regulations are rare, as actual policy proceeds incrementally. Rules for cars and appliances have faced periodic debates about relatively small changes in stringency. <sup>23</sup>

Third, linearization allows us to calibrate around an observed dataset with an existing energy efficiency standard or CAFE standard. Instead of choosing parameters that must pertain in all possible equilibria, we only specify value of parameters at the initial point. For example, the energy input share parameter is based on observed energy use in the equilibrium with current policy. A proposition below showing a condition on parameters is really about the initial conditions under which an increase in stringency will lead to negative rebound.

When we differentiate the equations above, we use a "hat" to denote a proportional change (e.g.,  $\hat{X} \equiv dX/X$ ). We then solve for the changes in quantities and prices that result from a small exogenous change in

technology  $(\hat{\epsilon})$  or policy  $(\hat{\eta})$ . The analysis of small changes does not mean results are small, however. Pre-existing standards act like taxes that raise the cost of appliances, so they create deadweight losses that rise disproportionately with the implicit tax rate. While a small standard or tax has only second-order effects on welfare, a small increase of a stringent standard or tax has first-order effects on income and welfare.

Specifically, total differentiation of production functions for the three goods shows how the change in each output is determined from changes in each set of inputs:

$$\hat{S} = \hat{\epsilon} + \hat{A} + \widehat{E}_{S} \tag{3.1}$$

$$\hat{X} = \theta_{KX}\widehat{K_X} + \theta_{EX}\widehat{E_X} \tag{3.2}$$

$$\hat{E} = \widehat{K_E}. \tag{3.3}$$

For sector X, the factor share for input K is  $\theta_{KX}$ , the factor share for energy is  $\theta_{EX}$ , and  $\theta_{KX}+\theta_{EX}=1$ . The elasticity of substitution in sector X between inputs to production is defined as  $\sigma_X$ , the percentage change in the input use ratio in response to 1% change in the input price ratio. For small changes, the definition of  $\sigma_X$  implies:

$$\widehat{K_X} - \widehat{E_X} = \sigma_X \left( \widehat{P_E} - \widehat{P_K} \right). \tag{3.4}$$

Since the marginal price of services is  $P_E/\epsilon A$ , differentiation yields the proportional change in that price as  $(\widehat{P_E} - \hat{A} - \hat{\epsilon})$ . The elasticity of substitution in utility between X and S is  $\sigma_U$ , defined as the percentage change in the ratio of those quantities for 1% change in the marginal price ratio. For small changes, we get:

$$\hat{X} - \hat{S} = \sigma_U \left( \left( \widehat{P_E} - \hat{A} - \hat{\epsilon} \right) - \widehat{P_X} \right). \tag{3.5}$$

Then, we differentiate  $K_A = B(A - A_0)^{\beta}$  and manipulate, to get:

$$\widehat{K_A} = \frac{\beta}{1 - \alpha} \hat{A},\tag{3.6}$$

where  $\alpha = A_0/A$  is minimum energy efficiency as a fraction of total initial energy efficiency. We assumed  $0 \le A_0 < A$  above in order to ensure that  $0 \le \alpha < 1$  and that  $\beta/(1-\alpha)$  is well defined. We interpret  $\beta/(1-\alpha)$  in this equation as the "cost elasticity of energy efficiency", because it is the percent change in cost for 1% more efficiency. Next, differentiate zero-profit Eqs. (2.6)-(2.7), and use the firm's FOCs:

$$\widehat{P_X} = \theta_{KX}\widehat{P_K} + \theta_{EX}\widehat{P_E} \tag{3.7}$$

$$\widehat{P_E} = \widehat{P_K} \tag{3.8}$$

Finally, total differentiation and linearization of the market-clearing condition in Eq. (2.8) and the resource constraint in Eq. (2.9) yield:

$$\hat{E} = \lambda_X \widehat{E_X} + \lambda_S \widehat{E_S} \tag{3.9}$$

$$0 = \gamma_X \widehat{K_X} + \gamma_A \widehat{K_A} + \gamma_E \widehat{K_E}$$
 (3.10)

where  $\lambda_i$  denotes the ratio of energy use in sector i to the total use of energy  $(\lambda_X + \lambda_S = 1)$ , and  $\gamma_i$  is the ratio of capital used in sector i to total capital in all sectors  $(\gamma_X + \gamma_A + \gamma_E = 1)$ . We define the primary factor K as numeraire, so  $\widehat{P_K} = 0$ .

# 4. Rebound from a costless technological shock with no binding mandate

Later sections analyze a CTS or EES in an economy like the U.S. with a pre-existing mandate that fixes energy efficiency A. Consumers do not get to choose A, and so we have no model of how they choose A. In

<sup>&</sup>lt;sup>22</sup> Copeland and Taylor (1995) use Cobb-Douglas forms to compare the equilibrium with no trade to a different equilibrium with perfect trade liberalization. In the rebound literature, Wei (2007) uses Cobb-Douglas forms and Lemoine (2018, 2020) uses Constant Elasticity of Substitution. They also assume each factor share is constant, whereas we use only the observed share at the initial equilibrium.

<sup>&</sup>lt;sup>23</sup> In Section 1, our main examples were vehicles and refrigerators. CAFE rules required new cars to average 18 mpg by 1978, then later 27.5 mpg by 1985, and eventually average 54.5 mpg by 2025 (DOT/EPA, 2012). For 28 types of refrigerators, the DOE set new standards in 1990, 1993, 2001, and again in 2014. For each, it shows a formula to calculate the *maximum* energy use in kWh/yr (https://www.ecfr.gov/cgi-bin/text-idx?SID=48f64e166fe3561666f871e521996e13&mc=true&node=se10.3.430\_132&rgn=div8). Our example is a refrigerator with 16 cubic feet (cu. ft.), a top-mounted freezer, automatic defrost, and no automatic icemaker. Its formula from 2001 until September 15, 2014, is 9.80 × (cu.ft.) + 276.0, which is 432.8 kWh/yr. The formula to apply after 2014 is 8.07 × (cu.ft.) + 233.7, which is 362.8 kWh/yr. This is only one example of how DOE debates and sets new rules to increase stringency incrementally on various appliances.

contrast, this section represents prior analyses of a CTS shock with no existing mandate. Prior CTS papers generally assume an exogenous costless increase in overall energy efficiency ( $\epsilon A$  in our model). Implicitly, they assume no change in spending on costly energy efficiency (A) but instead an exogenous increase in the scalar  $\epsilon$ . Hence, a CTS is represented here by a small exogenous change in technology,  $\epsilon > 0$ . This calculation is not "general equilibrium" in nature, because we also assume no change in A, but it means that all other features of our model including the level of A are comparable with our later results below. Yet, consumers in this section do react to greater energy efficiency by changing their fuel use, consumption of services, and other goods. This section also provides a walkthrough of how we decompose rebound into its components.

Thirteen changes  $(\hat{X}, \hat{S}, \hat{A}, \hat{E}, \widehat{E_X}, \widehat{E_S}, \widehat{K_X}, \widehat{K_A}, \widehat{K_E}, \widehat{P_X}, \widehat{P_E}, \widehat{P_K}, \hat{\epsilon})$  appear in the ten linearized Eqs. (3.1)–(3.10). This section takes  $\hat{\epsilon} > 0$  as exogenous and assumes that A is unchanged  $(\hat{A} = 0)$ . Thus, with K as numeraire  $(\widehat{P_K} = 0)$ , we can solve for the other ten outcomes (in proportional changes). In addition, we unpack the solution for  $\hat{E}$  into different terms. Each such term can show a particular rebound effect as a function of parameters (initial conditions) and of the exogenous increase in the efficiency scalar,  $\hat{\epsilon} > 0$ .

Some of the solutions are easy. Production of E uses only the numeraire primary factor, so our first result is  $\widehat{P_E} = \widehat{P_K} = 0$ . Production of X uses both K and E, but neither input price changes, so CRTS implies  $\widehat{P_X} = 0$ . Also, the two inputs change in the same proportion as output,  $\widehat{E_X} = \widehat{K_X} = \widehat{X}$ . The effective marginal price of S is  $P_E/\epsilon A$ , but  $\epsilon$  rises while A and  $P_E$  are unchanged. Therefore, the cost per additional unit of S falls  $(\widehat{P_E} - \hat{\epsilon} - \hat{A} = -\hat{\epsilon})$ . Appendix A shows derivations for all outcomes, but here are solutions for four of them:

$$\hat{X} = \left( -(\gamma_A + \gamma_E \lambda_S) \sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} \right) \hat{\epsilon}$$
(4.1)

$$\hat{S} = \left( (\gamma_X + \gamma_E \lambda_X) \sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} \right) \hat{\epsilon}$$
(4.2)

$$\widehat{E_S} = \left( (\gamma_X + \gamma_E \lambda_X) \sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - 1 \right) \hat{\epsilon} \tag{4.3}$$

$$\hat{\textit{E}} = \left( (\lambda_{\textit{S}} \gamma_{\textit{X}} - \lambda_{\textit{X}} \gamma_{\textit{A}}) \sigma_{\textit{U}} + \frac{(\gamma_{\textit{X}} + \gamma_{\textit{E}} \lambda_{\textit{X}}) \sigma_{\textit{U}} \gamma_{\textit{A}} + \lambda_{\textit{S}} \gamma_{\textit{E}}}{1 - \gamma_{\textit{A}}} - \lambda_{\textit{S}} \right) \hat{\boldsymbol{\varepsilon}} \tag{4.4}$$

The last equation for the overall change in energy use  $(\hat{E})$  will be rearranged to decompose it into key components, but we first gain some intuition by looking at the other outcomes.

The positive technology shock increases real incomes, so consumers buy more goods and services. <sup>24</sup> They also respond to the fall in the marginal cost of services by substituting toward S from X (in a way that depends on  $\sigma_U$ ). The simplicity of our model allows us to separate income from substitution effects for both services and the other good. Specifically, in Appendix B, we derive the substitution effect as the change in consumption when consumers face the new prices but are as happy as in the old equilibrium. The income effect is the remaining change in consumption. In fact, Appendix B shows that the first term in Eq. (4.1) is the substitution effect on X, which depends on  $\sigma_U$  and is negative. The income effect on X is the remaining term in Eq. (4.1) and is positive. Similarly, the first term in Eq. (4.2) is the positive substitution effect on S. The income effect on S is the remaining term in Eq. (4.2), and it matches the positive income effect on X (because of homothetic preferences).

Next, our model enables us to solve for the welfare gain from this shock as the change in utility from consumption (dU, ignoring the

**Table 1**Decomposition of the effect of energy efficiency on total energy use.

| Effects on energy use    | No existing policy (with $\hat{A} = 0$ )   | Pre-existing policy $(\operatorname{so} \hat{A} = -\hat{\epsilon})$ |   |     |  |  |
|--------------------------|--|---|---|-----|--|--|
|                          | (1)  | (2)   | (3)   | (4) |  |  |
| Direct efficiency effect | $-\lambda_S\hat{\epsilon}$   | (-)   | 0   | 0   |  |  |
| Direct rebound effect    |  | (+)   | (+)   | (+) |  |  |
| Income effect            | $\frac{\lambda_{s}((\gamma_{X}+\gamma_{E}\lambda_{X})\sigma_{U}\gamma_{A}+\lambda_{S}\gamma_{E})}{1-\gamma_{A}}\hat{\epsilon}$ $\lambda_{S}(\gamma_{X}+\gamma_{E}\lambda_{X})\sigma_{U}\hat{\epsilon}$ | (+)   | $\frac{\lambda_{S}\gamma_{A}\beta}{(1-\gamma_{A})(1-\alpha)}\hat{\epsilon}$ | (+) |  |  |
| Substitution effect      | $\lambda_S(\gamma_X + \gamma_E \lambda_X) \sigma_U \hat{\epsilon}$   | (+)   | 0   | 0   |  |  |
| Indirect rebound effect  |  | (+/-)   | (+)   | (+) |  |  |
| Income effect            | $\frac{\lambda_X((\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E)}{1 - \gamma_A} \hat{\epsilon} \\ -\lambda_X(\gamma_A + \gamma_E \lambda_S) \sigma_U \hat{\epsilon}$           | (+)   | $\frac{\lambda_X \gamma_A \beta}{(1-\gamma_A)(1-\alpha)} \hat{\epsilon}$    | (+) |  |  |
| Substitution effect      | $-\lambda_X(\gamma_A+\gamma_E\lambda_S)\sigma_U\hat{\epsilon}$   | (-)   | 0   | 0   |  |  |
| Panel B: efficiency      | mandate ( $\hat{\eta}$ >0)   |   |   |     |  |  |
| Effects on energy use    | Pre-existing policy (so $\hat{A}=\hat{\eta}{>}0$ )   |   |   |     |  |  |
|                          | (5)  |   | (6  | 5)  |  |  |
| Direct efficiency        | $-\lambda_{S}\hat{\eta}$   |   | (-  | -)  |  |  |

energy-use externality). It is divided by the marginal utility of income  $(\mu)$  to get the dollar value, and also divided by total income (I) to express it in relative terms. As shown in Appendix C, this measure of the change in welfare is:

$$\begin{split} \frac{dU}{\mu I} &= \frac{P_X X}{I} \hat{X} + \frac{P_K K_A + P_E E_S}{I} \hat{S} = (\gamma_X + \gamma_E \lambda_X) \hat{X} + (\gamma_A + \gamma_E \lambda_S) \hat{S} \\ &= \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} \hat{\epsilon} > 0 \end{split}$$

In the first line, the relative change in welfare is a weighted average of the changes in consumption of goods and services, where the weight for each is its share of income. Using solutions for  $\hat{X}$  and  $\hat{S}$  above, the closed-form solution for  $dU/(\mu I)$  is shown on the far right. As confirmation of these two derivations, we note that this relative change in welfare is identical to the earlier-derived income effect on goods and on services.

The substitution effect increases  $E_S$  by the first term in Eq. (4.3), and the income effect increases  $E_S$  by the second term in Eq. (4.3). But Eq. (4.3) for  $\widehat{E}_S$  has a third term that reduces energy use for appliance services (by  $-\hat{\epsilon}$ ). This last term is the direct effect of the CTS that allows consumers to produce the same services using less energy.

The solution for  $\hat{E}$  in Eq. (4.4) has an ambiguous sign, and it is hard to interpret, so we unpack it into three major components: a direct efficiency effect, a direct rebound, and an indirect rebound. We then further decompose both the direct and indirect rebound effects into income and substitution effects, in general equilibrium.<sup>25</sup> These components appear

 $<sup>^{24}\,</sup>$  Utility is homothetic, so both commodity X and services S are normal goods. See the end of Section 9 below.

<sup>&</sup>lt;sup>25</sup> Most PE empirical studies estimate only the uncompensated direct rebound effect, without separate income and substitution effects. Thomas and Azevedo (2013) and Borenstein (2015) use PE theory models to decompose direct and indirect rebound effects into substitution and income effects. We follow their lead, but in a GE model.

in the first column of Table 1, Panel A, which decomposes effects of a CTS on total energy use in this case (with no policy, and fixed A). The sum of all terms in the first column is  $\hat{E}$  in Eq. (4.4). The other columns and panel – discussed later – show GE effects of a CTS with a pre-existing standard, and effects of increased stringency of an energy efficiency standard (EES).

In column (1), with no policy, consumers could exploit the efficiency improvement to get the same services using less energy. That is, even if households were to choose unchanged X and S, the energy used to produce this bundle would fall. We define this energy saving as the "direct efficiency effect", identified in the first term of the column. This energy saving would be the only effect if relative prices and real incomes held constant – as it might be calculated by engineers. Thus, the direct efficiency effect reduces energy used *for services* by the full amount of the technology shock  $(-\hat{\epsilon})$ . It reduces total energy use in the economy by the shock times the fraction of total energy used for those services  $(-\lambda_S \hat{\epsilon} < 0)$ .

Next, in column (1) of Panel A, we define the "direct rebound effect" (DRE) as the change in E from the combined substitution and income effects on the demand for services S. The income effect component of that DRE is the next term in the column ( $\lambda_S$  times the income effect on  $E_S$  in Eq. 4.3). The real income gain allows consumers to get more S using more  $E_S$ . The DRE through the substitution effect is the next term down. It is  $\lambda_S$  times the substitution effect on  $E_S$  in Eq. (4.3), and it is also positive. Its size depends on  $\sigma_U$ . The DRE is the sum of the income and substitution effects and thus must be positive.

The "indirect rebound effect" (IRE) then refers to the effect of the shock on the equilibrium change in demand for the *other* good, X, and thus on *its* energy use,  $E_X$ . The second-to-last term in this first column of Panel A is the change in energy use from the income effect on demand for X. It is  $\lambda_X = E_X/E$  times the income effect on  $E_X$  in Eq. (4.1), and it is positive. The last term is the change in energy use through the substitution effect away from X. The total IRE is the sum of a positive income effect and negative substitution effect:

$$\frac{\lambda_X((\gamma_X+\gamma_E\lambda_X)\sigma_U\gamma_A+\lambda_S\gamma_E)}{1-\gamma_A}\hat{\varepsilon}-\lambda_X(\gamma_A+\gamma_E\lambda_S)\sigma_U\hat{\varepsilon}=\frac{\lambda_X\lambda_S\gamma_E(1-\sigma_U)}{1-\gamma_A}\hat{\varepsilon}.$$

The sign of this expression depends on the size of  $\sigma_U$ . If goods X and S are not substitutable enough ( $\sigma_U$ <1), then the income effect dominates, and indirect rebound is positive. If the substitution effect dominates ( $\sigma_U$ >1), however, then the IRE is negative.

Alternatively, we can decompose rebound into an "overall income effect" and an "overall substitution effect". The real income gain increases demands for both  $E_X$  and  $E_S$  and thus always adds positively to rebound. The sum of the two income effects in the first column of Table 1 is the total income effect, the second term in the solution for  $\hat{E}$  in Eq. (4.4). Similarly, we can calculate an overall substitution effect on rebound through both  $E_S$  and  $E_X$  by adding the two substitution effects. This sum is the first term in Eq. (4.4). It has an ambiguous sign because the substitution effect increases  $E_S$  but decreases  $E_X$ .

We can show that  $\lambda_S \gamma_X - \lambda_X \gamma_A > 0$  implies that appliance services are more energy-intensive than the composite good.<sup>26</sup> If this inequality holds, then the overall income effect and overall substitution effect both increase energy use. If S is less energy-intensive, however, then the overall substitution effect is a negative component of rebound, and the sign of total rebound depends on whether the income effect dominates the substitution effect.

These analytical results and their signs in the second column of Table 1 are consistent with prior results (reviewed below). When technological progress improves energy efficiency, its direct effect reduces energy use. Direct rebound through *S* is expected to be positive, for

both substitution and income effects, while indirect rebound in *X* can be either sign. We next distinguish these effects of a CTS with no mandate to effects of a CTS with a pre-existing and binding mandate (and later, to effects of increasing stringency of a binding EES).

# 5. Rebound from a costless technological shock with a binding mandate

Unlike existing papers, we now study general equilibrium rebound effects from a costless technology shock in the case with a pre-existing efficiency mandate that is both costly and binding. The CTS is still modeled as an increase in the technology parameter,  $\hat{\epsilon} > 0$ , but we assume that a small change in  $\epsilon$  does not make the policy non-binding.<sup>27</sup>

In general equilibrium, if consumers face a minimum energy efficiency  $\epsilon A \geq \eta$  and are given a free increase in  $\epsilon$ , then they can choose to spend less on A. With a costly mandate that remains binding, then consumers still have to purchase more A than desired, so they respond to the technology shock by cutting back as much as possible on purchase of A. If consumers spend only enough to satisfy the unchanged mandate, then the CTS results in no additional energy efficiency at all. The unchanged policy still requires  $\epsilon A \geq \eta$ , so consumers can reduce purchase of costly A such that  $\hat{A} = -\hat{\epsilon}$ , and energy efficiency  $\epsilon A$  is unchanged.

We use the ten linearized equations in Section 3 to solve for the same ten unknowns as above. The exogenous shock is still  $\hat{\epsilon} > 0$ , and the numeraire is still  $\widehat{P_K} = 0$ , but all outcomes are now different because  $\hat{A} = -\hat{\epsilon}$ . Appendix D derives a closed-form expression for each of these ten equilibrium changes resulting from a small CTS with a binding EES, including the change in total energy use,  $\hat{E}$ , but then we unpack that solution for  $\hat{E}$  into the same rebound effects as before, as shown in column (3) of Table 1.

Many of the results are both easy to solve and intuitive. As before,  $\widehat{P_E} = \widehat{P_K} = 0$ . Neither input price changes, so CRTS implies  $\widehat{P_X} = 0$ . Therefore, the two inputs change in the same proportion as output,  $\widehat{E_X} = \widehat{K_X} = \hat{X}$ . Next, the CTS does not change overall energy efficiency,  $\epsilon A$ , so  $\hat{S} = \widehat{E_S}$ . The effective marginal price of S is  $P_E/\epsilon A$ , where  $\widehat{P_E} = 0$ , and A falls by the same percentage that  $\epsilon$  rises. Therefore, the marginal cost of S does not change ( $\widehat{P_E} - \hat{\epsilon} - \hat{A} = 0$ ). Unchanged relative prices of services S and good S implies no substitution between these goods. Yet the CTS helps consumers to cut spending on S0 while they still meet the unchanged standard. Thus, it has a positive income effect that increases S1 and S2 by the same percentage as the income gain (as shown in Appendix D):

$$\hat{X} = \hat{S} = \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} \hat{\epsilon} > 0.$$

As a consequence, the relative welfare gain, measured by the dollar value of the change in utility divided by total income as in Appendix C, ic:

$$\frac{dU}{\mu I} = \frac{P_X X}{I} \hat{X} + \frac{P_K K_A + P_E E_S}{I} \hat{S} = \hat{X} = \hat{S} = \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} \hat{\epsilon}$$

Rearrange the inequality  $\lambda_S \gamma_X - \lambda_X \gamma_A > 0$  to get  $\frac{\lambda_S}{\gamma_A} \cdot \frac{\lambda_X}{\gamma_X}$ , or  $\frac{E_S/E}{K_A/K} \cdot \frac{E_X/E}{K_X/K}$ . Thus,  $\frac{E_S}{K_A} \cdot \frac{E_X}{K_X}$  which implies that the S sector is more energy intensive than sector X.

<sup>&</sup>lt;sup>27</sup> If an actual shock were large enough to make the policy non-binding, then the outcome would include some effects of this section with a binding mandate and some effects of the previous section with no binding mandate.

<sup>&</sup>lt;sup>28</sup> In a similar result, though not about rebound, Goulder et al. (2012) look at a subset of states that adopt a vehicle fuel efficiency standard that is more stringent than the federal standard requiring automakers to meet a nation-wide corporate average fuel economy (CAFE). Those automakers can sell more fuel efficient cars in the more-stringent states and sell less efficient cars in other states, with no overall change in average fuel efficiency.

<sup>&</sup>lt;sup>29</sup> Empirical evidence in Knittel (2012) suggests that improvements in fuel efficiency technology over time were offset by spending on other vehicle characteristics that reduce fuel efficiency, such as vehicle weight and power.

Increases in consumption of *X* and *S* lead to an increase in energy use by the same percentage:

$$\hat{E} = \widehat{E_X} = \widehat{E_S} = \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} \hat{\epsilon} > 0.$$

These positive rebound effects depend positively on the fraction of K used for energy efficiency  $(\gamma_A \equiv K_A/\overline{K})$ , and on the cost elasticity of additional energy efficiency  $[\beta/(1-\alpha)]$ . This elasticity depends on the curvature of the cost function  $\beta$  and the fraction  $\alpha \equiv A_0/A$ .

In fact, all effects on energy use in column (3) of Table 1 are either zero or positive. The economy stays at the same mandated efficiency after the technology shock, so the direct efficiency effect is zero. Rebound through substitution effects is zero, so the direct and indirect rebound effects have only positive income effects. The *total* effect of the CTS is to increase total energy use (as shown in Table 1). This result could be defined as a "backfire", since positive rebound swamps the zero direct effect. We state these results formally as:

**Proposition 1.** For a positive costless technology shock (CTS) in this model with a pre-existing energy efficiency mandate that is costly and still binding after the shock, then:

- (i) The direct efficiency effect is zero.
- (ii) Rebound substitution effects are zero.
- (iii) The only nonzero rebounds are through income effects and are positive.

The proof for Proposition 1 is the derivation above and in Appendix D. It follows from the assumption that the unchanged EES is still binding after the CTS.

These results are strikingly different from those in prior rebound literature. For the CTS with no pre-existing mandate, the prior literature shows that the direct efficiency effect is a reduction in energy use and can be offset by a positive direct rebound effect (as in the first two columns of Table 1). For the CTS with a pre-existing mandate, the next two columns show the CTS has no direct energy savings at all, and no rebound through substitution effects. In essence, the energy efficiency innovation reduces the cost of achieving the unchanged pre-existing standard. Then "direct rebound" is the positive income effect on energy for production of other goods, X.

A glance down column (3) shows that if the fraction of K in services  $(\lambda_S)$  is less than the fraction of K used in other goods  $(\lambda_X)$ , as could be expected, then the indirect rebound effect (on  $E_X$ ) exceeds the direct rebound effect (on  $E_S$ ). Most empirical PE models measure only direct rebound, using the elasticity of demand for services, but they ignore mandates. In contrast, our GE model can show indirect rebound. For the CTS with a pre-existing efficiency mandate, the positive indirect effect likely exceeds the positive direct rebound effect.

An additional possibility ignored here is that policy makers set standards based on costs and benefits, such that this reduction of costs could induce them eventually to tighten the standard. We do not analyze endogenous policy, but we do analyze a tighter standard (in the next section). We also ignore endogenous technology, but another possibility is that the costly policy can *induce* improvements in technology that then cause positive rebound effects.

# 6. Rebound effects from a change in the energy efficiency standard

We now solve for effects of a costly increase in EES stringency, decompose it into types of rebound, and compare these results to those above. Since the policy is represented by  $\epsilon A \ge \eta$ , we model the policy

shock as a small exogenous change,  $\hat{\eta} > 0$  (with no change in technology,  $\hat{\epsilon} = 0$ ). Because purchase of A is costly, and consumers are already required to purchase more than they would if unconstrained, they will not buy more than necessary to satisfy the new requirement. Therefore the chosen  $\hat{A}$  will exactly equal the required  $\hat{\eta}$ .

Here again, we use the ten linearized Eqs. (3.1)–(3.10) to solve for equilibrium changes in energy consumption,  $\hat{E}$ , and nine other unknowns. Then we decompose that effect on total energy use into separate terms. As before,  $\widehat{P_E} = \widehat{P_K} = 0$  and  $\widehat{P_X} = 0$ . Inputs change in the same proportion as output:  $\widehat{E_X} = \widehat{K_X} = \widehat{X}$ . The marginal cost of S falls in the same proportion as energy efficiency rises  $(\widehat{P_E} - \widehat{A} - \widehat{\epsilon} = -\widehat{\eta} < 0)$ . Appendix E explains in detail the derivations for all unknowns. Here, we show the general solutions for only four of them:

$$\hat{X} = \left(-(\gamma_A + \gamma_E \lambda_S)\sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)}\right) \hat{\eta} \ (6.1)$$

$$\hat{S} = \left( (\gamma_X + \gamma_E \lambda_X) \sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} \right) \hat{\eta} \ (6.2)$$

$$\widehat{E_S} = \left( (\gamma_X + \gamma_E \lambda_X) \sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} \mathbf{1} \right) \hat{\eta} \ (6.3)$$

$$\hat{E} = \left( (\lambda_S \gamma_X - \lambda_X \gamma_A) \sigma_U + \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} - \lambda_S \right) \hat{\eta} \quad (6.4)$$

These solutions in Eqs. (6.1)–(6.4) have mostly the same terms as in Eqs. (4.1)–(4.4), including the same direct efficiency effect, substitution effect, and the first term of the income effect. The only difference is the negative term added to the income effect,  $-\frac{\gamma_A\beta}{(1-\gamma_A)(1-\alpha)}\hat{\eta}$  <0, which represents the cost of added stringency. We then decompose rebound from the EES in the exact same way as in Section 4. In Panel B of Table 1, Column 5 shows rebound from this policy. This additional cost term in the income effect has a magnitude that depends on existing resource use for fuel efficiency  $(\gamma_A \equiv K_A/\overline{K})$ , curvature of its cost  $\beta$ , and the fraction  $\alpha$ .

The income effect is the main difference between the EES and CTS (with or without pre-existing mandate). The income effect for the CTS is always positive, as consumers get "free money" from reduced need to spend on A or  $E_S$ . Assuming the EES is binding and costly, however, the real income effect of the added cost of stringency must be negative.

As shown in Appendix C, using solutions for  $\hat{X}$  and  $\hat{S}$  above, we can measure the change in real income as:

$$\begin{split} \frac{dU}{\mu I} &= \frac{P_X X}{I} \hat{X} + \frac{P_K K_A + P_E E_S}{I} \hat{S} = (\gamma_X + \gamma_E \lambda_X) \hat{X} + (\gamma_A + \gamma_E \lambda_S) \hat{S} \\ &= \left( \frac{(\gamma_X + \gamma_E \lambda_X) \sigma_U \gamma_A + \lambda_S \gamma_E}{1 - \gamma_A} - \frac{\gamma_A \beta}{(1 - \gamma_A)(1 - \alpha)} \right) \hat{\eta} \end{split} \tag{6.5}$$

This long last expression for the income effect has two terms with opposite signs, but our assumption that the policy is costly and binding means that their sum must be negative. A slight re-arrangement of this condition implies:

**Proposition 2.** In this model, the pre-existing energy efficiency standard is costly and binding if and only if  $\gamma_A \beta > (1 - \alpha)((\gamma_X + \gamma_E \lambda_X)\sigma_U \gamma_A + \lambda_S \gamma_E)$ .

Our assumption that the EES is binding means that the inequality holds. It means we assume a large enough fraction of resources is used for energy efficiency ( $\gamma_A$ ), or a large enough curvature in its cost function ( $\beta$ ), or both. If so, then the total income effect is negative.

The direct rebound effect is the sum of substitution and income effects on  $E_s$ . The substitution effect on S and  $E_s$  is always positive, but the income effect is negative. Therefore, the DRE can be negative or positive. The indirect rebound effect on  $E_X$  includes a negative substitution effect and a negative income effect, so its sign is clearly negative.

<sup>&</sup>lt;sup>30</sup> Standards could be updated to keep shadow costs unchanged as technology advances. If so, the combination of both policy and technology changes might have no income effects. Thus, we do not imply that technology shocks increase actual energy use under binding standards. Here, we study each shock separately, all else equal.

Also, some prior studies find evidence for "backfire" from energy efficiency – where direct energy savings from increased energy efficiency are more than offset by positive rebound (Khazzoom, 1980; Brannlund et al., 2007). This backfire is less likely for a *mandated* increase in energy efficiency, because a mandate has negative income effects on rebound. Here, we show exact conditions under which backfire is impossible for the efficiency mandate.

**Proposition 3.** For increased stringency of an energy efficiency standard (EES) in this model with a pre-existing efficiency mandate that is costly and always binding:

- (i) The direct rebound is negative if and only if  $\gamma_A \beta > (1 \alpha)((\gamma_E \lambda_X + \gamma_X)\sigma_U + \gamma_E \lambda_S)$  (in which case,  $\gamma_A \beta$  is even larger than required for the EES to bind).
- (ii) The total rebound is negative if and only if  $\gamma_A \beta > (1 \alpha)(\lambda_S \gamma_X \sigma_U + \gamma_E \lambda_S)$  (less strict than in (i)). A sufficient condition is when X is energy-intensive  $(\lambda_S \gamma_X \lambda_X \gamma_A < 0)$ .
- (iii) Backfire is impossible if and only if  $\gamma_A \beta > (1 \alpha) \lambda_S \gamma_X (\sigma_U 1)$  (less strict than in (ii)). Sufficient conditions are X is energy-intensive  $(\lambda_S \gamma_X \lambda_X \gamma_A < 0)$ , or  $\sigma_U < 1$ .

Appendix F provides a detailed proof, but here we provide some intuition. The direct rebound effect on S is the sum of a positive substitution effect and a negative income effect, so it will be negative if the income effect dominates (a large enough  $\gamma_A$  or  $\beta$ , a small enough  $\sigma_U$ , or a combination of these). Total rebound is the sum of that direct rebound and unambiguously negative indirect rebound, so it can be negative under a looser condition. It is trivial to show that the condition in (i) is stricter than the one in (ii), and either is more strict than in (iii). If (ii) holds, so total rebound is negative, then direct energy savings are augmented by further energy savings, and backfire is impossible. The condition  $\sigma_U < 1$  is sufficient to rule out backfire, because the right side of the inequality in (iii) is negative, while the left side is positive. With  $\sigma_U < 1$ , the positive substitution effect is small and is swamped by the negative income effect, so the total rebound effect cannot offset direct energy savings.  $^{32}$ 

Empirical studies of rebound often ignore indirect rebound, i.e. effects in other markets captured only by GE models.<sup>33</sup> Yet we find above for the CTS that indirect rebound can exceed direct rebound. The same can hold for increased stringency of the EES. Proposition 3 shows that the condition under which direct rebound is positive is compatible with the condition where total rebound is negative. If so, then the magnitude of the negative indirect rebound must be greater that of the positive direct rebound. Moreover, Appendix F shows we can find conditions under which indirect rebound is more negative than direct rebound.

Finally, our model could be used to analyze and compare results for a tax on energy pollution or a subsidy to energy efficiency. After all, a tax has deadweight loss, and a subsidy must be funded by revenue from some other distorting tax. We do not analyze an energy tax here. First, pollution taxes are thoroughly studied already. Second, it would add length and detract from our focus on the effects of a mandate. Third, in any case, the mandate we analyze is a ratio requirement (e.g., reduced gallons per mile), which is already known to be equivalent to the revenue-neutral combination of a tax (per gallon) and subsidy (per mile).<sup>34</sup> That is, we already analyze a particular tax and

subsidy policy. Without non-optimizing behavior, neither tax nor mandate can achieve costless discovery of an effective new technology.

#### 7. Calibration

As shown above, rebound effects from an energy efficiency improvement depend on elasticities, shares, and cost parameters. To illustrate numerical magnitudes, we now choose plausible values for these parameters and use them in our formulas. Our benchmark dataset approximates the U.S. economy in 2015, the most recent year with relevant data. Our model has one primary factor, *K*, and two final outputs: appliance services *S* and a composite good *X*.

Two other outputs are used as intermediate inputs: energy, E, and energy efficiency features of appliances, A. We get residential energy expenditure data from State Energy Data System (SEDS), through which the Energy Information Administration (EIA) provides time series of energy production, consumption, prices, and expenditures across sectors. We get expenditure data on motor fuels from the Bureau of Economic Analysis (BEA). Household spending on energy for 2015 is 226 (\$B), and spending on motor fuels is 282 (\$B), for a total of 508 (\$B). We define a unit of E as the amount that costs \$1, so  $P_E$ =1 (dollars per unit of E). Energy  $E_S$  is the sum of all fuels used by households, so the initial  $E_S$ =508 billion units.

A problem is that observable spending on cars and appliances includes unobservable costs of many aesthetic features unrelated to energy efficiency that we wish to include in the other composite good X. Thus, we need to choose a new parameter,  $\phi$ , defined as the fraction of actual expenditures on cars and appliances that improve energy efficiency. Any accounting numbers for the explicit costs of energy efficiency would grossly understate the portion of total spending on cars and appliances that help increase energy efficiency, because firms can change designs and use lightweight structural materials to reduce fuel use, or add insulation to reduce energy use. Added costs to improve energy efficiency also vary across appliances.

For example, an energy-efficient LED light bulb lasts longer and uses less electricity, but it is about eight times the cost of an incandescent light bulb with the same light. If so, then 7/8 is the fraction of the capital cost paid for extra energy efficiency. For a vehicle, the fraction of cost that helps improve fuel economy is certainly smaller, since a car may include leather seats, moving sunroof, and stylish designs. But many parts of a car affect fuel use, including diverse features of its shape, weight, engine, transmission, and tires.<sup>35</sup> Furnaces and AC-units are installed outof-sight, so aesthetic features are less important, but all main parts of an AC affect its energy efficiency, including refrigerant, compressor, condenser coil, expansion valve, and evaporator coil. We find no estimate of this fraction  $\phi$  for any individual appliance, and certainly not for the aggregate or "average" appliance in our model. For a reasonably conservative but plausible single choice, we take  $\phi = 20\%$  as a fraction of all appliance costs that increase energy efficiency. But we put no particular emphasis on this choice and instead vary it from 10% to 80%. We show that smaller values of this parameter make income effects smaller and negative rebound less likely. But some appliances certainly have larger  $\phi$ >0.2, for which mandates will make negative income effects larger and rebound more negative.

To calculate the annual capital cost of appliances for energy efficiency,  $P_K K_A$ , we start with all household expenditures on new cars and appliances for 2015 from BEA (675 \$B).<sup>36</sup> We then perform an illustrative user cost calculation to arrive at the annualized cost, assuming annual

 $<sup>^{31}</sup>$  This condition on  $\sigma_U$  is consistent with findings in prior analytical literature (Sorrell, 2014)

 $<sup>^{32}</sup>$  A different special case also can guarantee that total rebound is negative. The costs of a more stringent EES certainly make the income effect negative, but if the X-sector is more energy-intensive than the S-sector, as well, then  $E_X$  falls by more than  $E_S$  rises, and the overall substitution effect must also be negative.

<sup>&</sup>lt;sup>33</sup> One difficulty for empirical studies is to define a complete set of substitutes and complements to a particular product. Second, most ignore indirect income effects and use only cross-price elasticity estimates.

<sup>&</sup>lt;sup>34</sup> See, e.g., Holland et al. (2009), Fullerton and Heutel (2010), and Goulder et al. (2016).

<sup>&</sup>lt;sup>35</sup> For example, a head gasket improves efficiency by sealing the combustion chamber, increasing compression, and containing exhaust gases. The average cost to replace a head gasket is around \$1200, and the average price of a new car in the U.S. is about \$36,000, so fixing a single head gasket costs >3% of the car price.

<sup>&</sup>lt;sup>36</sup> These assets include televisions, major household appliances (e.g., refrigerator, freezer, washer, dryer, stove, range, and air conditioners), small electric appliances (e.g., vacuum cleaner and electric kettle), equipment for house and garden (e.g., lawn mowers and propane grills), plus new motor vehicles and parts.

depreciation ( $\delta$ ) equal to 10% of total appliances and maintenance ( $\omega$ ) equal to another 5%. We ignore growth, so that all new appliance purchases are replacement investment (10% of the existing appliance stock each year). Then the existing stock is  $675 \times 10 = 6750$  (\$B). If the annual discount rate ( $\rho$ ) is 5%, then the annual user cost of all cars and appliances is  $6750 \times (\delta + \omega + \rho) = 1350$  (\$B).<sup>37</sup> With  $\phi$ =0.2 as the fraction representing energy efficiency features, the annual user cost of energy efficiency is  $(0.2) \times 1350 = P_K K_A = 270$  (\$B).

The goal here is not to calculate a definitive size of rebound, but to see illustrative magnitudes for the effects of parameters on results. All four parameter values  $(\delta, \omega, \rho,$  and  $\phi)$  could be debated and tested, but all four are used together only to obtain a single value for  $\gamma_A$ , the fraction of the annualized value of capital used for energy efficiency. To save space, we therefore vary just  $\gamma_A$  to see how any one of these choices could affect results.

Next, we use the EIA's SEDS to obtain economy-wide energy use for 2015 as \$1204 billion.<sup>38</sup> The ratio of energy use for cars and appliances to total energy use ( $\lambda_S = E_S/E$ ) is 508/1204 = 0.421. Then,  $\lambda_X = E_X/E = 1 - \lambda_S = 0.579$  is the fraction of energy used in production of the composite good X. Next, we define a unit of K as the amount that earns \$1 per year, so the initial  $P_K = 1$ , and national income is  $P_K \overline{K}$ . Using U.S. national income for 2015 from BEA, I = 15,783 (\$B).<sup>39</sup> The fractions of K used in production of energy and for energy efficiency are:

$$\gamma_E \equiv \frac{K_E}{\overline{K}} = \frac{P_K K_E}{P_K \overline{K}} = \frac{P_E E}{P_K \overline{K}} = \frac{1,204}{15,783} = 0.076$$

$$\gamma_A \equiv \frac{K_A}{\overline{K}} = \frac{P_K K_A}{P_K \overline{K}} = \frac{270}{15,783} = 0.017$$

Since the fractions of *K* used in all sectors add to one,  $\gamma_X = 1 - \gamma_A - \gamma_E = 0.907$ .

No one has estimated the elasticity of substitution in utility between appliance services and all other goods. To calibrate this  $\sigma_U$ , we start with an own-price demand elasticity for appliance services,  $\xi_{SS}$ . Following derivations in Baylis et al. (2014), which also apply here, these parameters are related to each other by  $\xi_{SS} = -(\xi_S + \sigma_U(1 - \xi_S))$ , where  $\xi_S$  is the share of income spent on services. Estimates range between -0.03 and -0.22 in most recent papers for the short- and medium-run price elasticity of demand for electricity, gasoline, and vehicle-miles-traveled. We use  $\xi_{SS} = -0.10$ , near the center of that range of estimates, to calculate our central value of 0.05 for  $\sigma_U$  (which we also vary in sensitivity analysis).

Our cost function  $K_A = B(A - A_0)^\beta$  needs no calibration of B, which drops out of the linear equation  $\widehat{K_A} = \frac{\beta}{1-\alpha} \hat{A}$  (where  $\alpha \equiv A_0/A$ ). In a simpler function where  $A_0$  and  $\alpha$  were zero,  $\beta$  could be calibrated from empirical studies that often find quadratic costs of energy efficiency  $(\beta=2)$ . But rather than *assume*  $A_0$  and  $\alpha$  are zero, our cost function has one more degree of freedom to allow a better fit with the data.

**Table 2** Calculated Values of  $\alpha$  and  $\beta$  for Particular Vehicles and Appliances.

| Vehicles                        | α    | β    |      | Appliances                | α    | β    |      |
|---------------------------------|------|------|------|---------------------------|------|------|------|
|                                 | 0.00 | 3.91 | 3.91 |                           | 0.00 | 4.66 | 4.66 |
|                                 | 0.27 | 3.11 | 4.26 |                           | 0.57 | 2.57 | 5.98 |
| Midsize cars (low               | 0.41 | 2.71 | 4.59 | Central air               | 0.64 | 2.31 | 6.42 |
| cost) <sup>a</sup>              | 0.55 | 2.32 | 5.16 | conditioners <sup>e</sup> | 0.71 | 2.05 | 7.07 |
|                                 | 0.00 | 2.84 | 2.84 |                           | 0.00 | 3.58 | 3.58 |
|                                 | 0.27 | 2.33 | 3.19 |                           | 0.54 | 2.18 | 4.74 |
| Midsize cars (high              | 0.41 | 2.08 | 3.53 |                           | 0.65 | 1.90 | 5.43 |
| cost) <sup>b</sup>              | 0.55 | 1.83 | 4.07 | Furnaces <sup>f</sup>     | 0.76 | 1.62 | 6.75 |
|                                 | 0.00 | 2.80 | 2.80 |                           | 0.00 | 5.82 | 5.82 |
|                                 | 0.28 | 2.30 | 3.19 |                           | 0.63 | 2.78 | 7.51 |
|                                 | 0.42 | 2.05 | 3.53 |                           | 0.68 | 2.53 | 7.91 |
| All passenger cars <sup>c</sup> | 0.56 | 1.79 | 4.07 | Refrigerator-freezersg    | 0.74 | 2.23 | 8.58 |
|                                 | 0.00 | 2.98 | 2.98 |                           |      |      |      |
|                                 | 0.26 | 2.43 | 3.28 |                           |      |      |      |
|                                 | 0.42 | 2.09 | 3.60 |                           |      |      |      |
| Light trucks <sup>d</sup>       | 0.58 | 1.75 | 4.17 |                           |      |      |      |

Data sources are as follows: (a) "low-technology cost scenario" in NRC (2015); (b) "high-technology cost scenario" in NRC (2015); (c) Lutsey et al. (2017); (d) Lutsey et al. (2017); (e) Department of Energy (2016a); (f) Department of Energy (2016b); (g) Department of Energy (2011). In every case, our use of those data to derive  $\alpha$  and  $\beta$  is described fully in Appendix G (online).

We jointly choose both  $\alpha$  and  $\beta$  using seven engineering studies of vehicles and appliances. Two studies are available for mid-size cars by the National Research Council (NRC, 2015). One study for cars and another for light trucks are from the International Council on Clean Transportation (Lutsey et al., 2017). We also have studies of central air conditioners from the Department of Energy (DOE, 2016a), furnaces from DOE (2016b), and refrigerator-freezers (DOE, 2011). Key for our purpose is that these studies provide an initial specific measure of energy efficiency A such as miles per gallon (mpg) and the extra cost of adding to it (in \$/mpg). These seven examples do not represent an average of all appliances, only a plausible range for our choice of  $\alpha$  and  $\beta$ .

Using each study, we choose  $A_0$  in the low-efficiency range and then calculate the implied  $\alpha$  and  $\beta$ . Appendix G provides detail of each calculation, and Table 2 shows results. For each studied car or appliance, we vary the initial choice of  $A_0$  (and thus  $\alpha$ ) to show how the resulting  $\alpha$  affects the calculated  $\beta$ . Table 2 shows that a higher  $\alpha$  always leads to a smaller  $\beta$ , and so these two parameters must be chosen in concert. If we were to choose  $\alpha$ =0, the values of  $\beta$  are very unstable (varying from 2.8 to 5.8). This is the reason we added the intercept  $A_0$  to the cost function. When  $\alpha$  is about one-half, the values of  $\beta$  vary much less (from about 1.8 to 2.6). The overall cost elasticity  $\frac{\beta}{1-\alpha}$  still ranges from 3.5 to 5.5 in Table 2, but that range implies costly mandates and large negative rebound. To be conservative, we choose  $\alpha$ =0.5 and  $\beta$ =1.5 as the "central" parameters to represent our aggregation of all cars and appliances, yielding only 3.0 for that overall cost elasticity. This choice is equivalent to the combination where  $\alpha$  is one-third and  $\beta$ =2, since the implied  $\frac{\beta}{1-\alpha}$  is still 3.0.

Finally, we check the condition in Proposition 2 to make sure that the EES is binding as our model assumes. The chosen parameter values imply  $\gamma_A\beta$  is 0.05, while  $(1-\alpha)\times[(\gamma_X+\gamma_E\lambda_X)(\sigma_U\gamma_A+\lambda_S\gamma_E)]$  is 0.03, so these values are indeed consistent with our assumption that the EES is binding and costly. Also, with all the calibrated parameters,  $\beta$  must be at least 0.96 to ensure that the EES is binding. <sup>43</sup> For sensitivity analysis, we vary  $\beta$  from 1 to 4.

<sup>&</sup>lt;sup>37</sup> Here, we assume perfect markets and ignore taxes, so the market rental price per year would be the same as the annualized cost to the owner of the car or appliance.

<sup>38</sup> https://www.eia.gov/state/seds/sep\_sum/html/pdf/sum\_ex\_tot.pdf.

<sup>&</sup>lt;sup>39</sup> Available at: https://apps.bea.gov/iTable/iTable.cfm?reqid=19&step=2#reqid=19&step=2&isuri=1&1921=survey.

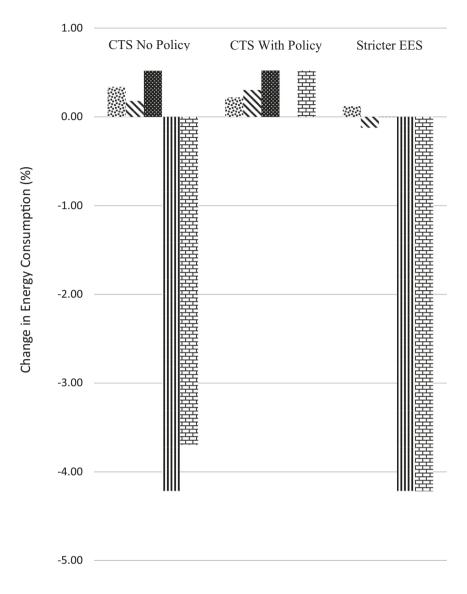
 $<sup>^{40}\,</sup>$  Hughes et al. (2008) find a short-run gasoline demand elasticity between  $-0.034\,\mathrm{and}$   $-0.077\,\mathrm{during}$  2001 to 2006. Gillingham (2014) finds a medium-run gasoline price elasticity of vehicle-miles-traveled about -0.22, while Allcott (2011) finds that the overall reduced-form price elasticity of demand for electricity is about  $-0.10.\,\mathrm{lto}$  (2014) finds short and medium run electricity demand elasticities are between  $-0.040\,\mathrm{and}$  -0.101. The short-run price elasticity of demand for electricity is about  $-0.09\,\mathrm{in}$  Deryugina et al. (2020).

<sup>&</sup>lt;sup>41</sup> Greene and DeCicco (2000) find that a quadratic cost function for fuel economy improvement can be a good fit to the data. Also see NRC (2002) for various estimates of fuel economy cost curves. Another relevant paper by Anderson and Sallee (2011) uses loopholes in CAFE standards to estimate the policy's marginal compliance cost from 1996 to 2006. Their estimates suggest nearly constant marginal cost ( $\beta$   $\cong$  1), but they explain that this result follows from the availability of loopholes and the fact that automakers have not exhausted those loopholes.

<sup>&</sup>lt;sup>42</sup> If the overall cost elasticity of 3.0 seems large, remember that it is the percent change in cost for a 1% increase in *total* energy efficiency. In the vehicle example, suppose the required mpg rises from  $\epsilon A$ =30.0 to 30.3 (a 1% increase in mpg). Manufacturers had no extra cost of achieving the initial "minimum" 15 mpg, so our cost function applies to the mpg in excess of  $A_0$ =15. The same increase of 0.3 mpg is a 2% increase in the *extra* 15 mpg (from 15 to 30 mpg). With our  $\beta$ =1.5, that 2% addition means a 3% increase in cost.

<sup>&</sup>lt;sup>43</sup> The calculations in seven studies for cars and appliances in Appendix G also can be used to show binding costs at current standards, as we do for two examples in Section 1 (midsize cars, and top-mount refrigerator-freezers).

«DRE ⊗IRE ■TRE □DEE \( \pm \text{TE} \)



DRE: Direct Rebound Effect

IRE: Indirect Rebound Effect

TE: Total Energy

TRE: Total Rebound Effect

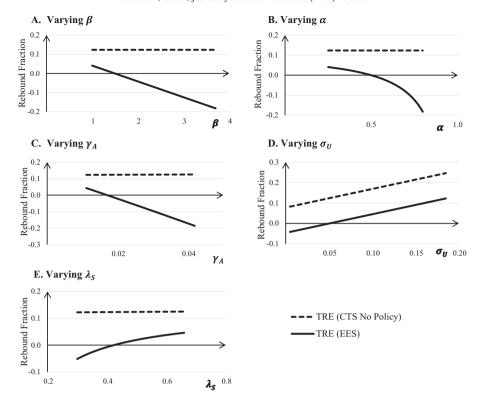
Fig. 1. Effects on energy use from the CTS and EES (central parameter values).

# 8. Numerical illustrations

For any costless technology shock, we assume a 10% increase in energy efficiency technology ( $\hat{\epsilon}=0.10$ ). In the CTS case with no policy and no change in purchased energy efficiency (where  $\hat{A}=0$ ), 10% better technology implies 10% better energy efficiency. For the EES, we assume the government raises the efficiency standard for all appliances by 10%, so it generates the same impact on energy efficiency. Inserting all calibrated parameters into each expression in Table 1 yields the size of each effect in percentage points. The direct efficiency effect (DEE) is calculated as if relative prices and incomes were constant. For example, a +10% change in  $\epsilon$  leads to a -4.2% DEE (because  $\lambda_S \equiv E_S/E = 0.42$ ). If the total rebound effect (TRE) is about +1%, then total use of energy E falls by the difference (-3.2%).

Using our calibrated parameters, we calculate and plot in Fig. 1 not only the DEE, but also the direct rebound effect (DRE), the indirect rebound effect (IRE), the TRE, and the total effect (TE). The first five bars on the left are the effects from the CTS with no mandate. The five bars in the middle show the CTS case with a binding EES, and the last five bars are the effects on energy use from increased stringency of the EES policy.

For the CTS with no policy, the fourth bar shows that the free 10% gain in energy efficiency leads to a -4.2% "direct effect" on total energy use. A little more than 12% of that energy saving is offset by the positive TRE in the third bar. Comparing the DRE in the first bar to the TRE in the third bar suggests that nearly two-third of total rebound is from appliance services. Though not shown in Fig. 1, both the income and substitution effects increase consumption of S. For the other goods in S, however, the substitution effect reduces consumption, while the



**Fig. 2.** Total rebound as a fraction of direct efficiency effect (CTS and EES). Notes: Panel A varies β, the curvature of the cost function for energy efficiency. Panel B varies  $α = A_0/A$ , the intercept in that cost function. Panel C varies  $γ_A$ , the fraction of composite inputs K used for energy efficiency. Panel D varies  $σ_U$ , the elasticity of substitution in utility. Panel E varies  $λ_S$ , the fraction of total energy used with cars and appliances to produce energy services.

income effect increases it. The IRE in the second bar is net positive, which means the positive income effect dominates the negative substitution effect on *X*. As intended, these numerical results are consistent with findings in the prior rebound literature that study the exogenous costless technology change.<sup>44</sup>

New in this paper is a pre-existing mandate that remains binding after the costless technology shock (in the middle bars of Fig. 1). Then the CTS allows consumers to pay less for energy efficiency but still meet the unchanged energy efficiency requirement,  $\epsilon A$ . The reduced spending on A perfectly offsets the increase in technology (so  $\hat{A} = -\hat{\epsilon}$ ). The savings can be spent on more energy for services ( $E_S$ ) and on other goods (X). Visually, in the middle bars for the CTS with existing policy, the bar for DEE seems missing, but energy efficiency stays at the original level, so the DEE is literally zero. The total effect on energy is exactly the total rebound effect, +0.52%. Both rebound effects are positive, but the IRE is slightly greater than the DRE. The income effect explains all rebound (i.e., no substitution effect).

The "stricter EES" case shows the effects on energy use when the pre-existing EES becomes 10% more stringent. As normalized, the CTS with no policy and the stricter EES have identical direct energy efficiency effects – as might be calculated by engineers. But the DRE from the EES (the dotted bar on the right) is one-third of the DRE from the CTS with no policy (the dotted bar on the left). Indirect rebound from the EES (the diagonal stripe bar on the right) is negative and slightly larger in magnitude than the positive DRE. Thus, total rebound is slightly negative: total energy saving in the last bar is greater than direct energy saving. Substitution effects are the same for both the EES and CTS with

no policy. Therefore, the real income effect is the main difference in numerical results between the CTS and EES shocks. The CTS provides an income gain, while policy presents a real income loss.

Since the income effect is a key factor that distinguishes the CTS and the EES, we next consider sensitivity of results first for  $\beta$  and  $\gamma_A$  and then for other parameters.

## 9. Sensitivity analysis

Fig. 2 investigates the sensitivity of rebound as we vary each key parameter (using central-case values for other parameters). For comparability with prior literature, the vertical axis shows total rebound as a fraction of positive energy savings from the direct efficiency effect (so the ratio has the same sign as rebound). The figure shows no line for the CTS with an existing policy, because that denominator would be zero. For that case, we already know from Proposition 1 that positive rebound is the total effect, because the mandate is still binding before and after the small costfree energy efficiency gain. Thus, each panel of Fig. 2 shows that rebound fraction only for the CTS with no policy (dashed line) and EES (solid line).

Fig. 2A investigates sensitivity of total rebound to the assumed curvature of the cost function for energy efficiency ( $\beta$ ). The line for the CTS with no policy shows total rebound is about 12% of the direct energy savings, and it is completely flat because the absence of a policy makes irrelevant the cost of policy. In contrast, with increased EES stringency, total rebound is slightly positive if  $\beta$ =1 but then turns negative and becomes more negative for larger  $\beta$ . Why? A larger  $\beta$  means that the additional required energy efficiency is more expensive, which causes a larger negative income shock and negative rebound.

Next, Fig. 2B shows results as the intercept in the cost function,  $\alpha = A_0/A$ , varies from 0.2 to 0.8. Results are almost identical to those in Fig. 2A, because  $\alpha$  and  $\beta$  affect only the overall added cost in  $\widehat{K_A} = \frac{\beta}{1-\alpha} \hat{A}$ . The difference is that  $\beta$  affects that added cost linearly, whereas  $\alpha$  affects it nonlinearly. Just as for  $\beta$  in Fig. 2A, the choice of  $\alpha$  in Fig. 2B does not

 $<sup>^{44}</sup>$  Empirical estimates of the direct rebound effect (DRE) from the CTS without policy vary widely in the prior literature. As a fraction of the direct efficiency effect, they range from 0% to nearly 90% (Sorrell et al., 2009), but most of those estimates are around 10–30%. Calculations of total rebound from computable GE models are often large, ranging from 30% to >100% (Sorrell, 2007; Chang et al., 2018). We calculate a DRE of 8.05%, and indirect rebound of 4.26%, so our total rebound effect is 12.32% for the CTS without policy.

affect the CTS. In both figures, when  $\frac{\beta}{1-\alpha}$  gets larger, total rebound is more negative for an EES. These sensitivity results show that our central values of  $\beta$ =1.5 and  $\alpha$ =0.5 appear to be both reasonable and conservative, as well as compatible with each other.

Next, we test sensitivity to the assumption that 20% of spending on appliances is for enhanced energy efficiency. This fraction  $\phi$  primarily affects  $\gamma_A \equiv K_A/\overline{K}$ , the fraction of K used for energy efficiency. The relationship between  $\phi$  and  $\gamma_A$  is monotonic, however, so sensitivity to  $\gamma_A$  is the same as sensitivity to  $\phi$ . Our calibration above has  $\gamma_A = 0.017$ , but Fig. 2C varies it from 0.01 to 0.04 (or equivalently,  $\phi$  varies from 0.1 to 0.5). As shown, total rebound from the CTS does not depend on  $\gamma_A$ , because again this parameter pertains to the cost of policy. In contrast, total rebound from the EES falls with  $\gamma_A$ . The first three panels are similar, as these parameters all

enter the policy cost  $(\frac{\gamma_A \beta}{(1-\gamma_A)(1-\alpha)}$  in Eq. (6.4)). Prior studies focus on the responsiveness of rebound to the elasticity

Prior studies focus on the responsiveness of rebound to the elasticity of substitution between energy services and other goods. To compare our results with prior literature, Fig. 2D plots rebound from the CTS and EES against the elasticity of substitution in utility ( $\sigma_U$ ). Total rebound effects rise with  $\sigma_U$  for both the EES shock and the CTS with no policy, a result that is both intuitive and consistent with prior literature. Ae Regardless of whether it is cost-free or costly, an increase in energy efficiency reduces the marginal cost of energy services and thus causes a strictly positive direct rebound substitution effect that rises with  $\sigma_U$ . The income effect is always negative for the EES shock and positive for the CTS shock, so total rebound is always smaller from the EES than the CTS, at every value of  $\sigma_U$  in Fig. 2D.

Next, Fig. 2E shows sensitivity to the fraction of total energy used with regulated appliances to produce energy services ( $\lambda_s$ ). This parameter is the only one that affects the denominator of our rebound ratio (direct energy saving is  $\lambda_s$  times either the CTS shock or the EES shock). But  $\lambda_s$  also enters the rebound effect for both shocks. The dashed line for the CTS is flat, so  $\lambda_s$  raises both rebound and energy savings similarly. The solid line for the EES rises with  $\lambda_s$ , however, so this parameter raises rebound more than energy savings. The reason is that the policy cost

$$(\frac{\gamma_A\beta}{(1-\gamma_A)(1-\alpha)}) \text{ is a negative portion of rebound that does not rise with } \lambda_S.$$

Finally, an important caveat. While this section varies each assumed parameter, we cannot vary other basic model assumptions. In particular, the tractability and simplicity of our analytical solutions is based on homothetic utility, but then every income elasticity of demand is one. A computational GE model could use separate income elasticities for each good, to obtain more specific and precise numerical calculations of rebound. But our paper is making a conceptual point, and the conceptual result already is clear: if the demand for vehicle miles or appliance services has an income elasticity greater than one (and larger than for other less energy-intensive goods), then a CTS with positive income effects has even-more positive rebound, and an EES with negative income effects will have even more negative rebound.

#### 10. Welfare implications

The costless technology improvement clearly raises welfare, with or without existing mandates, so here we discuss the more subtle welfare effects of increased mandate stringency. A stricter standard clearly has costs, but it also has benefits of reduced energy externalities – especially if negative rebound reduces energy use further. We can now ask whether and under what initial conditions a stricter EES raises welfare, considering both its costs and benefits.

For the change in EES, we derived an expression for welfare cost (and we showed it matches the negative income effect). The benefits of energy reductions are implicit in our utility function, U=U(S,X;nE). Here, we do not assume any value of marginal benefits from reduction in total energy use. Instead, we use these expressions to solve for the "threshold" value of the social cost of carbon (SCC) that barely makes the policy worthwhile. If a higher  $\beta$  raises the cost of additional stringency, for example, then it raises the threshold SCC (TSCC). The policy might raise net welfare, but only if the cost of carbon is above that threshold.

We use central values of all parameters to calculate the threshold SCC as follows. We assume that carbon emissions C are proportional to energy use, so  $\hat{C}$  is equal to  $\hat{E}$  in Eq. (6.4) for the 10% increase in EES stringency ( $\hat{\eta}=0.10$ ). We insert parameters to calculate  $\hat{C}=\hat{E}=0.04219$  (a 4.2% reduction in energy and emissions). For total C in the initial equilibrium, we use 5.423 billion metric tons, as reported for 2015 by the U.S. EPA.<sup>47</sup> Then, since  $\hat{C}\equiv dC/C$ , we calculate  $dC=5.423\times0.04219=0.2288$  billion tons. The dollar value of this small emission reduction is set equal to the dollar cost of the policy: SCC× $dC=dU/\mu$ , so that we can solve for the threshold SCC as \$129 per ton.<sup>48</sup>

This social cost of carbon is higher than the \$48–\$74 per ton cost for vehicle standards found by Anderson and Sallee (2011). This difference is consistent with the fact that our marginal cost curve slopes up ( $\beta$ = 1.5), while theirs is nearly flat ( $\beta$ ° 1.0) because of loopholes in those standards. Our TSCC is also greater than the U.S estimates of SCC. If the true SCC is less than our calculated threshold of \$129 per ton, then these results imply that energy standards are not an efficient way to reduce carbon emissions. In fact, they imply that welfare could be raised by reducing the stringency of those standards.

Which parameters most affect welfare, as opposed to rebound? We show the sensitivity of both of these outcomes to variation in two key parameters at a time in Fig. 3A, where a dashed line identifies the combination of the two parameters that leads to zero rebound. To the upper left is negative rebound with higher values of cost curvature ( $\beta$ ) on the vertical axis or lower values of the elasticity of substitution ( $\sigma_U$ ) on the horizontal axis. To the lower right is positive rebound (with lower  $\beta$  or higher  $\sigma_U$ ). Then the same figure shows contour lines for threshold SCC (where an increase in stringency has no net gain). This TSCC rises with the cost of energy efficiency mandates, as just described, but it does not much depend on  $\sigma_U$ . A round dot in the figure shows our chosen  $\beta$ = 1.5 and  $\sigma_U$ =0.05 (where TSCC = \$129).

Similarly, Fig. 3B shows sensitivity of the TSCC to variations in  $\gamma_A$  on the vertical axis and  $\lambda_S$  on the horizontal axis. A dot indicates the initial  $\gamma_A = 0.017$  and  $\lambda_S = 0.421$ . The contour lines show the TSCC is sensitive to both  $\gamma_A$  and  $\lambda_S$ . It increases with  $\gamma_A$  but decreases with  $\lambda_S$ . As discussed, sensitivity to  $\alpha$  matches that of  $\beta$ ; sensitivity to  $\phi$  matches that of  $\gamma_A$ .

Both figures generally show that both rebound and welfare are sensitive to variations in any of these parameters. Also, rebound per se is neither good nor bad: both figures show areas where positive rebound is associated with net welfare gain or loss, just as negative rebound can be associated with a net gain or loss. The figures show how parameters drive such results.

<sup>&</sup>lt;sup>45</sup> This assumption also affects  $\gamma_X = 1 - \gamma_A - \gamma_E$ . We directly observe  $\gamma_E = K_E/\overline{K}$ , but  $\gamma_X = K_X/\overline{K}$  is assumed to include the fraction of total appliance spending that is not for energy efficiency.

<sup>&</sup>lt;sup>46</sup> Rebound also rises with  $\sigma_U$  for the CTS with policy, but that rebound is 100% of the total effect on energy.

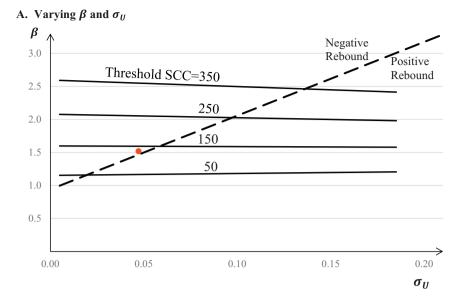
<sup>&</sup>lt;sup>47</sup> See https://www.epa.gov/ghgemissions/inventory-us-greenhouse-gas-emissions-and-sinks.

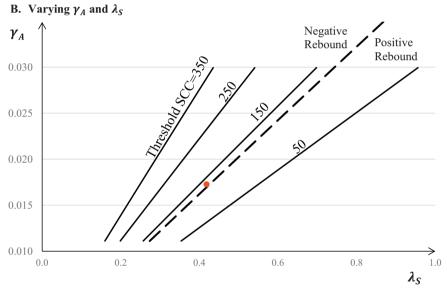
<sup>&</sup>lt;sup>48</sup> Eq. (6.5) provides  $(dU)/(\mu I)$ , which we calculate from parameters as 0.0018667, and our calibration above shows U.S. national income for 2015 as I=15,783 (\$B). Thus,  $dU/\mu$  is a \$29.46 billion loss. Finally, we solve for the threshold SSC as this  $dU/\mu$  divided by dC=0.2288 billion tons. to get \$129 per ton.

<sup>49</sup> See footnote 42 in our Section 7 on "Calibration".

<sup>&</sup>lt;sup>50</sup> See "Technical Support Document: Technical Update of the Social Cost of Carbon for Regulatory Analysis," by the Interagency Working Group, available at https://www.epa. gov/sites/production/files/2016-12/documents/sc\_co2\_tsd\_august\_2016.pdf.

<sup>&</sup>lt;sup>51</sup> This result may seem surprising, because a larger  $\sigma_U$  means larger rebound, which offsets more of the direct efficiency effect and reduces environmental benefits. But it also allows consumers to substitute more easily from X to take advantage of cheaper energy services – an effect that reduces the welfare cost of the EES.





**Fig. 3.** Welfare effects from an EES. Notes: Panel A varies  $\beta$ , the curvature of the cost function for energy efficiency, and  $\sigma_U$ , the elasticity of substitution in utility. Panel B varies  $\gamma_A$ , the fraction of composite inputs K used for energy efficiency, and  $\lambda_S$ , the fraction of total energy used with cars and appliances to produce energy services.

Thus, these calculations are important for policymakers to be able to make use of information about initial conditions to judge the likely success of their policy proposal in terms of both rebound and welfare. That success will depend on stringency of the existing standards on the appliances under consideration ( $\alpha$  and  $\beta$ ), on households' flexibility to substitute toward those services ( $\sigma_U$ ), on the initial fraction of energy used with those appliances ( $\lambda_S$ ), and on the energy efficiency costs of those appliances ( $\phi$  or  $\gamma_A$ ).

### 11. Extensions

Our model must be fairly simple to be able to solve for GE effects analytically, but the advantage is that mathematical expressions show exactly how each effect depends on parameters and policies. We explained several caveats along the way. In this section, we extend our model to relax four of our key assumptions and to see how they affect our main results: (a) negative effects of total energy use on productivity; (b) upward-sloping supply of energy; (c) mandates in the business sector, and (d) disaggregation.

First, we study the case where emissions from energy use reduce productivity (e.g., Williams III, 2002). If so, reducing energy use could have positive income effects on rebound. Then Eq. (2.9) of our basic model becomes  $G(E)\bar{K}=K_X+K_A+K_E$ , with  $\partial G/\partial E<0$ , so that more emissions reduce productivity of the primary factor K. In the initial equilibrium,  $G(E^0)=1$ , where  $E^0$  is initial energy use. We linearize that equation to get:

$$\delta \hat{E} = \gamma_{x} \widehat{K_{x}} + \gamma_{4} \widehat{K_{A}} + \gamma_{F} \widehat{K_{F}}$$
(11.1)

where  $\delta = \frac{\partial G}{\partial E} \frac{E}{G(E)} < 0$  is the elasticity of productivity with respect to energy use. Appendix H derives all outcomes for a stricter EES, but here we show only the effect on total energy,  $\hat{E}$ :

$$\hat{E} = -\lambda_{S}\hat{\eta} + \frac{1}{(1 - \gamma_{A} - \delta)} \left( \gamma_{X} \lambda_{S} \sigma_{U} + \lambda_{S} \gamma_{E} - \lambda_{S} \delta - \frac{\gamma_{A} \beta}{1 - \alpha} \right) \hat{\eta}$$
 (11.2)

The first term,  $-\lambda_5 \hat{\eta}$ , is the direct efficiency effect, and the rest is total rebound. The new parameter  $\delta$ <0 appears in two locations and therefore

affects rebound, but it does not affect the final term reflecting the cost of the policy – which generates a negative income effect with a magnitude that depends as before on the overall cost elasticity  $\beta/(1-\alpha)$  and the share  $\gamma_A$ .

The new condition for total rebound to be negative can be written as:

$$\gamma_A \beta > (1-\alpha)(\gamma_X \lambda_S \sigma_U + \lambda_S \gamma_E - \lambda_S \delta),$$
 (11.3)

which almost matches the condition in Proposition 3.ii from before, but adds  $-\lambda_S\delta > 0$  to the right hand side. This term makes it harder for total rebound to be negative. Clearly, the productivity gain from reducing energy consumption through the stricter EES generates a positive income effect and thus a positive effect on rebound. The sign of total rebound is still ambiguous, however. With a high enough cost of the mandate, rebound can still be negative.

For our second extension, an upward-sloping supply of energy means that an EES can cut demand for oil, reduce the equilibrium oil price, encourage other uses of oil, and thus have more positive indirect rebound (Gillingham et al., 2016). We replace the production function  $E = K_E$  with  $E = E(K_E, \bar{Q})$ , where  $\bar{Q}$  is a fixed natural resource. Then  $\sigma_E$  is the elasticity of substitution in the production of energy, and  $\theta_{QE}$  is the cost share of the fixed factor  $\bar{Q}$ . This extension introduces one new unknown variable  $P_Q$ , the return to the fixed factor  $\bar{Q}$ , and thus it raises the number of equations and unknowns from 10 to 11. All derivations are in Appendix I. The result for  $\hat{E}$  in the case with increased stringency of the EES is:

$$\hat{\textit{E}} = -\lambda_{\textit{S}}\hat{\eta} + \frac{1}{D} \bigg( \big(\sigma_{\textit{E}}\theta_{\textit{KE}} + \theta_{\textit{QE}}\lambda_{\textit{S}}\theta_{\textit{KX}}\big) \gamma_{\textit{X}}\lambda_{\textit{S}}\sigma_{\textit{U}} + \theta_{\textit{QE}}\lambda_{\textit{S}}\gamma_{\textit{X}}\sigma_{\textit{X}}(1 - \lambda_{\textit{S}}\theta_{\textit{KX}}) \\ + \lambda_{\textit{S}}\gamma_{\textit{E}}\sigma_{\textit{E}} - \sigma_{\textit{E}}\theta_{\textit{KE}}\frac{\gamma_{\textit{A}}\beta}{1 - \alpha} \bigg) \hat{\eta}$$

where 
$$D = \theta_{QE}(\sigma_E(\theta_{KE}\gamma_X + \gamma_E)/\theta_{QE} + \gamma_X\lambda_S\theta_{KX}\sigma_U + \gamma_X\sigma_X(1 - \lambda_S\theta_{KX})) > 0.$$
 (11.4).

The first term is the DEE,  $-\lambda_S \hat{\eta}$ , and the rest is the TRE (with both substitution and income effects from the reduced price of energy). Can the stricter EES still make the TRE negative? The last term in the big parenthesis shows the negative effect of the costly policy, but other terms are positive, so the sign of the TRE is ambiguous. The condition for a negative TRE is:

$$\gamma_{A}\beta > (1-\alpha) \left( \lambda_{S}\gamma_{X}\sigma_{U} + \gamma_{E}\lambda_{S} + \frac{\lambda_{S}\theta_{KX}\lambda_{S}\gamma_{X}\sigma_{U}\theta_{QE}}{\sigma_{E}\theta_{KE}} + \frac{\gamma_{E}\lambda_{S}\theta_{QE}}{\theta_{KE}} + \frac{\lambda_{S}(1-\lambda_{S}\theta_{KX})\gamma_{X}\sigma_{X}\theta_{QE}}{\sigma_{E}\theta_{KE}} \right) \tag{11.5}$$

which is stricter than the condition in Proposition 3.ii. Thus, adding slope to the energy supply curve makes it somewhat more difficult for negative rebound effects to outweigh positive rebound effects. But, it does not negate our point that the cost of the mandate introduces large negative effects on rebound. Thus, results of prior analyses of the costless technology shock should not be used directly to infer effects of energy efficiency policy.

Third, firms in sector X may also face energy standards on their business equipment and vehicles. A complete model of that fact would include several more equations, lengthy expressions, and more effects in our solutions. Using an ad hoc approach, however, we can approximate those complicated effects within our simple model. Our initial  $\lambda_S$ =0.421 reflects the fraction of total energy used with regulated household vehicles and appliances to produce energy services, but that parameter can be raised to reflect the case where business machinery also faces standards. Our earlier discussion of Fig. 2E explains why the rebound ratio rises with  $\lambda_S$ , so this extension might also make negative rebound less likely.

Fourth, disaggregation and heterogeneity could matter for several reasons: (a.) Diverse goods can have different cross-price effects and energy intensities (Chan and Gillingham, 2015). Thus, a change

**Table 3**Parameters for vehicles only and household appliances only.

|                 | Share of capital | Share of energy $\lambda_S \equiv E_S/E$ | Elasticity<br>parameters |     |                                   |
|-----------------|------------------|--|--------------------------|-----|-----------------------------------|
|                 |                  |  | α                        | β   | $\sigma_{\!\scriptscriptstyle U}$ |
| Both together   | 0.017            | 0.421                                    | 0.5                      | 1.5 | 0.05                              |
| Vehicles only   | 0.009            | 0.187                                    | 0.5                      | 1.5 | 0.05                              |
| Appliances only | 0.008            | 0.234                                    | 0.5                      | 1.5 | 0.05                              |

in the price of services from one appliance can raise or reduce demand for any other good, each with a positive or negative effect on energy use. <sup>52</sup> Indeed, our results highlight income effects, so different income elasticities can also matter. (b.) Disaggregation could capture details of policy. For example, U.S. vehicle CAFE standards implicitly subsidize fuel-efficient cars and tax fuel-inefficient cars. Here, we capture only the overall costs of such a policy on the average car with a single fuel efficiency. (c.) Heterogeneity would be important if high-income household expenditures differ from those of low-income households – where policy places more burden on one or the other.

We cannot disaggregate goods in GE without a computational model, but a GE model is not necessary to analyze a policy that applies only to one appliance or 0.1% of the economy; it would have tiny indirect effects on the other 99.9%. Again, however, we can use an ad hoc approach to see key differences between cars and appliances. Using calibration procedures analogous to those in Section 7 above, Appendix J derives share parameters  $(\gamma_A \text{ and } \lambda_S)$  only for household vehicles and energy use (motor fuel). Then it separately aggregates all household appliances other than vehicles, such as refrigerators, furnaces, and air conditioners. The calculated share parameters are shown in the first two columns of Table 3. We continue to use the previous cost function parameters for both cars and appliances.  $^{53}$ 

Using the parameters in Table 3 for vehicles separately from appliances, we calculate bar graphs just like Fig. 1 (showing bars for the DEE, DRE, IRE, TRE, and total energy use). Those bar graphs are in Appendix J, rather than here, because they look exactly like Fig. 1 but for the vertical scale. Table 3 shows that share parameters for vehicles are very nearly half the size of the shares for appliances and vehicles together (and so appliances alone are the other half). Thus, bars for the DEE extend down to about 2% less energy, instead of 4.2% less energy. Each type of positive rebound extends up to 0.25% more energy instead of 0.5% more energy; and each type of rebound is also half the size of those in Fig. 1.

The lesson here is that the size of the sector does not affect the relative size of effects. Both share parameters ( $\gamma_A$  and  $\lambda_S$ ) are halved, so all results are halved, and total rebound as a fraction of the direct efficiency effect is virtually unchanged. The relative sizes of the DEE would differ if  $only \ \lambda_S$  changed (as in Fig. 2E); the relative sizes of other effects would differ if only  $\gamma_A$  changed (as in Fig. 2C). The relative sizes of the effects for appliances vs. vehicles would also differ if those two categories faced different mandate stringency (i.e.,  $\beta$ , as in Fig. 2A), or have different demand elasticities (based on  $\sigma_U$ , varied in Fig. 2D). The point here is that the relative size of each rebound effect depends in a primary way on whether the improvement in energy efficiency derives from a technology shock or a mandate, and whether the technology shock takes place with or without a pre-existing mandate.

<sup>&</sup>lt;sup>52</sup> West et al. (2017) show that a change in fuel efficiency also changes important vehicle attributes such as size, safety, and performance. While the reduced cost per mile can increase driving, changes in those other attributes could reduce it. Conceptually, these other attributes are "other goods" in utility, with cross-price elasticities.

<sup>&</sup>lt;sup>53</sup> Table 2 shows that the calibration of  $\alpha$  and  $\beta$  are similar for vehicles in the first column and appliances in the second column. Also, Fig. 2 shows how  $\alpha$  or  $\beta$  affects results for either cars or appliances.

### 12. Conclusions

We use a simple analytical general equilibrium model to analyze and compare rebound effects from an EES and a CTS. For both shocks, an existing EES can be costly and binding. Also, we decompose each total effect on the use of energy into components, including a direct efficiency effect, a direct rebound effect, and an indirect rebound effect. Each rebound term is composed of substitution and income effects. Results show that the magnitude and sign of rebound for *both* the CTS and EES depend strongly on the costliness of efficiency mandates.

Unlike a CTS that raises real incomes, with positive rebound effects on energy use, standards have costs and therefore reduce income, reduce energy use, and reduce rebound. Our numerical illustrations use reasonable values for parameters such as elasticities, factor shares, and cost parameters. With these plausible parameter values, total rebound from a CTS is positive, but total rebound from an EES is slightly negative. These findings suggest that using a review of all rebound estimates to analyze a mandate would grossly overstate rebound.

Our results do not invalidate all prior empirical estimates of rebound from PE models, or those based on data from costless variations in energy efficiency, but instead guide the interpretation of those results. To provide such guidance, we would ask four questions.

First, what is the source of variation in the data? Energy efficiency can vary over time from costless innovation, costly innovation, costly mandates, or an unknown mix of all those changes. Thus, outcome variables can be affected not only by substitution effects from cheaper driving or cooling, but also by an unknown mix of income effects that are positive for costless technology, small for voluntary spending on new technology, or negative for increases in mandate stringency. Empirical design could be improved to focus just on costless variation or on policy variation. For example, data on energy use in locations subjected to a policy change could be compared to energy use in control locations with no policy change.

Second, what is the dependent variable? To study a new standard for air conditioners (AC), the dependent variable might be the indoor temperature setting or amount of cooling. That kind of study would likely find positive rebound for that narrow use of energy, but it ignores changes in other uses of electricity from negative income effects (increasingly expensive AC-units and other policy costs). Even if the dependent variable is all residential electricity, it would miss income effects on other kinds of energy and on business use of energy. These income effects on each particular use of energy would be small and likely not statistically significant, but those small changes add up over many other uses of energy. This paper demonstrates that, in aggregate, those indirect negative general equilibrium effects on other sectors likely swamp the positive rebound effect on AC-use of electricity.

Third, how can empirical estimates of rebound be used by policy analysts to calculate effects of a policy? An estimate of rebound based on variation from costless technology changes might be used as input into a simple analytical model of a costly mandate in three steps: (1) Calculate the real income gain from that technology and use an economywide income elasticity of demand for all energy to remove that positive income effect on rebound; (2) Calculate the extra voluntary costs to buy the new technology and use those costs to reduce rebound via the same income elasticity; (3) Calculate the income loss from distorting effects of mandates, and reduce rebound still further using the same income elasticity.

Fourth, can any rough calculation shortcut all those steps? Yes. Despite limitations of our GE model and especially of any particular parameter values, the theory makes clear that a costless shock has positive rebound both from the substitution effect and from the positive income effect. A mandate has the same positive substitution effect but a negative income effect, which can make total rebound zero or negative. The net effect depends on substitution elasticities and mandate stringency, but this logic makes clear that policy analysis cannot appropriately use a "rough number" for rebound like 15% or 20% that is based on estimates from

partial equilibrium models that do not account for economy-wide policy costs. For a simple rule of thumb to analyze a costly energy efficiency mandate, our results strongly suggest that the best rough number to assume for rebound is zero.

# Appendix A. Online Appendices A-J

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jpubeco.2020.104225.

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