An Incentive Scheme for Sensor Fusion With Strategic Sensors

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***Abstract*—Distributed estimation that recruits potentially large groups of humans to collect data about a phenomenon of interest has emerged as a paradigm applicable to a broad range of detection and estimation tasks. However, it also presents a number of challenges especially with regard to user participation and data quality, since the data resources may be strategic human agents instead of physical sensors. We consider a static estimation problem in which an estimator collects data from self-interested agents. Since it incurs cost to participate, mechanisms to incentivize the agents to collect and transmit data of desired quality are needed. Agents are strategic in the sense that they can take measurements with different levels of accuracy by expending different levels of effort. They may also misreport their information in order to obtain greater compensation, if possible. With both the measurements from the agents and their accuracy unknown to the estimator, we design incentive mechanisms that encourage desired behavior from strategic agents. Specifically, we solve an optimization problem at the estimator which minimizes the expected total compensation to the agents while guaranteeing a specified quality of the global estimate.**

***Index Terms*—Mechanism design, game theory, distributed estimation, crowdsourcing, knapsack problem.**

I. Introduction

DISTRIBUTED estimation theory to solve the problem of fusing data from a group of sensors to estimate a parameter or a random variable is a well-developed field. More recently, the emerging areas of social computing and crowdsourcing have enabled many large scale sensing and estimation tasks that leverage many humans (or human owned and operated devices) to collect data about phenomena of interest (see works such as [1], [2] for an overview). An example is that of aggregating information and opinions of a 'crowd' recruited using Amazon Mechanical Turk to perform tasks that are time consuming and difficult to scale such as image labeling. Similar applications have been proposed or demonstrated in the fields ranging from environmental monitoring [3], health data collection [4], traffic monitoring [5], and so on.

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Beyond already existing challenges in traditional distributed estimation or detection [6]–[8], new challenges arise in the design of such a crowdsensing system since data sources may not have any incentive to provide the data aggregator with the quality of data that it desires [9]. This might be due to the fact that agents may have to exert resources (e.g., time, power, or bandwidth) to produce an accurate measurement [10]. Further, even though the agents may have accurate data, they may still wish to corrupt data before transmission either to gain privacy or for some other selfish reason [11]. Early work in this field (e.g., [12]–[14]) ignored these issues and assumed that participants were voluntary recruits who would collect and provide high quality (and truthful) data. More recently, it has been recognized that without a suitable incentive, such voluntary providers of data may not be enough to generate an estimate of desired quality. As an illustrative example, [15] studied product reviews on Amazon.com and concluded that users with a moderate outlook are unlikely to report; thus, while controlled experiments on the same items reveal normally distributed opinions, voluntarily reported ratings often follow bi-modal, U-shaped distributions where most of the ratings are either very good or very bad.

A review of various incentive mechanisms, including both monetary and non-monetary incentives, is provided in [10], [16], [17]. Specifically, reverse auctions [18]–[21] have been proposed that design payments according to the agents' bids on the costs of solving the estimation problem (or the values of their information). To this end, the cost of each agent is typically assumed to be a constant that is unknown to the estimator. However, in practice, agents can exert less effort to incur lower cost and generate less accurate data, and vice versa. Accordingly, mechanisms such as those in [22]–[24] have been proposed to incentivize agents to exert sufficient effort by making payments depend on the accuracy of the data provided by the agents. In the case where the estimator cannot directly verify the data accuracy, the peer prediction mechanism [25] has been proposed to set up a game among the agents by designing the payment to each agent as a function of this agent's reported data and the reports from the other agents. Besides effort elicitation, the problem of truthfulness elicitation has also been studied using peer prediction methods with different information structures (see e.g., [26]–[28]) to incentivize the agents to report truthfully in a game-theoretic equilibrium.

In addition to effort elicitation and truthfulness elicitation, it is also of great interest for the estimator to consider how much total reward is to be paid, or the trade-off between the total payment and the estimation accuracy. A systematic theory that addresses the challenges of incentive mechanism design with the objective of optimizing the overall cost function at the estimator is not well studied. The main contribution of this paper is on the design of cost-efficient incentive mechanisms. Specifically, while using a payment structure similar to that of peer prediction method to achieve effort elicitation and truthfulness elicitation, we further solve an optimization problem of minimizing the expected total compensation to be paid to the strategic agents while guaranteeing a specified quality of global estimate. Compared to the existing works in the literature that also considered the overall cost of the estimator, the agent model that we consider has several assumptions that may be more practical:

1) Each agent can exert less effort to incur lower cost and generate less accurate data, and vice versa.
2) The actual level of effort exerted by each agent is unknown to the estimator.
3) The actual data obtained at each agent is unknown to the estimator.

Since the agents can act strategically to meet their own interests by not only exerting different levels of effort but also misreporting their data, the optimization problem at the estimator is thus difficult to solve.

The works that are closest to ours are [29]–[35]. [29] considered the optimization problem of minimizing the sum of estimation error and expected total payment. Their proposed incentive mechanism determines the compensation to the strategic agents by verifying their reported data with the ground truth of the phenomenon of interest, which was assumed to be available to the estimator after the estimation process. Minimizing the worst-case estimation error subject to a budget constraint was explored in [30] with the assumption that data reported from the agents is verifiable once revealed. In this paper, we do not require the availability of ground truth or data verification. [31] focused on minimizing a weighted sum of estimation error and expected total payment with the assumption that sensors do not misreport their data. With the same assumption, [32] extended [31] from the scenario of one-time data acquisition to the scenario of multi-time data acquisition, where the assignment of a future job opportunity was used as a part of the incentive. In this paper, we allow the agents to misreport their data to maximize their own utility. [33] also considered the problem of balancing estimation accuracy and expected total payment with the assumption that the actual effort costs of the agents are drawn from a known distribution. Although this assumption can be relaxed by learning the distribution in a sequential setting [34], their model is limited to a binary-answer task (e.g., an image contains either a certain object or not) and a binary effort model (e.g., either exert a fixed level of effort at a fixed cost or no effort at no cost at all). Instead, we focus on estimation tasks with continuous measurements and continuous effort models. Finally, unlike [35] where the model is limited to a specific effort cost function, we do not restrict the format of effort cost function.

The rest of the paper is organized as follows. Section II presents the problem statement and formulation. In Section III, we derive the solution to the optimization problem at the estimator which minimizes total compensation while guaranteeing a certain estimation accuracy. We then present in Section IV an optimal mechanism that achieves the desired behavior from the strategic agents in a Bayesian Nash Equilibrium (BNE) when the cost functions of the strategic agents satisfy a certain property. In Section V, we provide a feasibility-guaranteed sub-optimal mechanism when the cost functions of the strategic agents do not satisfy that property. Results of some simulation experiments are given in Section VI. Section VII concludes the paper.

Notation: Random variables are denoted by uppercase letters, and their realizations are denoted by the corresponding lowercase letters. $\mathbb{E}_X[f]$ denotes the expectation of function f taken with respect to the random variable X; when X is explained from the context, the notation is abbreviated as $\mathbb{E}[f]$. $C_{XX} = \mathbb{E}[(X-\mathbb{E}[X])(X-\mathbb{E}[X])]$ and $C_{XY} = \mathbb{E}[(X-\mathbb{E}[X])(Y-\mathbb{E}[Y])]$ denote, respectively, the variance of X and the covariance between X and Y. A Gaussian distribution with mean m and variance σ^2 is denoted by $\mathcal{N}(m,\sigma^2)$. A tuple of n elements is denoted with parentheses by $(e_1, e_2, \ldots, e_n)$.

II. Problem Statement

Estimation Setup: Consider a scalar-valued random variable X that is distributed according to a prior distribution $X \sim \mathcal{N}(0,\sigma_x^2)$ and takes an unknown value x in an experiment. An estimator (also called an aggregator) seeks to estimate the value x using measurements from N sensors (also called agents). The i-th sensor generates a measurement Y_i according to the relation

$$Y_i = X + V_i, \tag{1}$$

where V_i is the measurement noise with distribution $V_i \sim \mathcal{N}(0,\sigma_i^2)$. We assume that the measurement noises for the N sensors are mutually independent and further independent with the random variable X. For notational ease, we denote by ξ the reciprocal of the variance, i.e., $\xi_x = \sigma_x^{-2}$ and $\xi_i = \sigma_i^{-2}$.

For each sensor i, given the measurement $Y_i = y_i$, the minimum mean square error (MMSE) estimate $\hat{x}_i$ and the corresponding local mean squared error (MSE) Σ_i can be computed as

$$\hat{x}_i = \mathbb{E}[X] + C_{XY_i}C_{Y_iY_i}^{-1}(y_i - \mathbb{E}[Y_i]) = \frac{\sigma_x^2}{\sigma_x^2+\sigma_i^2}y_i, \tag{2}$$

$$\Sigma_i = C_{XX} - C_{XY_i}C_{Y_iY_i}^{-1}C_{Y_iX} = \frac{1}{\sigma_x^{-2}+\sigma_i^{-2}} = \frac{1}{\xi_x+\xi_i}, \tag{3}$$

where $C_{XX} = C_{XY_i} = C_{Y_iX} = \sigma_x^2$ and $C_{Y_iY_i} = \sigma_x^2+\sigma_i^2$. We denote $\hat{x}_i$ as the local estimate and Σ_i as the local MSE at the i-th sensor since these quantities are obtained based on the information at each sensor. These local estimates can be fused to obtain the global MMSE estimate $\hat{x}_g$ using the relation [36]

$$\Sigma_g^{-1}\hat{x}_g = \sum_{i=1}^{N}\Sigma_i^{-1}\hat{x}_i, \tag{4}$$

where Σ_g is the global MSE corresponding to $\hat{x}_g$ and can be calculated as

$$\Sigma_g^{-1} = \sum_{i=1}^{N}\Sigma_i^{-1} - (N-1)\sigma_x^{-2} = \xi_x + \sum_{i=1}^{N}\xi_i. \tag{5}$$

Effort Cost: The variance σ_i^2 affects the quality of the measurement at sensor i and is assumed to be a parameter that is under the control of the sensor. In other words, the sensor can put in less *effort* and increase the variance σ_i^2 while incurring a lower effort cost, and vice versa. The effort cost may represent usage of battery, time, or some other resource. For simplicity and without loss of generality, we assume that ξ_i is the effort level of agent i that incurs an effort cost $c_i(\xi_i)$. We make some weak assumptions on the cost function that describes the effort cost.

Assumption 1: The cost function of each sensor $c_i(\xi_i)$ satisfies the following properties:

- $c_i(\xi_i) \geq 0$, i.e., effort cost is non-negative;
- $\frac{\partial c_i(\xi_i)}{\partial \xi_i} > 0$, i.e., more effort cost is incurred to obtain a measurement with higher accuracy;
- $\xi_i \in [0, \xi_{iu}]$ and $c_i(\xi_i) \in [0, c_{iu}]$.

Note that when sensor i does not put in any effort, i.e., $\xi_i = 0$ and $\sigma_i^2 = \infty$, then the effort cost is zero, i.e., $c_i(0) = 0$ and its local MSE is equal to the variance of the prior distribution of X, i.e., $\Sigma_i = \sigma_x^2$.

Common Knowledge and Private Information: The common knowledge among the sensors is merely the statistical properties of the various random variables, i.e., the prior distribution of X and the mutual independence among the measurement noises from different sensors. The local measurement, local estimate, and local MSE at each sensor are its private information. The format of the cost function $c_i(.)$ of each sensor i is assumed to be known to the estimator while the actual effort ξ_i exerted by the sensor is unknown to the estimator. Therefore, the actual cost of each sensor is also unknown to the estimator.

Formulation as a Mechanism Design Problem: We are interested in a formulation in which the estimator and the sensors are all self-interested. The estimator is interested in generating a global estimate with a specified accuracy as measured by the global MSE. To do so, it must incentivize sensors to generate and transmit measurements with sufficiently low local MSE. On the other hand, the sensors do not gain directly from the estimator being able to generate an accurate global estimate. Since they incur effort costs to generate measurements with low local MSE, the estimator must compensate the sensors using a payment mechanism of some sort of some sort. For simplicity, we assume the payment is monetary, although money may be thought of as a proxy of some other resource such as battery charging. The problem we consider in this paper is to minimize the payment from the estimator to incentivize self-interested sensors to obtain and report measurements with sufficient accuracy that allow the global MSE to be below a specified level.

We now formulate this interaction as a mechanism design problem. The timeline of the interaction is as shown in Fig. 1. The estimator asks each sensor to report its measurement and local estimate. Note that reporting this pair is equivalent to reporting the local estimate and the local MSE. The strategy sets and the utility functions of each player are given as below.

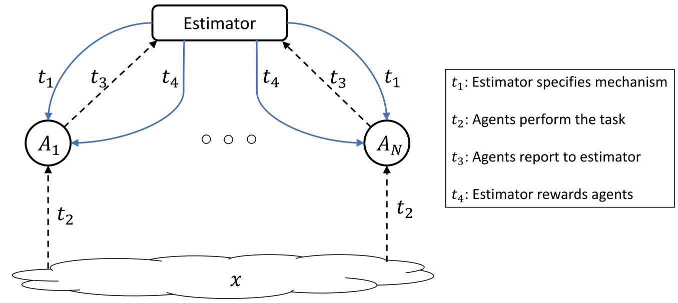

Fig. 1. Timeline and communication topology of the incentive mechanism design problem.

- *Strategy Sets:* Each sensor can choose the level of effort to exert and the values of its measurement and local estimate to report. For each sensor i, we define its strategy as choosing each element in the following tuple

$$s_i = (\xi_i, \hat{x}_{ri}, y_{ri}),$$

where $\hat{x}_{ri}$ is the reported local estimate and y_{ri} is the reported measurement. Denote the set of all feasible s_i's by S_i. With a slight abuse of standard notation in game theory, when sensor i adopts strategy s_i, denote by $s_{-i} = (s_1, s_2, \ldots, s_{i-1}, s_{i+1}, \ldots, s_N)$ the strategy profile of all the other sensors except for sensor i. The estimator decides how much payment each sensor i will obtain and how to fuse the reports from the sensors. Since the sensors may misreport their local estimates, (4) may not be the optimal way to fuse local reported estimates from the sensors. Thus, the strategy of the estimator includes the payment functions that map each strategy profile of the sensors to their payments and the fusion rule, i.e.,

$$s_e = (p_i(s_1, \ldots, s_N), \ell(s_1, \ldots, s_N)),$$

where $p_i(s_1, \ldots, s_N)$ denotes the payment made to sensor i which is in general a function of the strategies of all the sensors, and $\ell(s_1, \ldots, s_N)$ is the fusion rule used to obtain the global estimate. Note that the payment $p_i(s_1, s_2, \ldots, s_N)$ can also be expressed as $p_i(s_i, s_{-i})$. Denote the set of all feasible strategies s_e's by S_e.
- *Utility Functions:* The expected utility of each sensor i is given by

$$\mathbb{E}[U_i] = \mathbb{E}[p_i(s_i, s_{-i}) - c_i(\xi_i)], \tag{6}$$

where the expectation is taken over the uncertainties of the random variable X, measurement noises, and the strategies profile of all the other sensors. Thus each sensor i optimizes over the effort level and reports to maximize its expected utility,

$$\max_{s_i \in S_i} \mathbb{E}[U_i]. \tag{7}$$

On the other hand, the estimator is interested in minimizing the expected total payment while obtaining a global estimate with MSE less than a certain threshold. Formally, the optimization problem at the estimator is given as follows

$$\begin{aligned} \min_{s_e \in S_e} \quad & \mathbb{E}\left[\sum_{i=1}^{N} p_i(s_i, s_{-i})\right] \\ \text{s.t.} \quad & \Sigma_g \leq \Sigma_t, \\ & \mathbb{E}[p_i(s_i, s_{-i}) - c_i(\xi_i)] \geq 0, \forall i, \\ & s_i = \arg\max \mathbb{E}[p_i(s_i, s_{-i}) - c_i(\xi_i)], \forall i, \end{aligned} \tag{8}$$

where Σ_t is the specified threshold on the global MSE. The second constraint above ensures individual rationality, which is necessary for the sensors to participate.

In the sequel, we solve problem (8). Note that problem (8) specifies a Bayesian game among the sensors since the utility of each sensor i depends on not only its own strategy s_i but also the strategy profile of the other sensors s_{-i}. This further depends on the private information (such as local measurements and local estimates) of the other sensors which is unknown to sensor i. We will consider the solution of the optimization problem when the behavior of the sensors is specified according to a Bayesian Nash Equilibrium.

III. Optimization Problem at the Estimator

To understand why the problem (8) is difficult to solve, we note why some intuitive incentive mechanisms may not work.

- A payment scheme $p_i = c$ for a constant c that does not depend on the reports will lead to each sensor not making any effort and reporting some arbitrary value to the estimator. In economics, this is termed as the problem of *moral hazard*.
- A payment scheme that specifies p_i as a decreasing function of Σ_i can be considered to incentivize the sensors to exert effort and take accurate measurements. However, it will lead sensors reporting very low local MSE irrespective of the actual effort made. This is termed as the problem of *adverse selection*.

In either case, note that the actual measurements y_i, local estimates $\hat{x}_i$ and the local MSE Σ_i are all unknown to the estimator, fusing reported local estimates to obtain a global estimate that satisfies the constraint $\Sigma_g \leq \Sigma_t$ is also a nontrivial problem. The overall optimization problem (8) is even more difficult.

Our results are organized as shown in Fig. 2. Specifically, we show that the following technical condition on the effort cost functions plays an important role in the simplification of the problem:

$$-2\frac{\partial c_i(\xi_i)}{\partial \xi_i} - \frac{\partial^2 c_i(\xi_i)}{\partial \xi_i^2}(\xi_x + \xi_i) < 0, \forall \xi_i \text{ and } i. \tag{9}$$

Remark 1: If $c_i(\xi_i)$ is convex over ξ_i, the condition (9) holds for any ξ_i.

Remark 2: Condition (9) is a sufficient condition for our proposed optimal mechanism (presented in Section IV) to be able to induce the desired level of effort from each sensor. The intuition is that it guarantees the concavity of the expected utility function of each sensor over the effort, so that the desired level of effort will also be the unique utility maximizer for each sensor under the proposed optimal mechanism.

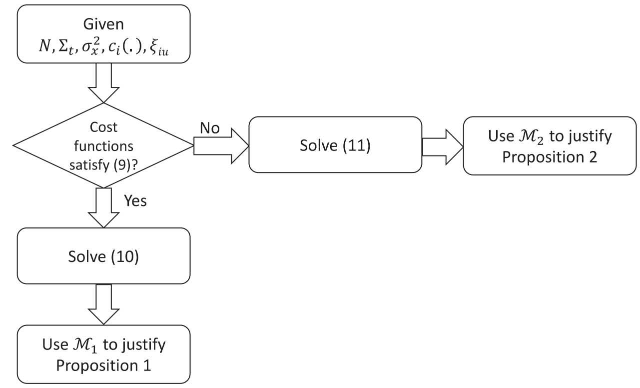

Fig. 2. The procedure of designing the incentive mechanism with strategic data sources.

Depending on whether (9) is satisfied or not, we have the following result.

Proposition 1: Consider the setup of problem (8). If condition (9) is satisfied, the estimator can specify a payment design such that the following three conditions hold as a BNE:

1) The selected sensors exert the effort levels specified by the estimator;
2) The selected sensors report truthfully about their measurements and local estimates;
3) The expected payment to each selected sensor is the effort cost of the sensor for the specified effort level.

Note that under these three conditions, the estimator can choose the optimal effort levels from agents that yield minimum payment while meeting the constraints in problem (8) with the fusion rule as shown in (4). Specifically in rewriting problem (8), the first condition enables the estimator to take the effort levels of all sensors as decision variables; the second condition yields that the global MSE Σ_g can be computed as shown in (5); and the third condition is sufficient to replace the expected payment term in the objective function with effort cost. Thus, in this case, the optimization problem (8) reduces to

$$\begin{aligned} \min_{\phi, \xi} \quad & \sum_{i=1}^{N} \phi_i c_i(\xi_i) \\ \text{s.t.} \quad & \frac{1}{\xi_x + \sum_{i=1}^{N} \phi_i \xi_i} \leq \Sigma_t, \\ & \phi_i \in (0, 1), \\ & \xi_i \in [0, \xi_{iu}], \end{aligned} \tag{10}$$

where $\xi = (\xi_1, \xi_2, \ldots, \xi_N)$ and $\phi = (\phi_1, \phi_2, \ldots, \phi_N)$. ϕ_i is an indicator about whether or not the estimator selects agent i: $\phi_i = 1$ represents the case where the estimator selects agent i and $\phi_i = 0$ represents the case where the estimator does not select agent i, which can be implemented by, for instance, setting $p_i = 0$.

In Section IV, we show that if the cost functions satisfy constraint (9), a mechanism $\mathcal{M}_1$ can be designed that specifies a payment design according to Proposition 1. Thus, problem (8) can be solved optimally. Otherwise if the cost functions do not satisfy constraint (9), we solve the problem in a sub-optimal way through the following proposition. This will be proved through the design of a mechanism $\mathcal{M}_2$ presented in Section V.

Proposition 2: Consider the setup of problem (8). If the condition (9) is not satisfied, the estimator can specify a payment design such that the following three conditions hold as a BNE:

1) The selected sensors exert their maximum effort levels;
2) The selected sensors report truthfully about their measurements and local estimates;
3) The expected payment to each selected sensor i is the effort cost of the sensor for its maximum effort level.

Note that under these three conditions, the estimator cannot choose the optimal effort levels from sensors as in Proposition 1. But the estimator can still select a subset of all agents given that the selected agents will exert their maximum efforts and the estimator will pay the corresponding costs incurred at maximum effort levels. Specifically in rewriting problem (8), the first condition states that the effort levels of the sensors cannot be considered as decision variables and are fixed to be their maximum levels; the second condition yields that the the global MSE Σ_g can be computed as shown in (5) with $\xi_i = \xi_{iu}$ for all i; and the third condition is sufficient to replace the expected payment term in the objective function with the maximum effort cost. Formally, problem (8) is transformed to the following well-defined binary knapsack problem (KP):

$$\begin{aligned} \min_{\phi} \quad & \sum_{i=1}^{N} \phi_i c_i(\xi_{iu}) \\ \text{s.t.} \quad & \frac{1}{\xi_x + \sum_{i=1}^{N} \phi_i \xi_{iu}} \leq \Sigma_t, \\ & \phi_i \in (0,1). \end{aligned} \tag{11}$$

This problem is NP-hard but can be solved exactly in pseudo-polynomial time through dynamic programming algorithms [37].

IV. Optimal Mechanism When (9) Holds

In this section, we address the cases where (9) holds. We first simplify the optimization problem (10) and present two interesting special cases. Then we present the optimal mechanism to prove Proposition 1.

A. Solving Problem (10)

(10) can be rewritten as

$$\begin{aligned} \min_{\phi,\xi} \quad & \sum_{i=1}^{N} \phi_i c_i(\xi_i) \\ \text{s.t.} \quad & \sum_{i=1}^{N} \phi_i \xi_i \geq \Sigma_t^{-1} - \xi_x, \\ & \phi_i \in (0,1), \\ & \xi_i \in [0, \xi_{iu}]. \end{aligned} \tag{12}$$

This is a mixed-integer nonlinear programming problem and specifically known as the general knapsack problem (GKP) with variable coefficients [38] [39], which is difficult to solve in general. However, since $c_i(0) = 0, \forall i$, we can transform problem (12) to problem (13) according to the following result.

Lemma 1: Problem (12) can be solved by constructing solution of the following optimization problem

$$\begin{aligned} \min_{\xi} \quad & \sum_{i=1}^{N} c_i(\xi_i) \\ \text{s.t.} \quad & \sum_{i=1}^{N} \xi_i \geq \Sigma_t^{-1} - \xi_x, \\ & \xi_i \in [0, \xi_{iu}]. \end{aligned} \tag{13}$$

Proof: Suppose that the minimum of (12), denoted by O_1, is achieved at (ξ^{O1}, ϕ^{O1}) and the minimum of (13), denoted by O_2, is achieved at ξ^{O2}. We have $O_1 \leq O_2$ because (13) is a special case of (12) by fixing all $\phi_i = 1$. On the other hand, $O_1 \geq O_2$, because any value achieved in (12) can be achieved in (13) by constructing ξ^{O2} from (ξ^{O1}, ϕ^{O1}) as

$$\xi_i^{O2} = \begin{cases} \xi_i^{O1}, & \text{for } \phi_i^{O1} = 1, \\ 0, & \text{for } \phi_i^{O1} = 0. \end{cases} \tag{14}$$

Thus, $O_1 = O_2$. In general, it is easier to solve (13) first and then construct (ξ^{O1}, ϕ^{O1}) from ξ^{O2} by setting

$$(\xi_i^{O1}, \phi_i^{O1}) = \begin{cases} (\xi_i^{O2}, 1), & \text{for } \xi_i^{O2} \neq 0, \\ (r, 0), & \text{for } \xi_i^{O2} = 0, \end{cases} \tag{15}$$

where r can be any number since $\phi_i^{O1} = 0$. ∎

We now present two interesting special cases.

1) Special Case: Continuous Quadratic Cost Function: A quadratic effort cost $c_i(\xi_i) = l\xi_i^2$ is quite popular, e.g., in control theory. In this case, then the optimization problem (13) becomes,

$$\begin{aligned} \min_{\xi} \quad & \sum_{i=1}^{N} l\xi_i^2 \\ \text{s.t.} \quad & \sum_{i=1}^{N} \xi_i \geq \Sigma_t^{-1} - \xi_x, \\ & \xi_i \in [0, \xi_{iu}], \end{aligned} \tag{16}$$

which is a standard Quadratic Programming (QP) problem. According to Cauchy-Schwarz inequality, the optimal solution of (16) is given by

$$\tilde{\xi}_1 = \tilde{\xi}_2 = \cdots = \tilde{\xi}_N = \frac{\Sigma_t^{-1} - \xi_x}{N}, \tag{17}$$

assuming for simplicity that $\frac{\Sigma_t^{-1} - \xi_x}{N} \leq \xi_{iu}$.

As stated in Remark 1, since the cost function is convex, the constraint (9) holds for any possible ξ_i. Under our optimal mechanism (presented in Section IV-B), there is a BNE where all agents select the effort level as $\frac{\Sigma_t^{-1} - \xi_x}{N}$ and report truthfully about their local estimates and their measurements. Meanwhile, the minimum expected total payment that can be achieved to ensure global MSE to be no greater than Σ_t is $\frac{l(\Sigma_t^{-1}-\xi_x)^2}{N}$.

2) Special Case: Discrete Linear Cost Function: A natural model is that each agent can increase the accuracy of its local estimate by taking more measurements and estimating based upon the sample mean. For instance, if agent i takes η_i number of measurements of the following

$$\begin{aligned} Y_i(1) &= X + V_i(1),\\ Y_i(2) &= X + V_i(2),\\ &\vdots\\ Y_i(\eta_i) &= X + V_i(\eta_i), \end{aligned} \tag{18}$$

where $V_i(k)$ follows i.i.d. Gaussian distribution $\mathcal{N}(0,\sigma_{io}^2)$. Denote the effort cost of taking each measurement by a cost of c_{io}. Then the noise level, effort level and effort cost of the sample mean $\bar{Y}_i = X + \bar{V}_i$ averaged from taking η_i measurements are respectively given by

$$\begin{aligned} \sigma_i^2 &= \frac{\sigma_{io}^2}{\eta_i},\\ \xi_i &= \eta_i\sigma_{io}^{-2},\\ c_i(\xi_i) &= \eta_i c_{io} = \sigma_{io}^2 c_{io}\xi_i. \end{aligned} \tag{19}$$

Therefore in this case, the effort cost function is a linear function and the effort level depends on the number of measurements taken. We denote the corresponding maximum number of measurements by η_i^m. The optimization problem (13) becomes,

$$\begin{aligned} \min_{\eta} \quad & \sum_{i=1}^{N} \eta_i c_{io}\\ \text{s.t.} \quad & \sum_{i=1}^{N} \eta_i\sigma_{io}^{-2} \geq \Sigma_t^{-1} - \xi_x,\\ & \eta_i \in (0,1,\ldots,\eta_i^m), \end{aligned} \tag{20}$$

which is known as the Bounded Knapsack Problem (BKP). It is NP-hard but it can be solved exactly in pseudo-polynomial time through dynamic programming algorithms [37] [40]. Denote by $\tilde{\eta} = (\tilde{\eta}_1, \tilde{\eta}_2, \ldots, \tilde{\eta}_N)$ the optimal solution of (20).

Under our optimal mechanism, there is a BNE where each agent takes $\tilde{\eta}_i$ number of measurements and report truthfully about the local estimate and measurement. Meanwhile, the minimum expected total payment that can be achieved to ensure global MSE to be no greater than Σ_t is $\sum_{i=1}^{N}\tilde{\eta}_i c_{io}$.

B. Optimal Mechanism

Denoting the optimal solution of problem (13) by $\tilde{\xi} = (\tilde{\xi}_1, \tilde{\xi}_2, \ldots, \tilde{\xi}_N)$, we now present mechanism $\mathcal{M}_1$ that fulfills Proposition 1, i.e., under Mechanism $\mathcal{M}_1$, all the agents exerting the desired effort levels and reporting their measurements and local estimates truthfully is a BNE. In addition, the expected payment to each agent i is the effort cost of the agent for the specified effort level.

In our proposed incentive mechanism $\mathcal{M}_1$, agents are asked to report two items $(\hat{x}_{ri}, y_{ri})$, where $\hat{x}_{ri}$ is the reported local estimate and y_{ri} is the reported measurement. Note that $\hat{x}_{ri} \neq \hat{x}_i$ and $y_{ri} \neq y_i$ in general since agents may falsify their reports to maximize their utilities. The payment function is given by

$$p_i(\hat{x}_{ri}, y_{rj}) = \gamma_i - \beta_i(\hat{x}_{ri} - y_{rj})^2, \tag{21}$$

where y_{rj} is the reported measurement from another agent $j \neq i$. As before, agents are interested in maximizing their expected utilities, i.e.,

$$\begin{aligned} s_i^* &= \arg\max_{s_i\in S_i} \mathbb{E}\left[U_i\right]\\ &= \arg\max_{s_i\in S_i} \mathbb{E}\left[p_i(\hat{x}_{ri}, y_{rj}) - c_i(\xi_i)\right]. \end{aligned} \tag{22}$$

Now, we state our results about the optimal mechanism $\mathcal{M}_1$.

Theorem 1: Consider the problem (8) when (9) holds. Let (21) be the payment function to each agent i with

$$\beta_i = \left.\frac{\partial c_i(\xi_i)}{\partial \xi_i}\right|_{\xi_i=\tilde{\xi}_i} \left(\xi_x + \tilde{\xi}_i\right)^2, \tag{23}$$

and

$$\gamma_i = \beta_i\left(\frac{1}{\xi_x+\tilde{\xi}_i} + \tilde{\xi}_j^{-1}\right) + c_i(\tilde{\xi}_i). \tag{24}$$

The strategy profile $s^* = (s_1^*, s_2^*, \ldots, s_N^*)$ with

$$s_i^* = \left(\xi_i = \tilde{\xi}_i, \hat{x}_{ri} = \hat{x}_i, y_{ri} = y_i\right) \tag{25}$$

is a BNE of the mechanism design problem (8). In addition, the expected payment to each agent is the effort cost, i.e., $\mathbb{E}[p_i] = c_i(\tilde{\xi}_i)$.

Proof: See Appendix A. ■

Intuitively, the payment is designed as a function of the difference between the reports from the agents, which motivates each agent to estimate the information of another agent based on its own information. The accuracy of the agent's estimate, and the corresponding expected payment, will depend on how much effort is exerted when the agent obtains its own information. Therefore, β_i can be designed as in (23) so that the effort level that the estimator wishes each agent to exert (i.e., $\tilde{\xi}_i$) will turn out to be exactly the optimal choice for agent i when all the other agents exert the effort levels desired by the estimator, i.e., $\xi_j = \tilde{\xi}_j, \forall j \neq i$. Further, γ_i can be designed as in (24) so that the expected payment is small but enough to cover the effort cost.

It is worth reminding that the estimation of the reports among the agents is in a Bayesian game setting, which means the desired strategy profile s^* from all the agents is obtained in a BNE sense. Further, if there exists an *'honest'* agent who reports its measurement and effort level truthfully, it is no longer needed to ask the strategic agents to report their measurements. In this case, the desired strategy profile s^* is *the unique BNE in strictly dominant strategies* in which every s_i^* is the strictly dominant strategy for agent i. We would further point out that honest agent need not be noiseless and can even be a noisy side measurement that the estimator has access to. The result is presented in the following corollary.

Corollary 1: Consider the setting of Theorem 1 with an honest agent h who reports its measurement and effort level truthfully, i.e., $Y_h = X + V_h$ with $V_h \sim \mathcal{N}(0, \xi_h^{-1})$, and $y_{rh} = y_h$ given the realization $Y_h = y_h$. Let the payment function to each agent i be specified by (21), (23) and (24) after replacing y_{rj} and $\tilde{\xi}_j$ with y_{rh} and ξ_h respectively. The strategy profile $s^{*\prime} = (s_1^{*\prime}, s_2^{*\prime}, \ldots, s_N^{*\prime})$ with

$$s_i^{*\prime} = \left(\xi_i = \tilde{\xi}_i, \hat{x}_{ri} = \hat{x}_i\right) \tag{26}$$

is the unique BNE in strictly dominant strategies.

Proof: The proof is similar to that of Theorem 1. The only difference in this case is that the strategic agents now estimate the measurement from the honest agent instead of estimating the measurement from another strategic agent. The utility of each strategic agent will no longer depend on other strategic agents, thus, the strategy in the BNE is the strictly dominant strategy for each agent and the BNE is the unique equilibrium. ■

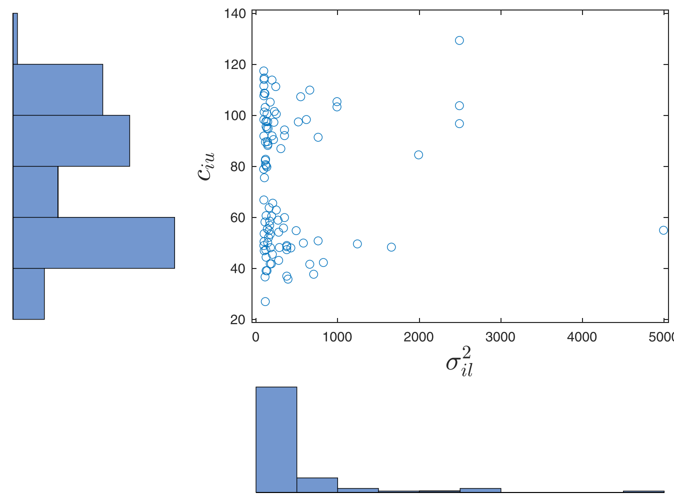

Fig. 3. Scatter plot and histograms of σ_{il}^2 and c_{iu}.

V. A Sub-Optimal Mechanism When (9) Does Not Hold

In this section, we provide a feasibility-guaranteed sub-optimal mechanism $\mathcal{M}_2$ for the cases where the constraint (9) can not be satisfied. $\mathcal{M}_2$ achieves truthful reporting and elicits maximum effort from the selected agents with expected payment to selected agent i being the effort cost $c_i(\xi_{iu})$ in a BNE.

Denote the optimal solution to (11) as $\tilde{\phi} = (\tilde{\phi}_1, \tilde{\phi}_2, \ldots, \tilde{\phi}_N)$. We present the incentive mechanism $\mathcal{M}_2$ that only selects the agents for which $\tilde{\phi}_i = 1$ and elicits their maximum efforts.

Theorem 2: Consider the problem (8) when (9) does not hold. Let the payment to each agent i with $\tilde{\phi}_i = 1$ be determined by comparing its reported local estimate with the reported measurement from another agent j with $\tilde{\phi}_j = 1$, i.e.,

$$p_i(\hat{x}_{ri}, y_{rj}) = \begin{cases} \gamma_i - \beta_i(\hat{x}_{ri} - y_{rj})^2, & \text{for } \tilde{\phi}_i = 1 \\ 0, & \text{for } \tilde{\phi}_i = 0 \end{cases} \tag{27}$$

with

$$\beta_i > \max_{\xi_i \in [0, \xi_{iu}]} \frac{\partial c_i(\xi_i)}{\partial \xi_i}(\xi_x + \xi_i)^2 \tag{28}$$

and

$$\gamma_i = \beta_i \left(\frac{1}{\xi_x + \xi_{iu}} + \xi_{ju}^{-1}\right) + c_i(\xi_{iu}). \tag{29}$$

The strategy profile $s^* = (s_1^*, s_2^*, \ldots, s_N^*)$ with

$$s_i^* = \begin{cases} (\xi_i = \xi_{iu}, \hat{x}_{ri} = \hat{x}_i, y_{ri} = y_i), & \text{for } \tilde{\phi}_i = 1 \\ (\xi_i = 0), & \text{for } \tilde{\phi}_i = 0 \end{cases} \tag{30}$$

is a BNE of the mechanism design problem (8). In addition, the expected payment to each selected agent is the effort cost for its maximum effort level, i.e., $\mathbb{E}[p_i] = c_i(\xi_{iu})$.

Proof: See Appendix B. ■

Remark 3: The payments to the agents with $\tilde{\phi}_i = 0$ are set as constant zero, which does not depend on their reports. By exerting no effort, the utilities of these agents are zero irrespective of their reporting strategies. For simplicity, they are not asked to report their local estimates or measurements.

Similarly to Corollary 1, if there exists an honest agent who reports its measurement and effort level truthfully, it is no longer needed to ask the strategic agents to report their measurements. In this case, the desired strategy profile is the unique BNE in strictly dominant strategies.

Corollary 2: Consider the setting in Theorem 2 with an honest agent h who reports its measurement and effort level truthfully, i.e., $Y_h = X + V_h$ with $V_h \sim \mathcal{N}(0, \xi_h^{-1})$, and $y_{rh} = y_h$ given the realization $Y_h = y_h$. Let the payment function to each agent i be specified by (27), (28) and (29) after replacing y_{rj} and ξ_{ju} with y_{rh} and ξ_h respectively. The strategy profile $s^{*\prime} = (s_1^{*\prime}, s_2^{*\prime}, \ldots, s_N^{*\prime})$ with

$$s_i^{*\prime} = \begin{cases} (\xi_i = \xi_{iu}, \hat{x}_{ri} = \hat{x}_i), & \text{for } \tilde{\phi}_i = 1 \\ (\xi_i = 0), & \text{for } \tilde{\phi}_i = 0 \end{cases} \tag{31}$$

is the unique BNE in strictly dominant strategies.

The proof is omitted since it is similar to that of Corollary 1.

VI. Simulation Experiments

In this section, we demonstrate our mechanisms with simulation experiments. We first consider the setting of the problem in Section IV-A2 and investigate the minimum payment at different threshold Σ_t. $N = 100$ agents are simulated and the variance of the prior distribution is selected as $\sigma_x^2 = 1000$. Further, fixing the minimum variance of each agent σ_{il}^2 and its corresponding maximum effort c_{iu} allow us to study the effect η_i^m, which can be interpreted as the quantization level of the effort cost of each agent. Without loss of generosity, we set $\eta_i^m = \eta^m$ for all i. To make the agents heterogeneous on their highest accuracies, we randomly generate $\sigma_{il}^{-2} \sim \mathcal{U}[0.0001, 0.01]$, which is selected such that roughly a half of σ_{il}^2 fall in the range $[100, 200]$ and the other half of σ_{il}^2 fall in the range $(200, 10000]$. c_{iu} is randomly generated from a mixture Gaussian distribution $c_{iu} \sim .5\mathcal{N}(50, 100) + .5\mathcal{N}(100, 100)$. The scatter plot and histograms of these two parameters are shown in Fig. 3.

The minimum payments with $\eta^m = 2$, $\eta^m = 4$, and $\eta^m = 100$ at different threshold Σ_t are shown in Fig. 4. In general, greater η^m yields smaller payment. On the other hand, we also study the effect of N. We use the same distributions to

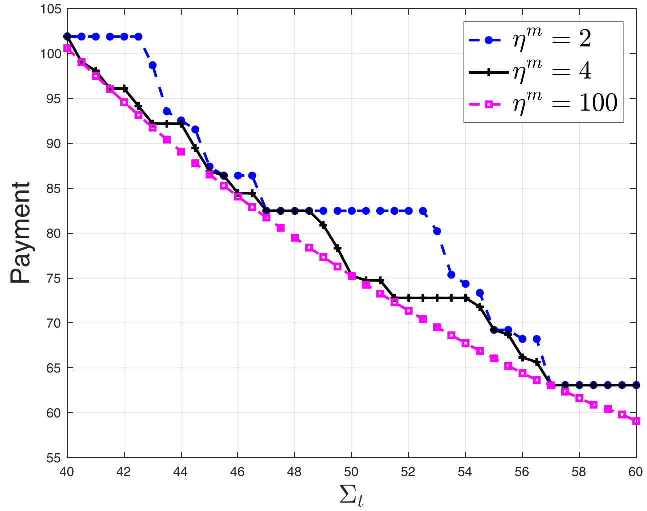

Fig. 4. Minimum payments with $N = 100$, and $\eta^m = 2$, $\eta^m = 4$, $\eta^m = 100$ respectively.

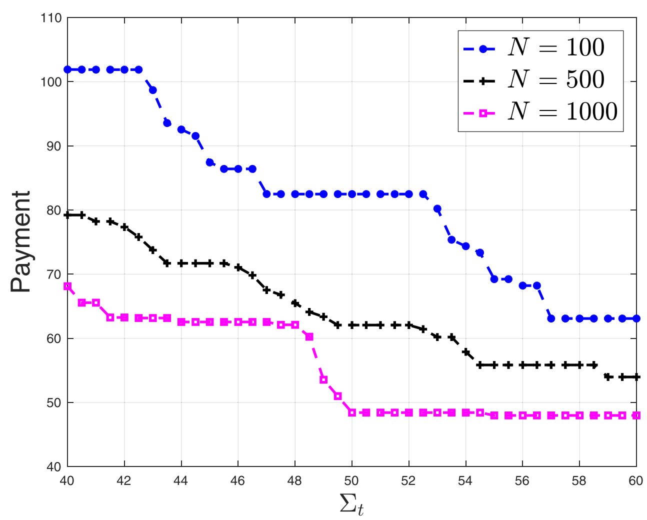

Fig. 5. Minimum payments with $\eta^m = 2$, and $N = 100$, $N = 500$, $N = 1000$ respectively.

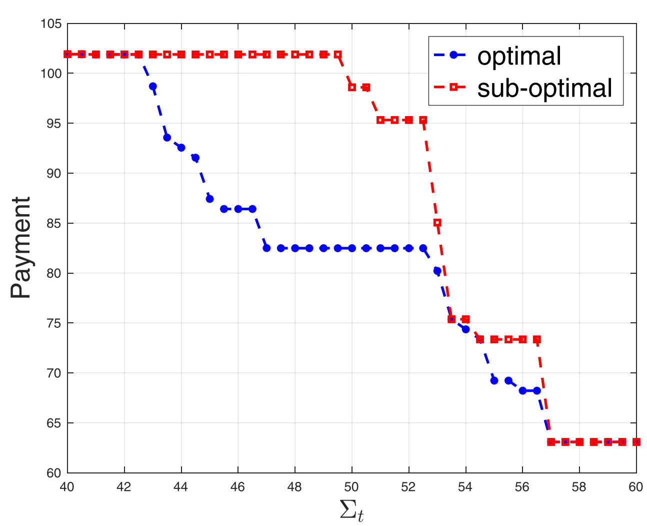

Fig. 6. Comparison of minimum payments in the sub-optimal case and the optimal case.

generate σ_{il}^2 and c_{iu}. η^m is fixed as $\eta^m = 2$. As shown in Fig. 5, more agents being available generally yields smaller payments. Lastly, we compare our sub-optimal case with the optimal case considered in Section IV-A2 under the same setting. Recall that in the sub-optimal solution, our mechanism $\mathcal{M}_2$ yields all selected agents exerting maximum effort. Using the same simulated parameters, the optimization problem (11) in the sub-optimal case can be viewed a problem similar to (20), but the decision variables are limited to be either 0 or η^m. In Fig. 6, we show the comparison of minimum payments between the sub-optimal case and the optimal case at different Σ_t with $\eta^m = 2$ and $N = 100$.

VII. Summary

In this paper, we designed incentive mechanisms for a static estimation problem where the data sources are strategic agents whose measurements and accuracies are both unknown to the estimator. The objective of the incentive mechanism is to minimize the expected total payment made to the agents with a guaranteed quality of global estimate. We formulated the problem in a very general setting without assuming any specific format of the agents' cost functions. Instead, we designed an optimal incentive mechanism for the cases where the cost functions satisfy certain property and provided a sub-optimal incentive mechanism for the other cases. We also demonstrated our mechanisms by two special cases with continuous quadratic cost function and discrete linear cost function. Both in the special case with the discrete linear cost function and in the sub-optimal case, the optimization problem were transformed to knapsack problems, which can be solved in pseudo-polynomial time by dynamic programming. Future work will include extending the results to dynamic estimation problems.

Appendix A
Proof of Theorem 1

It suffices to prove that if the strategy profile of all the other agents follow the stated equilibrium, denoted as $s_{-i} = s_{-i}^*$, agent i does not have another strategy which yields greater expected utility than s_i^*. Mathematically, when $s_{-i} = s_{-i}^*$, the optimal strategy for agent i is given by

$$\begin{aligned}(\xi_i^*, \hat{x}_{ri}^*, y_{ri}^*) &= \arg\max \mathbb{E}[U_i] \\ &= \arg\max \mathbb{E}\left[\gamma_i - \beta_i(\hat{x}_{ri} - Y_j)^2 - c_i(\xi_i)\right] \quad (32)\end{aligned}$$

First notice that the estimator cannot verify the reports $\hat{x}_{ri}^*$ and y_{ri}^* jointly, since

$$\hat{x}_i = \frac{\xi_x^{-1}}{\xi_x^{-1} + \xi_i^{-1}} y_i, \quad (33)$$

and ξ_i is unknown to the estimator. Therefore, the agent can optimize $\hat{x}_{ri}^*$ and y_{ri}^* independently. However, the expected utility is indifferent to y_{ri}, hence no other value can yield greater utility than $y_{ri}^* = y_i$.

Next, we prove that for any exerted effort ξ_i along with the corresponding obtained $\hat{x}_i$, the optimal $\hat{x}_{ri}^* = \hat{x}_i$. Since β_i and

γ_i are positive constants and ξ_i is fixed,

$$\begin{aligned}\hat{x}_{ri}^* &= \arg\min \mathbb{E}\left[(\hat{x}_{ri} - Y_j)^2 | \hat{x}_i\right] \\ &= \mathbb{E}[Y_j | \hat{x}_i] \\ &= C_{Y_j \hat{X}_i} C_{\hat{X}_i \hat{X}_i}^{-1} \hat{x}_i,\end{aligned} \tag{34}$$

where $C_{Y_j \hat{X}_i}$ and $C_{\hat{X}_i \hat{X}_i}$ are computed as

$$C_{Y_j \hat{X}_i} = \mathbb{E}\left[(X + V_j)\frac{\xi_x^{-1}}{\xi_x^{-1} + \xi_i^{-1}}(X + V_i)\right] = \frac{\xi_x^{-2}}{\xi_x^{-1} + \xi_i^{-1}}, \tag{35}$$

and

$$C_{\hat{X}_i \hat{X}_i} = \mathbb{E}\left[\left(\frac{\xi_x^{-1}}{\xi_x^{-1} + \xi_i^{-1}}(X + V_i)\right)^2\right] = \frac{\xi_x^{-2}}{\xi_x^{-1} + \xi_i^{-1}}, \tag{36}$$

since the measurement noises and X are all independent to each other. Therefore, after obtaining its local estimate at any effort level, truthfully reporting on the local estimate, i.e., $\hat{x}_{ri}^* = \hat{x}_i$, is the best response to $s_{-i} = s_{-i}^*$.

Now, we show that with β_i given by (23) and if the constraint (9) is satisfied, then the optimal $\xi_i^* = \tilde{\xi}_i$. Before choosing to exert its effort ξ_i to obtain a measurement, the local estimate is still random hence denoted by the uppercase letter $\hat{X}_i$. The expected utility with the optimal choice of $\hat{X}_{ri} = \hat{X}_i$ is given by

$$\mathbb{E}[U_i(\xi_i)] = \gamma_i - \beta_i \, \mathbb{E}[(\hat{X}_i - Y_j)^2] - c_i(\xi_i), \tag{37}$$

where

$$\begin{aligned}\mathbb{E}[(\hat{X}_i - Y_j)^2] &= \mathbb{E}\left[\left(\frac{\xi_x^{-1}}{\xi_x^{-1} + \xi_i^{-1}}(X + V_i) - (X + V_j)\right)^2\right] \\ &= \frac{\xi_i^{-1}\xi_x^{-1}}{\xi_x^{-1} + \xi_i^{-1}} + \tilde{\xi}_j^{-1} \\ &= \frac{1}{\xi_x + \xi_i} + \tilde{\xi}_j^{-1}.\end{aligned} \tag{38}$$

Thus, setting the first derivative of $\mathbb{E}[U_i(\xi_i)]$ over ξ_i to zero yields the unique maximum ξ_i^* if $\mathbb{E}[U_i(\xi_i)]$ is concave:

$$\frac{\partial \, \mathbb{E}[U_i(\xi_i)]}{\partial \xi_i} = \frac{\beta_i}{(\xi_x + \xi_i)^2} - \frac{\partial c_i(\xi_i)}{\partial \xi_i} = 0, \tag{39}$$

where the solution is $\tilde{\xi}_i$ if β_i is given by

$$\beta_i = \left.\frac{\partial c_i(\xi_i)}{\partial \xi_i}\right|_{\xi_i = \tilde{\xi}_i} \left(\xi_x + \tilde{\xi}_i\right)^2. \tag{40}$$

To guarantee the concavity of $\mathbb{E}[U_i(\xi_i)]$,

$$\frac{\partial^2 \, \mathbb{E}[U_i(\xi_i)]}{\partial \xi_i^2} = \frac{-2\beta_i}{(\xi_x + \xi_i)^3} - \frac{\partial^2 c_i(\xi_i)}{\partial \xi_i^2} < 0, \tag{41}$$

which implies the constraint (9) should be satisfied for any ξ_i.

Lastly, the maximum expected utility of agent i is given by

$$\mathbb{E}[U_i(\tilde{\xi}_i)] = \gamma_i - \beta_i\left(\frac{1}{\xi_x + \tilde{\xi}_i} + \tilde{\xi}_j^{-1}\right) - c_i(\tilde{\xi}_i). \tag{42}$$

Therefore, γ_i given by (24) is designed to satisfy individual rationality. Meanwhile, the expected payment is as small as the effort cost $c_i(\tilde{\xi}_i)$.

Appendix B
Proof of Theorem 2

The proof is similar to that of Theorem 1, except for that β_i is designed to ensure that the derivative over ξ_i as shown in (39) is always positive so that the selected agents would prefer to exert maximum effort.

References

[1] B. Guo, Z. Yu, X. Zhou, and D. Zhang, "From participatory sensing to mobile crowd sensing," in *Proc. IEEE Int. Conf. Pervasive Comput. Commun. Workshops*, 2014, pp. 593–598.

[2] H. Ma, D. Zhao, and P. Yuan, "Opportunities in mobile crowd sensing," *IEEE Commun. Mag.*, vol. 52, no. 8, pp. 29–35, Aug. 2014.

[3] M. N. K. Boulos *et al.*, "Crowdsourcing, citizen sensing and sensor web technologies for public and environmental health surveillance and crisis management: Trends, OGC standards and application examples," *Int. J. Health Geographics*, vol. 10, no. 1, 2011, Art. no. 67.

[4] R. Pryss, M. Reichert, J. Herrmann, B. Langguth, and W. Schlee, "Mobile crowd sensing in clinical and psychological trials—A case study," in *Proc. IEEE 28th Int. Symp. Comput.-Based Med. Syst.*, 2015, pp. 23–24.

[5] B. Pan, Y. Zheng, D. Wilkie, and C. Shahabi, "Crowd sensing of traffic anomalies based on human mobility and social media," in *Proc. 21st ACM SIGSPATIAL Int. Conf. Adv. Geographic Inf. Syst.*, 2013, pp. 344–353.

[6] Q. Berthet and V. Chandrasekaran, "Resource allocation for statistical estimation," *Proc. IEEE*, vol. 104, no. 1, pp. 111–125, Jan. 2016.

[7] F. S. Cattivelli and A. H. Sayed, "Diffusion LMS strategies for distributed estimation," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1035–1048, Mar. 2010.

[8] P. K. Varshney, *Distributed Detection and Data Fusion*. Berlin, Germany: Springer, 1997.

[9] M. Allahbakhsh, B. Benatallah, A. Ignjatovic, H. R. Motahari-Nezhad, E. Bertino, and S. Dustdar, "Quality control in crowdsourcing systems: Issues and directions," *IEEE Internet Comput.*, vol. 17, no. 2, pp. 76–81, Mar./Apr. 2013.

[10] H. Gao *et al.*, "A survey of incentive mechanisms for participatory sensing," *IEEE Commun. Surv. Tut.*, vol. 17, no. 2, pp. 918–943, Second Quarter 2015.

[11] Y. Gong, L. Wei, Y. Guo, C. Zhang, and Y. Fang, "Optimal task recommendation for mobile crowdsourcing with privacy control," *IEEE Internet Things J.*, vol. 3, no. 5, pp. 745–756, Oct. 2016.

[12] P. Mohan, V. N. Padmanabhan, and R. Ramjee, "Nericell: Rich monitoring of road and traffic conditions using mobile smartphones," in *Proc. 6th ACM Conf. Embedded Netw. Sensor Syst.*, 2008, pp. 323–336.

[13] A. Thiagarajan *et al.*, "Vtrack: Accurate, energy-aware road traffic delay estimation using mobile phones," in *Proc. 7th ACM Conf. Embedded Networked Sensor Syst.*, 2009, pp. 85–98.

[14] R. K. Rana, C. T. Chou, S. S. Kanhere, N. Bulusu, and W. Hu, "Ear-phone: An end-to-end participatory urban noise mapping system," in *Proc. 9th ACM/IEEE Int. Conf. Inf. Process. Sensor Netw.*, 2010, pp. 105–116.

[15] N. Hu, P. A. Pavlou, and J. Zhang, "Can online reviews reveal a product's true quality?: Empirical findings and analytical modeling of online word-of-mouth communication," in *Proc. 7th ACM Conf. Electron. Commerce*, 2006, pp. 324–330.

[16] L. G. Jaimes, I. J. Vergara-Laurens, and A. Raij, "A survey of incentive techniques for mobile crowd sensing," *IEEE Internet Things J.*, vol. 2, no. 5, pp. 370–380, Oct. 2015.

[17] F. Restuccia, S. K. Das, and J. Payton, "Incentive mechanisms for participatory sensing: Survey and research challenges," *ACM Trans. Sensor Netw.*, vol. 12, no. 2, 2016, Art. no. 13.

[18] J.-S. Lee and B. Hoh, "Sell your experiences: A market mechanism based incentive for participatory sensing," in *Proc. Int. Conf. Pervasive Comput. Commun.*, 2010, pp. 60–68.

[19] X. Zhang, G. Xue, R. Yu, D. Yang, and J. Tang, "Truthful incentive mechanisms for crowdsourcing," in *Proc. IEEE Conf. Comput. Commun.*, 2015, pp. 2830–2838.

[20] N. Cao, S. Brahma, and P. K. Varshney, "Target tracking via crowdsourcing: A mechanism design approach," *IEEE Trans. Signal Process.*, vol. 63, no. 6, pp. 1464–1476, Mar. 2015.

[21] K. Chen, V. Gupta, and Y.-F. Huang, "On auction design for crowd sensing," in *Proc. IEEE 19th Int. Conf. Inf. Fusion*, 2016, pp. 334–339.

[22] F. Farokhi, I. Shames, and M. Cantoni, “Optimal contract design for effort-averse sensors,” *Int. J. Control*, pp. 1–8, 2018. [Online]. Available: https://www.tandfonline.com/doi/full/10.1080/00207179.2018.1486041
[23] J. Witkowski, Y. Bachrach, P. Key, and D. C. Parkes, “Dwelling on the negative: Incentivizing effort in peer prediction,” in *Proc. 1st AAAI Conf. Human Comput. Crowdsourcing*, 2013, pp. 190–197.
[24] G. Radanovic, B. Faltings, and R. Jurca, “Incentives for effort in crowdsourcing using the peer truth serum,” *ACM Trans. Intell. Syst. Technol.*, vol. 7, no. 4, 2016, Art. no. 48.
[25] N. Miller, P. Resnick, and R. Zeckhauser, “Eliciting informative feedback: The peer-prediction method,” *Manage. Sci.*, vol. 51, no. 9, pp. 1359–1373, 2005.
[26] R. Jurca and B. Faltings, “Mechanisms for making crowds truthful,” *J. Artif. Intell. Res.*, vol. 34, no. 1, pp. 209–253, 2009.
[27] J. Witkowski and D. C. Parkes, “A robust Bayesian truth serum for small populations,” in *Proc. 26th AAAI Conf. Artif. Intell.*, 2012, pp. 1492–1498.
[28] G. Radanovic and B. Faltings, “A robust Bayesian truth serum for non-binary signals,” in *Proc. 27th AAAI Conf. Artif. Intell.*, 2013, pp. 833–839.
[29] Y. Luo, N. B. Shah, J. Huang, and J. Walrand, “Parametric prediction from parametric agents,” *Oper. Res.*, vol. 66, no. 2, pp. 313–326, 2018.
[30] Y. Chen, N. Immorlica, B. Lucier, V. Syrgkanis, and J. Ziani, “Optimal data acquisition for statistical estimation,” in *Proc. ACM Conf. Econ. Comput.*, 2018, pp. 27–44.
[31] Y. Cai, C. Daskalakis, and C. Papadimitriou, “Optimum statistical estimation with strategic data sources,” in *Proc. Conf. Learn. Theory*, 2015, pp. 280–296.
[32] Y. Liu and Y. Chen, “A bandit framework for strategic regression,” in *Proc. Adv. Neural Inf. Process. Syst.*, 2016, pp. 1821–1829.
[33] Y. Liu and Y. Chen, “Learning to incentivize: Eliciting effort via output agreement,” in *Proc. 25th Int. Joint Conf. Artif. Intell.*, 2016, pp. 3782–3788.
[34] Y. Liu and Y. Chen, “Sequential peer prediction: Learning to elicit effort using posted prices,” in *Proc. 31st AAAI Conf. Artif. Intell.*, 2017, pp. 607–613.
[35] D. G. Dobakhshari, N. Li, and V. Gupta, “An incentive-based approach to distributed estimation with strategic sensors,” in *Proc. IEEE 55th Conf. Decis. Control*, 2016, pp. 6141–6146.
[36] Y. Xu, V. Gupta, and C. Fischione, “Distributed estimation,” in *Academic Press Library in Signal Processing*, vol. 4. Amsterdam, The Netherlands: Elsevier, 2014, pp. 675–706.
[37] H. Kellerer, U. Pferschy, and D. Pisinger, *Knapsack Problems*. Berlin, Germany: Springer, 2004.
[38] H. Suzuki, “A generalized knapsack problem with variable coefficients,” *Math. Program.*, vol. 15, no. 1, pp. 162–176, 1978.
[39] K. Holmberg and K. Jörnsten, “Solving the generalized knapsack problem with variable coefficients,” *Nav. Res. Logistics*, vol. 43, no. 5, pp. 673–689, 1996.
[40] D. Pisinger, “A minimal algorithm for the bounded knapsack problem,” *INFORMS J. Comput.*, vol. 12, no. 1, pp. 75–82, 2000.

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