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On the Convergence of the Iterative Linear Exponential Quadratic Gaussian Algorithm to Stationary Points

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Abstract

A classical method for risk-sensitive nonlinear control is the iterative linear exponential quadratic Gaussian algorithm. We present its convergence analysis from a first-order optimization viewpoint. We identify the objective that the algorithm actually minimizes and we show how the addition of a proximal term guarantees convergence to a stationary point.

Introduction

We present a convergence analysis of the classical iterative linear quadratic exponential Gaussian controller (ILEQG) [Whittle, 1981] for finite-horizon risk-sensitive or safe nonlinear control. The ILEQG algorithm is particularly popular in robotics applications [Li and Todorov, 2007] and can be seen as a risk-sensitive counterpart of the iterative linear quadratic Gaussian (ILQG) algorithm . We adopt here the viewpoint of the modern complexity analysis of first-order optimization algorithms as done by Roulet et al. [2019b] for ILQG.

We address the following questions: (i) what is the convergence rate of ILEQG to a stationary point? (ii) how can we set the step-size to guarantee a decreasing objective along the iterations? The analysis we present here sheds light on these questions by highlighting the objective minimized by ILEQG which is a Gaussian approximation of a risk-sensitive cost around the linearized trajectory. We underscore the importance of the addition of a proximal regularization component for ILEQG to guarantee a worst-case convergence to a stationary point of the objective.

The main result of the paper is Theorem 2.5, where a sufficient decrease condition to choose the strength of the proximal regularization is given. The result also yields a complexity bound in terms of calls to a dynamic programming procedure. We illustrate the variant of the iterative regularized linear quadratic exponential Gaussian controller we recommend on simple risk-sensitive nonlinear control examples.

Related work. The linear exponential quadratic Gaussian algorithm is a fundamental algorithm for risk-sensitive or safe control [Whittle, 1981, Jacobson, 1973, Speyer et al., 1974]. The algorithm builds upon a risk-sensitive measure, a less conservative and more flexible framework than the H^∞ theory also used for robust control; see [Glover and Doyle, 1988, Hassibi et al., 1999, Helton and James, 1999] and references therein. An excellent review of the classical results in abstract dynamic programming and control theory, in particular for risk-sensitive control, was done by Bertsekas [2018]. Risk-measures were analyzed as instances of the optimized certainty equivalent applied to specific utility functions [Ben-Tal and Teboulle, 1986, 2007]. Risk-averse model predictive control was also studied to account for ambiguity in the knowledge of the underlying probability distribution [Sopasakis et al., 2019].

Algorithms for nonlinear control problems are usually derived by analogy to the linear case, which is solved in linear time with respect to the horizon by dynamic programming [Bellman, 1971]. In particular, the iterative linear quadratic regulator (ILQR) and iterative linear quadratic Gaussian (ILQG) algorithms are usually informally motivated as iterative linearization algorithms [Li and Todorov, 2007]. A risk-sensitive variant without theoretical guarantees was considered by Farshidian and Buchli [2015], Ponton et al. [2016].

On the first-order optimization front, optimization subproblems such as Newton or Gauss-Newton-steps were shown to be implementable by using dynamic programming in classical works [De O. Pantoja, 1988, Dunn and Bertsekas, 1989, Sideris and Bobrow, 2005]. Iterative linearized methods such as ILQR or ILQG were recently analyzed as Gauss-Newton-type algorithms and improved using proximal regularization and acceleration by extrapolation in [Roulet et al., 2019b]. This work shares the same viewpoint and establishes worst-case complexity bounds for iterative linear quadratic exponential Gaussian controller (ILEQG) algorithms.

The companion code is available at `https://github.com/vroulet/ilqc`. All proofs and notations are provided in the long version [Roulet et al., 2019a].